

Assignment 5

CS20BTECH11028

Download all python codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-5/codes>

Download latex-tikz codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-5>

1 PROBLEM UGC/MATH JUNE 2018 49

A standard fair die is rolled until some face other than 5 or 6 turns up. Let X denote the face value of the last roll. Let $A = \{X \text{ is even}\}$ and $B = \{X \text{ is at most } 2\}$. Then,

(A) $\Pr(A \cap B) = 0$ (C) $\Pr(A \cap B) = \frac{1}{4}$

(B) $\Pr(A \cap B) = \frac{1}{6}$ (D) $\Pr(A \cap B) = \frac{1}{3}$

2 SOLUTION

Given X is the face value of the last roll. So $X \in \{1, 2, 3, 4, 5, 6\}$. Given $A = \{X \text{ is even}\}$

$$\Rightarrow A = \{2, 4, 6\} \quad (2.0.1)$$

Given $B = \{X \text{ is at most } 2\}$

$$\Rightarrow B = \{1, 2\} \quad (2.0.2)$$

From (2.0.1) and (2.0.2) we can write,

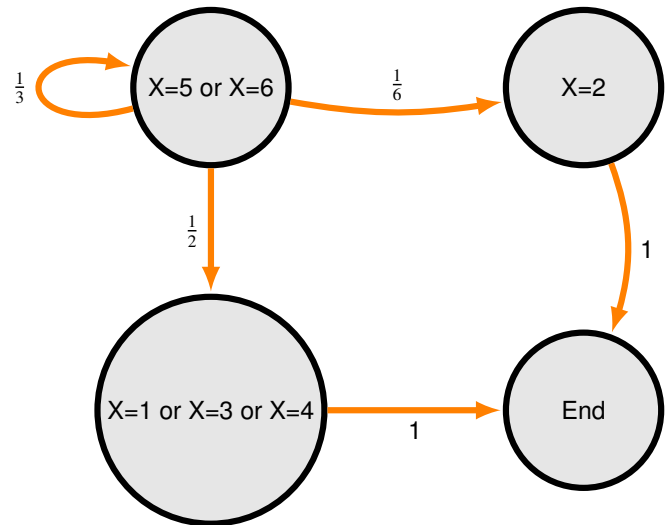
$$\Rightarrow AB = \{2\} \quad (2.0.3)$$

Given it is a fair die,

$$\Rightarrow \Pr(X = x) = \begin{cases} \frac{1}{6} & 1 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.4)$$

Given that we roll a die until a number other than 5 or 6 appears. So we have to add all cases, like

Fig. 4: Markov chain



getting a 2 in the first roll and getting 5 or 6 in first roll and 2 in the second roll and so on.

$$\begin{aligned} \Pr(AB) &= \Pr(X = 2) + \\ &\quad (\Pr((X = 5) + (X = 6))) \Pr(X = 2) + \\ &\quad (\Pr((X = 5) + (X = 6)))^2 \Pr(X = 2) + \dots + \infty \end{aligned} \quad (2.0.5)$$

Hence,

$$\Pr(AB) = \sum_{i=0}^{\infty} (\Pr((X = 5) + (X = 6)))^i \Pr(X = 2) \quad (2.0.6)$$

From (2.0.5) and Geometric progression we can write,

$$\Rightarrow \Pr(AB) = \frac{\Pr(X = 2)}{1 - (\Pr((X = 5) + (X = 6)))} \quad (2.0.7)$$

From (2.0.4) we can write,

$$\Rightarrow \Pr(X = 2) = \frac{1}{6} \quad (2.0.8)$$

As $\{X = 5\}$ and $\{X = 6\}$ are disjoint we can write

$$\Pr((X = 5) + (X = 6)) = \Pr(X = 5) + \Pr(X = 6) \quad (2.0.9)$$

$$= \frac{2}{6} \quad (2.0.10)$$

from (2.0.8) and (2.0.10) we can write,

$$\Pr(AB) = \frac{\frac{1}{6}}{1 - \frac{2}{6}} \quad (2.0.11)$$

$$= \frac{1}{4}. \quad (2.0.12)$$

\therefore option C is correct