Assignment 1

CS20BTECH11028

Download all python codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−1/codes

and latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−1

QUESTION (2.14)

A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?.

SOLUTION(2.14):

Let A be the event that the sum of the numbers appearing is 6 when a die is thrown twice.
Let B be the event such that the number 4 appears atleast once in the two throws.
We need the conditional probability of event B given that A has occurred.

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} \tag{0.0.1}$$

Let $X_i \in \{1,2,3,4,5,6\}, i = 1,2.$ be a random variable representing the outcome for each die.
The probability that A occur is same as the probability that $X_1 + X_2 = 6$.

$$\begin{aligned}\Pr(X_1 + X_2 = 6) &= \Pr(X_1 = 6 - X_2)\\ &= \sum_k \Pr(X_1 = 6-k)\Pr(X_2 = k)\end{aligned}$$

As $1 \le X_1, X_2 \le 6$, the equation simplifies to,

$$\Rightarrow \sum_k \Pr(X_1 = 6-k)\Pr(X_2 = k)$$

$$= \sum_{k=1}^{5} \Pr(X_1 = 6-k)\Pr(X_2 = k)$$

$$= (\frac{1}{6})(\frac{1}{6})\sum_{k=1}^{5}(1) = \frac{5}{36}.$$

(As the probability is 1/6 for $X_1, X_2 \in \{1,2,3,4,5,6\}$)
Hence,

$$\Rightarrow \Pr(A) = \frac{5}{36} \tag{0.0.2}$$

$\Pr(B)$=probability of getting a four atleast once
let X be a random variable which represents number of times 4 appears in two throws of a die.

$$\Rightarrow \Pr(B) = \Pr(X=1) + \Pr(X=2) \tag{0.0.3}$$

From binomial distribution we can write ,

$$\Pr(B) = \binom{2}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) + \binom{2}{2}\left(\frac{1}{6}\right)^2 \tag{0.0.4}$$

$$= \frac{11}{36} \tag{0.0.5}$$

The event AB is such that the sum should be six with atleast one 4. Therefore the other number must be 2.
There are only two possible cases {4,2},{2,4} out of 36 possible cases.
Hence,

$$\Pr(AB) = \frac{2}{36}. \tag{0.0.6}$$

Substituting equations (0.0.2),(0.0.6) in (0.0.1) , we get

$$\begin{aligned}\Pr(B|A) &= \frac{\frac{2}{36}}{\frac{5}{36}}\\ &= \frac{2}{5}.\end{aligned} \tag{0.0.7}$$

Hence the probability of occurring atleast one 4 when the sum of the numbers is 6 when a die is thrown twice is $\frac{2}{5}$.