Probability Assignment

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Question: Find $P(E|F)$ for

1) E: tail appears on one coin.
   F: head appears on one coin.
2) E: no tail appears.
   F: no head appears.

Solution: Define random variables X and Y as shown in Tables 1.

$X = 0$	First coin shows Tail.
$X = 1$	First coin shows Head.
$Y = 0$	Second coin shows Tail.
$Y = 1$	Second coin shows Head.

TABLE 1: Definition of X and Y.

Since the coins are fair.

$$P_{XY}(k,m) = \frac{1}{4} \tag{1}$$

where k belongs to $\{0,1\}$ and m belongs to $\{0,1\}$. So, total four different k,m combinations.

1) E: tail appears on one coin.So, one coin should be tail and obviously other will be head. We are required to find $\Pr(X+Y=1)$. Thus, from equation (1).

$$\Pr(E) = \Pr(X+Y=1) \tag{2}$$
$$= \Pr(X=0,\ Y=1) + \Pr(X=1,\ Y=0) \tag{3}$$
$$= \frac{1}{2} \tag{4}$$

F: head appears on one coin.So, one coin should be head and obviously other will be tail. We are required to find $\Pr(X+Y=1)$. Thus, from equation (1).

$$\Pr(F) = \Pr(X+Y=1) \tag{5}$$
$$= \Pr(X=0,\ Y=1) + \Pr(X=1,\ Y=0) \tag{6}$$
$$= \frac{1}{2} \tag{7}$$

EF: Here one coin is head and other is tail.We are required to find $\Pr(X+Y=1)$. Thus, from equation (1).

$$\Pr(EF) = \Pr(X+Y=1) \tag{8}$$
$$= \Pr(X=0,\ Y=1) + \Pr(X=1,\ Y=0) \tag{9}$$
$$= \frac{1}{2} \tag{10}$$
$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{11}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2}} \tag{12}$$
$$= 1 \tag{13}$$

2) E: no tail appears.We are required to find $\Pr(X \neq 0, Y \neq 0)$. Thus, from equation (1).

$$\Pr(E) = \Pr(X \neq 0, Y \neq 0) \tag{14}$$
$$= \Pr(X=1,\ Y=1) \tag{15}$$
$$= \frac{1}{4} \tag{16}$$

F: no head appears.We are required to find $\Pr(X \neq 1, Y \neq 1)$. Thus, from equation (1).

$$\Pr(F) = \Pr(X \neq 1, Y \neq 1) \tag{17}$$
$$= \Pr(X=0,\ Y=0) \tag{18}$$
$$= \frac{1}{4} \tag{19}$$

EF: coins should show neither head nor tail. From Table 1 we have coins showing head or tail. So, this is an impossible event

$$\Pr(EF) = 0 \tag{20}$$
$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{21}$$
$$= \frac{0}{\frac{1}{4}} \tag{22}$$
$$= 0 \tag{23}$$