# Assignment 5

CS20BTECH11028

Download all python codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−5/codes

Download latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−5

## 1 Problem UGC/MATH june 2018 49

A standard fair die is rolled until some face other than 5 or 6 turns up.Let X denote the face value of the last roll.Let A={X is even} and B={X is atmost 2} Then,

(A) $\Pr(A \cap B) = 0$

(B) $\Pr(A \cap B) = \frac{1}{6}$

(C) $\Pr(A \cap B) = \frac{1}{4}$

(D) $\Pr(A \cap B) = \frac{1}{3}$

## 2 Solution

Given $X$ is the face value of the last roll.So $X \in \{1, 2, 3, 4, 5, 6\}$. Given A={X is even}

$$\implies A = \{2, 4, 6\} \tag{2.0.1}$$

Given B={X is atmost 2}

$$\implies B = \{1, 2\} \tag{2.0.2}$$

From (2.0.1) and (2.0.2) we can write,

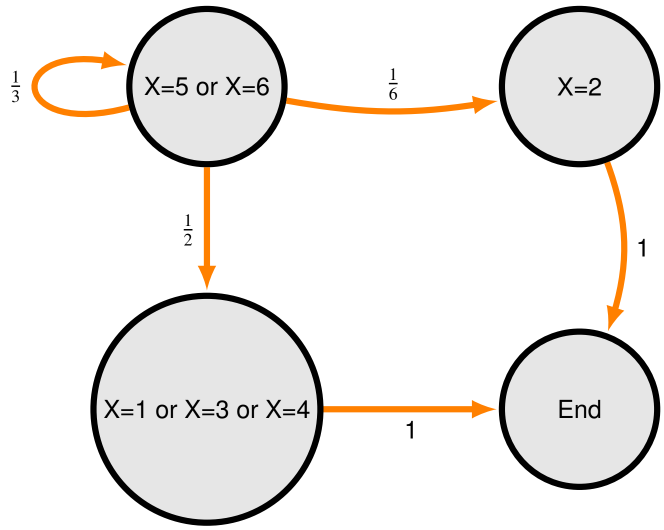
$$\implies AB = \{2\} \tag{2.0.3}$$

Given it is a fair die,

$$\implies \Pr(X = x) = \begin{cases} \frac{1}{6} & 1 \le x \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.4}$$

Given that we roll a die until a number other than 5 or 6 appears.So we have to add all cases, like getting a 2 in the first roll and getting 5 or 6 in first roll and 2 in the second roll and so on.

Fig. 4: Markov chain




$$\begin{aligned}\Pr(AB) = \Pr(X = 2) + \\ (\Pr((X = 5) + (X = 6)))\Pr(X = 2) + \\ (\Pr((X = 5) + (X = 6)))^2 \Pr(X = 2) + \cdots + \infty\end{aligned} \tag{2.0.5}$$

Hence,

$$\Pr(AB) = \sum_{i=0}^{\infty} (\Pr((X = 5) + (X = 6)))^i \Pr(X = 2) \tag{2.0.6}$$

From (2.0.5) and Geometric progression we can write,

$$\implies \Pr(AB) = \frac{\Pr(X = 2)}{1 - (\Pr((X = 5) + (X = 6)))} \tag{2.0.7}$$

From (2.0.4) we can write,

$$\implies \Pr(X = 2) = \frac{1}{6} \tag{2.0.8}$$

As $\{X = 5\}$ and $\{X = 6\}$ are disjoint we can write

$$\Pr((X = 5) + (X = 6)) = \Pr(X = 5) + \Pr(X = 6) \tag{2.0.9}$$

$$= \frac{2}{6} \tag{2.0.10}$$

from (2.0.8) and (2.0.10) we can write,

$$\Pr(AB) = \frac{\frac{1}{6}}{1 - \frac{2}{6}} \tag{2.0.11}$$

$$= \frac{1}{4}. \tag{2.0.12}$$

$\therefore$ **option C is correct**

## 3 Another method using Markov chain property

Let us assume the following, Let us represent the

TABLE 4

| state 1 | state 2 | state 3 | state 4 |
|---|---|---|---|
| $X = 5$ or $X = 6$ | $X = 2$ | $X = 1$ or $X = 3$ or $X = 4$ | end |

markov chain diagram in a matrix.Let $P_{ij}$ represent the element of a matrix which is in $i^{th}$ row and $j^{th}$ column.The value of $P_{ij}$ is equal to probability of transition from state $i$ to state $j$

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.0.1}$$

We need the probability that $X = 2$.Hence required probability is

$$P_{12} + (P_{12})^2 + \cdots + \infty \tag{3.0.2}$$

where $P_{12}^n$ represents the 1st row ,2nd column element in the $P^n$

$$P^2 = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.0.3}$$

$$= \begin{bmatrix} \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.0.4}$$

$$P^3 = (P^2)(P^1) \tag{3.0.5}$$

$$= \begin{bmatrix} \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.0.6}$$

$$= \begin{bmatrix} \frac{1}{27} & \frac{1}{54} & \frac{1}{18} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.0.7}$$

From above we can notice that each time $P_{12}$ reduces by $\frac{1}{3}$.Hence from (3.0.2),

$$\sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i \frac{1}{6} \tag{3.0.8}$$

From Geometric progression we ca write ,required probability $=\frac{1}{4}$