Intro to Financial Engineering

Markowitz portfolio optimization

Introduction

This report includes the details of the project **Markowitz portfolio optimization.** Markowitz portfolio optimization is a method used to find the optimal allocation of assets in a portfolio to achieve a desired return with minimum risk (or maximum return for a given level of risk). The key idea behind Markowitz's approach is to consider both the expected return and the risk (typically measured as the variance or standard deviation of returns) of the portfolio when selecting the asset allocation.

Assets Chosen

We have chosen the following 10 risky assets (stocks) :

- RELIANCE
- TCS
- HDFCBANK
- INFY
- ICICIBANK
- HINDUNILVR
- MRF
- TATAMOTORS
- SBIN
- IRCTC

As per the requirements of the assignment we have chosen a time period of 3 months (as per the instructions of the project). It is also to be noted that in 3 months there are only 62 business days which is the number of days the market is in play. Hence, the real data that we are using is for 62 days. These datasets are downloaded from yahoo finance using yfinance library.

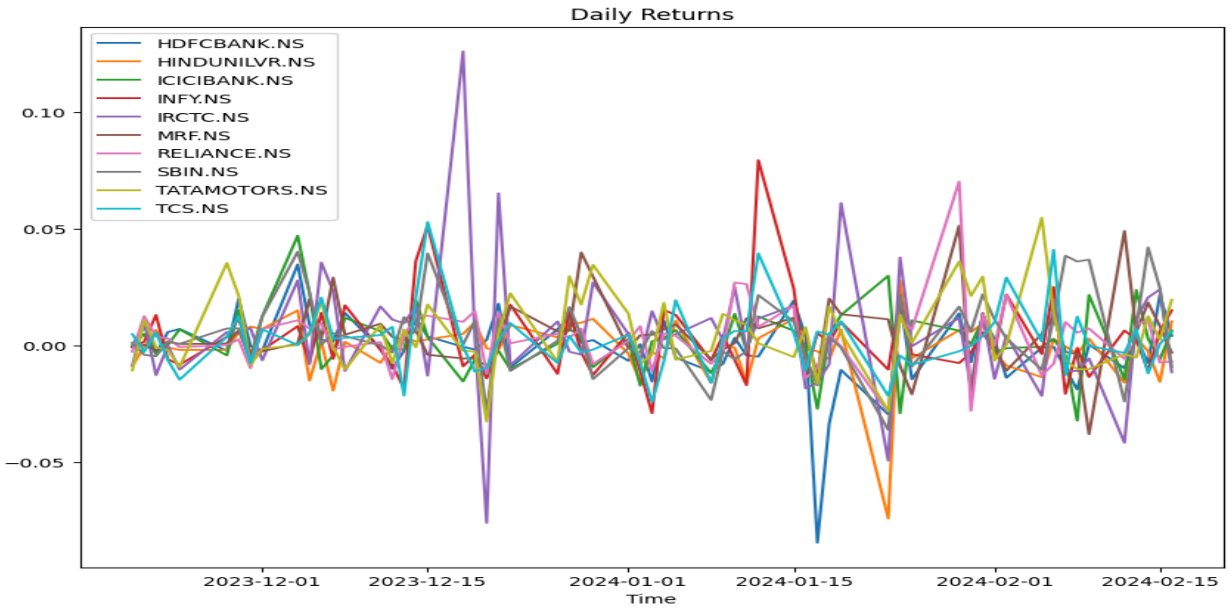
Returns and Risks for each asset

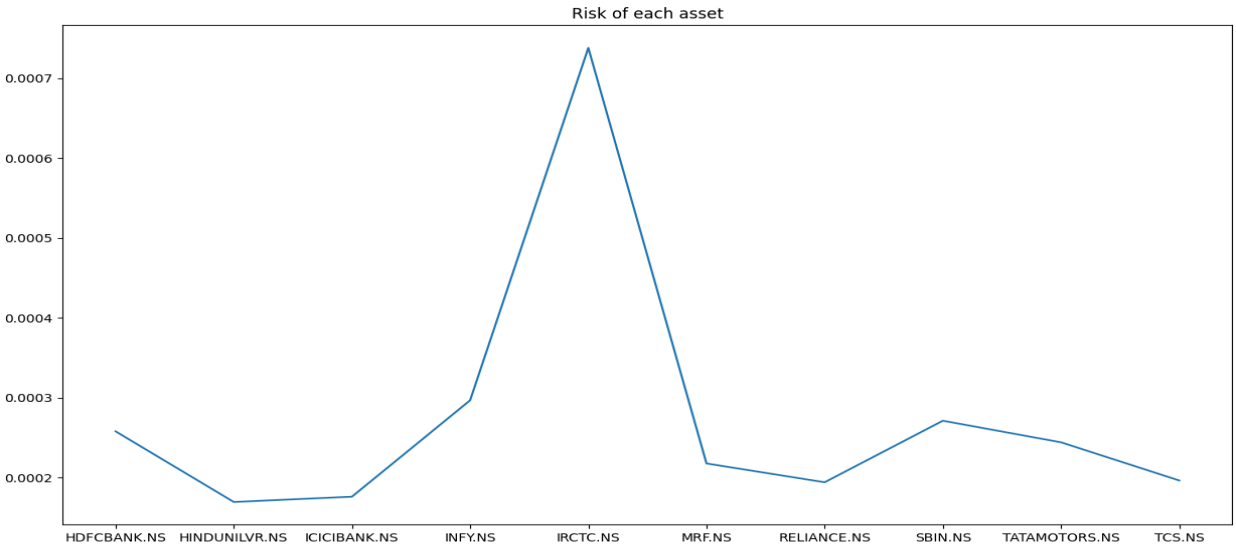
Return and risks are calculated using the below formulas:

$$r_j = \frac{P_{t+1} - P_t}{P_t} \equiv \frac{P_{t+1}}{P_t} - 1$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{N}$$

