

Chapter 16

Application of derivatives

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G : COMPREHENSION BASED QUESTIONS

PASSAGE-1

If a continuous function f defined on the real line R , assumes positive and negative values in R then the equation $f(x) = 0$ has a root in R . For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative then the equation $f(x) = 0$ has a root in R .

Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

1) The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at

- a) no point b) one point
c) two points d) more than two points

(2007-4marks)

2) The positive value of k for which $ke^x - x = 0$ has only one root is

- a) $\frac{1}{e}$ b) 1
c) e d) $\log_e 2$

(2007-4marks)

3) For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is

- a) $(0, \frac{1}{e})$ b) $(\frac{1}{e}, 1)$
c) $(\frac{1}{e}, \infty)$ d) $(0, 1)$

(2007-4marks)

PASSAGE-2

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t \right) f(t) dt$ for all $x \in (1, \infty)$.

4) Consider the statements:

P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1+x^2)$

Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1+x)$

Then

- a) both P and Q are true
b) P is true and Q is false
c) P is false and Q is true
d) both P and Q are false

(2012)

5) Which of the following is true?

- a) g is increasing on $(1, \infty)$
b) g is decreasing on $(1, \infty)$
c) g is increasing in $(1, 2)$ and decreasing on $(2, \infty)$
d) g is decreasing in $(1, 2)$ and increasing on $(2, \infty)$

(2012)

PASSAGE-3

Let $f(x) : [0, 1] \rightarrow \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies

$$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1].$$

6) Which of the following is true for $0 < x < 1$? (JEE Adv. 2013)

- a) $0 < f(x) < \infty$ b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
c) $-\frac{1}{4} < f(x) < 1$ d) $-\infty < f(x) < 0$

7) If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = \frac{1}{4}$, which of the following is true?

(JEE Adv. 2013)

- 1) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$

- 2) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$
- 3) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$
- 4) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

I: INTEGER VALUE CORRECT TYPE

- 1) The maximum value of the function
 $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set
 $A = \{x | x^2 + 20 \leq 9x\}$ is
 (2009)
- 2) Let $p(x)$ be a polynomial of degree 4 having
 extremum at $x = 1, 2$ and $\lim_{x \rightarrow 0} \left(1 + \frac{p(x)}{x^2}\right) = 2$.
 Then the value of $p(2)$ is (2009)
- 3) Let f be a real-valued differentiable function
 on \mathbf{R} (the set of all real numbers) such that
 $f(1) = 1$. If the y-intercept of the tangent at
 any point $P(x, y)$ on the curve $y = f(x)$ is equal
 to the cube of the abscissa of P , then find the
 value of $f(-3)$
 (2010)