Assignment 1

CS20BTECH11028

Download all python codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−1/codes

and latex-tikz codes from

https://github.com/Harsha24112002/AI1103/tree/main/Assignment−1

QUESTION (2.14)

A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?.

SOLUTION(2.14):

Let $A \in \{2,3,4,5,6,7,8,9,10,11,12\}$ be a random variable representing the sum of outcomes when a die is thrown twice.
Let $B \in \{0,1,2\}$ be a random variable that represents the number of times 4 occurs in two throws.
We need the conditional probability of event $(B \geq 1)$ given that $(A = 6)$ has occurred.

$$\Pr\left((B \geq 1)\,|\,(A = 6)\right) = \frac{\Pr\left((A = 6) \cap (B \geq 1)\right)}{\Pr\left(A = 6\right)} \tag{0.0.1}$$

We have that,

$$\Pr\left(A = n\right) = \begin{cases} 0 & n \leq 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 8 \leq n \leq 12 \\ 0 & n \geq 13 \end{cases} \tag{0.0.2}$$

Therefore using equation (0.0.2) we can write that,

$$\Pr\left(A = 6\right) = \frac{5}{36} \tag{0.0.3}$$

From binomial distribution we can write ,

$$\Pr\left(B \geq 1\right) = \Pr\left(B = 1\right) + \Pr\left(B = 2\right) \tag{0.0.4}$$

$$= \binom{2}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) + \binom{2}{2}\left(\frac{1}{6}\right)^2 \tag{0.0.5}$$

$$= \frac{11}{36} \tag{0.0.6}$$

The event $((A = 6) \cap (B \geq 1))$ is such that the sum should be six 6 with atleast one 4.
There are only two possible cases {4,2},{2,4} out of 36 possible cases.
Hence,

$$\Pr\left((A = 6) \cap (B \geq 1)\right) = \frac{2}{36}. \tag{0.0.7}$$

Substituting equations (0.0.4),(0.0.8) in (0.0.1) , we get

$$\Pr((B \geq 1)|(A = 6)) = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}. \tag{0.0.8}$$

Hence the probability of occurring atleast one 4 when the sum of the numbers is 6 when a die is thrown twice is $\frac{2}{5}$.