Project 4: Option Pricing

Harsh Sharma (B20CS017)

Dataset

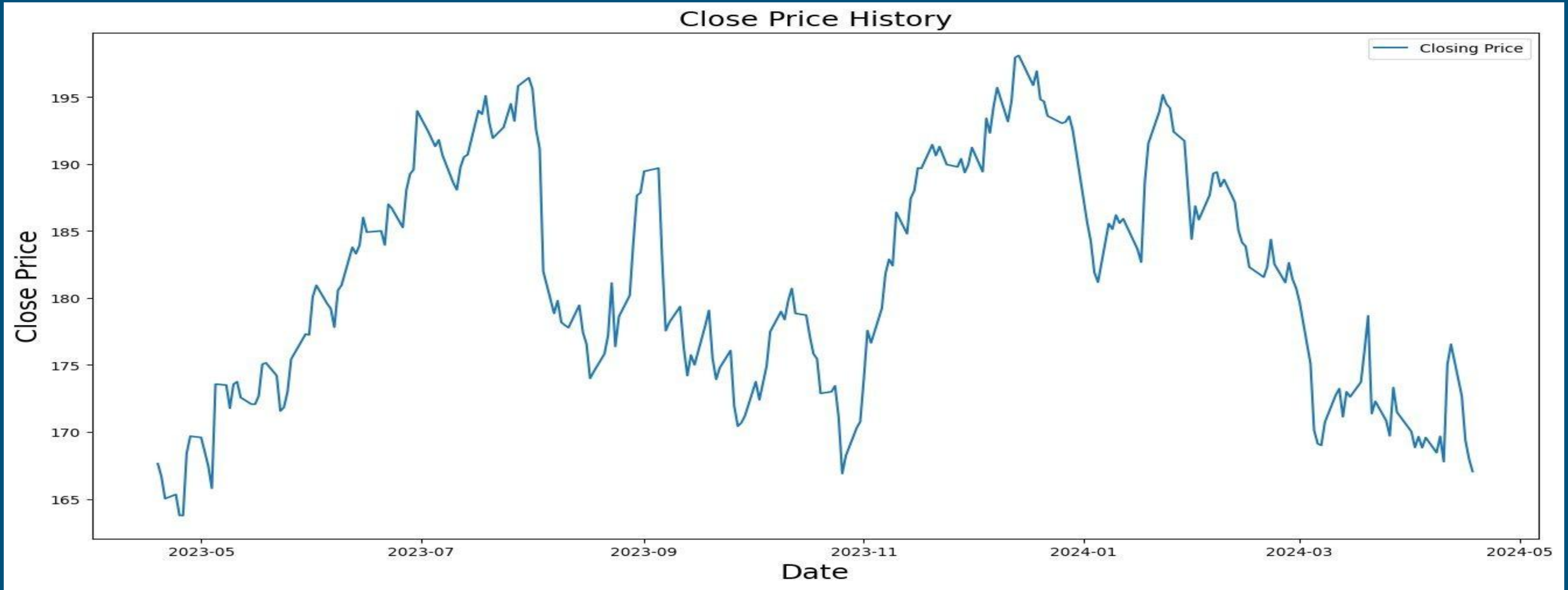
Description : Company Chosen Apple

Asset: Stocks

Time Period: 4-19-2023 to 4-19-2024

Assets Chosen

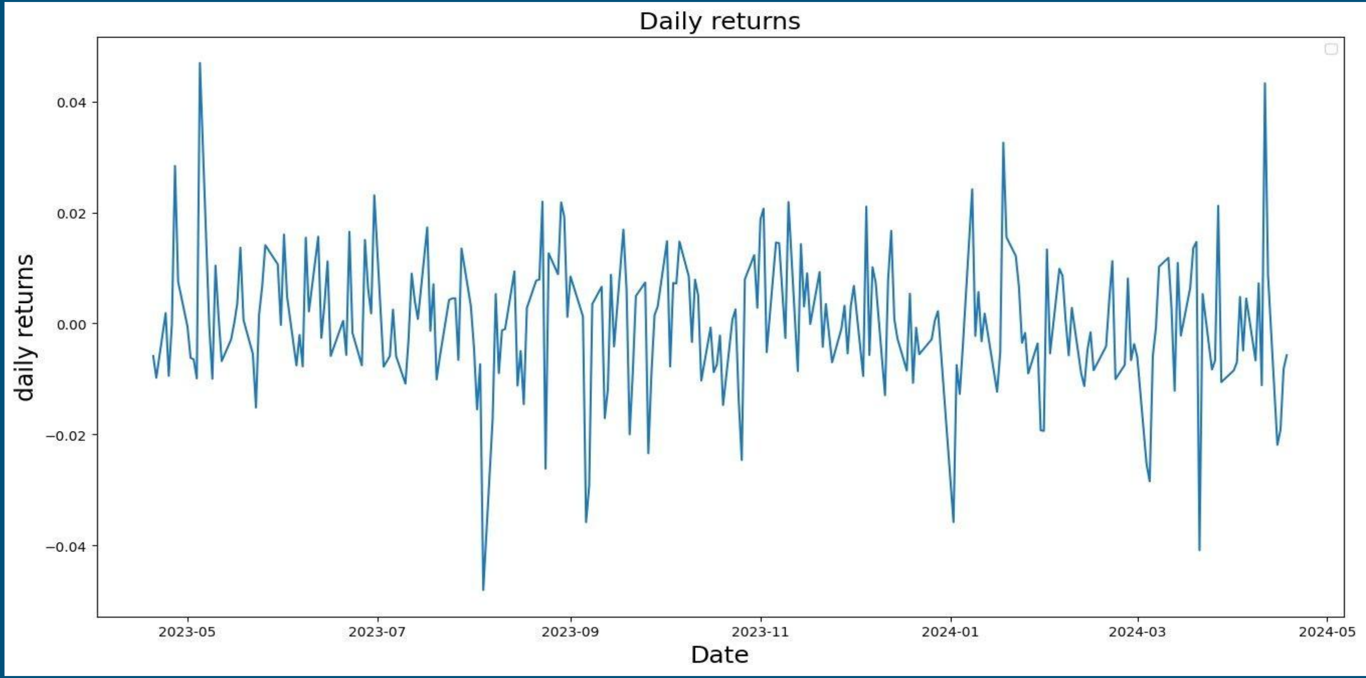
We have chosen the following stocks and options data of Apple for this particular project. As per the requirements of the assignment we have chosen a time period of 1 year (as per the instructions of the project).



Daily Returns

Return is calculated by using below formula :

$$r_j = \frac{P_{t+1} - P_t}{P_t} \equiv \frac{P_{t+1}}{P_t} - 1$$



Calculating the Annual volatility for asset

The annual volatility of an asset is calculated based on historical price data and represents the standard deviation of the asset's returns over a specific time frame, usually one year. It provides investors with insights into the level of risk associated with holding the asset and helps them make informed decisions about portfolio allocation and risk management strategies.

Annual Volatility is calculated by using below formula :

$$\text{Annual Volatility Formula} = \sqrt{252} \times \sqrt{\text{Variance}}$$

The value of annual volatility is **0.1961299**

Binomial Option pricing Model.

The Binomial Option Pricing Model is a popular numerical method used to value options. It's based on the concept of constructing a binomial tree to represent the possible price paths of the underlying asset over time. To implement we have follow the following steps:

- Construct the Binomial Tree: Divide the time to expiration into discrete time steps and construct a binomial tree representing the possible price paths of the underlying asset. At each node of the tree, calculate both the up and down movements of the asset's price.
- Calculate Option Values at Expiration: At the terminal nodes of the tree (at expiration), calculate the option values based on the payoff function (e.g., for a call option, the payoff is $\max(0, S-K)$).

