Assignment No 5

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EE20B084

March 11, 2022

1 Introduction

We wish to solve for the currents in a resistor. The currents depend on the shape of the resistor and we also want to know which part of the resistor is likely to get hottest.

2 Equations

A cylindrical wire is soldered to the middle of a copper plate and its voltage is held at 1 Volt. One side of the plate is grounded, while the remaining are floating. The plate is 1 cm by 1 cm in size.
Conductivity

$$\vec{j} = \sigma \vec{E} \tag{1}$$

Electric field is the gradient of the potential

$$\vec{E} = -\nabla \phi \tag{2}$$

Continuity of charge

$$\nabla \vec{j} = -\frac{\partial \rho}{\partial t} \tag{3}$$

Combining the equations and for DC currents, the rightside in (3) is zero and we get

$$\nabla^2 \phi = 0 \tag{4}$$

3 Defining parameters and Initialising potential

We choose a 25x25 zero 2-D array and then a list of coordinates lying within the radius 8 is generated and these points are initialized to 1V. We also choose to run the difference equation for 1500 iterations by default.

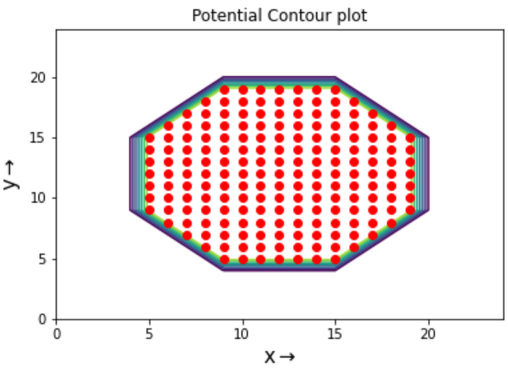

Figure 1: Contour plot of Potential

4 Performing iterations

4.1 Updating the potential

We convert the differential equation(4) to the following difference equation.

$$\phi_{i,j} = \frac{\phi_{i-1,j} + \phi_{i,j-1} + \phi_{i+1,j} + \phi_{i,j+1}}{4} \tag{5}$$

4.2 Enforcing Boundary Conditions

The bottom boundary is grounded. The other 3 boundaries have a normal potential difference zero.

4.3 Calculating error after each iteration

We will find what is the maximum error in the block of potential in each iteration

4.4 Code performed in iteration

```
errors = np.zeros(Niter)
for k in range(Niter):
oldphi = phi.copy()
phi[1:-1,1:-1]=0.25*(phi[1:-1,0:-2]+phi[1:-1,2:]+phi[0:-2,1:-1]+phi[2:,1:-1])
phi[1:-1,0]=phi[1:-1,1]#left
phi[1:-1,Nx-1]=phi[1:-1,Nx-2]#right
phi[0,1:-1]=phi[1,1:-1]#top
phi[ii]=1.0
errors[k]=(abs(phi-oldphi)).max()
```

4.5 Plotting Error

Plot the errors on semi-log and log-log plots.
A general comment on the algorithm: This method of solving Laplace's Equation is known to be one of the worst available. This is because of the very slow coefficient with which the error reduces.

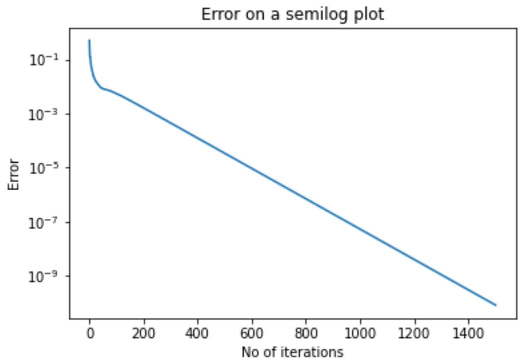

Figure 2: Semilog plot of errors

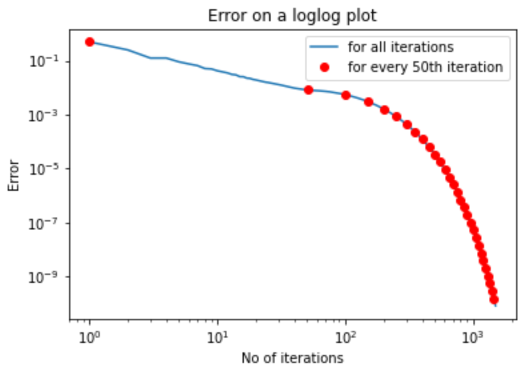

Figure 3: Loglog plot of errors

5 Fitting the error

The error is a decaying potential for higher iterations. We attempt to fit a function

$$y = Ae^{Bx} \tag{6}$$

$$log(y) = log(A) + Bx \tag{7}$$

We estimate log(A) and B with the least squares method. fit1 is considering all the iterations and fit2 is considering iterations from 500 onwards.

