

Assignment 1

CS20BTECH11028

Download all python codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-1/codes>

and latex-tikz codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-1>

QUESTION(2.13):

Let A be the event that the sum of the numbers appearing is 6 when a die is thrown twice.

Let B be the event such that the number 4 appears atleast once in the two throws.

We need the conditional probability of event B given that A has occurred.

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} \quad (0.0.1)$$

Let $X_i \in \{1, 2, 3, 4, 5, 6\}, i = 1, 2$. be a random variable representing the outcome for each die.

The probability that A occur is same as the probability that $X_1 + X_2 = 6$.

$$\begin{aligned} \Pr(X_1 + X_2 = 6) &= \Pr(X_1 = 6 - X_2) \\ &= \sum_k \Pr(X_1 = 6 - k) \Pr(X_2 = k) \end{aligned}$$

As $1 \leq X_1, X_2 \leq 6$, the equation simplifies to,

$$\begin{aligned} &\Rightarrow \sum_k \Pr(X_1 = 6 - k) \Pr(X_2 = k) \\ &= \sum_{k=1}^5 \Pr(X_1 = 6 - k) \Pr(X_2 = k) \\ &= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) \sum_{k=1}^5 (1) = \frac{5}{36}. \end{aligned}$$

(As the probability is $1/6$ for $X_1, X_2 \in \{1, 2, 3, 4, 5, 6\}$)

Hence,

$$\Rightarrow \Pr(A) = \frac{5}{36} \quad (0.0.2)$$

The event AB is such that the sum should be six with atleast one 4. Therefore the other number must be 2.

There are only two possible cases $\{4, 2\}, \{2, 4\}$ out of 36 possible cases.

Hence,

$$\Pr(AB) = \frac{2}{36}. \quad (0.0.3)$$

Substituting equations (0.0.2), (0.0.3) in (0.0.1), we get

$$\begin{aligned} \Pr(B|A) &= \frac{\frac{2}{36}}{\frac{5}{36}} \\ &= \frac{2}{5}. \end{aligned} \quad (0.0.4)$$

Hence the probability of occurring atleast one 4 when the sum of the numbers is 6 when a die is thrown twice is $\frac{2}{5}$.