Probability Assignment

EE22BTECH11022-G.SAI HARSHITH*

Question: Let X be a positive valued continuous random variable with finite mean μ . If Y = [X], the largest integer less than or equal to X, then which of the following statements is/are true?

- (A) $Pr(Y \le \mu) \le Pr(X \le \mu)$ for all $\mu \ge 0$
- (B) $Pr(Y \ge \mu) \le Pr(X \ge \mu)$ for all $\mu \ge 0$
- (C) E(X) < E(Y)
- (D) E(X) > E(Y)

Solution: Given that X is a positive valued random variable and Y = [X]. So,

$$X = Y + Z \tag{1}$$

Here, Z is an uniform distrubtion.

$$Z \sim U[0,1) \tag{2}$$

$$F_Z(x) = x \tag{3}$$

$$E(Z) = \frac{1}{2} \tag{4}$$

Consider

1)

$$\Pr(Y \le \mu) = \Pr(X - Z \le \mu)$$
 (5)
= $\Pr(Z \ge X - \mu)$ (6)
= $E(1 - F_Z(X - \mu))$ (7)

$$= E(1 - X + \mu)$$
(8)
= 1 - E(X) + \mu (9)

$$-1 \quad E(\Omega) \mid \mu \qquad (2)$$

(10)

From option (A), we have $1 \le \Pr(X \le \mu)$. Option (A) is wrong since probability can't be greater than 1.

= 1

2)

$$\Pr(Y \ge \mu) = \Pr(X - Z \ge \mu) \tag{11}$$

$$= \Pr\left(Z \le X - \mu\right) \tag{12}$$

$$= E(F_Z(X - \mu)) \tag{13}$$

$$= E(X - \mu) \tag{14}$$

$$= E(X) - \mu \tag{15}$$

$$=0 (16)$$

From option B, we have $Pr(X \le \mu) \ge 0$. Option (B) is correct.

3)

$$E(Y) = E(X - Z) \tag{17}$$

$$= E(X) - E(Z) \tag{18}$$

$$=\mu-\frac{1}{2}\tag{19}$$

$$= E(X) - \frac{1}{2} \tag{20}$$

E(X) > E(Y). Option (D) is correct and (C) is wrong.

Steps for Simulation:

- 1) Taking n samples, Generate n exponential random variable(*X*) samples.
- 2) Generate n samples of Y = [X] by floor to every sample of X.
- 3) Find number of samples of X where $X \le \mu$ and $X \ge \mu$ and divide with n to get $\Pr(X \le \mu)$ and $\Pr(X \ge \mu)$ respectively.
- 4) Find number of samples of Y where $Y \le \mu$ and $Y \ge \mu$ and divide with n to get $\Pr(Y \le \mu)$ and $\Pr(Y \ge \mu)$ respectively.
- 5) Sum the n samples of X and Y and divide with n to get E(X) and E(Y).

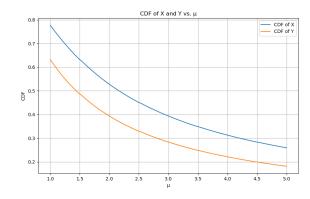


Fig. 5. CDF'S of X and Y for varying μ at x=1.5

Note: At $x \in$ integers, Y = X, so, CDF curves of Y and X are same. At non-integers we can see some difference in CDF curves in X and Y.