Assignment6

Mudadla Harshavardhan

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1 Time response of a spring

In this assignment we are about to find output of given function in the s-domain using impulse response of given function. Consider the forced oscillatory system given by the equation:$x(0) = 0, \dot{x}(0) = 0$

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

where

$$f(t) = cos(1.5t)e^{-0.5t} * u(t) \tag{2}$$

whose laplace transform is

$$F(s) = (s + 0.5)/((s + 0.5)^2 + 2.25) \tag{3}$$

Solving for $X(s)$ in Laplace domain we get,

$$X(s) = \frac{s + 0.5}{(s + 0.5^2) + 2.25)(s^2 + 2.25)} \tag{4}$$

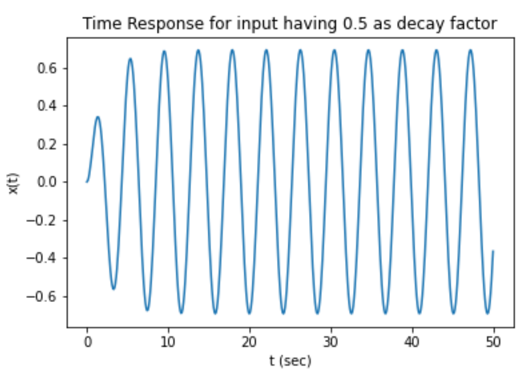

Figure 1: x(t) vs t at decay constant 0.5

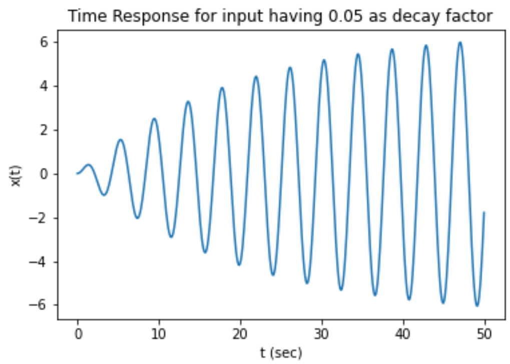

Figure 2: x(t) vs t at decay constant 0.05

2 Time response of a spring using transfer function

Obtain system transfer function $X(s)/F(s)$ for above $x(t), f(t)$. And vary ω from 1.4 to 1.6 with 0.05 gapping in $f(t)$.Use *signal.lsim* and find resulting responses and plot them. From the given equation, we can see that the natural response of the system has the frequency w = 1.5 rad/s . Thus, as expected the maximum amplitude of oscillation is obtained when the frequency of $f(t)$ is 1.5 rad/s, due to resonance.