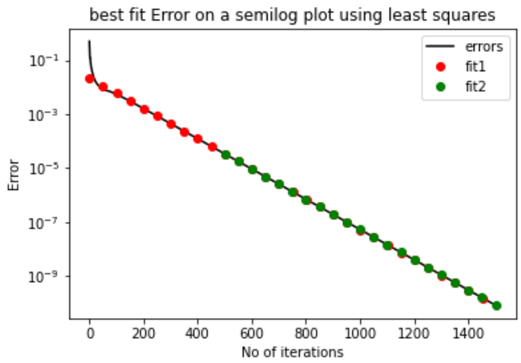

Figure 4: Best fit of errors

There is very little difference between the two fits.

6 Plotting maximum error

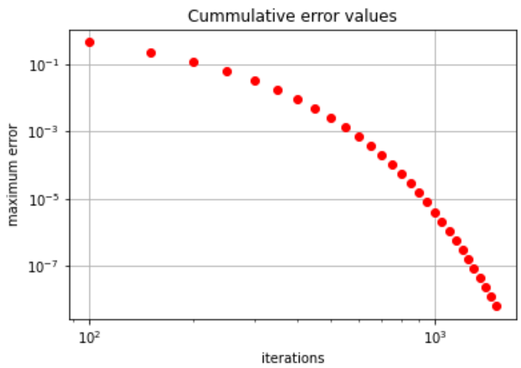

Figure 5: Cumulative error values on a log log scale

7 Plotting Potential

8 Calculating current density

$$J_{x,ij} = \frac{\phi_{i,j-1} - \phi i, j+1}{2} \tag{8}$$

$$J_{y,ij} = \frac{\phi_{i-1,j} - \phi i+1, j}{2} \tag{9}$$

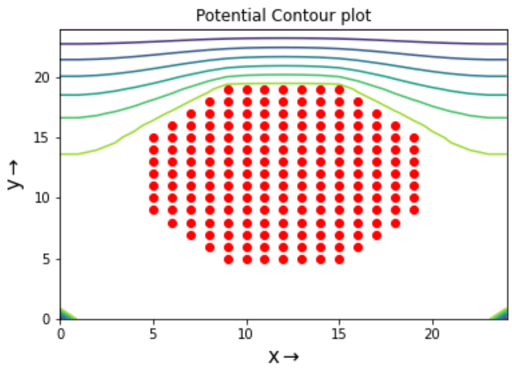

Figure 6: 2d plot of potential

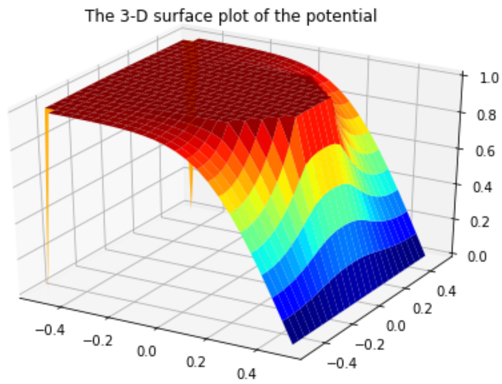

Figure 7: 3d plot of potential

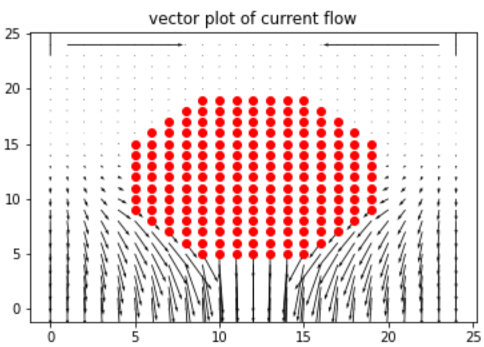

Figure 8: Vector plot of current

From the current density plot, we can see that hardly any current flows through the top part of the wire. We can conclude that the lower surface being grounded, the easiest way for charge carriers to flow from the electrode would be directly through the lower half of the wire, thus avoiding a longer, more resistive path through the top half of the wire.

9 Conclusion

We can find solution to Laplace's equation for a given system using a finite differentiation approximation. The error is seen to decay at a gradual pace. Thus the chosen method of solving Laplace's equation is inefficient. On analysing the vector plot of the currents, we can conclude that the current was mostly restricted to the bottom of the wire.