Probability Assignment

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Determine the probability p,for each of following events.

1) An odd number appears in a single roll of dice.
2) Atleast one head appears in two tosses of fair coin.
3) A king,9 of hearts or 3 of spades appears in drawing a single card from a well shuffled deck of 52 cards.
4) The sum of 6 appears in single toss of a pair of fair dice.

Solution:

1) Let the random variable X be defined as:

Random Variable	Values	Description
X	$1 \leq X \leq 6$	Number appeared on a roll

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1,2,3,4,5,6\} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Let E be event occuring odd number on single roll. Since, the dice rolls are mutually exclusive. From (1).

$$\Pr(E) = p_X(1) + p_X(3) + p_X(5) \tag{2}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \tag{3}$$
$$= \frac{1}{2} \tag{4}$$

2) Let 1 be Head and 0 be Tail. Consider random variable X_i where $i \in \{1,2\}$ as

Random Variable	Values	Description
X_i	$\{0,1\}$	Result appeared on "i"th coin

Let $Y = X_1 + X_2$ be a binomial distribution with parameters

$$n = 2 \qquad p = \frac{1}{2} \tag{5}$$

Probabilty of getting k Head in 2 tosses is

$$p_Y(k) = {}^2C_k p^k (1-p)^{2-k} \tag{6}$$

Now, Let F be event of getting alteast one head.

$$\Pr(F) = p_Y(1) + p_Y(2) \tag{7}$$
$$= {}^2C_1 \left(\frac{1}{2}\right)^1 \left(1 - \frac{1}{2}\right)^{2-1} + {}^2C_2 \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^{2-2} \tag{8}$$
$$= \frac{3}{4} \tag{9}$$

3) Let the random variables X and Y be defined as:

Random Variable	Values	Description
X	$1 \leq X \leq 4$	Shape of Card
Y	$1 \leq Y \leq 13$	Number on Card

Let For $X \in \{1,2,3,4\}$ represents Diamonds, Clubs, Hearts, Spades respectively.

$$p_X(k) = \begin{cases} \frac{1}{13} & \text{if } 1 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$
$$p_Y(k) = \begin{cases} \frac{1}{4} & \text{if } 1 \leq k \leq 13 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$
$$p_{XY}(k,m) = \begin{cases} \frac{1}{52} & \text{if } 1 \leq k \leq 4 \text{ and } 1 \leq m \leq 13 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Let $Y = 13$ represent king Card. So, Let G be event to get 4 kings, 9 of hearts,3 of spades. From (11) and (12).

$$\Pr(G) = p_Y(13) + p_{XY}(3,9) + p_{XY}(4,3) \tag{13}$$
$$= \frac{1}{13} + \frac{1}{52} + \frac{1}{52} \tag{14}$$
$$= \frac{3}{26} \tag{15}$$

4) Let random variables X_i where $i \in \{1,2\}$ be defined as

Random Variable	Values	Description
X_i	$1 \leq X_i \leq 6$	Number appeared on "i"th dice

$$p_{X_1}(k) = \begin{cases} \frac{1}{6} & \text{if } 1 \le k \le 6 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

$$p_{X_2}(k) = \begin{cases} \frac{1}{36} & \text{if } 1 \le k \le 6 \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

Consider an H for which sum of both dice is six. Since the dice are independent using MGF method.

$$Y = X_1 + X_2 \quad (18)$$

$$M_Y(z) = P_{X_1}(z)P_{X_2}(z) \quad (19)$$

$$M_Y(z) = \left(\frac{z^{-1}(1-z^{-6})}{6(1-z^{-1})}\right)\left(\frac{z^{-1}(1-z^{-6})}{6(1-z^{-1})}\right) \quad (20)$$

$$M_Y(z) = \left(\frac{z^{-1}(1-z^{-6})}{6(1-z^{-1})}\right)^2 \quad (21)$$

As $M_Y(z) = \sum_{i=-\infty}^{\infty} p_Y(i)$. On camparing coefficients, it yeilds

$$p_Y(k) = \begin{cases} \frac{k-1}{36} & \text{if } 1 \le k \le 7 \\ \frac{13-k}{36} & \text{if } 8 \le k \le 13 \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

$$\Pr(H) = p_Y(6) \quad (23)$$

$$= \frac{5}{36} \quad (24)$$

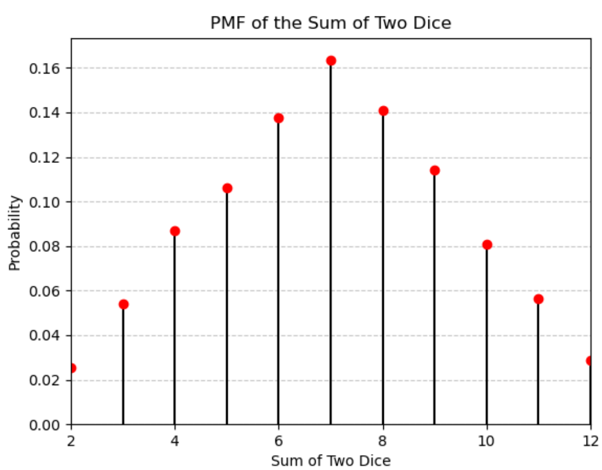

Fig. 4. Figure indications pmf of sum of two dice