

# 8.1.11

EE24BTECH11063 - Y. Harsha Vardhan Reddy

## Question:

Find the area of the region bounded by the curve  $y^2 = 4x$  and the line  $x = 3$

## Solution:

Variable	Description	values
<b>V</b>	Quadratic form of the matrix	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
<b>u</b>	Linear coefficient vector	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
<b>f</b>	constant term	0
<b>m</b>	The direction vector of line	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
<b>h</b>	Point on line	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

TABLE 0: Variables used

## Theoretical Solution:

The point of intersection of the line with the circle is  $x_i = h + k_i m$ , where,  $k_i$  is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^T V m} \left( -m^T (V h + u) \pm \sqrt{[m^T (V h + u)]^2 - g(h) (m^T V m)} \right)$$

Substituting the input parameters into  $k_i$ ,

$$k_i = \frac{1}{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \left( -\begin{pmatrix} 0 & 1 \end{pmatrix} \left( \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right) \right) \pm \sqrt{\left[ \begin{pmatrix} 0 & 1 \end{pmatrix} \left( \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right) \right]^2 - g(h) \left( \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)} \quad (0.1)$$

We get,

$$k_i = \sqrt{12}, -\sqrt{12}$$

Substituting  $k_i$  into  $x_i = h + k_i m$  we get

$$x_1 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + (\sqrt{12}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.2)$$

$$\Rightarrow x_1 = \begin{pmatrix} 3 \\ \sqrt{12} \end{pmatrix} \quad (0.3)$$

$$x_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + (-\sqrt{12}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.4)$$

$$\Rightarrow x_2 = \begin{pmatrix} 3 \\ -\sqrt{12} \end{pmatrix} \quad (0.5)$$

Area of the region bounded by  $y^2 = 4x$  and  $x = 3$ ,

$$2 \times \int_0^3 (\sqrt{4x}) \cdot dx \quad (0.6)$$

$$= 2 \times 2 \times \left[ \frac{x^{3/2}}{3/2} \right]_0^3 \quad (0.7)$$

$$= \frac{8}{3} \times \sqrt{27} \quad (0.8)$$

$$= 13.856 \quad (0.9)$$

The area of the region bounded between the curve  $y^2 = 4x$  and  $x = 3$  is 13.856 sq.units

### Computational Solution:

Taking trapezoid-shaped strips of small area and adding them all up. Say we have to find the area of  $y_x$  from  $x = x_0$  to  $x = x_n$ , discretize the points on the  $x$  axis  $x_0, x_1, x_2, \dots, x_n$  such that they are equally spaced with the step size  $h$ .

Sum of all trapezoidal areas is given by,

$$A = \frac{1}{2}h(y(x_1) + y(x_0)) + \frac{1}{2}h(y(x_2) + y(x_1)) + \dots + \frac{1}{2}h(y(x_n) + y(x_{n-1})) \quad (0.10)$$

$$= h \left[ \frac{1}{2}(y(x_0) + y(x_n)) + y(x_1) + \dots + y(x_{n-1}) \right] \quad (0.11)$$

Let  $A(x_n)$  be the area enclosed by the curve  $y(x)$  from  $x = x_0$  to  $x = x_n$ ,  $(x_0, x_1, \dots, x_n)$  be equidistant points with step-size  $h$ .

$$A(x_n + h) = A(x_n) + \frac{1}{2}h(y(x_n + h) + y(x_n)) \quad (0.12)$$

We can repeat this till we get required area.

Discretizing the steps, making  $A(x_n) = A_n, y(x_n) = y_n$  we get,

$$A_{n+1} = A_n + \frac{1}{2}h(y_{n+1} + y_n) \quad (0.13)$$

We can write  $y_{n+1}$  in terms of  $y_n$  using first principle of derivative.  $y_{n+1} = y_n + hy'_n$

$$A_{n+1} = A_n + \frac{1}{2}h((y_n + hy'_n) + y_n) \quad (0.14)$$

$$A_{n+1} = A_n + \frac{1}{2}h(2y_n + hy'_n) \quad (0.15)$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \quad (0.16)$$

$$x_{n+1} = x_n + h \quad (0.17)$$

In the given question,  $y_n = \sqrt{4x_n}$  and  $y'_n = -\frac{4}{\sqrt{x}}$

The general difference equation will be given by

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \quad (0.18)$$

$$A_{n+1} = A_n + h\left(\sqrt{4x_n}\right) + \frac{1}{2}h^2\left(-\frac{4}{\sqrt{x}}\right) \quad (0.19)$$

$$x_{n+1} = x_n + h \quad (0.20)$$

Iterating from  $x_n = 0$  to  $x_n = 3$  will return required area.(Upper half region)

The final result is multiplied by two include both half regions

Area obtained computationally: 13.856 sq. units

Area obtained theoretically: 13.856 sq. units

