Probability Assignment

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Question: Let X be a positive valued continuous random variable with finite mean μ. If $Y = [X]$, the largest integer less than or equal to X, then which of the following statements is/are true?

(A) $\Pr(Y \le \mu) \le \Pr(X \le \mu)$ for all $\mu \ge 0$
(B) $\Pr(Y \ge \mu) \le \Pr(X \ge \mu)$ for all $\mu \ge 0$
(C) E(X) < E(Y)
(D) E(X) > E(Y)

Solution: Given that X is a positive valued random variable. As uniform distribution is a continuous distributiion. Every property from uniform distribution can be generalized to continuous distrbution and viceversa. Lets consider X as uniform distribution. Let 0 < x < 1.

$$X \sim U[0,1) \tag{1}$$
$$\mu = E(X) \tag{2}$$
$$= \frac{1}{2} \tag{3}$$
$$p_X(x) = 1 \tag{4}$$
$$F_X(x) = \Pr(X \le x) \tag{5}$$
$$= x \tag{6}$$

GIven $Y = [X]$, So, $Y = 0$.

$$Y = 0 \tag{7}$$
$$\Pr(Y = a) = \Pr(a \le X < a+1) \tag{8}$$
$$= F_X(a+1) - F_X(a) \tag{9}$$
$$= a + 1 - a \tag{10}$$
$$= 1 \tag{11}$$
$$E(Y) = \sum_{y=0}^{0} y p_Y(y) \tag{12}$$
$$= 0(1) \tag{13}$$
$$= 0 \tag{14}$$

From (3) and (14),

$$E(X) > E(Y) \tag{15}$$

Now,

$$\Pr(Y \le \mu) = \Pr\left(Y \le \frac{1}{2}\right) \tag{16}$$
$$= \Pr(Y = 0) \tag{17}$$
$$= 1 \tag{18}$$
$$\Pr(X \le \mu) = \Pr\left(X \le \frac{1}{2}\right) \tag{19}$$
$$= F_X\left(\frac{1}{2}\right) \tag{20}$$
$$= \frac{1}{2} \tag{21}$$
$$\Pr(Y \ge \mu) = \Pr\left(Y \ge \frac{1}{2}\right) \tag{22}$$
$$= 0 \tag{23}$$
$$\Pr(X \ge \mu) = \Pr\left(X \ge \frac{1}{2}\right) \tag{24}$$
$$= 1 - \Pr\left(X \le \frac{1}{2}\right) \tag{25}$$
$$= 1 - F_X\left(\frac{1}{2}\right) \tag{26}$$
$$= 1 - \frac{1}{2} \tag{27}$$
$$= \frac{1}{2} \tag{28}$$

From (18) and (21),

$$\Pr(Y \le \mu) \ge \Pr(X \le \mu) \tag{29}$$

From (23) and (28),

$$\Pr(Y \ge \mu) \le \Pr(X \ge \mu) \tag{30}$$

Option (B) and (D) are correct.