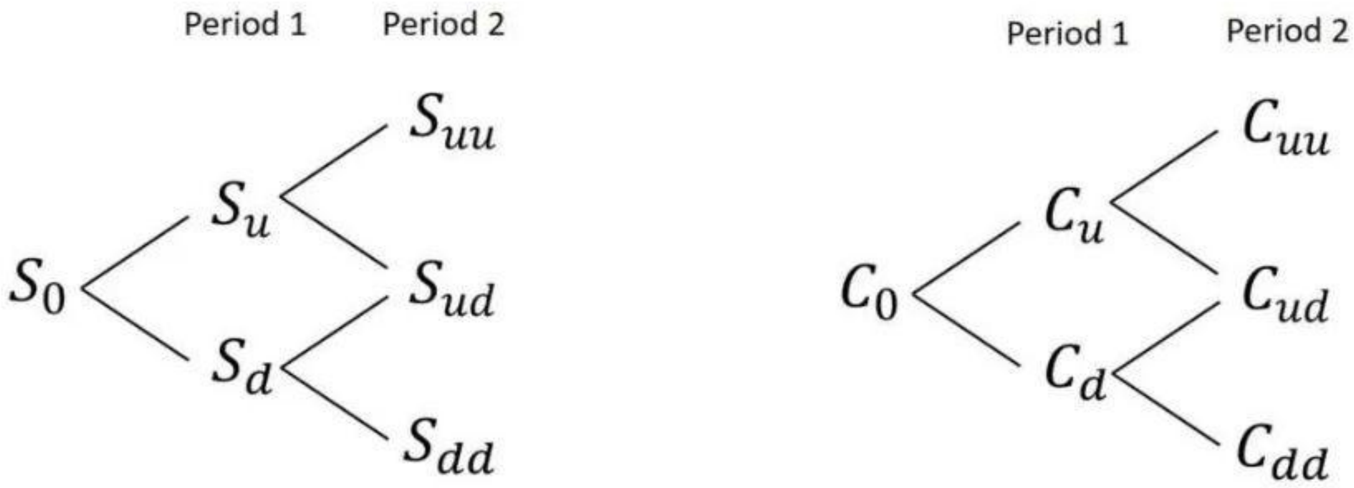


Figure 1. Two-period binomial tree

- Backward Induction: Starting from the terminal nodes, recursively calculate the option values at earlier nodes of the tree. At each node, the option value is the discounted expected value of the option's future cash flows.

$$V = e^{-r\Delta t}(qP_u + (1-q)P_d)$$

where q is the risk neutral probability as determined by

$$q = \frac{e^{-r\Delta t} - d}{u - d}$$

- We took US treasury rate as risk-free rate which equals 0.0464.
- Option Valuation: Once we have calculated the option values at the initial (current) node of the tree, we have the present value of the option.

Black Scholes Formula

The Black-Scholes formula is a mathematical model used for pricing European-style options, taking into account factors such as the current price of the underlying asset, the option's strike price, the time to expiration, the risk-free interest rate, and the volatility of the underlying asset's returns.

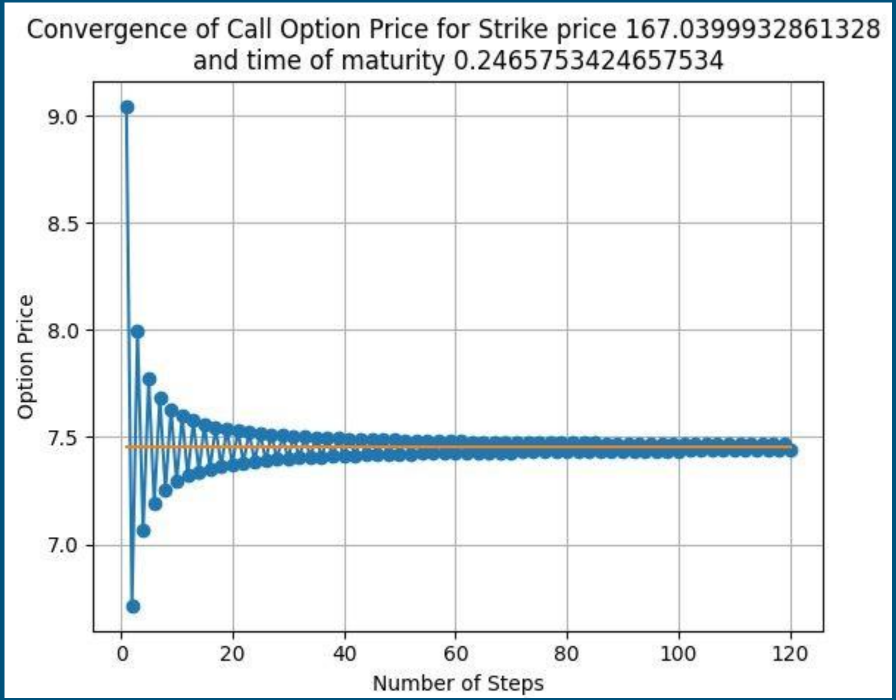
$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

