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**Project Title:** *Minimum-Energy Trajectory Control*

# 1. Problem Statement

Autonomous vehicles must perform lane-change maneuvers smoothly, safely, and efficiently. A key requirement is to generate a lateral trajectory that moves the vehicle from its current lane to a target lane without abrupt steering or violating limits. This project computes such a trajectory optimally by modeling the vehicle's lateral motion as a dynamic system and determining control inputs that achieve the maneuver with minimum energy.

We use a discrete-time lateral motion model where the state contains lateral position and velocity, and the control input is lateral acceleration. The vehicle must start at the center of the current lane and reach the center of the adjacent lane within a fixed horizon while satisfying acceleration and velocity constraints. The goal is to compute the optimal control sequence and resulting trajectory that meets all constraints and minimizes energy. This naturally forms a constrained optimal control problem solvable using convex optimization.

# 2. Planned Method

The lateral dynamics are modeled as a linear discrete-time system:

$$x_{k+1} = Ax_k + Bu_k, \quad x_k = [y_k, v_{y,k}]^T,$$

where $y_k$ is lateral position, $v_{y,k}$ is lateral velocity, and $u_k$ is lateral acceleration. Matrices $A$ and $B$ follow from the chosen sampling time $\Delta t$. The optimization variables are the state sequence $x_0, \ldots, x_N$ and control inputs $u_0, \ldots, u_{N-1}$. Constraints include system dynamics, boundary conditions, and bounds on velocity and acceleration.

The cost to minimize is:

$$\min \sum_{k=0}^{N-1} u_k^2,$$

which promotes smooth, efficient control. Since the cost is quadratic and constraints are linear, the problem becomes a Convex Quadratic Program (QP).

KKT/Lagrangian analysis is used to study the optimal solution structure and confirm convexity. The final trajectory is computed numerically using Python and CVXPY, which solves the QP and provides the optimal lateral path and control inputs for the lane-change maneuver.