Probability Assignment

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Question: The frequency response $H(f)$ of of a linear time-invariant syatem has magnitude as shown in Fig. 0
Statement 1: The system is necessarily a pure delay system for inputs which are bandlimited to $-a \leq f \leq a$.
Statement 2: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_X(f) = S_Y(f)$ for $-a \leq f \leq a$.

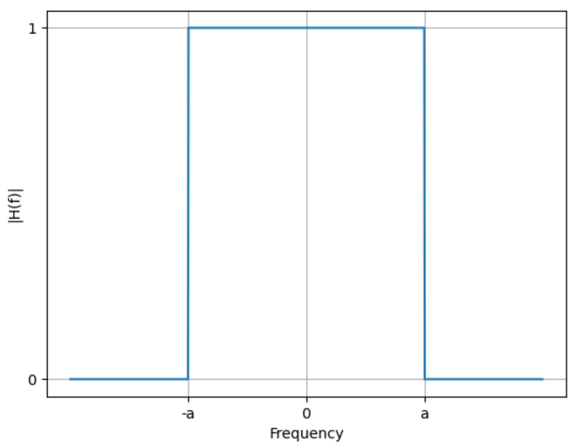

Fig. 0. $|H(f)|$ vs frequency

Solution: A system where output signal is a delayed version of input signal with no other transformations or operations is called a pure delay syatem.

1) Let us consider a pure delay LTI system with x(t) and y(t) as input and output signals in time domian. Let T_d be delay between input and output. So,

$$y(t) = x(t - T_d) \tag{1}$$

Here $Y(f)$ and $X(f)$ are output and input signals in frequency domian. Let $H(f)$ be

$$H(f) = \frac{Y(f)}{X(f)} \tag{2}$$
$$= e^{-2\pi f j T_d} \tag{3}$$
$$= (1)e^{-2\pi f j T_d} \tag{4}$$

Comparing (4) with

$$H(f) = |H(f)|e^{j\angle H(f)} \tag{5}$$
$$|H(f)| = 1 \tag{6}$$
$$\angle H(f) = -2\pi f T_d \tag{7}$$

$\angle H(f)$ is directly propotional to frequency and time delay. The system will acts as pure delay sytem. Now, if we take f^2 as frequency, the system doesn't act as pure delay system.
Example : Consider

$$H(f) = e^{-2\pi f^2 j T_d} \tag{8}$$

Appling inverse fourier transform to get time responce.

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{2\pi j f t} df \tag{9}$$
$$= \sqrt{\frac{T_d}{it}} e^{-\left(\frac{i\pi t}{T_d}\right)} \neq \delta(t - T_d) \tag{10}$$

Therfore system doesn't necessarily be pure delay. So, Statement 1 is incorrect.

2) For wide-sense stationary LTI sytem, Spectral power density of a signal describes the power present in the signal as a function of frequency, per unit frequency. In frequency domain,

$$S_X(f) = |X(f)|^2 \tag{11}$$
$$S_Y(f) = |Y(f)|^2 \tag{12}$$
$$S_Y(f) = \frac{|Y(f)|^2}{|X(f)|^2} S_X(f) \tag{13}$$
$$= \left|\frac{Y(f)}{X(f)}\right|^2 S_X(f) \tag{14}$$
$$= |H(f)|^2 S_X(f) \tag{15}$$

From (3), for $-a \leq f \leq a$,

$$S_Y(f) = (1)^2 S_X(f) \tag{16}$$

$$S_Y(f) = S_X(f) \tag{17}$$

Statement 2 is correct.