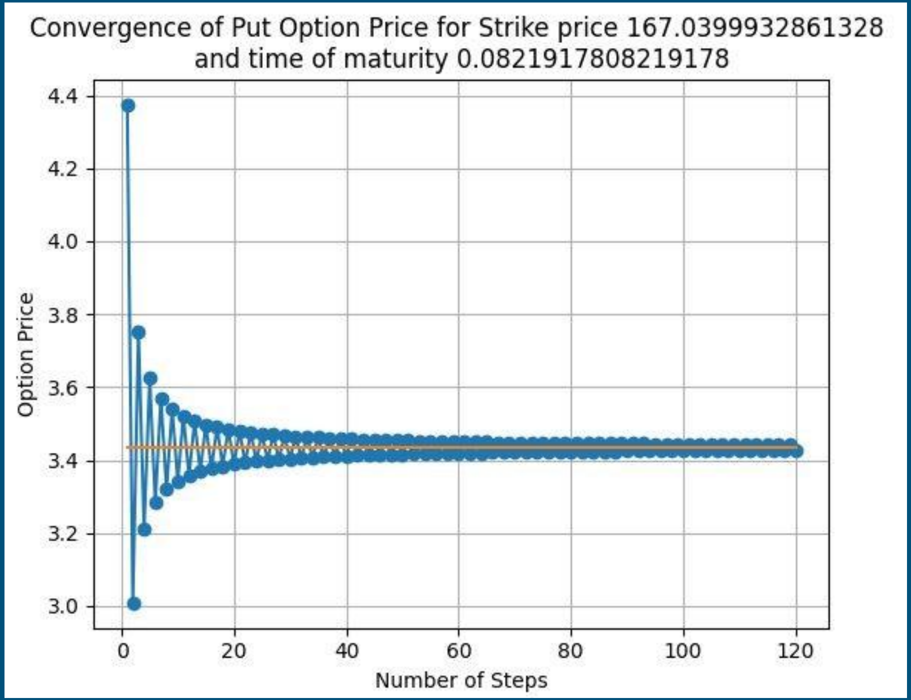
$$\text{where } d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

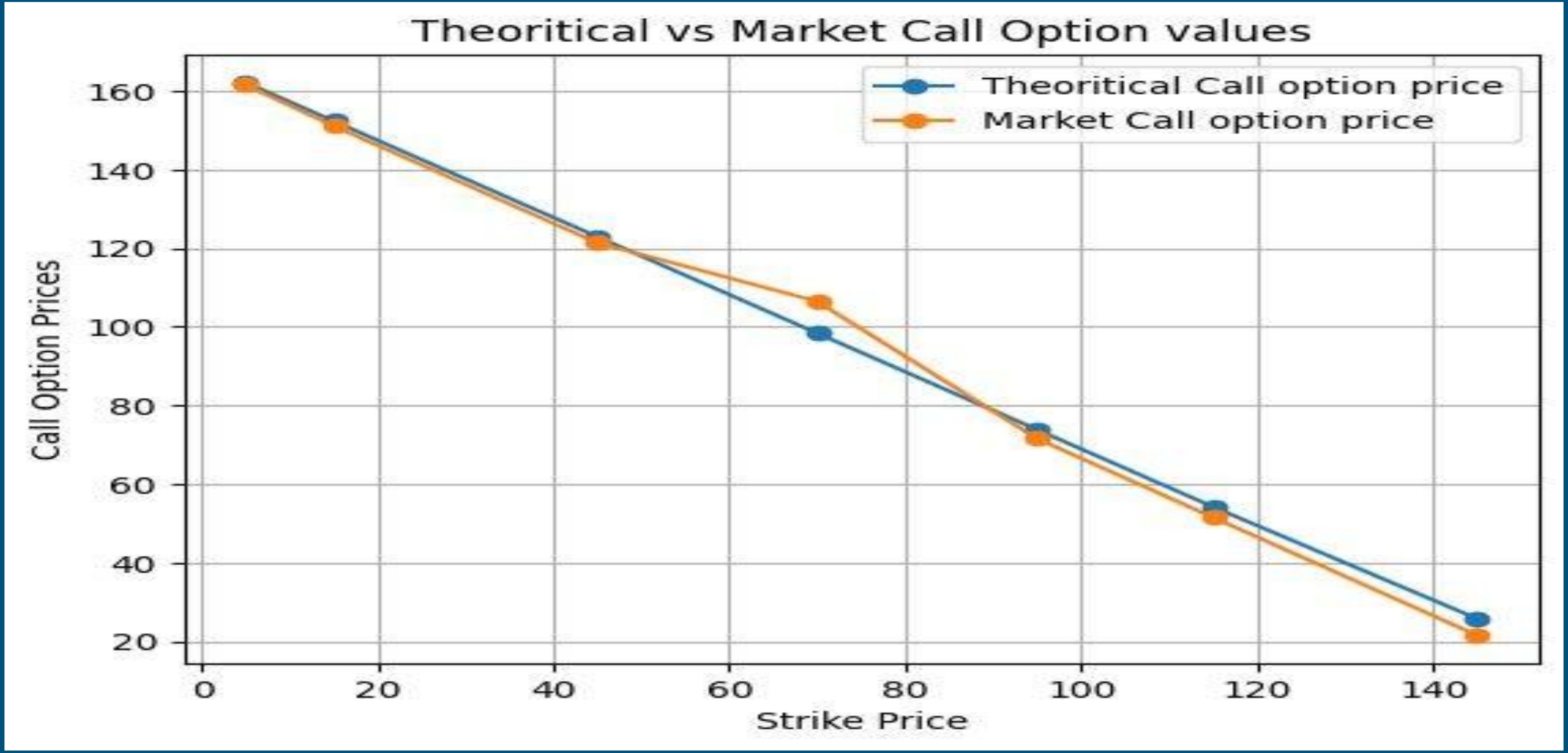
- N(d) is the cumulative distribution function of the standard normal distribution.

