Analysis of Convolution with Original, Modified, and Shifted Kernels on Inverse Trigonometric Functions

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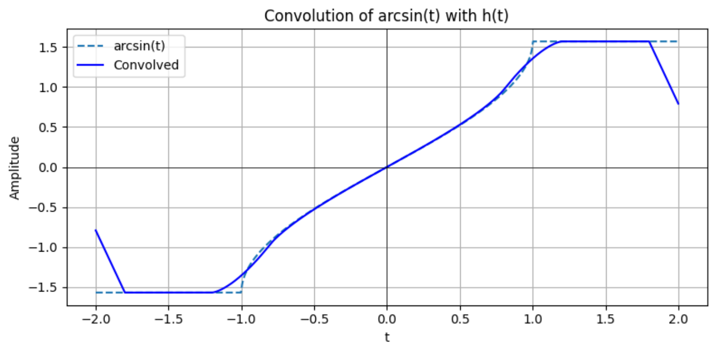
1. Original Kernel

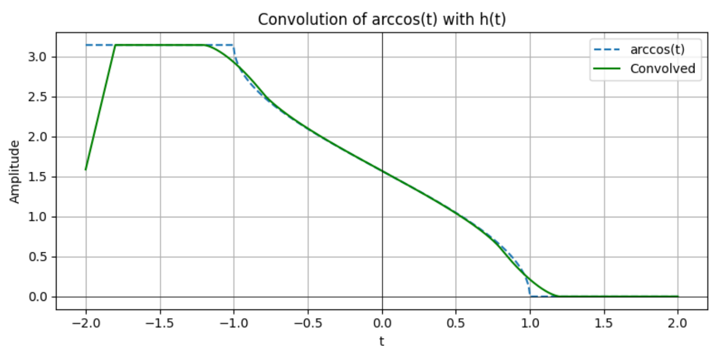
The **original symmetric kernel** $h(t)$ defined as:

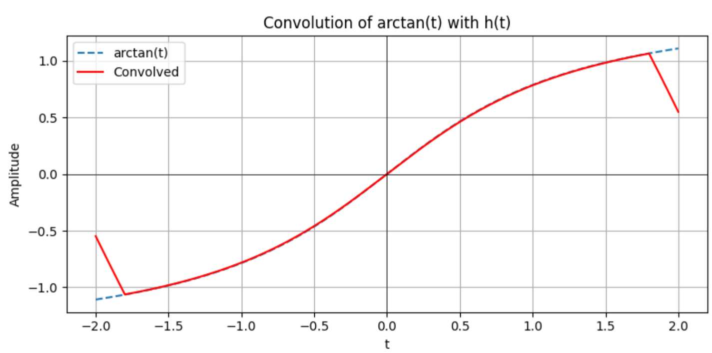
$$h(t) = \begin{cases} 1, & \text{for } -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

The symmetric kernel includes both past and future values around time t. When this kernel is convolved with the inverse trigonometric functions:

- The result is a **smoothed** version of the original signal.
- The kernel acts like a moving average centered at each point.
- For $\arcsin(t)$ and $\arccos(t)$, the undefined regions outside $[-1, 1]$ are managed using `np.clip`, leading to saturation outside that interval.
- The convolution retains the central trend and symmetry of the original signal.


Convolution of arcsin(t) with h(t)
arcsin(t)
Convolved
Amplitude
t


Convolution of arccos(t) with h(t)
arccos(t)
Convolved
Amplitude
t


Convolution of arctan(t) with h(t)
arctan(t)
Convolved
Amplitude
t

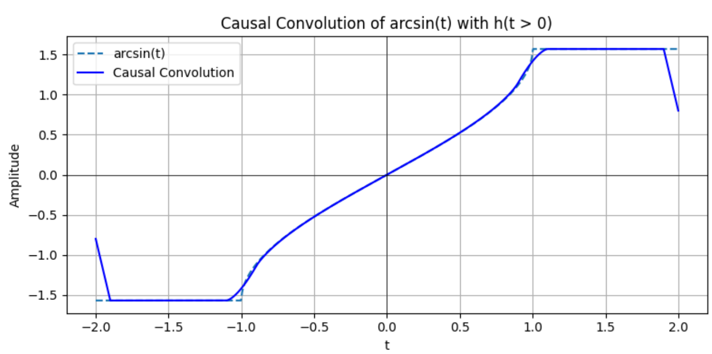
2. Modified One-Sided Kernel

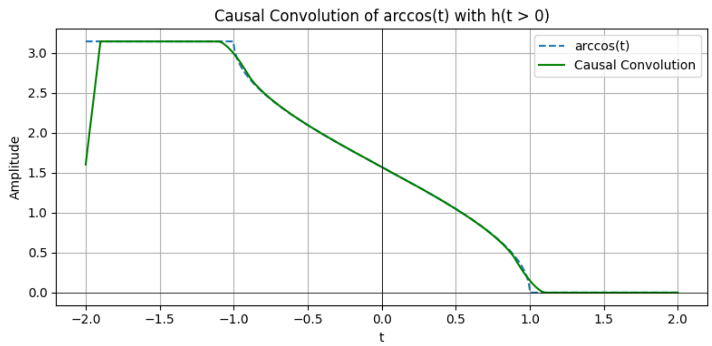
The **modified one-sided kernel** that only considers $t > 0$:

