Probability Assignment

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Question: The frequency response $H(f)$ of of a linear time-invariant syatem has magnitude as shown in figure Fig. 0
Statement 1: The system is necessarily a pure delay system for inputs which are bandlimited to $-a \leq f \leq a$.
Statement 2: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_X(f) = S_Y(f)$ for $-a \leq f \leq a$.
Solution:

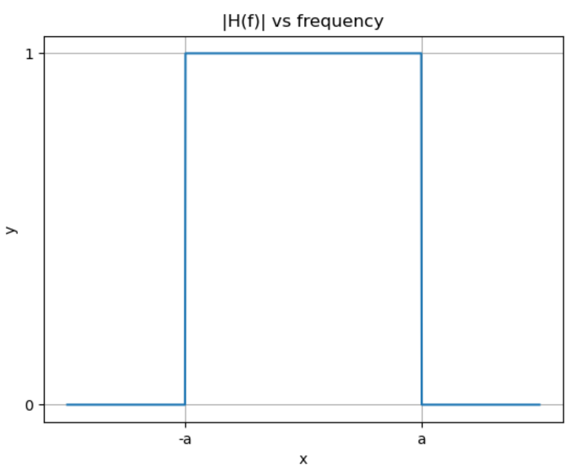

Fig. 0. $|H(f)|$ vs frequency

1) Let us consider a pure delay LTI system with x(t) and y(t) as input and output signals in time domian. Let T_d be delay between input and output. So,

$$y(t) = x(t - T_d) \tag{1}$$

Appling fourier transform for converting to Frequency response.

$$\int_{-\infty}^{\infty} y(t)e^{-2\pi f jwt}dt = \int_{-\infty}^{\infty} x(t - T_d)e^{-2\pi f jt}dt \tag{2}$$

$$= \int_{-\infty}^{\infty} x(t)e^{-2\pi f j(t+T_d)}d(t + T_d) \tag{3}$$

$$= e^{-2\pi f jT_d} \int_{-\infty}^{\infty} x(t)e^{-2\pi f jt}dt \tag{4}$$

$$Y(f) = e^{-2\pi f jT_d}X(f) \tag{5}$$

Here $Y(f)$ and $X(f)$ are output and input signals in frequency domian. Let $H(f)$ be tranfer function such that

$$|H(f)| = \left|\frac{Y(f)}{X(f)}\right| \tag{6}$$

$$= \left|e^{-2\pi f jT_d}\right| \tag{7}$$

$$= 1 \tag{8}$$

Here, $|H(f)| = 1$ for all frequencies. But defination of $|H(f)|$ in question was

$$|H(f)| = \begin{cases} 1 & \text{if } -a \leq k \leq a \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

From (8) and (9), $|H(f)|$ is different.So, system will not act as pure delay system. Statement 1 is incorrect.

2) For wide sense stationary LTI sytem, input power spectral density $S_X(f)$ and output power spectral density $S_Y(f)$ are related as

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{10}$$

Fron (9), for $-a \leq k \leq a$, $|H(f)| = 1$

$$S_Y(f) = (1)^2 S_X(f) \tag{11}$$

$$S_Y(f) = S_X(f) \tag{12}$$

STatement 2 is correct.