11.16.3.3.4

EE24BTECH11063 - Y. Harsha Vardhan Reddy

Question:
Find the probability that no toss results in a tail when coin is tossed thrice(independent tosses)
Solution:
Computational solution:

INTRODUCTION

This document explains the computational process of determining the probability distribution of the number of tails when a coin is tossed three times. The implementation involves two components:

- A **C program** to perform the simulation, calculate the probabilities (PMF), and the cumulative distribution function (CDF).
- A **Python** to use the results from the C program and generate a plot of the probability.

DEFINITIONS

Let the random variable X represent the number of tails when a fair coin is tossed three times. The possible values of X are:

$$X \in \{0, 1, 2, 3\}.$$

- **Probability Mass Function (PMF):** The PMF of X is defined as:

$$P(X = x) = \frac{\text{Number of occurrences of } x}{\text{Total number of trials}},$$

  where $x \in \{0, 1, 2, 3\}$.
- **Cumulative Distribution Function (CDF):** The CDF of X is defined as:

$$F(X = x) = \sum_{k=0}^{x} P(X = k),$$

  which gives the cumulative probability of obtaining k or fewer tails.

C PROGRAM IMPLEMENTATION

The C program performs the following steps:

1) Simulate n trials of tossing a coin three times using the `rand()` function to generate random outcomes (head or tail).
2) Count the number of occurrences of 0, 1, 2, and 3 tails across all trials.

3) Compute the PMF:

$$P(X = x) = \frac{\text{Number of occurrences of } x}{n}, \quad \text{for } x \in \{0, 1, 2, 3\}.$$

4) Compute the CDF using the PMF:

$$F(X = x) = \sum_{k=0}^{x} P(X = k).$$

5) Expose the results (PMF and CDF) as arrays for use in Python.

Python Script Implementation

The Python script:
1) Loads the shared object file (`.so`) created by the C program.
2) Calls the `calculate_probabilities` function in the C program, passing the number of trials (n) and retrieving the PMF and CDF as arrays.
3) Plots the PMF as a stem plot, showing the probabilities for each outcome (0, 1, 2, and 3 tails).

Use of CDF in Computation

The CDF is computed in the C program using the PMF:

$$F(X = x) = F(X = x - 1) + P(X = x), \quad \text{for } x \geq 1,$$

with the initial condition:

$$F(X = 0) = P(X = 0).$$

The Python script does not directly use the CDF but focuses on visualizing the PMF.

Visualization

The Python script generates a stem plot of the PMF, illustrating the probabilities for each outcome:

$$P(X = 0), P(X = 1), P(X = 2), P(X = 3).$$

Conclusion

This computational process combines the efficiency of C for numerical simulation and the versatility of Python for visualization. The CDF plays a key role in ensuring the correctness of cumulative probabilities, which validates the PMF computation.
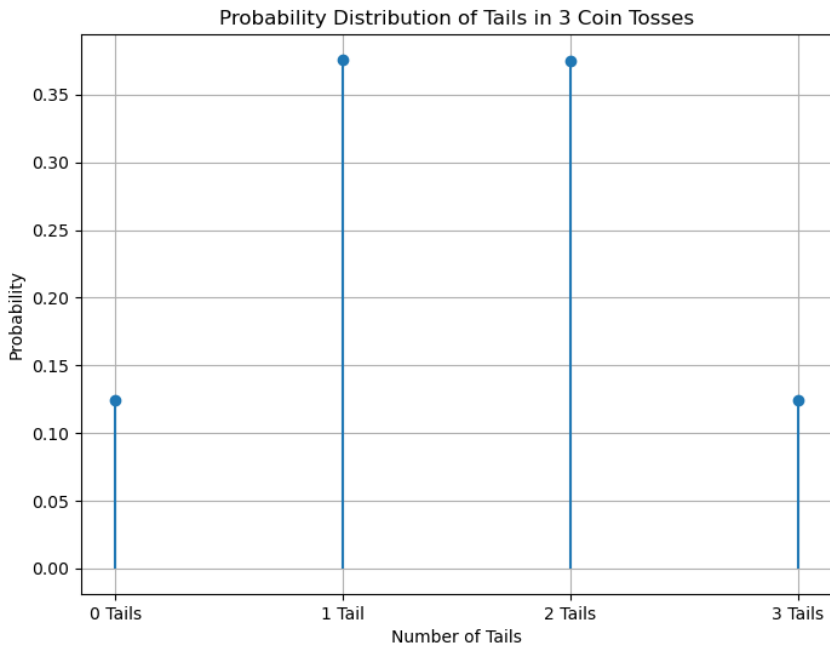
Fig. 3.1: Solution of the system of linear equations