Assignment 3

CS20BTECH11028

Download all python codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−3/codes

and latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−3

1 Problem GATE MA 2012 30

The probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{(-\frac{x}{\lambda})}, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{1.0.1}$$

where $\lambda > 0$. For testing the hypothesis $H_0 : \lambda = 3$ against $H_1 : \lambda = 5$, a test is given as "Reject H_0 if $X \ge 4.5$".The probability of type 1 error and power of the test are respectively:

(A) 0.1353 and 0.4966 (C) 0.2021 and 0.4493

(B) 0.1827 and 0.379 (D) 0.2231 and 0.4066

2 Solution

Definition 2.1. *A type 1 error occurs if the null hypothesis H_0 is rejected even if it is true.*

Definition 2.2. *The probability that the alternative hypothesis H_1 is true is defined to be Power of a given test.*

Given,

$$f_X(x) = \begin{cases} \frac{1}{\lambda} e^{(-\frac{x}{\lambda})}, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{2.0.1}$$

Let cumulative distribution function be $F_X(x)$ for a given λ.
Hence,

$$F_X(x) = \int_{-\infty}^{x} f_X(a)da \tag{2.0.2}$$

From the probability density function,

$$\implies F_X(4.5) = \int_{-\infty}^{x} f_X(a)da \tag{2.0.3}$$

$$= \int_{0}^{4.5} \frac{1}{\lambda} e^{(-\frac{a}{\lambda})} da \tag{2.0.4}$$

$$= 1 - e^{-\frac{4.5}{\lambda}} \tag{2.0.5}$$

We need the probability for $X \ge 4.5$,hence

$$\text{Required probability} = 1 - F_X(4.5) \tag{2.0.6}$$

$$= e^{-\frac{4.5}{\lambda}} \tag{2.0.7}$$

Now the probability that the given null hypothesis(H_0) is true is ,
From (2.0.7)

$$\text{Required probability} = e^{-\frac{4.5}{3}} \tag{2.0.8}$$

$$= 0.2231. \tag{2.0.9}$$

Therefore the probability that we are rejecting a null hypothesis which is true for $X \ge 4.5$ is 0.2231. Hence the **probability of type 1 error is 0.2231**.

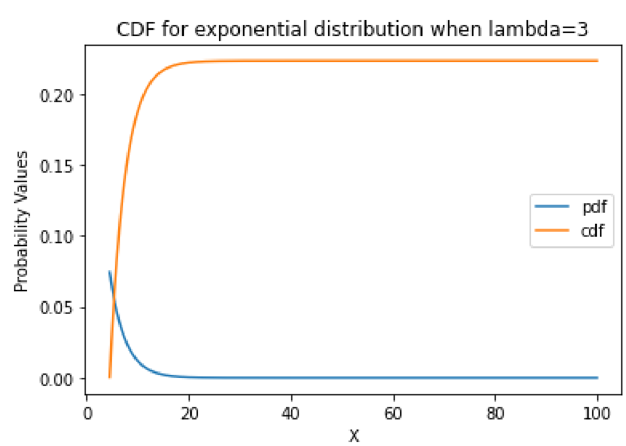

Fig. 4: Probability of type 1 error

Now the probability that the given alternative hypothesis(H_1) is true is, From (2.0.7)

$$\text{Required probability} = e^{-\frac{4.5}{5}} \tag{2.0.10}$$
$$= 0.4066 \tag{2.0.11}$$

Hence the probability that the given alternative hypothesis is true for $X \geq 4.5$ is 0.4066.
Thus,**The power of the test is 0.4066**

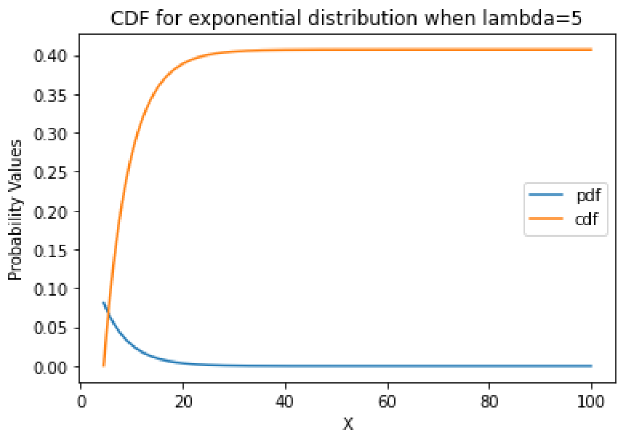

Fig. 4: Power of test