Assignment8

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EE20B084

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1 Aim

The aim of this assignment is to examine the DFT of various functions using the fft library in numpy.

2 Sinusoid

In this part we study the spectrum of $sin(5t)$. As expected, we get 2 peaks at +5 and -5 with height 0.5. The phase at the peaks are $\pi/2$ and $-\pi/2$.

$$y = sin(5t) = 0.5(\frac{e^{5t}}{j} - \frac{e^{-5t}}{j}) \tag{1}$$

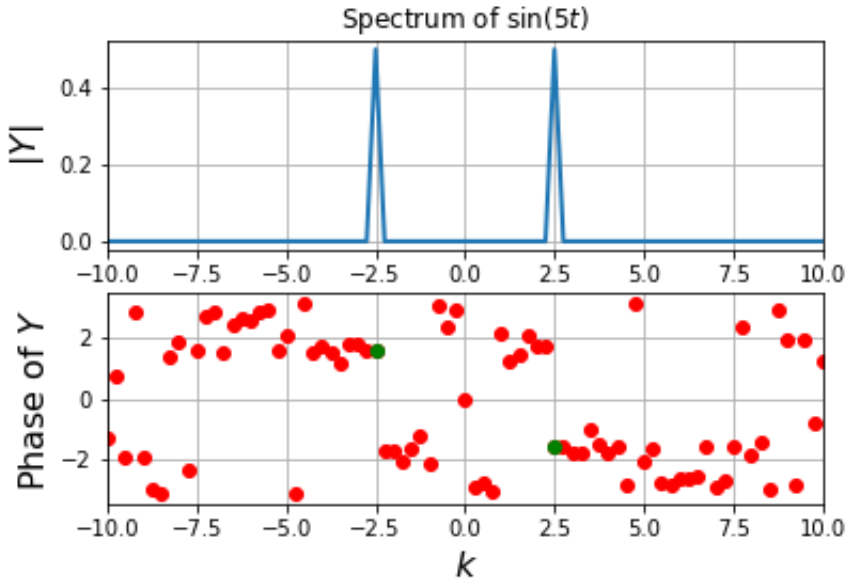

Figure 1: DFT of $sin(5t)$

3 Amplitude Modulated Signal

The amplitude modulated signal is,

$$f(t) = (1 + 0.1\cos(t))\cos(10t) \tag{2}$$

By using a larger range and higher number of samples, we can see the three peaks clearly. At all three peaks the phase is zero.

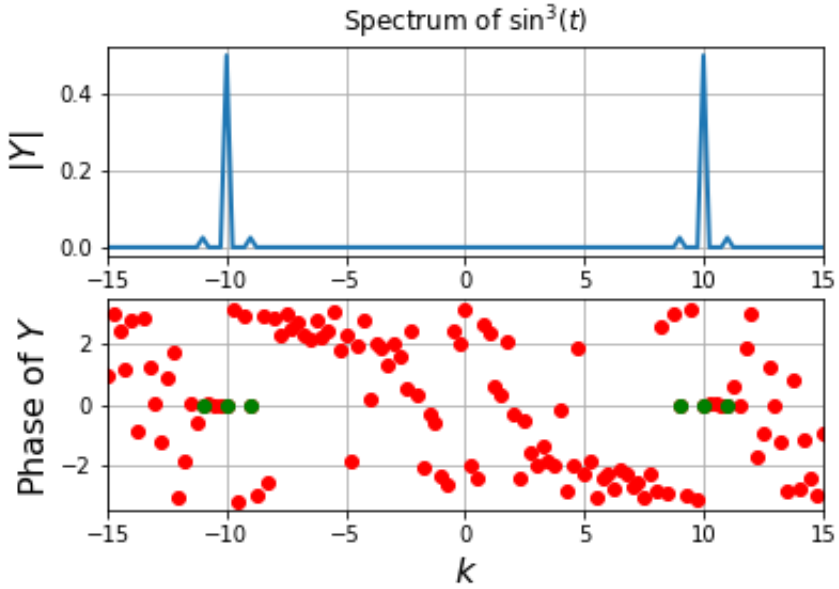

Figure 2: DFT of Amplitude modulated signal

4 Spectrum of $sin^3(t)$ and $cos^3(t)$

The signals can be represented as follows:

$$\sin^3(t) = \frac{3}{4}\sin(t) - \frac{1}{4}\sin(3t) \tag{3}$$

$$\cos^3(t) = \frac{3}{4}\cos(t) + \frac{1}{4}\cos(3t) \tag{4}$$

Thus there will be 4 impulses in the frequency spectrum. Following are the spectrums obtained.

