11.16.3.3.4

EE24BTECH11063 - Y. Harsha Vardhan Reddy

Question:
Find the probability that no toss results in a tail when coin is tossed thrice(independent tosses)
Solution:
Theoretical solution:
To calculate the probability of getting all three tails when a coin is tossed three times:

- The probability of getting a tail (T) in a single toss is:
$$P(T) = \frac{1}{2}.$$
- Since the tosses are independent, the probability of getting three tails (TTT) is given by:
$$P(\text{Three Tails}) = P(T) \times P(T) \times P(T).$$
- Substituting $P(T) = \frac{1}{2}$, we have:
$$P(\text{Three Tails}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Thus, the probability of getting all three tails is:

$$\boxed{\frac{1}{8}}$$

Computational solution:

Z-Transform Computational Method for Coin Toss PMF

1. Z-Transform Expansion

The Z-transform for the number of tails in n coin tosses is given by:

$$T(z) = (p + (1-p)z)^n \tag{0.1}$$

where:

- p is the probability of tails ($p = 0.5$ for a fair coin),
- $1 - p$ is the probability of heads ($1 - p = 0.5$ for a fair coin),
- n is the number of tosses ($n = 3$).

2. Expansion of $T(z)$

Expand the expression $(p + (1 - p)z)^n$ using the binomial theorem:

$$T(z) = \sum_{k=0}^{n} \binom{n}{k} p^{n-k} (1 - p)^k z^k \tag{0.2}$$

where:

- $\binom{n}{k}$ is the binomial coefficient, computed as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{0.3}$$

- The coefficient of z^k in $T(z)$ gives the probability $P(X = k)$, where X is the number of tails.

3. Probability Mass Function (PMF)

The PMF is computed as:

$$P(X = k) = \binom{n}{k} p^{n-k} (1 - p)^k \tag{0.4}$$

Substitute $p = 0.5$ and $1 - p = 0.5$ for a fair coin:

$$P(X = k) = \binom{n}{k} (0.5)^n \tag{0.5}$$

4. Computational Steps

For each $k \in \{0, 1, \ldots, n\}$:

1) Compute the binomial coefficient:

$$\binom{n}{k} = \prod_{i=0}^{k-1} \frac{n-i}{k-i} \tag{1.1}$$

2) Multiply by $(0.5)^n$ to compute $P(X = k)$:

$$P(X = k) = \binom{n}{k} \cdot (0.5)^n \tag{2.1}$$

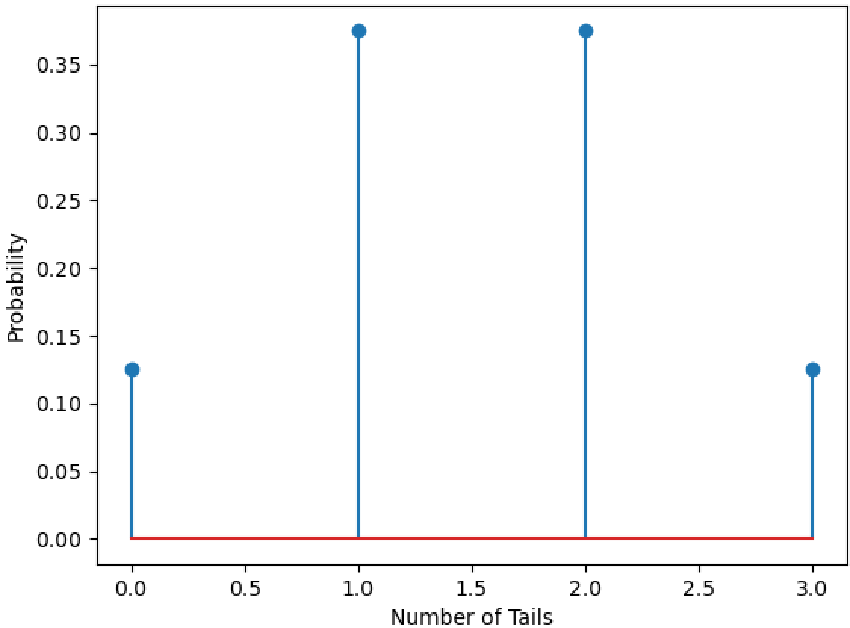
5. Result for $n = 3$

Substituting $n = 3$ and expanding:

$$T(z) = (0.5 + 0.5z)^3 = 0.125 + 0.375z + 0.375z^2 + 0.125z^3 \tag{2.2}$$

The PMF values are:

$$P(X = 0) = 0.125, \quad P(X = 1) = 0.375, \quad P(X = 2) = 0.375, \quad P(X = 3) = 0.125 \tag{2.3}$$



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Probability
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Number of Tails