Assignment 2

CS20BTECH11028

Download latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−2

1 Problem GATE EC 56

Let X and Y be jointly distributed random variables such that the conditional distribution of Y, given X =x, is uniform on the interval $(x-1, x+1)$. Suppose $E(X) = 1$ and $Var(X) = \frac{5}{3}$.
The mean of the random variable Y is

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 2

2 Solution GATE EC 56

We know that,

$$f_{Y|X=x}(y) = \frac{f(x,y)}{f_X(x)} \tag{2.0.1}$$

Given that $f_{Y|X=x}(y)$ is uniform over the interval (x-1,x+1).

$$\Rightarrow f_{Y|X=x}(y) = \begin{cases} \frac{1}{2} & y \in (x-1, x+1) \\ 0 & \text{otherwise} \end{cases} \tag{2.0.2}$$

Given $E(X) = 1$

$$\Rightarrow \int_{-\infty}^{\infty} x f_X(x) dx = 1 \tag{2.0.3}$$

Now consider $E(Y|X = x)$,

$$E(Y|X = x) = \int_{-\infty}^{\infty} y f_{Y|X=x}(y) dy \tag{2.0.4}$$

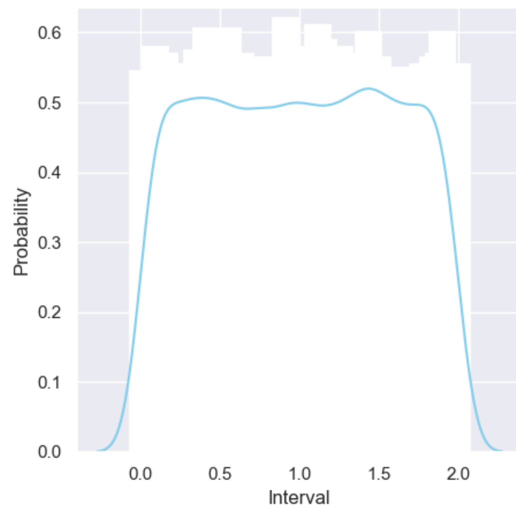

Fig. 4: Distribution of $f_{(Y|X=1)}(y)$

From (2.0.2) it simplifies to,

$$\Rightarrow E(Y|X = x) = \int_{-\infty}^{x-1} y f_{Y|X=x}(y) dy + \int_{x-1}^{x+1} y f_{Y|X=x}(y) dy + \int_{x+1}^{\infty} y f_{Y|X=x}(y) dy \tag{2.0.5}$$

$$\Rightarrow E(Y|X = x) = \int_{x-1}^{x+1} y \left(\frac{1}{2}\right) dy \tag{2.0.6}$$

$$= x \tag{2.0.7}$$

Now we can write ,

$$E(Y) = \int_{-\infty}^{\infty} E(Y|X = x) f_X(x) dx \tag{2.0.8}$$

$$= \int_{-\infty}^{\infty} x f_X(x) dx \tag{2.0.9}$$

$$= E(X) \tag{2.0.10}$$

From (2.0.3) we get

$$E(Y) = 1. \tag{2.0.11}$$

$\therefore$ **Option B is true**