Probability Assignment

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Question: It is known that 10% of certain articles manufactered are defective. What is probability that a random sample space of 12 such articles,9 are defective?

Solution: Let X be random variable defined as

Random Variable	Values	Description
X	$1 \leq X \leq 12$	Number of defective in 12 articles

X has a binomial distribution with parameters

$$n = 12 \qquad p = \frac{10}{100} = \frac{1}{10} \tag{1}$$

Pmf of X for $1 \leq k \leq 12$ is

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{2}$$

Probability that a random sample space of 12 such articles,9 are defective is

$$p_X(9) = {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(1 - \frac{1}{10}\right)^{12-9} \tag{3}$$

$$= \frac{12!}{9!3!} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 \tag{4}$$

$$= 220 \left(\frac{1}{10^9}\right) \left(\frac{9^3}{10^3}\right) \tag{5}$$

$$= 22 \left(\frac{9^3}{10^{11}}\right) \tag{6}$$

$$= 1.603773(10^{-7}) \tag{7}$$

Let Y be goussian variable

$$\mu = np \tag{8}$$

$$= \frac{6}{5} \tag{9}$$

$$\sigma^2 = np(1-p) \tag{10}$$

$$= \frac{27}{25} \tag{11}$$

Using Normal distribution at X=9.

$$Z = \frac{X - \mu}{\sigma} \tag{12}$$

$$= \frac{9 - \frac{6}{5}}{\sqrt{\frac{27}{25}}} \tag{13}$$

$$= 7.50555 \tag{14}$$

For pdf calculation

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{15}$$
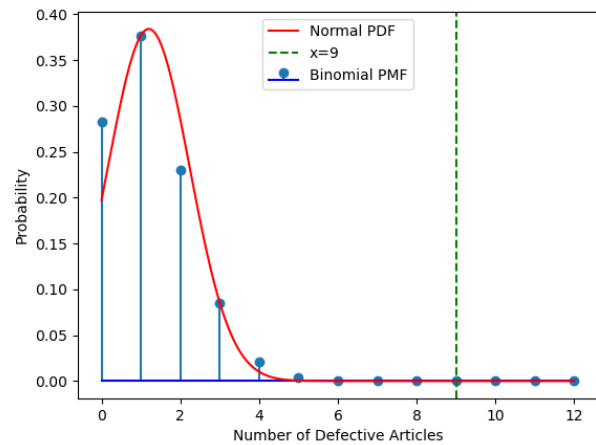
From the plot, pmf is close to normal distribution

Fig. 0. Binomial pmf vs Gaussian pdf

pdf.

$$p_Y(9) = p_Z(7.5055) \tag{16}$$

$$= 1.6109(10^{-7}) \tag{17}$$

From (7) and (17),

$$p_X(9) \approx p_Y(9) \tag{18}$$