Probability Assignment

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Question: Find $P(E|F)$ for

1) E: tail appears on one coin.
   F: head appears on one coin.
2) E: no tail appears.
   F: no head appears.

Solution: Define random variables X and Y as shown in Tables 1 and 2. Since the coins are fair. Form the joint

$X = 0$	First coin shows Tail.
$X = 1$	First coin shows Head.

TABLE 1: Definition of X.

	$X = 0$	$X = 1$
$Y = 0$	$\frac{1}{4}$	$\frac{1}{4}$
$Y = 1$	$\frac{1}{4}$	$\frac{1}{4}$

TABLE 2: Definition of Y.

pmf as in Table 3.

$Y = 0$	Second coin shows Tail.
$Y = 1$	Second coin shows Head.

TABLE 3: Joint pmf of X and Y.

1) E: tail appears on one coin.So, one coin should be tail and obviously other will be head. We are required to find $\Pr(X + Y = 1)$. Thus, from Table 3,

$$\Pr(E) = \Pr(X + Y = 1) \tag{1}$$
$$= \Pr(X = 0, \ Y = 1) + \Pr(X = 1, \ Y = 0) \tag{2}$$
$$= \frac{1}{2} \tag{3}$$

F: head appears on one coin.So, one coin should be head and obviously other will be tail. We are required to find $\Pr(X + Y = 1)$. Thus, from Table 3,

$$\Pr(F) = \Pr(X + Y = 1) \tag{4}$$
$$= \Pr(X = 0, \ Y = 1) + \Pr(X = 1, \ Y = 0) \tag{5}$$
$$= \frac{1}{2} \tag{6}$$

EF: Here one coin is head and other is tail.We are required to find $\Pr(X + Y = 1)$. Thus, from Table 3,

$$\Pr(EF) = \Pr(X + Y = 1) \tag{7}$$
$$= \Pr(X = 0, \ Y = 1) + \Pr(X = 1, \ Y = 0) \tag{8}$$
$$= \frac{1}{2} \tag{9}$$
$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{10}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2}} \tag{11}$$
$$= 1 \tag{12}$$

2) E: no tail appears.We are required to find $\Pr(X \neq 0, Y \neq 0)$. Thus, from Table 3,

$$\Pr(E) = \Pr(X \neq 0, Y \neq 0) \tag{13}$$
$$= \Pr(X = 1, \ Y = 1) \tag{14}$$
$$= \frac{1}{4} \tag{15}$$

F: no head appears.We are required to find $\Pr(X \neq 1, Y \neq 1)$. Thus, from Table 3,

$$\Pr(F) = \Pr(X \neq 1, Y \neq 1) \tag{16}$$
$$= \Pr(X = 0, \ Y = 0) \tag{17}$$
$$= \frac{1}{4} \tag{18}$$

EF: coins should show neither head nor tail. From 1 and 2 we have coins showing head or tail. So, this is an impossible event

$$\Pr(EF) = 0 \tag{19}$$
$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{20}$$
$$= \frac{0}{\frac{1}{4}} \tag{21}$$
$$= 0 \tag{22}$$