

Assignment 3

CS20BTECH11028

Download all python codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-3/codes>

and latex-tikz codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-3>

1 PROBLEM GATE MA 2012 30

The probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{(-\frac{x}{\lambda})}, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (1.0.1)$$

where $\lambda > 0$. For testing the hypothesis $H_0 : \lambda = 3$ against $H_1 : \lambda = 5$, a test is given as "Reject H_0 if $X \geq 4.5$ ". The probability of type 1 error and power of the test are respectively:

(A) 0.1353 and 0.4966 (C) 0.2021 and 0.4493

(B) 0.1827 and 0.379 (D) 0.2231 and 0.4066

2 SOLUTION

Definition 2.1. A type 1 error occurs if the null hypothesis H_0 is rejected even if it is true.

Definition 2.2. The probability that the alternative hypothesis H_1 is true is defined to be Power of a given test.

Given,

$$f_X(x) = \begin{cases} \frac{1}{\lambda} e^{(-\frac{x}{\lambda})}, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (2.0.1)$$

Let cumulative distribution function be $F_X(x)$ for a given λ . Hence,

$$F_X(x) = \int_{-\infty}^x f_X(a) da \quad (2.0.2)$$

From the probability density function,

$$\Rightarrow F_X(4.5) = \int_{-\infty}^x f_X(a) da \quad (2.0.3)$$

$$= \int_0^{4.5} \frac{1}{\lambda} e^{(-\frac{a}{\lambda})} da \quad (2.0.4)$$

$$= 1 - e^{-\frac{4.5}{\lambda}} \quad (2.0.5)$$

We need the probability for $X \geq 4.5$, hence required probability is,

$$1 - F_X(4.5) = e^{-\frac{4.5}{\lambda}} \quad (2.0.6)$$

From (2.0.6) we get probability that the given null hypothesis (H_0) is true is,

$$e^{-\frac{4.5}{3}} = 0.2231. \quad (2.0.7)$$

\therefore The **probability of type 1 error is 0.2231.**

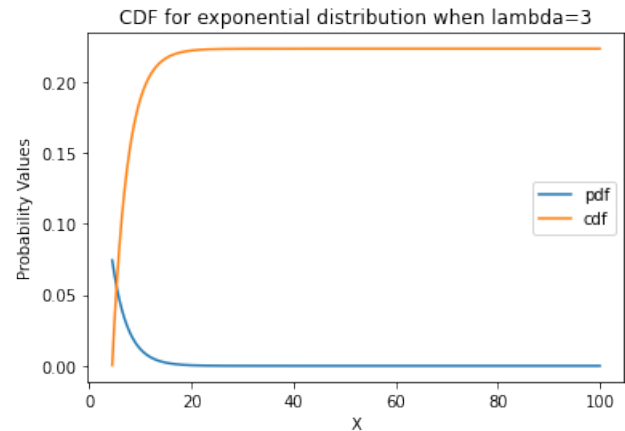


Fig. 4: Probability of type 1 error

From (2.0.6), we get the required probability that the given alternative hypothesis (H_1) is true is,

$$e^{-\frac{4.5}{5}} = 0.4066 \quad (2.0.8)$$

\therefore The **power of the test is 0.4066**

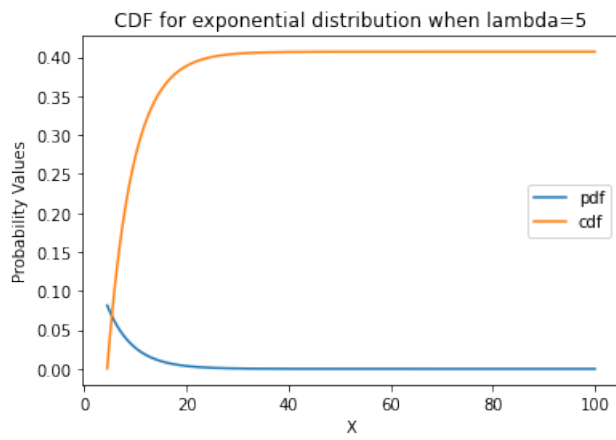


Fig. 4: Power of test