Convergence Graph





Compare with Actual Market Data



Delta Neutral Portfolio

- The principle of delta hedging is based on embedding derivative securities in a portfolio, the value of which does not alter too much when S varies.
- This can be achieved by ensuring that the delta of the portfolio is equal to zero. Such a
- portfolio is called delta neutral.
- We take a portfolio composed of stock, bonds and the hedged derivative security, its value given by V (S) = xS + y + zD(S) with z=-1.
- The delta of the option is given by $\frac{\mathrm{d}}{\mathrm{d}S} C^{E}(S) = N(d_1),$
- The portfolio (x, y, z) =(N(d1), y, −1), where the position in stock N(d1) is computed for the initial stock price S = S(0), has delta equal to zero for any money market position y.
- Consequently, its value V (S) = N(d1)S + y – CE(S)
- For delta neutral portfolio y= CE(S) – N(d1)S
- By using Black–Scholes formula for CE(S) will give y = $-Xe^{(-Tr)}$N(d2)
- For our portfolio value of CE = 98.36 , value of x = 0.99 and value of y = -68.67

Implied Volatility

- Implied volatility is a measure of the market's expectation of future volatility of an underlying asset, such as a stock. It is inferred from the current market price of an option using an option pricing model, such as the Black-Scholes model.
- A higher implied volatility suggests that the market expects larger price swings in the underlying asset, making the option more valuable (and vice versa for lower implied volatility).
- The Newton Raphson method is a widely used algorithm for calculating the implied volatility of an option.
- The steps for the algorithm are in next slide

- First defined the function as $f(\sigma) = V_{BS_\sigma} - V_{market}$
- Choose an initial guess: Start with an initial guess for the implied volatility. This guess can be based on historical volatility, previous iterations, or any other reasonable estimate.
- Calculate theoretical option price: Use the chosen implied volatility to calculate the theoretical price of the option using the option pricing model (e.g., Black-Scholes formula).
- Compute the error: Calculate the error between the theoretical option price and the market price observed in the market data.
- Update the guess: Use the error and its derivative with respect to volatility to update the guess for implied volatility using the Newton-Raphson formula.

$$\sigma_{n+1} = \sigma_n - \frac{V_{BS_\sigma} - V_{market}}{\frac{\partial V_{BS_\sigma}}{\partial \sigma}}$$

- Repeat steps 2-4 until the error is sufficiently small or a maximum number of iterations is reached.

Thanks

References:

https://finance.yahoo.com/quote/AAPL/options

https://www.investopedia.com/

https://quant-next.com/implied-volatility-calculation-with-newton-raphson-algorithm/