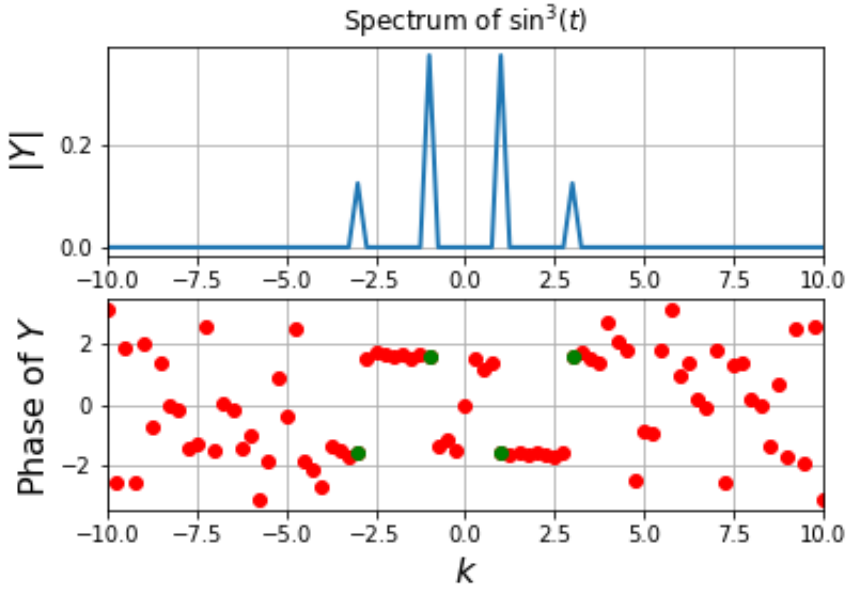

Figure 3: DFT of $cos^3(t)$

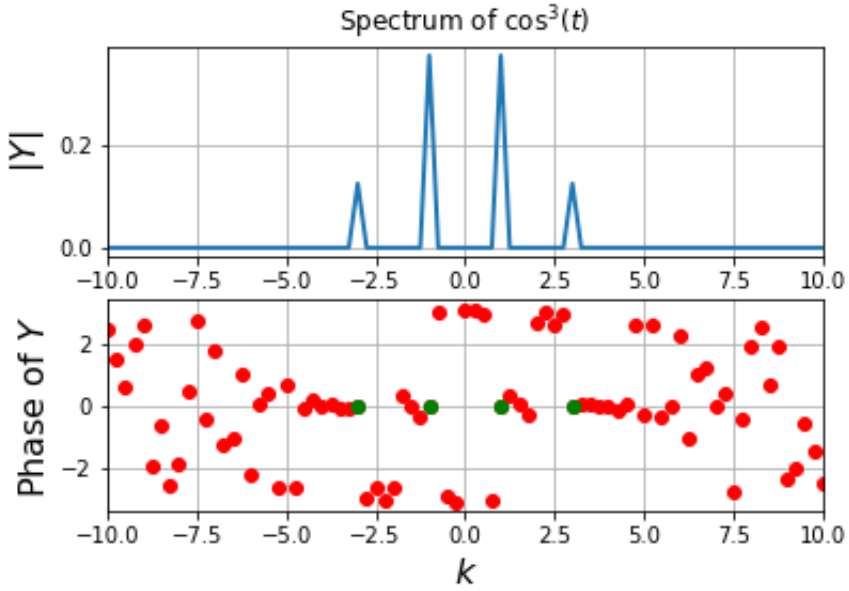

Figure 4: DFT of $sin^3(t)$

5 Frequency Modulation

Consider the frequency modulated signal, $\cos(20t + 5\cos(t))$

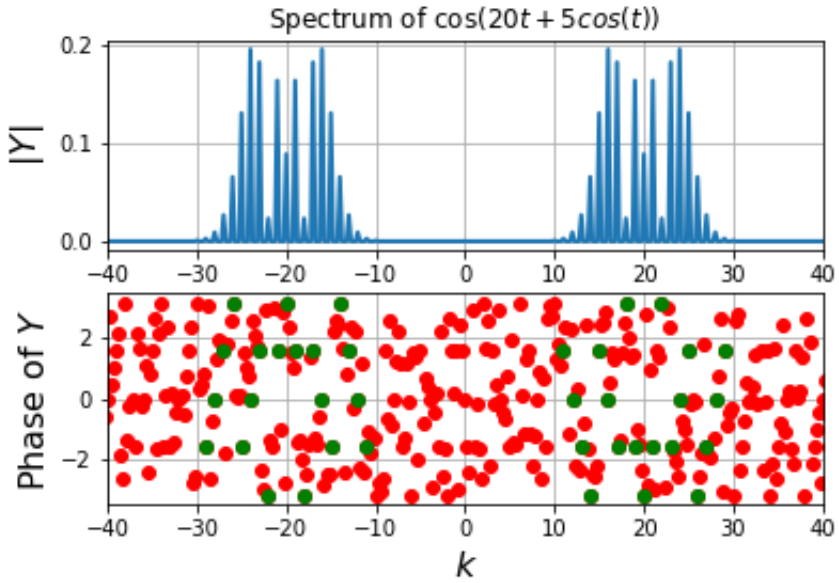

Figure 5: DFT of Frequency modulated signal

6 Gaussian

The Gaussian function $f(x) = e^{-x^2/2}$ is not band limited as the frequency spectrum has non zero values even for large frequencies. The value of error varies with different time ranges and the sampling rate. For sampling rate = 512 and time range = 8π s, the error is found to be around 10^{-15}. As the sampling rate increases, the peak sharpens. Also, it broadens for greater time ranges.

