3-3.3-10

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:
Construct a right triangle ABC with AB=6cm, BC =8cm and $\angle B = 90°$. Draw BD, the perpendicular from B on AC. Draw the circle through B, C and D and construct the tangents from A to this circle.
Solution: Let us place B at origin, A along x-axis and C along the y-axis i.e,

Variable	Description
a	length of side-BC
b	length of side-CA
c	length of side-AB
A	co-ordinates of vertex-1
B	co-ordinates of vertex-2
C	co-ordinates of vertex-3
D	co-ordinates of perpendicular from B on AC

TABLE 0: Variables Used

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.1}$$

$$A = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{0.2}$$

$$C = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \tag{0.3}$$

Now let us find the co-ordinates of D, Equation of AC is given by,

$$4x + 3y = 8 \tag{0.4}$$

$$P = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, n = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, c = 8 \tag{0.5}$$

$$\tag{0.6}$$

the desired foot of perpendicular is given by,

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} Q = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \tag{0.7}$$

$$D = \begin{pmatrix} 3.84 \\ 2.88 \end{pmatrix} \tag{0.8}$$

By using the co-ordinates of B, C, D circle can be drawn and it's equation is given by,

$$(x-0)^2 + (y-4)^2 = 16 \tag{0.9}$$
$$x^2 + y^2 - 8y = 0 \tag{0.10}$$

Now let us find the equation of tangent from A to circle
Let, Q be the conic matrix of the circle. Then Q is given by,

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & -4 & 0 \end{pmatrix} \tag{0.11}$$

The direction vector of tangent from $P(x_1, y_1)$ is given by,

$$\begin{pmatrix} x_1 & y_1 & 1 \end{pmatrix} Q \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \tag{0.12}$$
$$x1 = 6, y1 = 0 \tag{0.13}$$
$$\begin{pmatrix} 6 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & -4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \tag{0.14}$$
$$3x - 2y = 0 \tag{0.15}$$

The tangent passes through $A(6, 0)$ and is given by,

$$3x - 2y = c \tag{0.16}$$

By substituting A in line we get the equation of tangent to be

$$3x - 2y = 18 \tag{0.17}$$

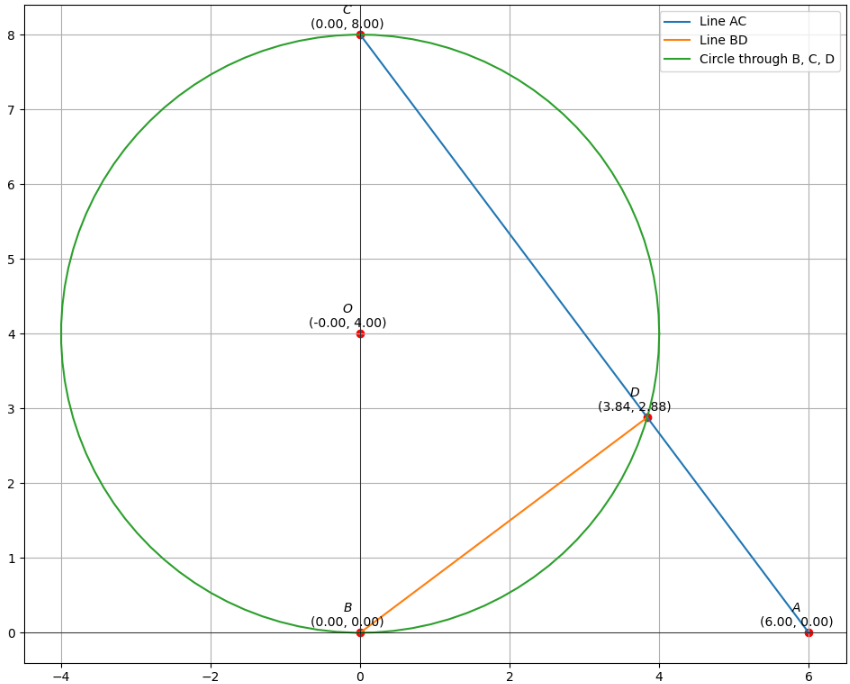
Line AC
Line BD
Circle through B, C, D
C
(0.00, 8.00)
O
(-0.00, 4.00)
D
(3.84, 2.88)
B
(0.00, 0.00)
A
(6.00, 0.00)
8
7
6
5
4
3
2
1
0
−4
−2
0
2
4
6

Fig. 0.1