9-9.5-1

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:
Find the area of the region

$$\{(x, y) : x^2 + y^2 \leq 16a^2 \text{ and } y^2 \leq 6ax\}$$

Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola
V_2, u_2, f_2	Parameters of circle
P_1, P_2	Points of intersection
A	Area between the conics

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ u_1 = \begin{pmatrix} -3a \\ 0 \end{pmatrix}, \ f_1 = 0 \tag{0.1}$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ u_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ f_2 = -16a^2 \tag{0.2}$$

The intersection of two conics with parameters $V_i, u_i, f_i, \ i = 1, 2$ is defined as

$$x^T (V_1 + \mu V_2) x + 2 (u_1 + \mu u_2)^T x + (f_1 + \mu f_2) = 0 \tag{0.3}$$

Solving this the points of intersection are

$$\begin{pmatrix} 2a \\ \sqrt{12}a \end{pmatrix}, \begin{pmatrix} 2a \\ -\sqrt{12}a \end{pmatrix} \tag{0.4}$$

Area between the curves is,

$$\int_0^{2a} \left(\sqrt{16a^2 - x^2} - \sqrt{6ax} \right) dx \tag{0.5}$$

$$= \left(2\sqrt{3} - \frac{8\sqrt{6}}{3} + \frac{4\pi}{3} \right) \tag{0.6}$$

By solving the integration, we get area is equal to $1.10a^2$ sq.units

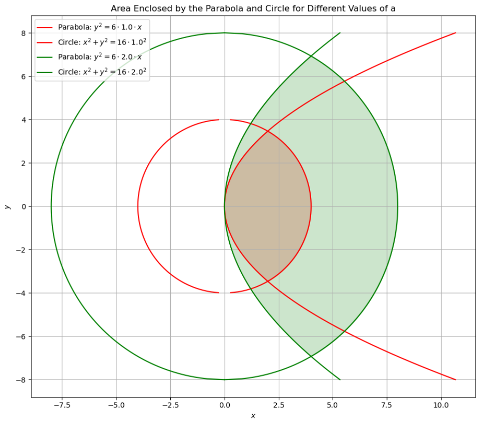
Area Enclosed by the Parabola and Circle for Different Values of a
Parabola: $y^2 = 6 \cdot 1.0 \cdot x$
Circle: $x^2 + y^2 = 16 \cdot 1.0^2$
Parabola: $y^2 = 6 \cdot 2.0 \cdot x$
Circle: $x^2 + y^2 = 16 \cdot 2.0^2$
y
x
8
6
4
2
0
−2
−4
−6
−8
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

Fig. 0.1