Chapter 12
Differentiation

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E : SUBJECTIVE PROBLEMS

1) Let f be a twice differentiable function such that $f''(x) = -f(x)$, and $f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2$. Find $h(10)$ if $h(5) = 11$.
(1982-3 Marks)
2) If α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x), B(x)$ and $C(x)$ be polynomials of degree 3,4 and 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, where prime denotes the derivatives.
(1984-4 Marks)
3) If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that $\left(x^2+4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2+4\right)$
(1989-2 Marks)
4) Find $\frac{dy}{dx}$ at $x = -1$, when $(\sin y)^{\sin(\frac{\pi}{2}x)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x\tan(\ln(x+2)) = 0$
(1991- 4 Marks)
5) If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + 1$, prove that $\frac{y'}{y} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$
(1998- 8 Marks)

H : Assertion & Reason Type Questions

1) Let $f(x) = 2 + \cos x$ for all real x.
STATEMENT - 1:For each real t, there exists a point c in [t,t+π] such that $f'(c) = 0$ because
STATEMENT - 2: $f(t) = f(t+2\pi)$ for each real t.
(2007-3 Marks)
   (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
   (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (c) Statement-1 is True, Statement-2 is False
   (d) Statement-1 is False, Statement-2 is True
2) Let f and g be real valued functions defined on interval (-1,1) such that $g''(x)$ is continuous, $g(0) \neq 0. g'(0) = 0, g''(0) \neq 0$, and $f(x) = g(x)\sin x$
STATEMENT-1:
$\lim_{x\to 0}[g(x)\cot x - g(0)\csc x] = f''(0)$ and
STATEMENT-2: $f'(0) = g(0)$ (2008)
   (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
   (b) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
   (c) Statement-1 is True, Statement-2 is False
   (d) Statement-1 is False, Statement-2 is True

I:Integer Value Correct Type

1) If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is
(2009)
2) Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is
(2011)