Probability Assignment

EE22BTECH11022-G.SAI HARSHITH*

Question: Let X be a positive valued continuous random variable with finite mean μ. If $Y = [X]$, the largest integer less than or equal to X, then which of the following statements is/are true?

(A) $\Pr(Y \leq \mu) \leq \Pr(X \leq \mu)$ for all $\mu \geq 0$
(B) $\Pr(Y \geq \mu) \leq \Pr(X \geq \mu)$ for all $\mu \geq 0$
(C) E(X) < E(Y)
(D) E(X) > E(Y)

Solution: Given that X is a positive valued random variable and $Y = [X]$.So,

$$X = Y + Z \tag{1}$$

Here, Z is an uniform distrubtion.

$$Z \sim U[0, 1] \tag{2}$$
$$F_Z(x) = x \tag{3}$$
$$E(Z) = \frac{1}{2} \tag{4}$$

Consider

1)

$$\Pr(Y \leq \mu) = \Pr(X - Z \leq \mu) \tag{5}$$
$$= \Pr(Z \geq X - \mu) \tag{6}$$
$$= E(1 - F_Z(X - \mu)) \tag{7}$$
$$= E(1 - X + \mu) \tag{8}$$
$$= 1 - E(X) + \mu \tag{9}$$
$$= 1 \tag{10}$$

From option (A), we have $1 \leq \Pr(X \leq \mu)$. Option (A) is wrong since probability can't be greater than 1.

2)

$$\Pr(Y \geq \mu) = \Pr(X - Z \geq \mu) \tag{11}$$
$$= \Pr(Z \leq X - \mu) \tag{12}$$
$$= E(F_Z(X - \mu)) \tag{13}$$
$$= E(X - \mu) \tag{14}$$
$$= E(X) - \mu \tag{15}$$
$$= 0 \tag{16}$$

From option B, we have $\Pr(X \leq \mu) \geq 0$. Option (B) is correct.

3)

$$E(Y) = E(X - Z) \tag{17}$$
$$= E(X) - E(Z) \tag{18}$$
$$= \mu - \frac{1}{2} \tag{19}$$
$$= E(X) - \frac{1}{2} \tag{20}$$

$E(X) > E(Y)$. Option (D) is correct and (C) is wrong.

Steps for Simulation:

1) Taking n samples, Generate n positive Gaussian random variable(X) samples.
2) Generate n samples of $Y = [X]$ by floor to every sample of X.
3) Find number of samples of X where $X \leq \mu$ and $X \geq \mu$ and divide with n to get $\Pr(X \leq \mu)$ and $\Pr(X \geq \mu)$ respectively.
4) Find number of samples of Y where $Y \leq \mu$ and $Y \geq \mu$ and divide with n to get $\Pr(Y \leq \mu)$ and $\Pr(Y \geq \mu)$ respectively.
5) Sum the n samples of X and Y and divide with n to get $E(X)$ and $E(Y)$.