11.16.3.3.4

EE24BTECH11063 - Y. Harsha Vardhan Reddy

Question:
A die is rolled, Find the probability that a number greater than 6 will appear
Solution:
Textual solution:
Probability of a given event 'A'(A: Outcome is greater than 6),

$$P(A) = \frac{0}{6} = 0 \tag{0.1}$$

Computational solution:

COMPUTATION OF PROBABILITIES FOR ROLLING A DIE

To compute the probability of obtaining specific outcomes when rolling a six-sided die, we rely on two key concepts: the **Probability Mass Function (PMF)** and the **Cumulative Distribution Function (CDF)**.

Definitions

Probability Mass Function (PMF): The PMF represents the probability of each individual outcome in the sample space S. For a six-sided die:

$$S = \{1, 2, 3, 4, 5, 6\},$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{6}, & x \in S, \\ 0, & x \notin S. \end{cases}$$

Cumulative Distribution Function (CDF): The CDF represents the cumulative probability of outcomes up to a given value x, defined as:

$$F(x) = P(X \le x) = \sum_{k=1}^{x} P(X = k).$$

For a six-sided die:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{x}{6}, & x \in \{1, 2, 3, 4, 5, 6\}, \\ 1, & x > 6. \end{cases}$$

Simulation Process

We simulate the rolling of a die using the following steps:

1) A six-sided die produces outcomes in the set:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

2) For each simulated roll, a random integer X is generated such that $X \in S$, using a random number generator function:

$$X = (\text{rand()} \mod 6) + 1.$$

3) The number of occurrences of each outcome is tracked over N trials, where N is the total number of simulations.

4) Both the PMF and CDF are computed:
   - **PMF**: The frequency of each outcome is divided by the total trials to compute the probability of each face.
   - **CDF**: The cumulative probabilities are calculated as the running total of the PMF values.

Calculation of Probabilities

Probability of Each Outcome (PMF): The probability of rolling each face i ($i \in \{1, 2, 3, 4, 5, 6\}$) is computed as:

$$P(i) = \frac{\text{Number of rolls resulting in } i}{N}.$$

Cumulative Probability (CDF): The cumulative probability up to face i is:

$$F(i) = \sum_{k=1}^{i} P(k).$$

Probability of Rolling $X > 6$: The probability of rolling a number greater than 6 is:

$$P(X > 6) = \frac{\text{Number of rolls resulting in } X > 6}{N}.$$

For a standard six-sided die, $P(X > 6) = 0$.

Output Representation

The computed probabilities are represented in two forms:
   - **PMF**: The probabilities of rolling each face $\{1, 2, 3, 4, 5, 6\}$, as well as the probability of $X > 6$.
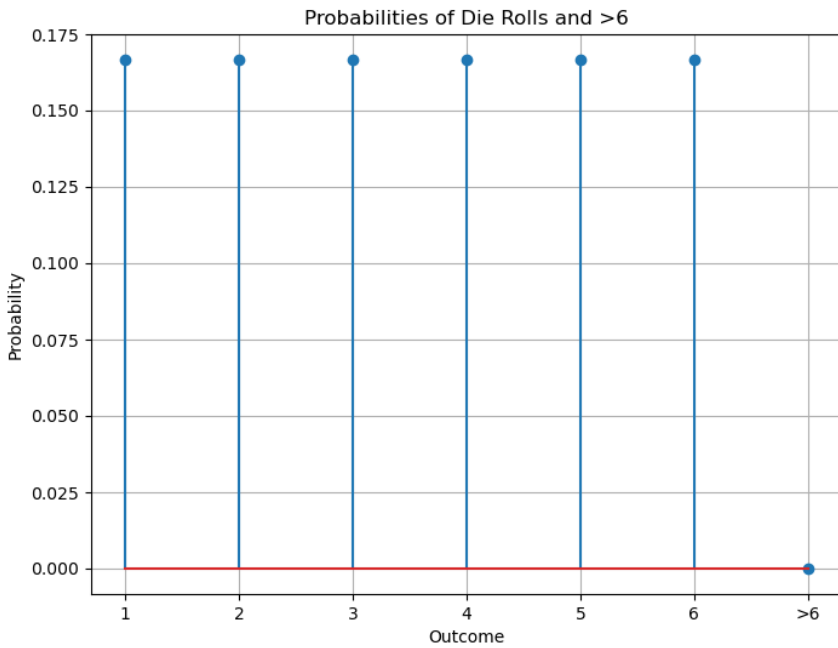   - **CDF**: The cumulative probabilities up to each face, $\{1, 2, 3, 4, 5, 6\}$, showing the cumulative likelihood of outcomes.

Fig. 4.1: Solution of the system of linear equations