

Probability Assignment

EE22BTECH11022-G.SAI HARSHITH*

Question: Let X be a positive valued continuous random variable with finite mean μ . If $Y = [X]$, the largest integer less than or equal to X , then which of the following statements is/are true?

- (A) $\Pr(Y \leq \mu) \leq \Pr(X \leq \mu)$ for all $\mu \geq 0$
- (B) $\Pr(Y \geq \mu) \leq \Pr(X \geq \mu)$ for all $\mu \geq 0$
- (C) $E(X) < E(Y)$
- (D) $E(X) > E(Y)$

Solution: Given that X is a positive valued random variable and $Y = [X]$. So,

$$X = Y + Z \quad (1)$$

Here, Z is an uniform distribution.

$$Z \sim U[0, 1) \quad (2)$$

$$F_Z(x) = x \quad (3)$$

$$E(Z) = \frac{1}{2} \quad (4)$$

Consider

1)

$$\Pr(Y \leq \mu) = \Pr(X - Z \leq \mu) \quad (5)$$

$$= \Pr(Z \geq X - \mu) \quad (6)$$

$$= E(1 - F_Z(X - \mu)) \quad (7)$$

$$= E(1 - X + \mu) \quad (8)$$

$$= 1 - E(X) + \mu \quad (9)$$

$$= 1 \quad (10)$$

From option (A), we have $1 \leq \Pr(X \leq \mu)$. Option (A) is wrong since probability can't be greater than 1.

2)

$$\Pr(Y \geq \mu) = \Pr(X - Z \geq \mu) \quad (11)$$

$$= \Pr(Z \leq X - \mu) \quad (12)$$

$$= E(F_Z(X - \mu)) \quad (13)$$

$$= E(X - \mu) \quad (14)$$

$$= E(X) - \mu \quad (15)$$

$$= 0 \quad (16)$$

From option B, we have $\Pr(X \leq \mu) \geq 0$. Option (B) is correct.

3)

$$E(Y) = E(X - Z) \quad (17)$$

$$= E(X) - E(Z) \quad (18)$$

$$= \mu - \frac{1}{2} \quad (19)$$

$$= E(X) - \frac{1}{2} \quad (20)$$

$E(X) > E(Y)$. Option (D) is correct and (C) is wrong.

Steps for Simulation:

- 1) Taking n samples, Generate n positive Gaussian random variable(X) samples.
- 2) Generate n samples of $Y = [X]$ by floor to every sample of X .
- 3) Find number of samples of X where $X \leq \mu$ and $X \geq \mu$ and divide with n to get $\Pr(X \leq \mu)$ and $\Pr(X \geq \mu)$ respectively.
- 4) Find number of samples of Y where $Y \leq \mu$ and $Y \geq \mu$ and divide with n to get $\Pr(Y \leq \mu)$ and $\Pr(Y \geq \mu)$ respectively.
- 5) Sum the n samples of X and Y and divide with n to get $E(X)$ and $E(Y)$.

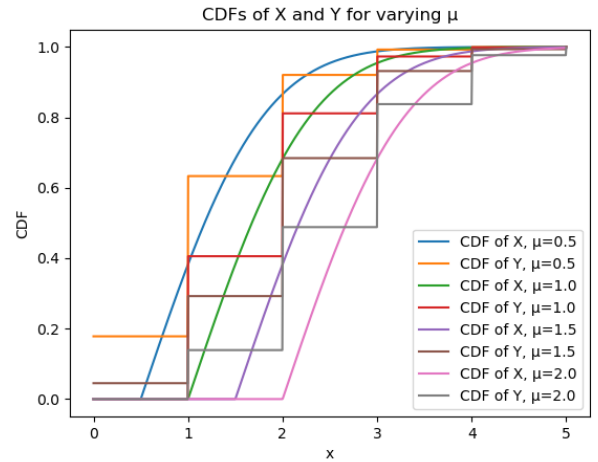


Fig. 5. CDF'S of X and Y for varying μ