

# Chapter 16

## Application of derivatives

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### G : COMPREHENSION BASED QUESTIONS

#### PASSAGE-1

If a continuous function  $f$  defined on the real line  $R$ , assumes positive and negative values in  $R$  then the equation  $f(x) = 0$  has a root in  $R$ . For example, if it is known that a continuous function  $f$  on  $R$  is positive at some point and its minimum value is negative then the equation  $f(x) = 0$  has a root in  $R$ .

Consider  $f(x) = ke^x - x$  for all real  $x$  where  $k$  is a real constant.

- 1) The line  $y = x$  meets curve  $y = ke^x$  for  $k \leq 0$  at (2007-4marks)

- a) no point                      b) one point  
c) two points                      d) more than two points

- 2) The positive value of  $k$  for which  $ke^x - x = 0$  has only one root is (2007-4marks)

- a)  $\frac{1}{e}$                                   b) 1  
c)  $e$                                       d)  $\log_e 2$

- 3) For  $k > 0$ , the set of all values of  $k$  for which equation  $ke^x - x = 0$  has two distinct roots is (2007-4marks)

- a)  $(0, \frac{1}{e})$                               b)  $(\frac{1}{e}, 1)$   
c)  $(\frac{1}{e}, \infty)$                               d)  $(0, 1)$

#### PASSAGE-2

Let  $f(x) = (1-x)^2 \sin^2 x + x^2$  for all  $x \in \mathbb{R}$  and let  $g(x) = \int_1^x \left( \frac{2(t-1)}{t+1} - \ln t \right) f(t) dt$  for all  $x \in (1, \infty)$ .

- 4) Consider the statements:

$P$  : There exists some  $x \in \mathbb{R}$  such that  $f(x) + 2x = 2(1+x^2)$

$Q$  : There exists some  $x \in \mathbb{R}$  such that  $2f(x) + 1 = 2x(1+x)$

Then (2012)

- a) both  $P$  and  $Q$  are true  
b)  $P$  is true and  $Q$  is false  
c)  $P$  is false and  $Q$  is true  
d) both  $P$  and  $Q$  are false

- 5) Which of the following is true? (2012)

- a)  $g$  is increasing on  $(1, \infty)$   
b)  $g$  is decreasing on  $(1, \infty)$   
c)  $g$  is increasing in  $(1, 2)$  and decreasing on  $(2, \infty)$   
d)  $g$  is decreasing in  $(1, 2)$  and increasing on  $(2, \infty)$

#### PASSAGE-3

Let  $f(x) : [0, 1] \rightarrow \mathbb{R}$  (the set of all real numbers) be a function. Suppose the function  $f$  is twice differentiable,  $f(0) = f(1) = 0$  and satisfies

$$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]. \quad (1)$$

- 6) Which of the following is true for interval

$$0 < x < 1$$

? (JEE Adv. 2013)

- a)  $0 < f(x) < \infty$                       b)  $-\frac{1}{2} < f(x) < \frac{1}{2}$   
c)  $-\frac{1}{4} < f(x) < 1$                       d)  $-\infty < f(x) < 0$

- 7) If the function  $e^{-x}f(x)$  assumes its minimum in the interval  $[0, 1]$  at  $x = \frac{1}{4}$ , which of the following is true?

(JEE Adv. 2013)

- 1)  $f'(x) < f(x)$ ,  $\frac{1}{4} < x < \frac{3}{4}$

- 2)  $f'(x) > f(x)$  ,  $0 < x < \frac{1}{4}$
- 3)  $f'(x) < f(x)$  ,  $0 < x < \frac{1}{4}$
- 4)  $f'(x) < f(x)$  ,  $\frac{3}{4} < x < 1$

I: INTEGER VALUE CORRECT TYPE

- 1) The maximum value of the function  
 $f(x) = 2x^3 - 15x^2 + 36x - 48$  on the set  
 $A = \{x | x^2 + 20 \leq 9x\}$  is  
 (2009)
- 2) Let  $p(x)$  be a polynomial of degree 4 having  
 extremum at  $x = 1, 2$  and  $\lim_{x \rightarrow 0} \left(1 + \frac{p(x)}{x^2}\right) = 2$ .  
 Then the value of  $p(2)$  is (2009)
- 3) Let  $f$  be a real-valued differentiable function  
 on  $\mathbf{R}$  (the set of all real numbers) such that  
 $f(1) = 1$ . If the y-intercept of the tangent at  
 any point  $P(x, y)$  on the curve  $y = f(x)$  is equal  
 to the cube of the abscissa of  $P$ , then find the  
 value of  $f(-3)$   
 (2010)