Integrations (Mode Decomposition exercise)

December 24, 2024

1 Code structure

- Definition of three functions f_1, f_2, and f_3
- Definition of the function $simpson3\_8$ for implementing Simpson's 3/8 rule
  - if $MOD(m, 3) = 0$ then apply Simpson's 3/8 rule over the whole interval
  - if $MOD(m, 3) = 1$ then apply Simpson's 3/8 rule up to the third to the last subinterval; then apply Simpson's 1/3 rule to the last two subintervals.
  - if $MOD(m, 3) = 2$ then apply Simpson's 3/8 rule up to the penultimate subinterval; then apply the trapezoidal rule for the last subinterval.
  - the errors are computed accordingly
- Definition of a function for implementing the Gauss-Legendre quadrature of order 2.
- Definition of three functions needed to calculate the errors ($error3\_8$, $error1\_3$, $error\_trap$)
  - We choose to calculate the derivatives at x_0 such that $x_0 = x_vec(index_0)$ where $index_0 = \text{int}(n/2)$ if $n < 30$ and $index_0 = \text{int}(n/2 - 10)$ if $n \geq 30$ to deal with the piecewise-defined function.
- Regarding the error associated with Gauss-Legendre quadrature we take the difference between our approximation and exact result.
- Definition of a subroutine for creating documents.
- We store the errors and log(h) values, and create the related documents.

We make use of a python code to plot the data.

2 Results

We get the following numerical results:

- Simpson's 3/8 rule
  - Integral 1: 1.905097
  - Integral 2: 19.540702
  - Integral 3: 3.229884
- Gauss-Legendre quadrature
  - Integral 1: 1.905239
  - Integral 2: 19.717657
  - Integral 3: 3.249390