Assignment 2

CS20BTECH11028

and latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−2

1 Problem GATE EC 56

Let X and Y be jointly distributed random variables such that the conditional distribution of Y, given X =x, is uniform on the interval $(x-1, x+1)$. Suppose $E(X) = 1$ and $Var(X) = \frac{5}{3}$.
The mean of the random variable Y is

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 2

2 Solution GATE EC 56

We know that,

$$\Pr(Y = y|X = x) = \frac{f(x,y)}{f_1(x)} \tag{2.0.1}$$

where $f(x,y) = \Pr(X = x, Y = y)$.,$f_1(x)$ is the marginal probability for X=x.
Given that $\Pr(Y|X = x)$ is uniform over the interval (x-1,x+1).

$$\Rightarrow \Pr(Y = y|X = x) = \frac{1}{(x+1)-(x-1)} \tag{2.0.2}$$

$$= \frac{1}{2} \tag{2.0.3}$$

$$\Rightarrow \Pr(Y = y|X = x) = \begin{cases} \frac{1}{2} & y \in (x-1, x+1) \\ 0 & \text{otherwise} \end{cases} \tag{2.0.4}$$

Given $E(X) = 1$

$$\Rightarrow \int_{-\infty}^{\infty} x f_1(x) dx = 1 \tag{2.0.5}$$

Now consider $E(Y)$,

$$E(Y) = \int_{-\infty}^{\infty} y f_1(y) dy \tag{2.0.6}$$

As $f_1(y)$ is a marginal probability of Y=y we can write as,

$$E(Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f(x,y) dx dy \tag{2.0.7}$$

From 2.0.1 we can write ,

$$E(Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y \Pr(Y = y|X = x) f_1(x) dx dy \tag{2.0.8}$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} y \Pr(Y = y|X = x) dy \right) f_1(x) dx \tag{2.0.9}$$

From 2.0.4 we can simplify as,

$$E(Y) = \int_{-\infty}^{\infty} \left(\int_{(x-1)}^{(x+1)} y \Pr(Y = y|X = x) dy \right) f_1(x) dx \tag{2.0.10}$$

$$= \int_{-\infty}^{\infty} \left(\int_{(x-1)}^{(x+1)} y \left(\frac{1}{2}\right) dy \right) f_1(x) dx \tag{2.0.11}$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{2}\right) \left(\frac{(x+1)^2 - (x-1)^2}{2} \right) f_1(x) dx \tag{2.0.12}$$

$$= \int_{-\infty}^{\infty} x f_1(x) dx \tag{2.0.13}$$

$$= E(X) \tag{2.0.14}$$

Hence from 2.0.5 it can be seen that

$$E(Y) = 1 \tag{2.0.15}$$

$\therefore$ **Option B is true**