

Probability Assignment

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Question: The frequency response $H(f)$ of a linear time-invariant system has magnitude as shown in Fig. 0

Statement 1: The system is necessarily a pure delay system for inputs which are bandlimited to $-a \leq f \leq a$.

Statement 2: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_X(f) = S_Y(f)$ for $-a \leq f \leq a$.

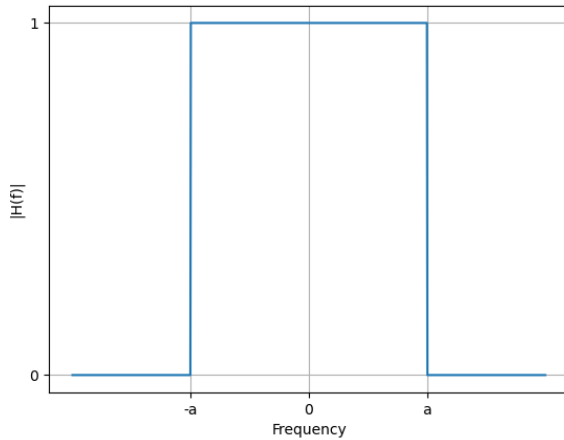


Fig. 0. $|H(f)|$ vs frequency

Solution: A system where output signal is a delayed version of input signal with no other transformations or operations is called a pure delay system.

- 1) Let us consider a pure delay LTI system with $x(t)$ and $y(t)$ as input and output signals in time domain. Let T_d be delay between input and output. So,

$$y(t) = x(t - T_d) \quad (1)$$

Applying fourier transform,

$$\int_{-\infty}^{\infty} y(t) e^{-2\pi f j \omega t} dt = \int_{-\infty}^{\infty} x(t - T_d) e^{-2\pi f j t} dt \quad (2)$$

$$= \int_{-\infty}^{\infty} x(t) e^{-2\pi f j (t+T_d)} d(t + T_d) \quad (3)$$

$$= e^{-2\pi f j T_d} \int_{-\infty}^{\infty} x(t) e^{-2\pi f j t} dt \quad (4)$$

$$Y(f) = e^{-2\pi f j T_d} X(f) \quad (5)$$

Here $Y(f)$ and $X(f)$ are output and input signals in frequency domain. Let $H(f)$ be

$$H(f) = \frac{Y(f)}{X(f)} \quad (6)$$

$$= e^{-2\pi f j T_d} \quad (7)$$

$$= (1) e^{-2\pi f j T_d} \quad (8)$$

Comparing (8) with

$$H(f) = |H(f)| e^{j\angle H(f)} \quad (9)$$

$$|H(f)| = 1 \quad (10)$$

As input is bandlimited in $-a \leq f \leq a$. We have

$$|H(f)| = \begin{cases} 1 & \text{if } -a \leq f \leq a \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

From (11) and Fig. 0, $|H(f)|$ is same. So, system will act as pure delay system. Statement 1 is correct.

- 2) For wide-sense stationary LTI system,

$$S_Y(f) = |H(f)|^2 S_X(f) \quad (12)$$

From (11), for $-a \leq f \leq a$,

$$S_Y(f) = (1)^2 S_X(f) \quad (13)$$

$$S_Y(f) = S_X(f) \quad (14)$$

Statement 2 is correct.