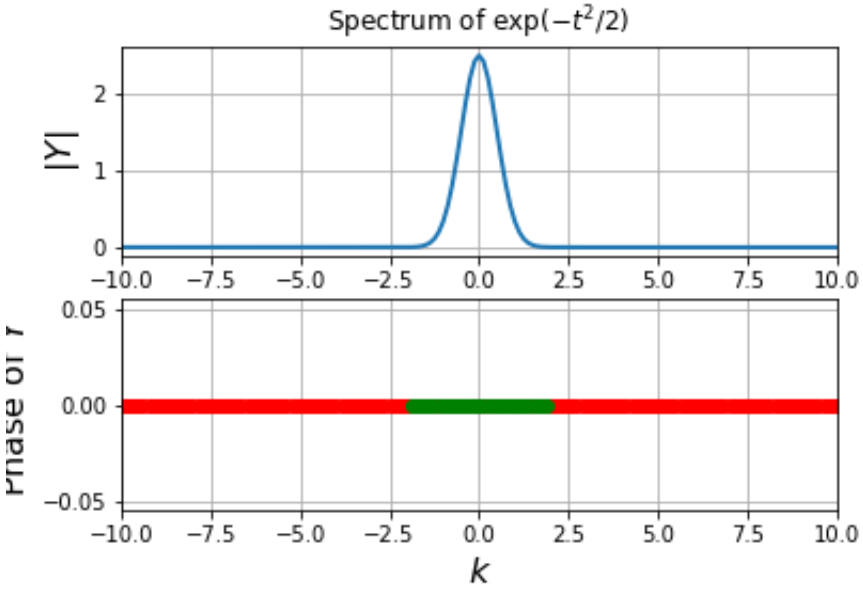

Figure 6: DFT of gaussian for time range = 4π and sampling rate = 512

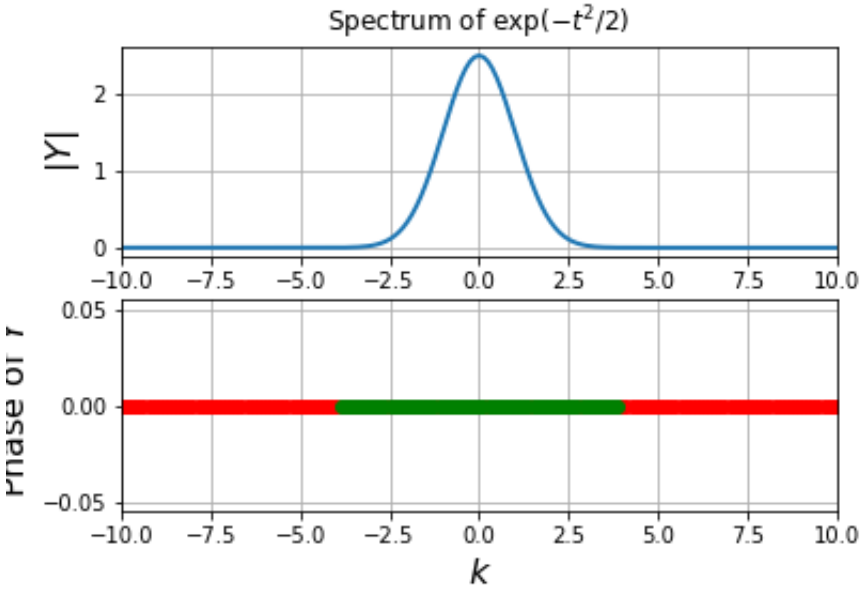

Figure 7: DFT of gaussian for time range = 8π and sampling rate = 512

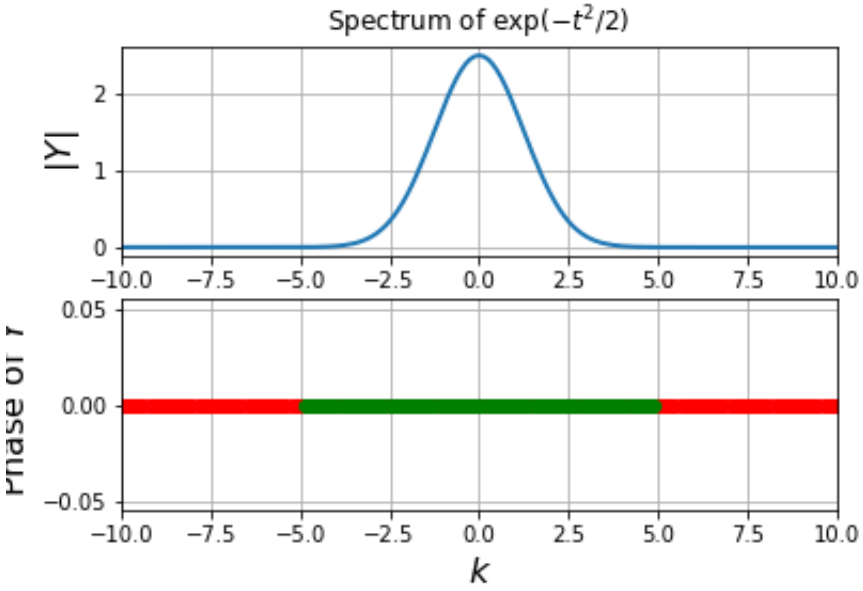

