

Programming Assignment

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Problem Statement :

With the provided intrinsic camera matrix K ("Intrinsic Matrix K.txt"), consider two photos I_1 ("im1.jpg") and I_2 ("im2.jpg") of a static scene taken by a single camera.

- ☐ **Objective 1:** With any of the current implementations, find a collection of ground-truth correspondences $\{(p_i, p'_i)\}_{i=1}^n$. Ascertain that $n = 100$ true correspondences are present.
- ☐ **Objective 2:** Suppose that the camera location's coordinate system and the global coordinate system are in alignment. Use the method that was given in class to locate the Essential matrix E .
- ☐ **Objective 3:** Decompose the obtained Essential matrix E into the camera motion rotation matrix R and the translation vector t .
- ☐ **Objective 4:** Let P_i be the corresponding 3D point for the pixel pair (p_i, p'_i) . Find $P_i, \forall i \in \{1, 2, \dots, n\}$ using the triangulation approach learned in the class.
- ☐ **Objective 5:** Plot the obtained $P_i, \forall i \in \{1, 2, \dots, n\}$ and the camera center t .

Python and its dependencies :

Libraries	Functions
CV2	Operations on images.
Numpy	Matrix Math etc.
Matplotlib	3D Graphs plots etc.

Objective 1:

Approach

- **SIFT (Scale-Invariant Feature Transform)** method is used to find correspondences between images.

Implementation Explanation:

- To find a set of ground-truth correspondences using the SIFT method, we can follow these steps:
 - Extract SIFT features from both images. This involves detecting and describing local invariant features in each image using the SIFT algorithm.
 - Match the SIFT features between the two images. This involves finding corresponding features between the two images based on their descriptors.
 - Through BF matcher matching is found between descriptors.

Correspondence points(Images):



Objective 2:

Approach/Implementation:

- First we have normalized the correspondence as following:
$$x = K^{-1} * p_i \text{ and } y = K^{-1} * p'_i.$$
- Now using camera equations for p_i and p'_i and doing some manipulation in the equations we can come to the following result:

$$y_i^T [t]_x R x_i = 0$$

$$y_i^T E x_i = 0 \quad \text{where } E = [t]_x R$$

- Now doing some further manipulations we can get the following equation :

$$Ae = 0$$

- e is the column matrix of of dimension 9×1
- We need to find e such that $Ae=0$ and $e^t e=1$ that is $\|Ae\|_2$ is minimized
- We can do this by finding eigen vector of $A^T A$ corresponding to smallest eigen value of A
- The E we got from this does not satisfy the general properties of the essential matrix .
- Then, to enforce this property, we update the essential matrix as follows:

$$E = U \text{diag}([\sigma \ \sigma \ 0]) V^T \text{ with } \sigma = (\sigma_1 + \sigma_2)/2.$$

Essential Matrix:

$$E = \begin{bmatrix} 0.00797246 & -0.18799947 & -0.11662825 \\ 0.1261202 & -0.0248958 & 0.68643113 \\ 0.10079511 & -0.67300107 & -0.01200153 \end{bmatrix}$$

Objective 3:

Approach/Implementation:

- Now, we can decompose E as $[t]_x R$:

$$\begin{aligned} R &= UR_z^{+}(\pi/2) V^T & R &= UR_z^{-}(\pi/2) V^T \\ [t]_x &= UR_z^{+}(\pi/2) \Sigma U^T & [t]_x &= UR_z^{-}(\pi/2) \Sigma U^T \end{aligned}$$

- There are four possible solutions . We have chosen any one of them .\

Rotation Matrix:

$$R = \begin{bmatrix} -0.8538945 & -0.30002817 & 0.42526143 \\ -0.29091454 & 0.9526931 & 0.08800337 \\ 0.43154712 & 0.04856914 & 0.90078195 \end{bmatrix}$$

Translation Vector:

$$T = [-0.9497246 \ -0.15662339 \ 0.27109464]$$

Objective 4:

Approach/Implementation:

- For finding the 3D points we have used triangulation method as follows :

$$\begin{aligned} \lambda_1 \hat{p}_1 &= MP^{\wedge} \Rightarrow [p_1]_x MP^{\wedge} = 0 \\ \lambda_2 \hat{p}_2 &= NP^{\wedge} \Rightarrow [p_2]_x NP^{\wedge} = 0 \end{aligned}$$

$$\begin{bmatrix} [p^1 1]_{xM} \\ [p^1 1]_{xN} \end{bmatrix} \hat{P} = 0 \Rightarrow AP^{\wedge} = 0.$$

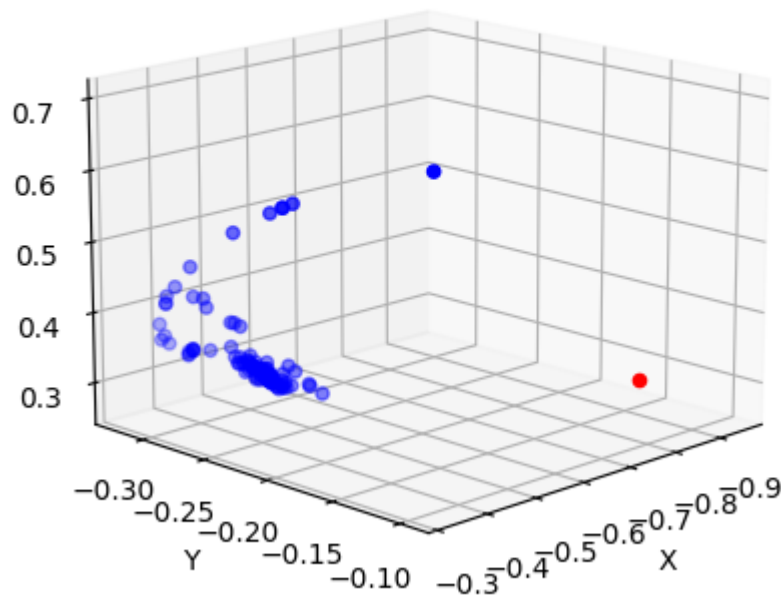
- The equation is similar to $Ae=0$. We can find P by calculating the eigen vector of A^tA corresponding to the smallest eigen value .

Objective 5:

Approach/Implementation:

- After finding the 3D points we have plotted them using a 3D plot with the help of matplotlib .

Camera Center and 3D Points



The red point is the camera and blue are the 3D points we got from the triangulation method . There is one blue point which is separated from the cluster because in finding correspondence there was one outlier . There was one correspondence which was not present in the main image of the book . We can see that correspondence clearly in the correspondence image (on the blue background of the image)