Probability Assignment

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Question: Find $P(E|F)$ for

1) E: tail appears on one coin.
   F: head appears on one coin.
2) E: no tail appears.
   F: no head appears.

Solution: The random variables X_1 and X_2 are shown in Table 1.

$X_1 = 0$	First coin shows Tail.
$X_1 = 1$	First coin shows Head.
$X_2 = 0$	Second coin shows Tail.
$X_2 = 1$	Second coin shows Head.

TABLE 1: Definition of X_1 and X_2.

Since the coins are fair.

$$P_{X_1X_2}(k,m) = \frac{1}{4} \tag{1}$$

In (1), $k, m \in \{0, 1\}$. So, total four different k,m combinations.

1) E: Here one coin is tail and other is head. We are required to find $\Pr(X_1 + X_2 = 1)$. Thus, from (1).

$$\Pr(E) = \Pr(X_1 + X_2 = 1) \tag{2}$$
$$= \Pr(X_1 = 0,\ X_2 = 1) + \Pr(X_1 = 1,\ X_2 = 0) \tag{3}$$
$$= \frac{1}{2} \tag{4}$$

F: Here one coin is head and other is tail. We are required to find $\Pr(X_1 + X_2 = 1)$. Thus, from (1).

$$\Pr(F) = \Pr(X_1 + X_2 = 1) \tag{5}$$
$$= \Pr(X_1 = 0,\ X_2 = 1) + \Pr(X_1 = 1,\ X_2 = 0) \tag{6}$$
$$= \frac{1}{2} \tag{7}$$

EF: Here one coin is head and other is tail. We are required to find $\Pr(X_1 + X_2 = 1)$. Thus, from (1).

$$\Pr(EF) = \Pr(X_1 + X_2 = 1) \tag{8}$$
$$= \Pr(X_1 = 0,\ X_2 = 1) + \Pr(X_1 = 1,\ X_2 = 0) \tag{9}$$
$$= \frac{1}{2} \tag{10}$$
$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{11}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2}} \tag{12}$$
$$= 1 \tag{13}$$

2) E: no tail appears.We are required to find $\Pr(X_1 \neq 0, X_2 \neq 0)$. Thus, from (1).

$$\Pr(E) = \Pr(X_1 \neq 0, X_2 \neq 0) \tag{14}$$
$$= \Pr(X_1 = 1,\ X_2 = 1) \tag{15}$$
$$= \frac{1}{4} \tag{16}$$

F: no head appears.We are required to find $\Pr(X_1 \neq 1, X_2 \neq 1)$. Thus, from (1).

$$\Pr(F) = \Pr(X_1 \neq 1, X_2 \neq 1) \tag{17}$$
$$= \Pr(X_1 = 0,\ X_2 = 0) \tag{18}$$
$$= \frac{1}{4} \tag{19}$$

EF: coins should show neither head nor tail. From Table 1, we have coins showing head or tail. So, this is an impossible event

$$\Pr(EF) = 0 \tag{20}$$
$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{21}$$
$$= \frac{0}{\frac{1}{4}} \tag{22}$$
$$= 0 \tag{23}$$