MCA-301: Design and Analysis of Algorithms
Master of Computer Applications
Semester Third, Nov/Dec-2017

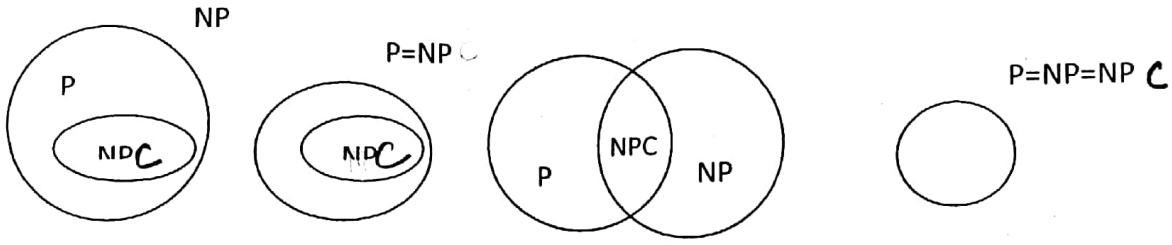
Max. Marks: 70

Time: Three Hours

Q.1 a) Let S be a NP-Complete problem and Q, R are two other problems known not to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. What can you conclude about problem Q and R? 2

b) Teena and Meena have been asked to show that a certain problem A is NP-complete. Teena shows a polynomial time reduction from the 3-SAT problem to A, and Meena shows a polynomial time reduction from A to 3-SAT. What can be inferred about A from these reductions? 2

c) Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of the following represents the correct Venn diagram of the complexity classes P, NP and NP Complete (NPC) and Why? 2



Q.2 a) Derive and solve the recurrence relation of Heapify algorithm.

b) Prove that build Heap requires O (n) time. 3+2

Q.3 a) Show that average time complexity of Quick Sort is O (nlogn) (don't consider luck and unluck).

b) Suppose you implement quicksort by always choosing the central element of the array as the pivot. Then what is the tightest upper bound for the worst case performance of this modified Quick Sort? 4+1

Q.4 A B+ tree of order *d* is a tree in which each internal node has between *d* and *2d* key values. An internal node with *M* key values has *M+1* children. The root (if it is an internal node) has between *1* and *2d* key values. The distance of a node from the root is the length of the path from the root to the node. All leaves are at the same distance from the root. The height of the tree is the distance of a leaf from the root.

a). What is the total number of key values in the internal nodes of a B+ tree with *l* leaves *(l≥2)*? 2

b). What is the maximum number of internal nodes in a B+ tree of order 4 with 52 leaves? 1

c). What is the minimum number of leaves in a B+ tree of order *d* and height *h (h≥1)*? 2

Q.5 a) Consider the matrices P, Q, R and S which are 20 x 15, 15 x 30, 30 x 5 and 5 x 40 matrices respectively. What is the minimum number of multiplications required to multiply the four matrices using Dynamic Programming? 5

b) A sub-sequence of a given sequence is just the given sequence with some elements (possibly none or all) left out. We are given two sequences X [i] and Y [j] of lengths i and j, respectively with indexes of X and Y starting from 0. We wish to find the length of the longest common sub-sequence (LCS) of X [i] and Y[j] as l(i,j), where an incomplete recursive definition for l(i,j) to compute the length of the LCS of X[i] and Y[j] is given below:

5

6

52 leaves Nodes $x = 52$

M+1 children $x(M+1) = 52$

$x(5+1) = 52$

l(i,j) = expr0, if either i=0 or j=0
= expr1, if i,j > 0 and X[i-1] = Y[j-1]
= expr2, if i,j > 0 and X[i-1] != Y[j-1]

i) What is required in place of expr0, expr1 and expr2?

ii) Consider two strings A = "qpqrr" and B = "pqprqrp". Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let y be the number of such longest common subsequences between A and B. Calculate the value of 2x + 5y. What is the time and space complexity of LCS? 2+5

Q.6 A networking company uses a compression technique to encode the message before transmitting over the network. Suppose the message contains the following characters with their frequency:

Character	Frequency
a	0.19
b	0.05
c	0.17
d	0.08
e	0.40
f	0.11

Note : Each character in input message takes 1 byte.

a)If the compression technique used is Huffman Coding, how many bits will be saved in the message?

b)What is the time and space complexity of Huffman Coding? 4+1

Q.7 a) What is the worst case time complexity of following implementation of the subset sum problem. 4

```
// Returns true if there is a subset of set[] with sun equal to a given sum
bool isSubsetSum(int set[], int n, int sum)
{
  if (sum == 0)
    return true;
  if (n == 0 && sum != 0)
    return false;
  if (set[n-1] > sum)
    return isSubsetSum(set, n-1, sum);
  return isSubsetSum(set, n-1, sum) || isSubsetSum(set, n-1, sum-set[n-1]);
}
```

T(n-1)

b) Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n)=T(n-a)+T(a)+cn$ where $a\geq 1$ and $c>0$ are constants. 5

c) The recurrence relation $T(n)=7T(n/2)+n^2$ describes the running time of an algorithm A. An another algorithm B has running time $aT(n/4)+n^2$. What is the largest integer value for 'a' such that B is asymptotically faster than A? 4

Q.8 a) An undirected graph G(V, E) contains n (n > 2) nodes named v1 , v2 ,....vn. Two nodes vi , vj are connected if and only if 0 < |i – j| <= 2. Each edge (vi, vj) is assigned a weight i + j. What will be the cost of the minimum spanning tree (MST) of such a graph with n nodes?
What will be the length of the from v5 to v6 in the MST if n=10? 3+2

b) Find the optimal schedule and maximum profit for the following jobs with n=9, profit (p1,p2.....p9)= (15,20,30,18,18,10,23,16,25) and dead lines (d1,d2.....d9)=(7,2,5,3,4,5,2,7,3)? 3

Q9 a) Discus Strassen's Matrix Multiplication Algorithm using Divide and Conquer. That means, given two n x m matrices, A and B, compute C=A×B, where the elements of C are given by 7

$$Cij = \sum_{k=0}^{n-1} Aik\ Bkj$$

b) Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry Wij in the matrix W below is the weight of the edge {i, j}. Compute the minimum weight spanning tree T in this graph such that vertex 0 is a leaf node in the tree T. 4

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$