Probability Assignment

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Question: Let X be a positive valued continuous random variable with finite mean μ. If $Y = [X]$, the largest integer less than or equal to X, then which of the following statements is/are true?

(A) $\Pr(Y \le \mu) \le \Pr(X \le \mu)$ for all $\mu \ge 0$
(B) $\Pr(Y \ge \mu) \le \Pr(X \ge \mu)$ for all $\mu \ge 0$
(C) E(X) < E(Y)
(D) E(X) > E(Y)

Solution: Given that X is a positive valued random variable and $Y = [X]$.So,

$$X = Y + Z \tag{1}$$

Here, Z is an uniform distrubtion.

$$Z \sim U[0, 1) \tag{2}$$
$$F_Z(x) = x \tag{3}$$
$$E(Z) = \frac{1}{2} \tag{4}$$

Consider

1)

$$\begin{aligned} \Pr(Y \le \mu) &= \Pr(X - Z \le \mu) \quad (5)\\ &= \Pr(Z \ge X - \mu) \quad (6)\\ &= E(1 - F_Z(X - \mu)) \quad (7)\\ &= E(1 - X + \mu) \quad (8)\\ &= 1 - E(X) + \mu \quad (9)\\ &= 1 \quad (10) \end{aligned}$$

From option (A), we have $1 \le \Pr(X \le \mu)$. Option (A) is wrong since probability can't be greater than 1.

2)

$$\begin{aligned} \Pr(Y \ge \mu) &= \Pr(X - Z \ge \mu) \quad (11)\\ &= \Pr(Z \le X - \mu) \quad (12)\\ &= E(F_Z(X - \mu)) \quad (13)\\ &= E(X - \mu) \quad (14)\\ &= E(X) - \mu \quad (15)\\ &= 0 \quad (16) \end{aligned}$$

From option B, we have $\Pr(X \le \mu) \ge 0$. Option (B) is correct.

3)

$$\begin{aligned} E(Y) &= E(X - Z) \quad (17)\\ &= E(X) - E(Z) \quad (18)\\ &= \mu - \frac{1}{2} \quad (19)\\ &= E(X) - \frac{1}{2} \quad (20) \end{aligned}$$

$E(X) > E(Y)$. Option (D) is correct and (C) is wrong.

Steps for Simulation:

1) Taking n samples, Generate n exponential random variable(X) samples.
2) Generate n samples of $Y = [X]$ by floor to every sample of X.
3) Find number of samples of X where $X \le \mu$ and $X \ge \mu$ and divide with n to get $\Pr(X \le \mu)$ and $\Pr(X \ge \mu)$ respectively.
4) Find number of samples of Y where $Y \le \mu$ and $Y \ge \mu$ and divide with n to get $\Pr(Y \le \mu)$ and $\Pr(Y \ge \mu)$ respectively.
5) Sum the n samples of X and Y and divide with n to get $E(X)$ and $E(Y)$.

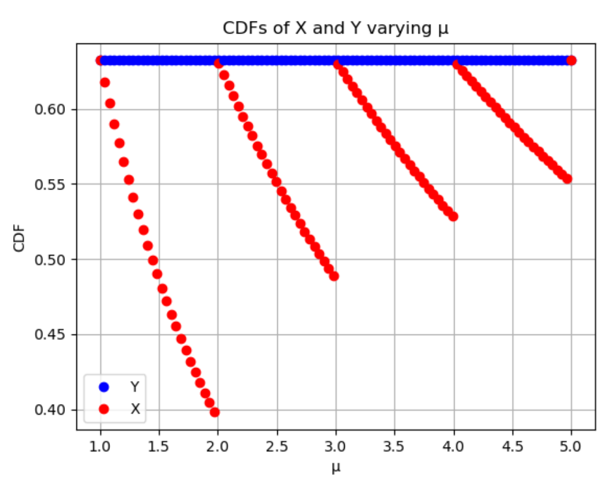

Fig. 5. CDF'S of X and Y for varying μ