Probability Assignment

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Question: Let X be a positive valued continuous random variable with finite mean μ. If $Y = [X]$, the largest integer less than or equal to X, then which of the following statements is/are true?

(A) $\Pr(Y \le \mu) \le \Pr(X \le \mu)$ for all $\mu \ge 0$
(B) $\Pr(Y \ge \mu) \le \Pr(X \ge \mu)$ for all $\mu \ge 0$
(C) $E(X) < E(Y)$
(D) $E(X) > E(Y)$

Solution: Given that X is a positive valued random variable and $Y = [X]$.So,

$$X = Y + Z \tag{1}$$

Here, Z is an uniform distrubtion.

$$Z \sim U[0, 1) \tag{2}$$
$$F_Z(x) = x \tag{3}$$
$$E(Z) = \frac{1}{2} \tag{4}$$

Consider
1)

$$\Pr(Y \le \mu) = \Pr(X - Z \le \mu) \tag{5}$$
$$= \Pr(Z \ge X - \mu) \tag{6}$$
$$= E(1 - F_Z(X - \mu)) \tag{7}$$
$$= E(1 - X + \mu) \tag{8}$$
$$= 1 - E(X) + \mu \tag{9}$$
$$= 1 \tag{10}$$

From option (A), we have $1 \le \Pr(X \le \mu)$. Option (A) is wrong since probability can't be greater than 1.
2)

$$\Pr(Y \ge \mu) = \Pr(X - Z \ge \mu) \tag{11}$$
$$= \Pr(Z \le X - \mu) \tag{12}$$
$$= E(F_Z(X - \mu)) \tag{13}$$
$$= E(X - \mu) \tag{14}$$
$$= E(X) - \mu \tag{15}$$
$$= 0 \tag{16}$$

From option B, we have $\Pr(X \le \mu) \ge 0$. Option (B) is correct.

3)

$$E(Y) = E(X - Z) \tag{17}$$
$$= E(X) - E(Z) \tag{18}$$
$$= \mu - \frac{1}{2} \tag{19}$$
$$= E(X) - \frac{1}{2} \tag{20}$$

$E(X) > E(Y)$. Option (D) is correct and (C) is wrong.

Steps for Simulation:
1) Taking n samples, Generate n exponential random variable(X) samples.
2) Generate n samples of $Y = [X]$ by floor to every sample of X.
3) Find number of samples of X where $X \le \mu$ and $X \ge \mu$ and divide with n to get $\Pr(X \le \mu)$ and $\Pr(X \ge \mu)$ respectively.
4) Find number of samples of Y where $Y \le \mu$ and $Y \ge \mu$ and divide with n to get $\Pr(Y \le \mu)$ and $\Pr(Y \ge \mu)$ respectively.
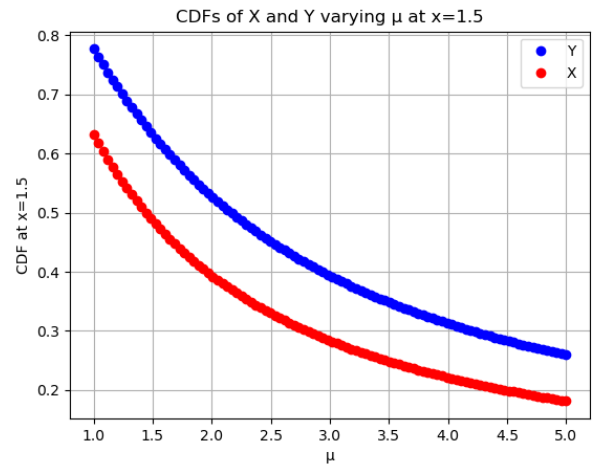5) Sum the n samples of X and Y and divide with n to get $E(X)$ and $E(Y)$.

Fig. 5. CDF'S of X and Y for varying μ at x=1.5

Note: At $x \in$ integers, $Y = X$, so, CDF curves of Y and X are same. At non-integers we can see some difference in CDF curves in X and Y.