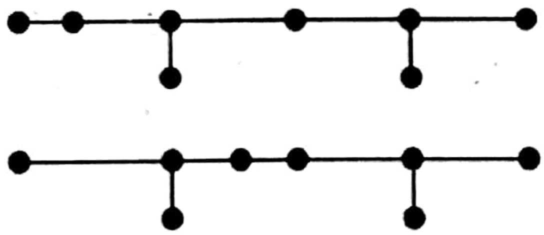
MCA-202: Discrete Mathematics
Master of Computer Applications
Semester II, May-2018

Time: Three Hours **Max. Marks: 70**

1. Fill in the blanks. (5)
   i. The edge-connectivity of a complete graph on n vertices is ……………………
   ii. The vertex induced graph obtained by deleting one vertex from K_n is …………
   iii. The chromatic number of a cycle graph with 2n+1 nodes is ……………………
   iv. The number of ways in which two integers can be selected from the integers 1, 2, 3……50 such that their difference is exactly 5 ……………………
   v. Let P(x,y) be the proposition "x passes course y", where the universe of discourse is the set of people in a certain college. Then, the statement "there is a course that no one has passed" is expressed as ……………………

2. State whether each of the following statements is true/false. Justify your answer. (10)
   i. A cycle graph on n vertices is 1-edge connected.
   ii. The edge-connectivity of a graph can be upper bounded by its vertex-connectivity.
   iii. An Euler path in a directed graph can have a vertex with indegree 3 and outdegree 5
   iv. The composite of two functions is invertible iff the two functions are invertible.
   v. The negation of the statement $\forall x : x^2 > 2$ is given by $\forall x$: NOT$(x^2 > 2)$.

3. Are the following graphs isomorphic? Explain. (3)



4. Consider a forest G with *n* vertices and *k* connected components. How many edges does G have? (3)

5. Suppose a simple planar graph has 20 vertices, each of degree 3. Into how many regions does the planar representation of this graph split the plane? (3)

6. A tree is given to have 2 vertices of degree 2, 1 vertex of degree 3 and 3 vertices of degree 4. How many degree 1 vertices are there in the tree? (3)

7. Give an example of each of the following: (3)
   i. a graph having both a Hamiltonian circuit and Euler circuit
   ii. a graph having a Hamiltonian circuit but not an Euler circuit

8. For each of the following recurrences state whether or not the Master's Theorem applies. If [redacted] not, give reasons, else use it to solve the recurrence. (4)

i. $T(n) = 2^n T(n/2) + n^n$

ii. $T(n) = 4T(n/2) + \log n$

9. Show that the function $f(x) = mx + b$ from R to R is invertible[redacted] find the inverse. (4)

10. Let R be an equivalence relation on set A. Show that if two elements a and b in A are related under R, their equivalence classes are identical. (4) pot-

11. Find an explicit formula for Fibonacci numbers. (4)

12. Show that a polynomial of degree k given by $P(x) = a_k.x^k + a_{k-1}.x^{k-1} + \ldots.. + a_1x + a_0$, where $a_i > 0$ for all i, is $\Theta(n^k)$. (4)

13. Show that among any n+1 numbers one can find two numbers so that their difference is divisible by n.
(5)

a − b

14. Prove using the Principle of Mathematical Induction that $2^n > n$ for $n \geq 1$. (5)

remainder

15. Consider two functions f: A → B and g: B → A such that fog = I_B. Then, show that f is a surjection and g is an injection. (5)

16. Prove by contradiction that an undirected graph has an even number of vertices of odd degree. (5)

−1 −1 +1

$\boxed{r \geq 0}$

$r^3 - r^2 - r = 0$

$r(r^2 - r) - r = 0$

$r(r^2 - r - 1) = 0$

$r^2 - r - 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{1 \pm \sqrt{1+4}}{2}$

$\frac{1 \pm \sqrt{5}}{2}$

3 3 4 6 7 1 3 2 n 5 1000