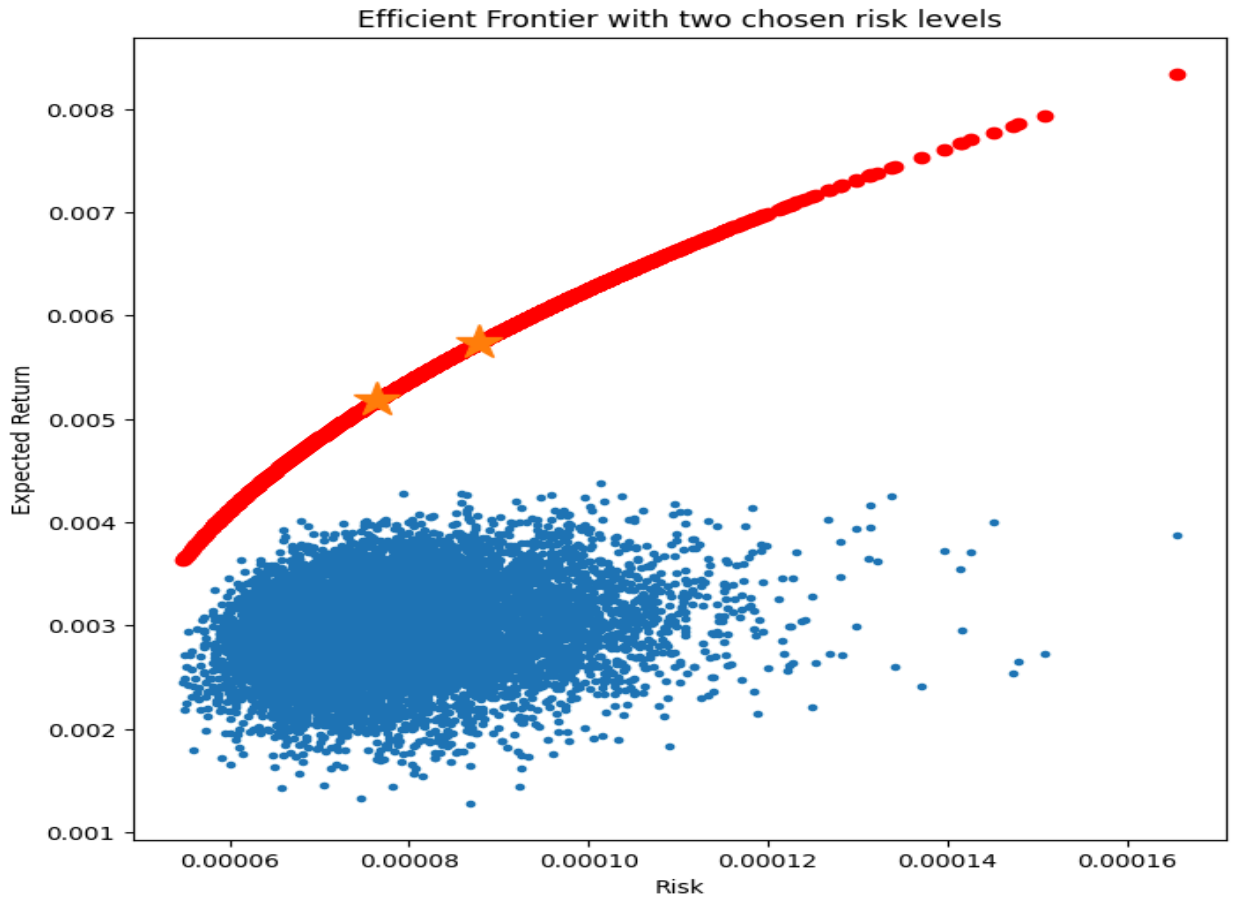
Return **Risk**





A graph showing the Markowitz efficient frontier and the two chosen points

Initially, we obtain 10,000 randomly generated portfolios with random weights, risks, and returns. Next, we attempt to identify an optimal frontier for each of these risks. That is, there is no portfolio with less risk and the same return or the same risk and higher return if we choose any portfolio on the efficient frontier.To do this, we create an optimization problem with the constraint that the sum of the weights must equal one in order to maximize the returns associated with each sigma (risk). We are then left with a Markowitz efficient frontier. Furthermore we have chosen two random risk levels lying on the efficient frontier and calculated the weights corresponding to maximum returns using the optimization problem with constraints.



The optimal portfolio weights for each chosen point on the efficient frontier

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Weight corresponding to risk 1 corresponding to maximum return :
[-0.24819279 -0.07762403  0.24626867  0.08914907  0.06656949  0.29357099
  0.08667909  0.26879669  0.23493493  0.03984787]
```

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Weight corresponding to risk 2 corresponding to maximum return :
[-0.21413747 -0.02149465  0.25171081  0.06682778  0.05635353  0.28346699
  0.07858808  0.23131718  0.20308858  0.06427917]
```

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Maximum return corresponding to risk 1 :
0.0057345529195805 97
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Maximum return corresponding to risk 2 :
0.0051810291258990 31
```

A brief discussion of the trade-off between risk and return in your portfolio choices.

The trade-off between risk and return is a fundamental concept in investing and portfolio management. It refers to the relationship where higher potential returns are typically associated with higher levels of risk, and lower risk investments tend to offer lower potential returns. This we can clearly see in the above graph of efficient frontier obtained as well. We can see that on increasing the risk there is a potential chance of increase in returns as well. The lower the risk the less is the return.

