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Shift-1

EE24BTECH11063 - Y.Harsha Vardhan Reddy

SINGLE CORRECT

- 1) Let x_1, x_2, \dots, x_{100} be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \leq i \leq 100$, then the mean of y_1, y_2, \dots, y_{100} is :

a) 10051.50
b) 10100
c) 10101.50
d) 10049.50

- 2) The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3 \cos^4 \theta - 5 \cos^2 \theta - 2 \sin^6 \theta + 2 = 0\}$ is:

a) 10
b) 9
c) 8
d) 12

- 3) The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x (\log_e (e^x + \sqrt{1 + e^{2x}})) dx$ is equal to

a) $\log_e \frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} + \frac{\sqrt{5}}{2}$

b) $\log_e \frac{2(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} - \frac{\sqrt{5}}{2}$

c) $\log_e \frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} + \frac{\sqrt{5}}{2}$

d) $\log_e \frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} - \frac{\sqrt{5}}{2}$

- 4) Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \leq i, j \leq 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then $P(A)$ is equal to:

a) $\frac{16}{27}$
b) $\frac{50}{81}$
c) $\frac{47}{81}$
d) $\frac{49}{81}$

- 5) Let $f : [2, 4] \rightarrow R$ be a differentiable function such that $(x \log_e x) f'(x) + (\log_e x) f(x) + f(x) \geq 1$, $x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$. Consider the following two statements:
 (A) : $f(x) \leq 1$, for all $x \in [2, 4]$
 (B) : $f(x) \geq \frac{1}{8}$, for all $x \in [2, 4]$
 Then,

a) Only statement (B) is true

b) Only statement (A) is true

c) Neither statement (A) nor statement (B) is true

d) Both the statements (A) and (B) are true

- 6) Let A be a 2×2 matrix with real entries such that $A^T = \alpha A + I$, where $\alpha \in R - \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to:

- a) 0 b) $\frac{5}{2}$ c) 2 d) $\frac{3}{2}$

7) The number of integral solutions x of $\log_{(x+\frac{7}{2})} \left(\frac{x-7}{2x-1} \right)^2$

- a) 5 b) 7 c) 8 d) 6

8) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $0 < |a_i| < 1$, $i = \{1, 2, 3\}$

Statement-A : $|a| \geq \max\{|a_1|, |a_2|, |a_3|\}$

Statement-B : $|a| < 3\max\{|a_1|, |a_2|, |a_3|\}$

- a) Both A and B are true
b) Both A and B are false
c) A is true, B is false
d) A is false, B is true

9) The number of triplets (x, y, z) , where x, y, z are distinct non-negative integers satisfying $x + y + z = 15$, is :

- a) 136 b) 114 c) 80 d) 92

10) Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B . Then the sum of the mean and variance of the elements of C is

- a) 36 b) 40 c) 32 d) 38

11) Area of the region $\{(x, y) : x^2 + (y - 2)^2 \leq 4, x^2 \geq 2y\}$ is :

- a) $\pi + \frac{8}{3}$ b) $2\pi + \frac{16}{3}$ c) $2\pi - \frac{16}{3}$ d) $\pi - \frac{8}{3}$

12) Let R be a rectangle given by the line $x = 0, x = 2, y = 0$ and $y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta)$, $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the mid-point of AB lies on a :

- a) straight line b) parabola c) circle d) hyperbola

13) Let \vec{a} be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector \vec{a} and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to:

- a) $(\frac{\pi}{3}, 6)$ b) $(\frac{\pi}{4}, 3\sqrt{6})$ c) $(\frac{\pi}{3}, 3\sqrt{6})$ d) $(\frac{\pi}{4}, 6)$

14) Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anti-clockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction, Then the principal argument of $w_1 - w_2$ is equal to :

- a) $\pi - \tan^{-1} \frac{8}{9}$ b) $-\pi + \tan^{-1} \frac{8}{9}$ c) $\pi - \tan^{-1} \frac{33}{5}$ d) $-\pi + \tan^{-1} \frac{33}{5}$

15) Consider ellipse $E_k : kx^2 + ky^2 = 1, k = 1, 2, \dots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k . If r_k is the radius of the circle C_k then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

- a) 3320 b) 3210 c) 3080 d) 2870