8.6-6.5-1.1

EE24BTECH11063 - Y. Harsha Vardhan Reddy

Question:
Find the minimum value of the function

$$f(x) = (2x-1)^2 + 3$$

Solution:

Theoritical solution:
Given,

$$\frac{dy}{dx} = 4(2x-1) = 0 \tag{0.1}$$

$$\implies x = \frac{1}{2} \tag{0.2}$$

$$\frac{d^2y}{dx^2} = 8 \tag{0.3}$$

$$\tag{0.4}$$

Since, $\frac{d^2y}{dx^2} > 0$, at $x = \frac{1}{2}$ there exists minimum
Therefore, $f\left(\frac{1}{2}\right) = 3$ is the minimum value of the function
Computational Solution Using Gradient Descent
To verify the analytical results, we use gradient descent to find the local minimum
Gradient Descent for local minimum :
- Start with $x_0 = 4$
- Update x iteratively using

$$x_{n+1} = x_n - \eta \cdot f'(x_n) \tag{0.5}$$

where :

$$\eta = 0.1 \tag{0.6}$$

$$f'(x) = 4(2x-1) \tag{0.7}$$

$$x_{n+1} = x_n - \eta \cdot (4(2x_n - 1)) \tag{0.8}$$

Computational Results
- Local minimum

$$x \approx 0.5, \; g(x) \approx 3.000 \tag{0.9}$$

Computational Solution Using Quadratic programming problem

Find the point lying on the line y = 1, which is nearest to origin.
We can formulate the problem as follows:

$$\min_{\mathbf{x}} \left\| e_2^{\top} \mathbf{x} \right\|^2 \tag{0.10}$$

$$\text{s.t.} \tag{0.11}$$

$$\mathbf{x}^{\top} V \mathbf{x} + 2\mathbf{u}^{\top} \mathbf{x} + f = 0 \tag{0.12}$$

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.13}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{0.14}$$

$$f = 4 \tag{0.15}$$

In the current form, the constraint is non-convex since the constraint defines a set which is not convex, since points on the line joining any 2 points on the curve don't belong to the set. However, if we become lenient and make the constraint

$$\mathbf{x}^{\top} V \mathbf{x} + 2\mathbf{u}^{\top} \mathbf{x} \leq 0 \tag{0.16}$$

the constraint becomes convex. Using cvxpy to solve this convex optimization problem, we get

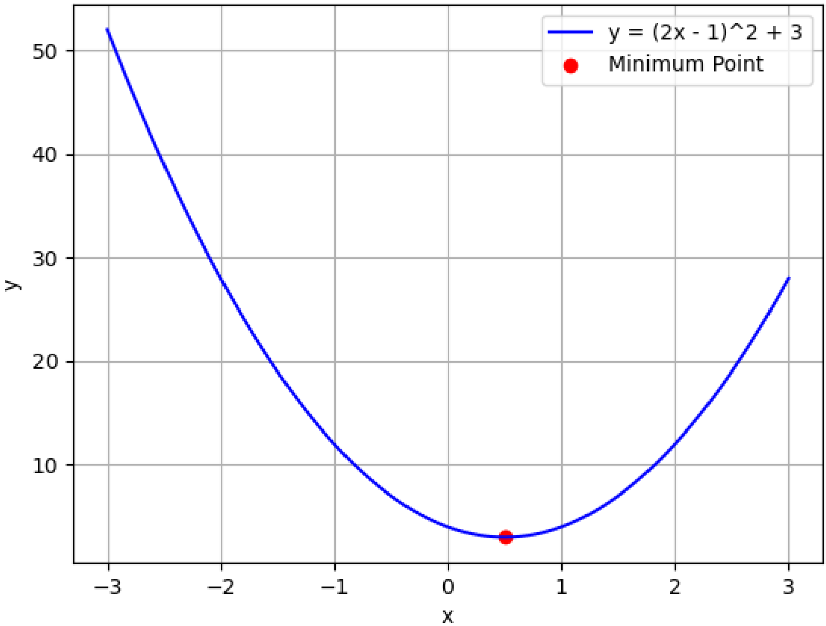
Computational solution

The minimum value of the function is:

$$\boxed{3}$$

and it occurs at:

$$\boxed{x = 0.5}$$


y = (2x - 1)^2 + 3
Minimum Point
50
40
30
20
10
y
−3
−2
−1
0
1
2
3
x