

Probability Assignment

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Question: The frequency response $H(f)$ of a linear time-invariant system has magnitude as shown in Fig. 0

Statement 1: The system is necessarily a pure delay system for inputs which are bandlimited to $-a \leq f \leq a$.

Statement 2: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_X(f) = S_Y(f)$ for $-a \leq f \leq a$.

Solution:

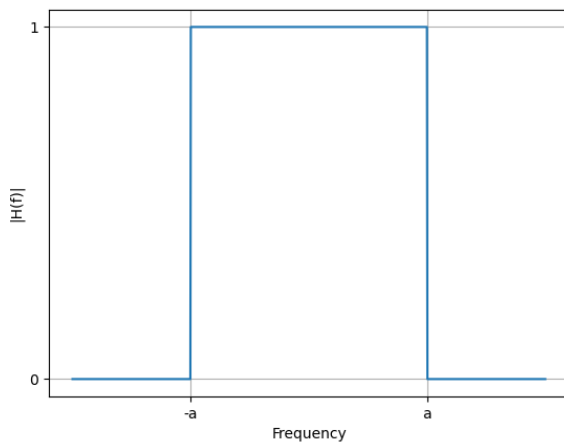


Fig. 0. $|H(f)|$ vs frequency

- 1) Let us consider a delay LTI system with $x(t)$ and $y(t)$ as input and output signals in time domain. Let T_d be delay between input and output. So,

$$y(t) = x(t - T_d) \quad (1)$$

Frequency response.

$$\int_{-\infty}^{\infty} y(t) e^{-2\pi f j \omega t} dt = \int_{-\infty}^{\infty} x(t - T_d) e^{-2\pi f j t} dt \quad (2)$$

$$= \int_{-\infty}^{\infty} x(t) e^{-2\pi f j (t+T_d)} d(t + T_d) \quad (3)$$

$$= e^{-2\pi f j T_d} \int_{-\infty}^{\infty} x(t) e^{-2\pi f j t} dt \quad (4)$$

$$Y(f) = e^{-2\pi f j T_d} X(f) \quad (5)$$

Here $Y(f)$ and $X(f)$ are output and input signals in frequency domain. Let $H(f)$ be transfer function such that

$$|H(f)| = \left| \frac{Y(f)}{X(f)} \right| \quad (6)$$

$$= |e^{-2\pi f j T_d}| \quad (7)$$

$$= 1 \quad (8)$$

Here, $|H(f)| = 1$ for all frequencies. But definition of $|H(f)|$ in question was

$$|H(f)| = \begin{cases} 1 & \text{if } -a \leq k \leq a \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

From (8) and (9), $|H(f)|$ is different. So, system will not act as pure delay system. Statement 1 is incorrect.

- 2) For wide-sense stationary LTI system, input power spectral density $S_X(f)$ and output power spectral density $S_Y(f)$ are related as

$$S_Y(f) = |H(f)|^2 S_X(f) \quad (10)$$

From (9), for $-a \leq k \leq a$, $|H(f)| = 1$

$$S_Y(f) = (1)^2 S_X(f) \quad (11)$$

$$S_Y(f) = S_X(f) \quad (12)$$

Statement 2 is correct.

Applying Fourier transform for converting to