

9-9.5-8

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides $x = 0$, $x = 4$, $y = 4$, and $y = 0$ into three equal parts.

Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola-1
V_2, u_2, f_2	Parameters of Parabola-2
P_1, P_2	Points of intersection
$A_1, A_2, A_{overlap}$	Areas swept by the parabolas

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, u_1 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f_1 = 0 \quad (0.1)$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, u_2 = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f_2 = 0 \quad (0.2)$$

Area of square is 16 sq.units Intersection points are,

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (0.3)$$

Let, A_1 =Area under $y^2 = 4x$, A_2 =Area under $x^2 = 4y$

$$A_1 = \int_0^4 (2\sqrt{x}) dx = \frac{32}{3} \quad (0.4)$$

$$A_2 = \int_0^4 \left(\frac{x^2}{4}\right) dx = \frac{16}{3} \quad (0.5)$$

$$A_{overlap} = A_1 - A_2 = \frac{16}{3} \quad (0.6)$$

By removing overlapped portion from A_1 we get $\frac{16}{3}$

Therefore, the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square into 3 equal parts.

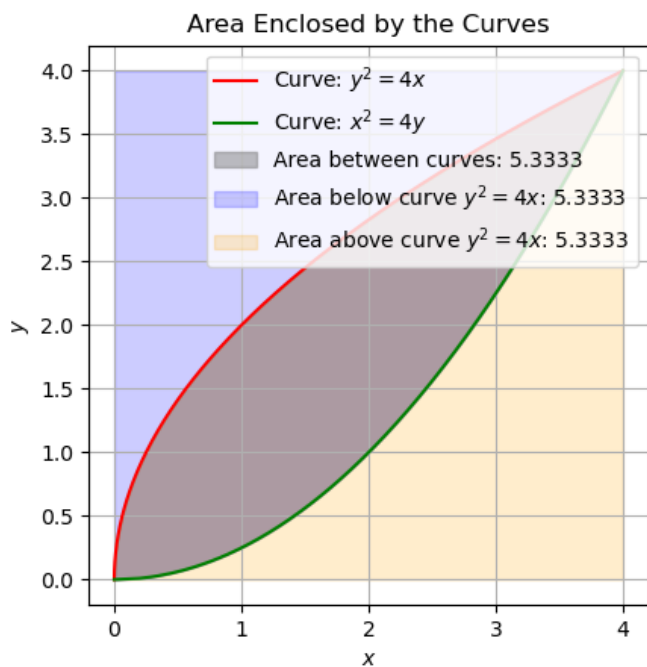


Fig. 0.1