Chapter 21
Probability

EE24BTECH11063 - Y.Harsha Vardhan Reddy

D:MCQs with One or More than One Correct

1) Let X and Y be two events such that $\Pr(X|Y) = \frac{1}{2}$, $\Pr(Y|X) = \frac{1}{3}$ and $\Pr(X \cap Y) = \frac{1}{6}$.Which of the following is (*are*) correct ? (2012)
   a) $\Pr(A \cup B) = \frac{2}{3}$
   b) X and Y are independent
   c) X and Y are not independent
   d) $\Pr\left(X^C \cap Y\right) = \frac{1}{3}$
2) Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is (JEE Adv. 2013)

   a) $\frac{235}{256}$ b) $\frac{21}{256}$ c) $\frac{3}{256}$ d) $\frac{253}{256}$

3) Let X and Y be two that $\Pr(X) = \frac{1}{3}$, $\Pr(X|Y) = \frac{1}{2}$ and $\Pr(Y|X) = \frac{2}{5}$.Then (JEE Adv. 2017)

   a) $\Pr(Y) = \frac{4}{15}$ b) $\Pr(X'|Y) = \frac{1}{2}$

   c) $\Pr(X \cap Y) = \frac{1}{5}$ d) $\Pr(X \cup Y) = \frac{2}{5}$

4) There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct? (JEE Adv. 2019)
   a) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$
   b) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$
   c) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$
   d) Probabilty that the chosen ball is green equals $\frac{39}{80}$

E : Subjective Problems

1) Balls are drawn one-by-one without replacement from a box containing 2 black, 4 white and 3 red balls till all the balls are drawn. Find the probability that the balls drawn are in the order 2 black, 4 white 3 red. (1978)
2) Six boys and six girls sit in a row randomly. Find the probability that

   a) the six girls sit together

   b) the boys and girls sit alternately.

   (1979)

3) An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.3, 0.1 respectively. What is the probability that the gun hits the plane? (1981-2Marks)
4) A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting slected is 0.9? (1982-2 Marks)
5) Cards are drawn one by one at random from a well-shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the numbert of cards required to be drawn, then show that

$$P_r\{N = n\} = \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$$

where $2 \leq n \leq 50$ (1983-3 Marks)
6) A, B, C are events such that

$$\Pr(A) = 0.3, \Pr(B) = 0.4, \Pr(C) = 0.8$$

$$\Pr(AB) = 0.08, \Pr(AC) = 0.28, \Pr(ABC) = 0.09$$

If $\Pr(A \cup B \cup C) \geq 0.75$, then show that $\Pr(BC)$ lies in the interval $0.23 \leq x \leq 0.48$
7) In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is known that 30% of those who read A but not B look into advertisements and 40% of those who read A but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population that reads an advertisement? (1984-4 Marks)
8) In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answers at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the questions. (1985-5 Marks)
9) A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6 . Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelth testing. (1986-5 Marks)
10) A man takes a step forward with probability 0.4 and backwards with probability 0.6 . Find the probability that at the end of eleven steps he is exactly one step away from the starting point. (1987-3 Marks)
11) A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number $N(\geq 2)$ of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise (1988-3 Marks)