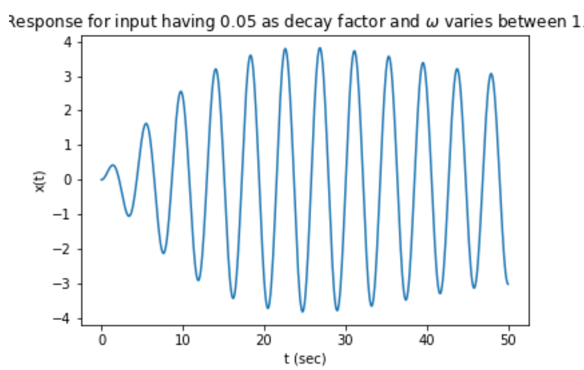

Figure 3: responses at 1.4 frequency

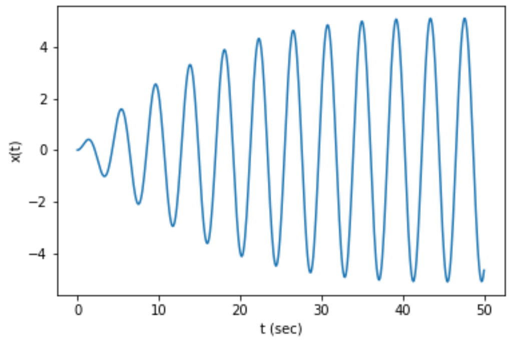

Figure 4: responses at 1.45 frequency

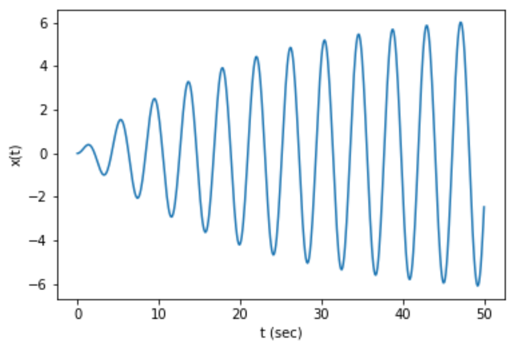

Figure 5: responses at 1.5 frequency

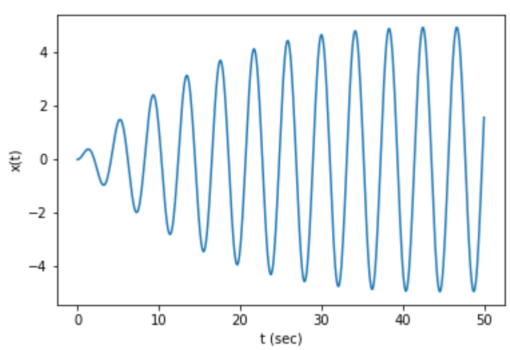

Figure 6: responses at 1.55 frequency

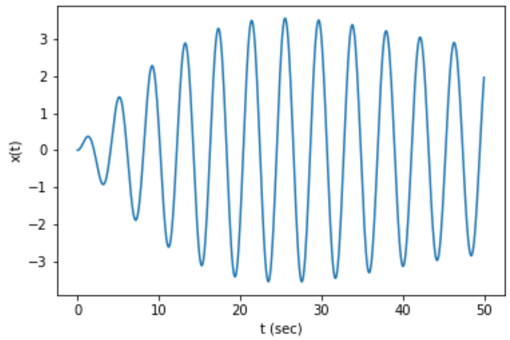

Figure 7: responses at 1.6 frequency

3 Coupled spring problem

We now consider a coupled Differential system

$$\ddot{x} + (x - y) = 0 \tag{5}$$

and

$$\ddot{y} + 2(y - x) = 0 \tag{6}$$

with the initial conditions: $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$. Taking Laplace Transform and solving for $X(s)$ and $Y(s)$, We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{7}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{8}$$

And with time being constrained between 0 and 20 we need to plot both $x(t), y(t)$ with respect to time.

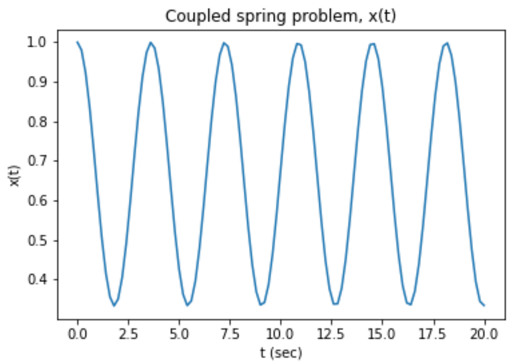

Figure 8: plot of $x_t incoupledspring$

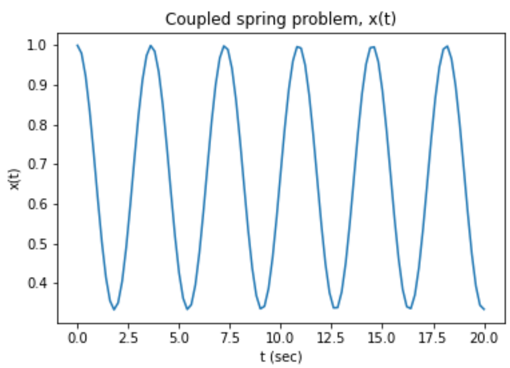

Figure 9: plot of $y_t incoupledspring$

4 Two-port Network

We need to obtain Magnitude and phase response of the steady state transfer function of given 2 port network. The Steady-State transfer function of the given circuit is given by

$$H(s) = \frac{10^6}{s^2/10^6 + 100s + 10^6} \tag{9}$$

5 Low pass filter response

Low pass filter is given by below equation:

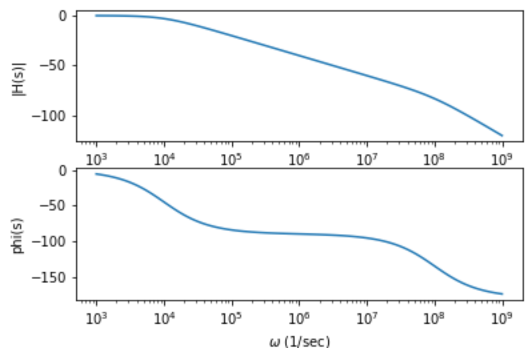

Figure 10: Magnitude response and phase response

$$V_i(t) = (cos(10^3t) - cos(10^6t))u(t)$$

we need to find output for $0 < t < 30\mu s$ and $0 < t < 10ms$ using $signal.lsim$ nd then plot it w.r.t input. we can find all by using $t, y, svec = sp.lsim(H, u, t)$ where y is output, u is input, H is transfer function of given 2 port network.

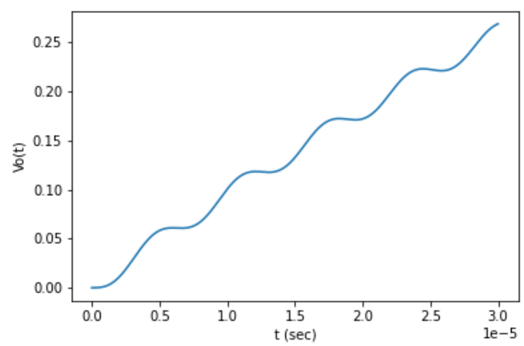

Figure 11: for $0 < t < 30\mu s$

6 Conclusion

The scipy.signal library provides a useful toolkit of functions for circuit analysis. The toolkit was used for the analysis of LTI systems in various domains.
The forced response of a simple spring body system was obtained over various frequencies of the applied force, and highest amplitude was observed at resonant frequency.
A coupled spring problem was solved using the sp.impulse function to obtain

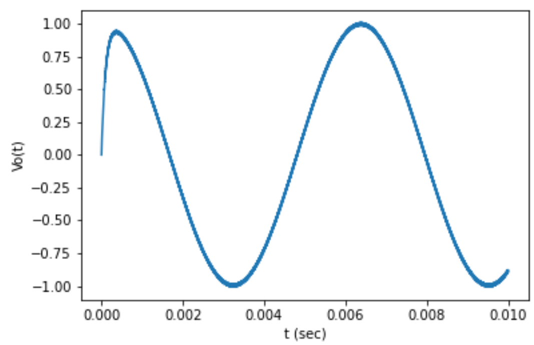

Figure 12: for $0 < t < 10ms$

two sinusoids of the same frequency.
A two-port network, functioning as a low-pass filter was analysed and the output was obtained for a mixed frequency input.