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Shift-1

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Single correct

1) Let $x_1, x_2, \cdots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i\left(x_i - i\right), 1 \leq i \leq 100$, then the mean of $y_1, y_2, \cdots y_{100}$ is :

   a) 10051.50 b) 10100 c) 10101.50 d) 10049.50

2) The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is:

   a) 10 b) 9 c) 8 d) 12

3) The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x \left(\log_e\left(e^x + \sqrt{1 + e^{2x}}\right)\right) dx$ is equal to

   a) $\log_e \frac{\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}} + \frac{\sqrt{5}}{2}$
   b) $\log_e \frac{2\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}} - \frac{\sqrt{5}}{2}$
   c) $\log_e \frac{\sqrt{2}\left(3-\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}} + \frac{\sqrt{5}}{2}$
   d) $\log_e \frac{\sqrt{2}\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}} - \frac{\sqrt{5}}{2}$

4) Let $S = \{M = \left[a_{ij}\right], a_{ij} \in \{0, 1, 2\}, 1 \leq i, j \leq 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then $P(A)$ is equal to:

   a) $\frac{16}{27}$ b) $\frac{50}{81}$ c) $\frac{47}{81}$ d) $\frac{49}{81}$

5) Let $f : [2, 4] \to R$ be a differentiable function such that $\left(x \log_e x\right) f'(x) + \left(\log_e x\right) f(x) + f(x) \geq 1, x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$. Consider the following two statements:
   $(A) : f(x) \leq 1$, for all $x \in [2, 4]$
   $(B) : f(x) \geq \frac{1}{8}$, for all $x \in [2, 4]$
   Then,
   a) Only statement (B) is true
   b) Only statement (A) is true
   c) Neither statement (A) nor statement (B) is true
   d) Both the statements (A) and (B) are true

6) Let A be a 2×2 matrix with real entries such that $A^T = \alpha A + I$, where $a \in R - \{-1, 1\}$. If det $\left(A^2 - A\right) = 4$, then the sum of all possible values of α is equal to:

a) 0 b) $\frac{5}{2}$ c) 2 d) $\frac{3}{2}$

7) The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-1}\right)^2$

a) 5 b) 7 c) 8 d) 6

8) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $0 < |a_i| < 1$, $i = \{1, 2, 3\}$
Statement-A : $|a| \geq \max\{|a_1|, |a_2|, |a_3|\}$
Statement-B : $|a| < 3\max\{|a_1|, |a_2|, |a_3|\}$
a) Both A and B are true
b) Both A and B are false
c) A is true, B is false
d) A is false, B is true

9) The number of triplets (x, y, z), where x,y,z are distinct non-negative integers satisying $x+y+z = 15$, is :

a) 136 b) 114 c) 80 d) 92

10) Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is

a) 36 b) 40 c) 32 d) 38

11) Area of the region $\{(x, y) : x^2 + (y-2)^2 \leq 4, x^2 \geq 2y\}$ is :

a) $\pi + \frac{8}{3}$ b) $2\pi + \frac{16}{3}$ c) $2\pi - \frac{16}{3}$ d) $\pi - \frac{8}{3}$

12) Let R be a rectangle given by the line $x = 0, x = 2, y = 0$ and $y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta)$, $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the mid-point of AB lies on a :

a) straight line b) parabola c) circle d) hyperbola

13) Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i}+\hat{j}, \hat{i}+\hat{k}$ and $\hat{i}-\hat{j}, \hat{j}-\hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then ordered pair $\left(\theta, |\vec{a} \times \vec{b}|\right)$ is equal to:

a) $\left(\frac{\pi}{3}, 6\right)$ b) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$ c) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$ d) $\left(\frac{\pi}{4}, 6\right)$

14) Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anti-clockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction, Then the principal argument of $w_1 - w_2$ is equal to :

a) $\pi - \tan^{-1}\frac{8}{9}$ b) $-\pi + \tan^{-1}\frac{8}{9}$ c) $\pi - \tan^{-1}\frac{33}{5}$ d) $-\pi + \tan^{-1}\frac{33}{5}$

15) Consider ellipse $E_k : kx^2 + ky^2 = 1, k = 1, 2, \cdots, 20$. Let C_k be the circle which touches the four chords joining the end points(one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

a) 3320 b) 3210 c) 3080 d) 2870