

# Probability Assignment

EE22BTECH11022-G.SAI HARSHITH\*

Determine the probability  $p$ , for each of following events.

- 1) An odd number appears in a single roll of dice.
- 2) Atleast one head appears in two tosses of fair coin.
- 3) A king, 9 of hearts or 3 of spades appears in drawing a single card from a well shuffled deck of 52 cards.
- 4) The sum of 6 appears in single toss of a pair of fair dice.

## Solution:

- 1) Let the random variable  $X$  be defined as:

Random Variable	Values	Description
$X$	$1 \leq X \leq 6$	Number appeared on a roll

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Let  $E$  be event occurring odd number on single roll. Since, the dice rolls are mutually exclusive.

$$\Pr(E) = p_X(1) + p_X(3) + p_X(5) \quad (2)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

- 2) Let 1 be Head and 0 be Tail. Consider random variables as

Random Variable	Values	Description
$X_1$	$\{0, 1\}$	Result appeared on first coin
$X_2$	$\{0, 1\}$	Result appeared on second coin

$$p_{X_1 X_2}(k, m) = \begin{cases} \frac{1}{4} & \text{if } k \in \{0, 1\} \text{ and } m \in \{0, 1\} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Let  $F$  be event of occurring atleast one head. Since, the coin tosses are mutually exclusive.

$$\Pr(F) = \Pr(X_1 + X_2 \geq 1) \quad (6)$$

$$= p_{X_1 X_2}(0, 1) + p_{X_1 X_2}(1, 0) + p_{X_1 X_2}(1, 1) \quad (7)$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad (8)$$

$$= \frac{3}{4} \quad (9)$$

- 3) Let the random variables  $X$  and  $Y$  be defined as:

Random Variable	Values	Description
$X$	$1 \leq X \leq 4$	Shape of Card
$Y$	$1 \leq Y \leq 13$	Number on Card

$$p_{XY}(k, m) = \begin{cases} \frac{1}{52} & \text{if } 1 \leq k \leq 4 \text{ and } 1 \leq m \leq 13 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Let For  $X \in \{1, 2, 3, 4\}$  represents Diamonds, Clubs, Hearts, Spades respectively. Let  $Y = 13$  represent king Card. So, Let  $G$  be event to get 4 kings, 9 of hearts, 3 of spades.

$$\Pr(G) = p_{XY}(1, 13) + p_{XY}(2, 13) + p_{XY}(3, 13) + p_{XY}(4, 13) \quad (11)$$

$$= \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} \quad (12)$$

$$= \frac{3}{26} \quad (13)$$

- 4) Let random variables  $X_1$  and  $X_2$  be defined as

Random Variable	Values	Description
$X_1$	$1 \leq X \leq 6$	Number appeared on first dice
$X_2$	$1 \leq X \leq 6$	Number appeared on second dice

$$p_{X_1 X_2}(k, m) = \begin{cases} \frac{1}{36} & \text{if } 1 \leq k \leq 6 \text{ and } 1 \leq m \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

Consider an H for which sum of both dice is six.

$$\Pr(E) = \Pr(X_1 + X_2 = 6) \quad (15)$$

$$= p_{X_1X_2}(1, 5) + p_{X_1X_2}(2, 4) + p_{X_1X_2}(3, 3) + p_{X_1X_2}(5, 1) + p_{X_1X_2}(4, 2) \quad (16)$$

$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} \quad (17)$$

$$= \frac{5}{36} \quad (18)$$