```
### Question 1 ###

set.seed(123)
n <- 10000
sigma_optimal <- 1
samples <- rnorm(n, mean = 0, sd = sigma_optimal)
weights <- dnorm(samples, mean = 0, sd = 1) / dnorm(samples, mean = 0, sd =
sigma_optimal)
expectation <- mean(weights * samples)
print(paste("Estimated expectation:", expectation))

### Question 2 ###

set.seed(123)
n_samples <- 10000
alpha_target <- 2
beta_target <- 4
alpha_importance <- 3
beta_importance <- 3.5

samples_importance <- rgamma(n_samples, shape = alpha_importance, scale =
beta_importance)
weights <- dgamma(samples_importance, shape = alpha_target, scale = beta_target) /
  dgamma(samples_importance, shape = alpha_importance, scale = beta_importance)

sample_variances <- (samples_importance - mean(samples_importance))^2
weighted_variances <- weights * sample_variances
estimated_variance <- sum(weighted_variances) / sum(weights)
variance_of_estimator <- var(weighted_variances) / n_samples

cat("Estimated Variance of Gamma(2, 4):", estimated_variance, "\n")
cat("Variance of Importance Sampling Estimator:", variance_of_estimator, "\n")
```