

9-9.7-1.3

EE24BTECH11063 - Y. Harsha Vardhan Reddy

Question:

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$

Solution:

Let us first solve find the area theoretically and then verify it computationally

Theoretical solution:

Given curve,

$$y^2 = 4x \quad (0.1)$$

Area of the region bounded by $y^2 = 4x$ and $x = 3$,

$$2 \times \int_0^3 (\sqrt{4x}) \cdot dx \quad (0.2)$$

$$= 2 \times 2 \times \left[\frac{x^{3/2}}{3/2} \right]_0^3 \quad (0.3)$$

$$= \frac{8}{3} \times \sqrt{27} \quad (0.4)$$

$$= 13.856 \quad (0.5)$$

The area of the region bounded between the curve $y^2 = 4x$ and $x = 3$ is 13.856 sq.units

Computational Solution: Trapezoidal method

The difference equation for any general integral is as follows

$$\int_a^b f(x) dx \approx \frac{h}{2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right] \quad (0.6)$$

For n=1000,

$$\int_0^3 f(x) dx \approx \frac{h}{2} \left[(2 \sqrt{x_0}) + 2 \sum_{i=1}^{999} (2 \sqrt{x_i}) + (2 \sqrt{x_{1000}}) \right] \quad (0.7)$$

'x' varies from 0 to 3 and y varies accordingly.

The code sums up the required values iteratively for 'n'(say 1000) intervals

By computing it iteratively(computationally) we get area as 13.856 sq. units

Hence, the area we calculated theoretically is verified