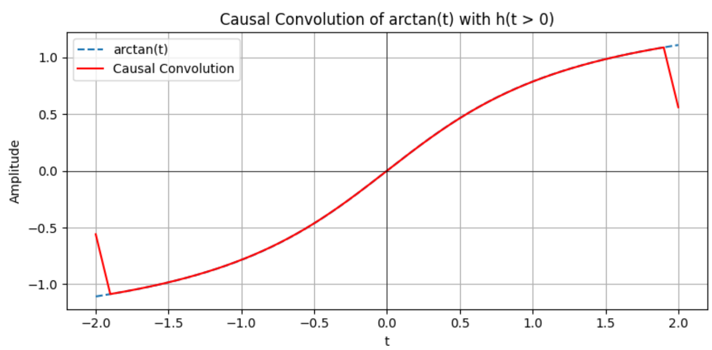
$$h(t) = \begin{cases} 1, & \text{for } 0 < t \leq T \\ 0, & \text{otherwise} \end{cases}$$

The one-sided kernel (nonzero only for $t > 0$) introduces an inherent **asymmetry** into the convolution:

- It models a **causal** system—one that responds only to the present and past.
- The output becomes more biased toward **past values**, creating a smoothing effect that **lags behind** the original function.
- For $\arcsin(t)$ and $\arccos(t)$, the convolution output shows a shift toward the left and becomes less symmetric.
- For $\arctan(t)$, the effect is more subtle but still causes a smoothing that trails the input.



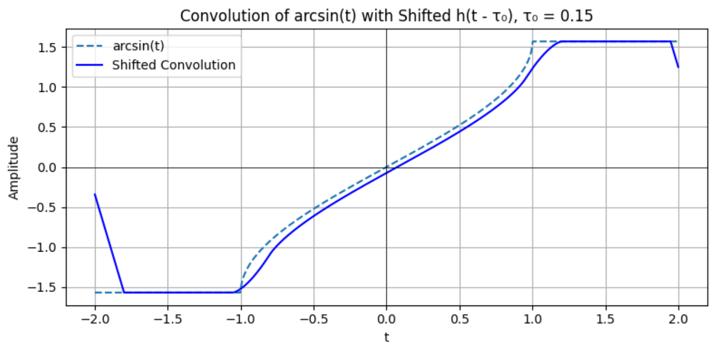


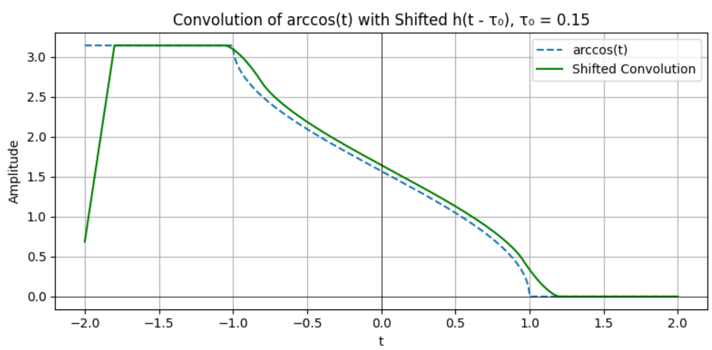


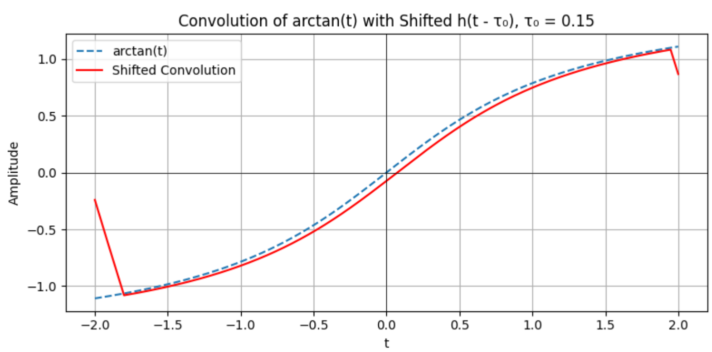
3. Shifted Kernel

Convolving with $h(t - \tau_0)$ introduces a time delay of τ_0 into the output:

- The **shape** of the convolution result remains essentially unchanged.
- The entire output is **delayed** by τ_0, modeling systems with fixed time delay (e.g., transmission delays).
- This shift preserves symmetry and smoothness, unlike the one-sided kernel.
- Such modeling is useful in time-delayed systems in control and signal transmission.







Conclusion

Each type of kernel plays a different role:

- The **original kernel** performs symmetric smoothing.

- The **modified kernel** introduces causality and asymmetry.
- The **shifted kernel** adds a pure delay without distorting the function shape.