8.6-6.5-1.1

EE24BTECH11063 - Y. Harsha Vardhan Reddy

Question:
Find the minimum value of the function

$$f(x) = (2x-1)^2 + 3$$

Solution:

Theoritical solution:
Given,

$$\frac{dy}{dx} = 4(2x-1) = 0 \tag{0.1}$$

$$\implies x = \frac{1}{2} \tag{0.2}$$

$$\frac{d^2y}{dx^2} = 8 \tag{0.3}$$

$$\tag{0.4}$$

Since, $\frac{d^2y}{dx^2} > 0$, at $x = \frac{1}{2}$ there exists minimum
Therefore, $f\left(\frac{1}{2}\right) = 3$ is the minimum value of the function
Computational Solution Using Gradient Descent
To verify the analytical results, we use gradient descent to find the local minimum
Gradient Descent for local minimum :
- Start with $x_0 = 4$
- Update x iteratively using

$$x_{n+1} = x_n - \eta \cdot f'(x_n) \tag{0.5}$$

where :

$$\eta = 0.1 \tag{0.6}$$

$$f'(x) = 4(2x-1) \tag{0.7}$$

$$x_{n+1} = x_n - \eta \cdot (4(2x_n - 1)) \tag{0.8}$$

Computational Results
- Local minimum

$$x \approx 0.5, \ g(x) \approx 3.000 \tag{0.9}$$

Computational Solution Using Quadratic programming problem

We aim to find the minimum value of the quadratic function:

$$f(x) = (2x - 1)^2 + 3$$

Expanding the terms, we get:

$$f(x) = 4x^2 - 4x + 1 + 3 = 4x^2 - 4x + 4$$

Formulating the Problem as Quadratic Programming The general form of a quadratic programming problem is:

$$\text{Minimize } \frac{1}{2}x^\top Qx + c^\top x$$

where:

- Q is the coefficient matrix for the quadratic term,
- c is the coefficient vector for the linear term.

For the given function:

$$f(x) = 4x^2 - 4x + 4$$

we identify:

$$Q = 4, \quad c = -4$$

The constant term +4 does not affect the minimization process but will be added back to compute the final minimum value.

Steps to Solve Using cvxpy

We use the Python library `cvxpy` to solve this quadratic programming problem. The steps are as follows:

1) Define the variable x to be optimized.
2) Write the objective function $\frac{1}{2}Qx^2 + cx$ in terms of x.
3) Solve the problem using `cvxpy`.
4) Add the constant term +4 to the resulting minimum value of the objective function.

Computational solution

From the code, the solution is:

- The value of x at the minimum is:

$$x = \frac{-c}{2Q} = \frac{-(-4)}{2(4)} = \frac{4}{8} = 0.5$$

- The minimum value of the function is:

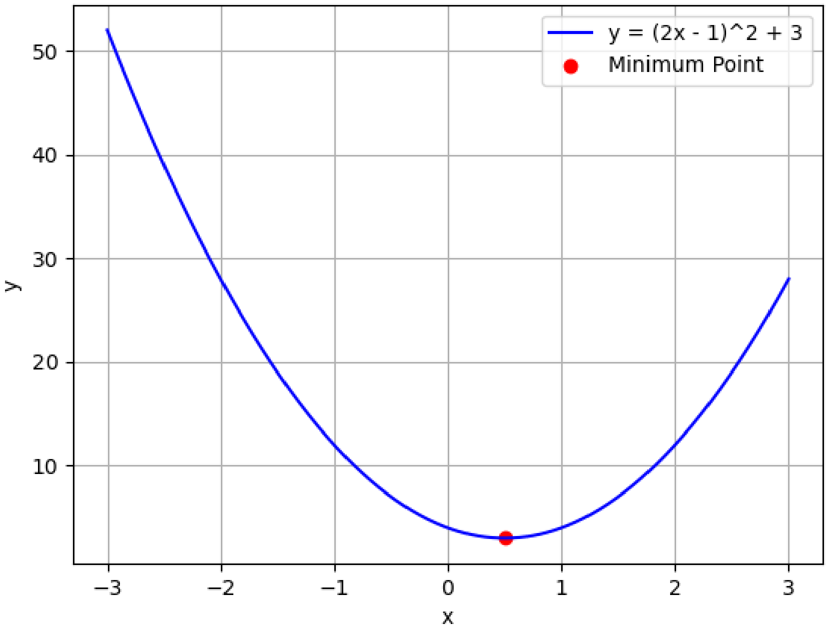
$$f(0.5) = 4(0.5)^2 - 4(0.5) + 4 = 1 - 2 + 4 = 3$$

Thus, the minimum value of the function is:

$$\boxed{3}$$

and it occurs at:

$$\boxed{x = 0.5}$$


y = (2x - 1)^2 + 3
Minimum Point
50
40
30
20
10
y
−3
−2
−1
0
1
2
3
x