Assignment 5

CS20BTECH11028

Download all python codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−5/codes

Download latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−5

1 Problem UGC/MATH june 2018 49

A standard fair die is rolled until some face other than 5 or 6 turns up.Let X denote the face value of the last roll.Let A={X is even} and B={X is atmost 2} Then,

(A) $\Pr(A \cap B) = 0$

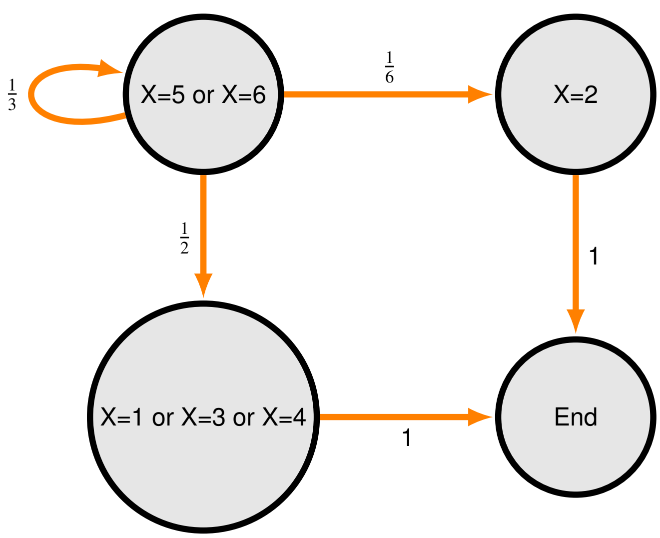
(B) $\Pr(A \cap B) = \frac{1}{6}$

(C) $\Pr(A \cap B) = \frac{1}{4}$

(D) $\Pr(A \cap B) = \frac{1}{3}$

2 Solution

Fig. 4: Markov chain



Let us assume the following table. Let us represent the markov chain diagram in a matrix.Let P_{ij} represent the element of a matrix which is in i^{th} row and j^{th} column.The value of P_{ij} is equal to probability of transition from state i to state j

TABLE 4

state 1	state 2	state 3	state 4
$X = 5$ or $X = 6$	$X = 2$	$X = 1$ or $X = 3$ or $X = 4$	end

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.0.1}$$

We need the probability that $X = 2$.Hence required probability is

$$P_{12} + (P_{12})^2 + \cdots + \infty \tag{2.0.2}$$

where P^n_{12} represents the 1st row ,2nd column element in the P^n

$$P^2 = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.0.3}$$

$$= \begin{bmatrix} \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.0.4}$$

$$P^3 = (P^2)(P^1) \tag{2.0.5}$$

$$= \begin{bmatrix} \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.0.6}$$

$$= \begin{bmatrix} \frac{1}{27} & \frac{1}{54} & \frac{1}{18} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.0.7}$$

From above we can notice that each time P_{12} reduces by $\frac{1}{3}$.Hence from (2.0.2),

$$\sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i \frac{1}{6} \tag{2.0.8}$$

From Geometric progression we can write ,required probability $=\frac{1}{4}$ $\therefore$ **option C is correct**