Manual Convolution Analysis

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I. Manual Convolution (with Step Signal and Rectangular Kernel)

To start the analysis, I chose a very simple signal: the unit step function defined as:

$$f(t) = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

The kernel we are convolving with is a rectangular pulse:

$$h(t) = \begin{cases} 1, & -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

The convolution formula in continuous time is:

$$y(t) = (f * h)(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau$$

Since $u(\tau) = 0$ for $\tau < 0$, we can update the limits:

$$y(t) = \int_{0}^{\infty} h(t-\tau)d\tau$$

Now, the function $h(t-\tau)$ is non-zero only when:

$$-T \leq t - \tau \leq T \Rightarrow t - T \leq \tau \leq t + T$$

Combining with the lower bound of the step function ($\tau \geq 0$), we get:

$$\tau \in [\max(0, t-T), t+T]$$

The integrand is 1, so evaluating the integral:

$$y(t) = \int_{\max(0,t-T)}^{t+T} 1 \, d\tau = t + T - \max(0, t-T)$$

A. Cases

Lets break it into different time ranges to understand the output better:

- If $t < -T$: No overlap, so $y(t) = 0$
- If $-T \leq t < 0$: Partial overlap, area increases: $y(t) = T + t$
- If $0 \leq t < T$: Kernel continues to grow on step: $y(t) = T + t$
- If $T \leq t \leq 2T$: Full overlap: $y(t) = 2T$
- If $2T < t \leq 3T$: Overlap starts reducing: $y(t) = 3T - t$
- If $t > 3T$: No overlap again: $y(t) = 0$

The result is a trapezoidal waveform:

$$y(t) = \begin{cases} 0, & t < -T \\ T + t, & -T \leq t < T \\ 2T, & T \leq t \leq 2T \\ 3T - t, & 2T < t \leq 3T \\ 0, & t > 3T \end{cases}$$