Chapter 16
Application of derivatives

EE24BTECH11063 - Y.Harsha Vardhan Reddy

G : Comprehension Based Questions

PASSAGE-1

If a continuous function f defined on a real line R, assumes positive and negative values in R then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

1) The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at
   (a) no point
   (b) one point
   (c) two points
   (d) more than two points

(2007-4marks)

2) The positive value of k for which $ke^x - x = 0$ has only one root is
   (a) $\frac{1}{e}$
   (b) 1
   (c) e
   (d) $\log_e 2$

(2007-4marks)

3) For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is
   (a) $(0,\frac{1}{e})$
   (b) $(\frac{1}{e},1)$
   (c) $(\frac{1}{e},\infty)$
   (d) (0,1)

(2007-4marks)

PASSAGE-2

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\,dt$ for all $x \in (1,\infty)$.

4) Consider the statements:
   P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$
   Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$
   Then
   (a) both P and Q are true
   (b) P is true and Q is false
   (c) P is false and Q is true
   (d) both P and Q are true

(2012)

5) Which of the following is true?
   (a) g is increasing on $(1,\infty)$
   (b) g is decreasing on $(1,\infty)$
   (c) g is increasing in (1,2) and decreasing on $(2,\infty)$
   (d) g is decreasing in (1,2) and increasing on $(2,\infty)$

(2012)

PASSAGE-3

Let $f(x) : [0,1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable , $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0,1]$.

6) Which of the following is true for $0 < x < 1$?

(JEE Adv. 2013)

   (a) $0 < f(x) < \infty$
   (b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
   (c) $-\frac{1}{4} < f(x) < 1$
   (d) $-\infty < f(x) < 0$

7) If the function $e^{-x}f(x)$ assumes its minimum in the interval [0,1] at $x = \frac{1}{4}$, which of the following is true?

(JEE Adv. 2013)

   (a) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$
   (b) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$

(c) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$

(d) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

I:Integer Value Correct Type

1) The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x|x^2 + 20 \leq 9x\}$ is

(2009)

2) Let $p(x)$ be a polynomial of degree 4 having extremum at x=1,2 and $\lim_{x\to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2$. Then the value of $p(2)$ is
3) Let f be a real-valued differentiable function on $\mathbf{R}$(the set of all real numbers) such that $f(1) = 1$. If the y-intercept of the tangent at any point P(x,y) on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then find the value of $f(-3)$

(2010)