

Probability Assignment

EE22BTECH11022-G.SAI HARSHITH*

Question: It is known that 10% of certain articles manufactured are defective. What is probability that a random sample space of 12 such articles, 9 are defective?

Solution: Let X be random variable defined as

Random Variable	Values	Description
X	$1 \leq X \leq 12$	Number of defective in 12 articles

X has a binomial distribution with parameters

$$n = 12 \quad p = \frac{10}{100} = \frac{1}{10} \quad (1)$$

Pmf of X for $1 \leq k \leq 12$ is

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (2)$$

Probability that a random sample space of 12 such articles, 9 are defective is

$$p_X(9) = {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(1 - \frac{1}{10}\right)^{12-9} \quad (3)$$

$$= \frac{12!}{9!3!} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \quad (4)$$

$$= 220 \left(\frac{1}{10^9}\right) \left(\frac{9^3}{10^3}\right) \quad (5)$$

$$= 22 \left(\frac{9^3}{10^{11}}\right) \quad (6)$$

$$= 1.603773(10^{-7}) \quad (7)$$

Let Y be gaussian variable

$$\mu = np \quad (8)$$

$$= \frac{6}{5} \quad (9)$$

$$\sigma^2 = np(1-p) \quad (10)$$

$$= \frac{27}{25} \quad (11)$$

Using Normal distribution at $X=9$.

$$Z = \frac{X - \mu}{\sigma} \quad (12)$$

$$= \frac{9 - \frac{6}{5}}{\sqrt{\frac{27}{25}}} \quad (13)$$

$$= 7.50555 \quad (14)$$

For pdf calculation

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (15)$$

From the plot, pmf is close to normal distribution

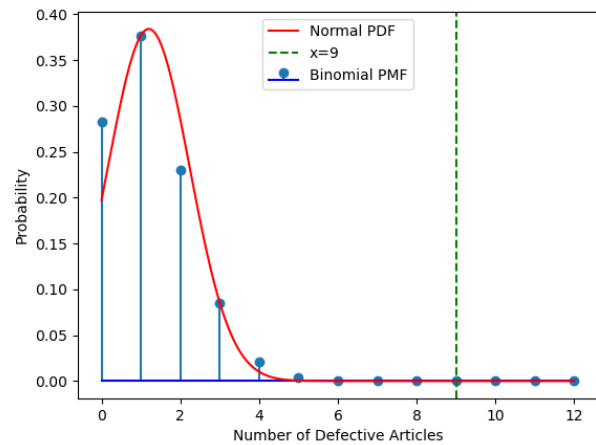


Fig. 0. Binomial pmf vs Gaussian pdf

pdf.

$$p_Y(9) = p_Z(7.5055) \quad (16)$$

$$= 1.6109(10^{-7}) \quad (17)$$

From (7) and (17),

$$p_X(9) \approx p_Y(9) \quad (18)$$