Figure 8: DFT of gaussian for time range = 12π

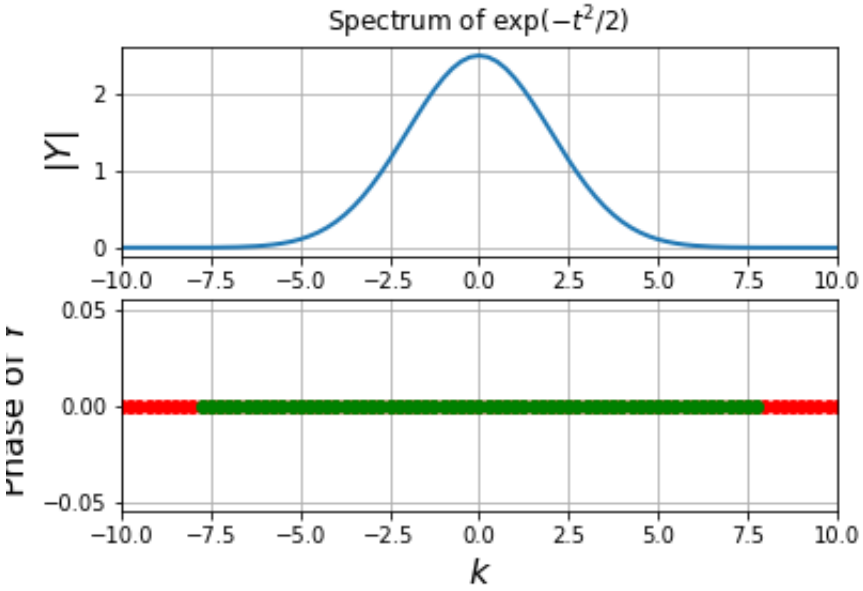

Figure 9: DFT of gaussian for sampling rate = 256

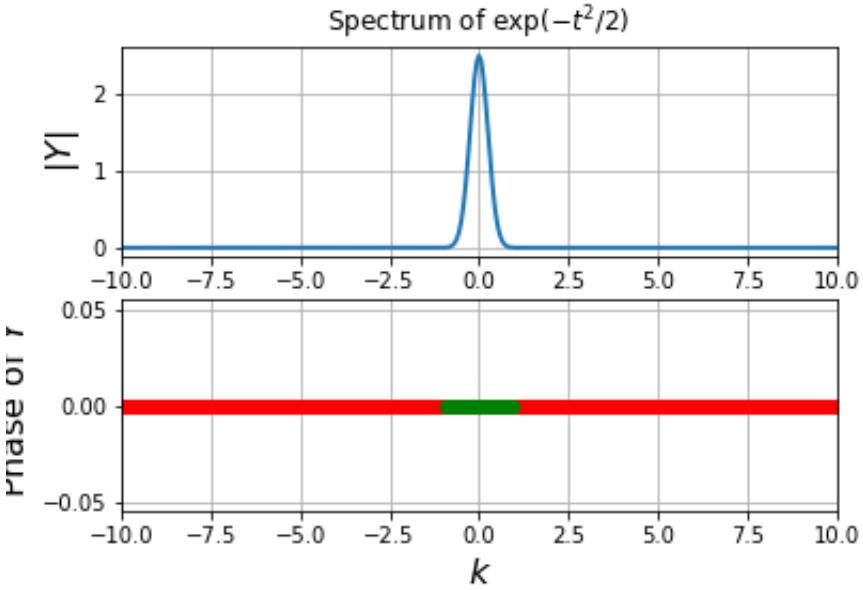

Figure 10: DFT of gaussian for sampling rate = 1024