

Assignment 3

CS20BTECH11028

Download all python codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-3/codes>

and latex-tikz codes from

<https://github.com/Harsha24112002/AI1103/tree/main/Assignment-3>

1 PROBLEM GATE MA 2012 30

The probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (1.0.1)$$

where $\lambda > 0$. For testing the hypothesis $H_0 : \lambda = 3$ against $H_1 : \lambda = 5$, a test is given as "Reject H_0 if $X \geq 4.5$ ". The probability of type 1 error and power of the test are respectively:

(A) 0.1353 and 0.4966 (C) 0.2021 and 0.4493

(B) 0.1827 and 0.379 (D) 0.2231 and 0.4066

2 SOLUTION

Let the probability that $X \geq 4.5$ is $F(X \geq 4.5)$ for a given λ .

where $F(X \geq a) = \int_{x=a}^{\infty} f(x)$

From the probability density function,

$$\Rightarrow F(X \geq 4.5) = \int_{x=4.5}^{\infty} f(x) \quad (2.0.1)$$

$$= \int_{x=4.5}^{\infty} \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \quad (2.0.2)$$

$$= e^{-\frac{4.5}{\lambda}} \quad (2.0.3)$$

Now the probability that the given null hypothesis(H_0) is true is ,

$$F(X \geq 4.5) = e^{-\frac{4.5}{3}} \quad (2.0.4)$$

$$= 0.2231 \quad (2.0.5)$$

Therefore the probability that we are rejecting a null hypothesis which is true for $X \geq 4.5$ is 0.2231. Hence the **probability of type 1 error is 0.2231**.

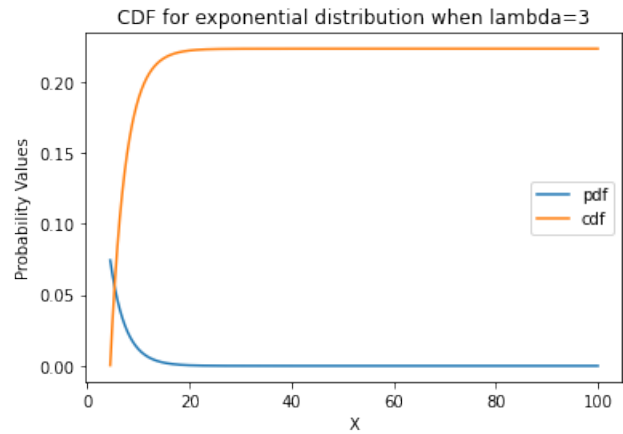


Fig. 4: $\Pr(4.5 \leq X \leq 100(\text{some large number}))$

Now the probability that the given alternative hypothesis is true is,

$$F(X \geq 4.5) = e^{-\frac{4.5}{5}} \quad (2.0.6)$$

$$= 0.4066 \quad (2.0.7)$$

Hence the probability that the given alternative hypothesis is true for $X \geq 4.5$ is 0.4066.

Thus, **The power of the test is 0.4066**

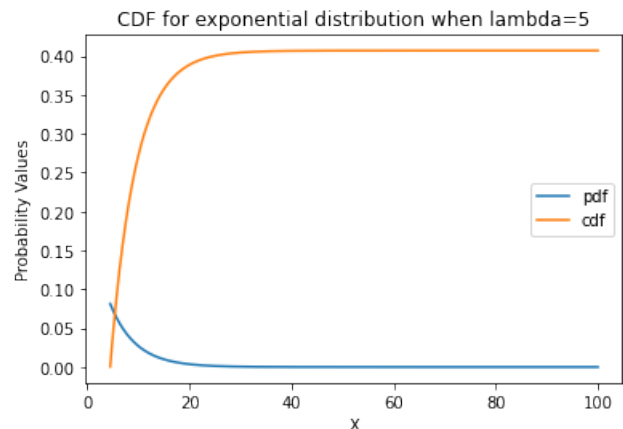


Fig. 4: $\Pr(4.5 \leq X \leq 100(\text{some large number}))$