

9-9.5-1

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:

Find the area of the region

$$\{(x, y) : x^2 + y^2 \leq 16a^2 \text{ and } y^2 \leq 6ax\}$$

Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola
V_2, u_2, f_2	Parameters of circle
P_1, P_2	Points of intersection
A	Area between the conics

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, u_1 = \begin{pmatrix} 3a \\ 0 \end{pmatrix}, f_1 = 0 \quad (0.1)$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, u_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f_2 = -16a^2 \quad (0.2)$$

The intersection of two conics with parameters $V_i, u_i, f_i, i = 1, 2$ is defined as

$$x^T (V_1 + \mu V_2) x + 2(u_1 + \mu u_2)^T x + (f_1 + \mu f_2) = 0 \quad (0.3)$$

Solving this the points of intersection are

$$\begin{pmatrix} 2a \\ \sqrt{12}a \end{pmatrix}, \begin{pmatrix} 2a \\ -\sqrt{12}a \end{pmatrix} \quad (0.4)$$

Area between the curves is,

$$\int_0^{2a} (\sqrt{16a^2 - x^2} - \sqrt{6ax}) dx \quad (0.5)$$

$$= \left(2\sqrt{3} - \frac{8\sqrt{6}}{3} + \frac{4\pi}{3} \right) \quad (0.6)$$

By solving the integration, we get area is equal to $1.10a^2$ sq.units

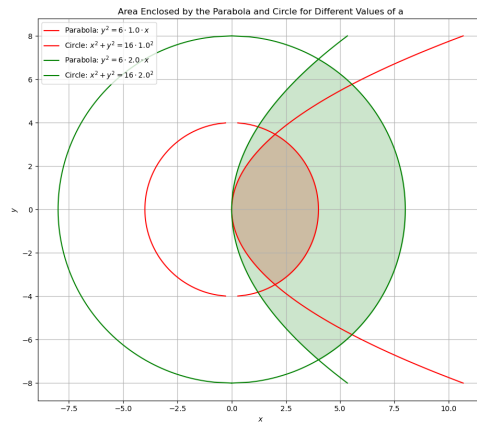


Fig. 0.1