

# Probability Assignment

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Question: Find  $P(E|F)$  for

- 1)  $E$ : tail appears on one coin.  
 $F$ : head appears on one coin.
- 2)  $E$ : no tail appears.  
 $F$ : no head appears.

**Solution:** Define random variables  $X$  and  $Y$  as shown in Tables 1 and 2. Since the coins are fair. Form the joint

$X = 0$	First coin shows Tail.
$X = 1$	First coin shows Head.

TABLE 1: Definition of  $X$ .

$Y = 0$	Second coin shows Tail.
$Y = 1$	Second coin shows Head.

TABLE 2: Definition of  $Y$ .

pmf as in Table 3.

	$X = 0$	$X = 1$
$Y = 0$	$\frac{1}{4}$	$\frac{1}{4}$
$Y = 1$	$\frac{1}{4}$	$\frac{1}{4}$

TABLE 3: Joint pmf of  $X$  and  $Y$ .

- 1)  $E$ : tail appears on one coin. So, one coin should be tail and obviously other will be head. We are required to find  $\Pr(X + Y = 1)$ . Thus, from Table 3,

$$\Pr(E) = \Pr(X + Y = 1) \quad (1)$$

$$= \Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0) \quad (2)$$

$$= \frac{1}{2} \quad (3)$$

$F$ : head appears on one coin. So, one coin should be head and obviously other will be tail. We are required to find  $\Pr(X + Y = 1)$ . Thus, from Table 3,

$$\Pr(F) = \Pr(X + Y = 1) \quad (4)$$

$$= \Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0) \quad (5)$$

$$= \frac{1}{2} \quad (6)$$

$EF$ : Here one coin is head and other is tail. We are required to find  $\Pr(X + Y = 1)$ . Thus, from Table 3,

$$\Pr(EF) = \Pr(X + Y = 1) \quad (7)$$

$$= \Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0) \quad (8)$$

$$= \frac{1}{2} \quad (9)$$

$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \quad (10)$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}} \quad (11)$$

$$= 1 \quad (12)$$

- 2)  $E$ : no tail appears. We are required to find  $\Pr(X \neq 0, Y \neq 0)$ . Thus, from Table 3,

$$\Pr(E) = \Pr(X \neq 0, Y \neq 0) \quad (13)$$

$$= \Pr(X = 1, Y = 1) \quad (14)$$

$$= \frac{1}{4} \quad (15)$$

$F$ : no head appears. We are required to find  $\Pr(X \neq 1, Y \neq 1)$ . Thus, from Table 3,

$$\Pr(F) = \Pr(X \neq 1, Y \neq 1) \quad (16)$$

$$= \Pr(X = 0, Y = 0) \quad (17)$$

$$= \frac{1}{4} \quad (18)$$

$EF$ : coins should show neither head nor tail. From 1 and 2 we have coins showing head or tail. So, this is an impossible event

$$\Pr(EF) = 0 \quad (19)$$

$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \quad (20)$$

$$= \frac{0}{\frac{1}{4}} \quad (21)$$

$$= 0 \quad (22)$$