9-9.3-14

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:
Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.
Solution: The parameters of given conic,

Variable	Description
V, u, f	Parameters of conic
h, m	Parameters of line
c	length of side-AB
P_1, P_2	Points of intersection

TABLE 0: Variables Used

$$V = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \tag{0.1}$$

$$u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

for the line ,

$$h = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.4}$$

$$m = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{0.5}$$

By Solving we get points of intersection as

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.6}$$

Area can be evaluated as

$$\int_0^3 (f(x) - g(x)) \, dx \tag{0.7}$$

$$\int_0^3 \left(2\sqrt{1 - \frac{x^2}{9}} - \left(-\frac{2}{3}x + 2 \right) \right) dx \tag{0.8}$$

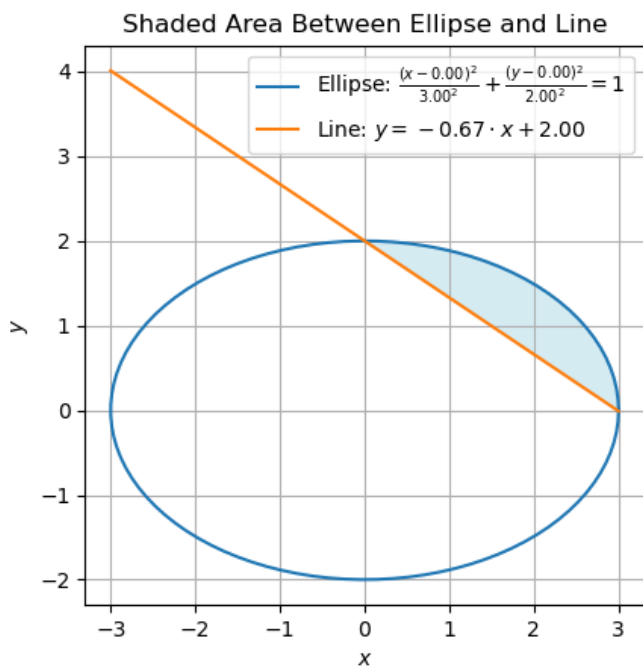By simplifying area enclosed comes to be 1.71 sq.units

Fig. 0.1