The limitations of Markowitz optimization

- The Markowitz Model relies heavily on historical data, which may not reliably predict future market trends. Past performance is no guarantee of future results.
- The model's assumptions are based on normally functioning markets. In highly volatile and unpredictable markets, the model may lose relevance.
- Mean-variance theory assumes normally distributed returns. Assets that do not follow this distribution will not work well with the Markowitz Model.

- The model assumes all portfolio assets are allocated to a single timeframe, which is rarely the case in reality.
- Combining the Markowitz Model with a comprehensive understanding of market dynamics and trends is important.
- Efficient frontier optimization may be difficult to achieve in practice and does not guarantee intended portfolio performance.
- During market crashes, correlations often go to one, rendering uncorrelated assets ineffective in protecting the portfolio.
- Implementing the Markowitz Model correctly in practice can be challenging due to inaccuracies or misrepresentations in data.

Markowitz optimization real-world applications

- Portfolio Construction: To create diversified portfolios with a goal of striking a balance between risk and return, investors and portfolio managers frequently employ Markowitz optimisation.
- Asset Allocation: Based on the projected returns and risk characteristics of several asset classes, including cash, bonds, and stocks, this technique is used to find the best way to allocate assets.
- Risk management: By spreading investments among uncorrelated assets, Markowitz optimization can assist in identifying and reducing risks in a portfolio.
- Performance evaluation is the process of comparing a portfolio's actual performance to a benchmark and determining the effects of various investment methods.