

3-3.3-10

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:

Construct a right triangle ABC with $AB=6\text{cm}$, $BC=8\text{cm}$ and $\angle B = 90^\circ$. Draw BD , the perpendicular from B on AC . Draw the circle through B , C and D and construct the tangents from A to this circle.

Solution: Given, $a=8\text{cm}$ and $c=6\text{cm}$.

Variable	Description
a	length of side-BC
b	length of side-CA
c	length of side-AB
A	co-ordinates of vertex-1
B	co-ordinates of vertex-2
C	co-ordinates of vertex-3
D	co-ordinates of perpendicular from B on AC

TABLE 0: Variables Used

Let us place B at origin, A along x-axis and C along the y-axis i.e.,

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.1)$$

$$A = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (0.2)$$

$$C = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \quad (0.3)$$

Now let us find the co-ordinates of D ,

Equation of AC is given by,

$$4x + 3y = 8 \quad (0.4)$$

$$P = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, n = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, c = 8 \quad (0.5)$$

$$(0.6)$$

the desired foot of perpendicular is given by,

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} Q = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \quad (0.7)$$

By solving system of equations we get,

$$D = \begin{pmatrix} 3.84 \\ 2.88 \end{pmatrix} \quad (0.8)$$

By using the co-ordinates of B , C , D circle can be drawn and it's equation is given by,

$$(x - 0)^2 + (y - 4)^2 = 16 \quad (0.9)$$

$$x^2 + y^2 - 8y = 0 \quad (0.10)$$

Now let us find the equation of tangent from A to circle

Let, Q be the conic matrix of the circle. Then Q is given by,

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & -4 & 0 \end{pmatrix} \quad (0.11)$$

The direction vector of tangent from $P(x_1, y_1)$ is given by,

$$\begin{pmatrix} x_1 & y_1 & 1 \end{pmatrix} Q \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \quad (0.12)$$

$$x_1 = 6, y_1 = 0 \quad (0.13)$$

$$\begin{pmatrix} 6 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & -4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \quad (0.14)$$

$$3x - 2y = 0 \quad (0.15)$$

The tangent passes through $A(6, 0)$ and is given by,

$$3x - 2y = c \quad (0.16)$$

By substituting A in line we get the equation of tangent to be

$$3x - 2y = 18 \quad (0.17)$$

