

Probability Assignment

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Determine the probability p , for each of following events.

- 1) An odd number appears in a single roll of dice.
- 2) Atleast one head appears in two tosses of fair coin.
- 3) A king, 9 of hearts or 3 of spades appears in drawing a single card from a well shuffled deck of 52 cards.
- 4) The sum of 6 appears in single toss of a pair of fair dice.

Solution:

- 1) Let the random variable X be defined as:

| Random Variable | Values | Description |
|-----------------|-------------------|---------------------------|
| X | $1 \leq X \leq 6$ | Number appeared on a roll |

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Let E be event occurring odd number on single roll. Since, the dice rolls are mutually exclusive. From (1).

$$\Pr(E) = p_X(1) + p_X(3) + p_X(5) \quad (2)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

- 2) Let 1 be Head and 0 be Tail. Consider random variable X_i where $i \in \{1, 2\}$ as

| Random Variable | Values | Description |
|-----------------|------------|-----------------------------------|
| X_i | $\{0, 1\}$ | Result appeared on " i "th coin |

Let $Y = X_1 + X_2$ be a binomial distribution with parameters

$$n = 2 \quad p = \frac{1}{2} \quad (5)$$

Probability of getting k Head in $n = 2$ coins is

$$p_Y(k) = {}^2C_k p^k (1-p)^{2-k} \quad (6)$$

Now, Let F be event of getting atleast one head.

$$\begin{aligned} \Pr(F) &= p_Y(1) + p_Y(2) \quad (7) \\ &= {}^2C_1 \left(\frac{1}{2}\right)^1 \left(1 - \frac{1}{2}\right)^{2-1} + {}^2C_2 \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^{2-2} \quad (8) \\ &= \frac{3}{4} \quad (9) \end{aligned}$$

- 3) Let the random variables X and Y be defined as:

| Random Variable | Values | Description |
|-----------------|--------------------|----------------|
| X | $1 \leq X \leq 4$ | Shape of Card |
| Y | $1 \leq Y \leq 13$ | Number on Card |

Let For $X \in \{1, 2, 3, 4\}$ represents Diamonds, Clubs, Hearts, Spades respectively.

$$p_X(k) = \begin{cases} \frac{1}{13} & \text{if } 1 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

$$p_Y(k) = \begin{cases} \frac{1}{4} & \text{if } 1 \leq k \leq 13 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

$$p_{XY}(k, m) = \begin{cases} \frac{1}{52} & \text{if } 1 \leq k \leq 4 \text{ and } 1 \leq m \leq 13 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Let $Y = 13$ represent king Card. So, Let G be event to get 4 kings, 9 of hearts, 3 of spades. From (11) and (12).

$$\Pr(G) = p_Y(13) + p_{XY}(3, 9) + p_{XY}(4, 3) \quad (13)$$

$$= \frac{1}{13} + \frac{1}{52} + \frac{1}{52} \quad (14)$$

$$= \frac{3}{26} \quad (15)$$

- 4) Let random variables X_i where $i \in \{1, 2\}$ be defined as

| Random Variable | Values | Description |
|-----------------|---------------------|-----------------------------------|
| X_i | $1 \leq X_i \leq 6$ | Number appeared on " i "th dice |

Let $Y = X_1 + X_2$. Then

$$p_Y(k) = \begin{cases} \frac{k-1}{36} & \text{if } 1 \leq k \leq 7 \\ \frac{13-k}{36} & \text{if } 8 \leq k \leq 13 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Consider an H for which sum of both dice is six. Since the dice are independent.

$$\Pr(H) = p_Y(6) \quad (17)$$

$$= \frac{5}{36} \quad (18)$$

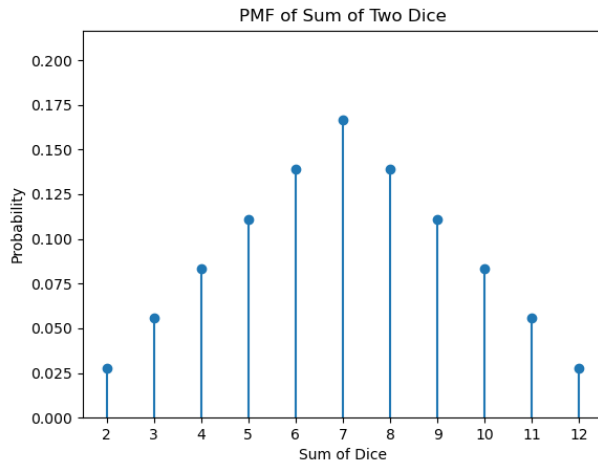


Fig. 4. Figure indicates pmf of sum of two dice