Assignment-2

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Chapter 15
Matrices and Determinants
Single Correct Type

1) Let k be an integer such that triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area $28 sq.units$. Then the orthocentre of the triangle is at the point : [JEE M 2017]

   a) $\left(2, \frac{1}{2}\right)$
   b) $\left(2, -\frac{1}{2}\right)$
   c) $\left(1, \frac{3}{4}\right)$
   d) $\left(1, -\frac{3}{4}\right)$

2) Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k \tag{1}$$

   , then k is equal to: [JEE M 2017]

   a) 1
   b) -z
   c) z
   d) -1

3) If A= $\begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$, then $adj\left(3A^2 + 12A\right)$ is equal to: [JEE M 2017]

   a) $\begin{pmatrix} 72 & -63 \\ -84 & 51 \end{pmatrix}$
   b) $\begin{pmatrix} 72 & -84 \\ -63 & 51 \end{pmatrix}$
   c) $\begin{pmatrix} 51 & 63 \\ 84 & 72 \end{pmatrix}$
   d) $\begin{pmatrix} 51 & 84 \\ 63 & 72 \end{pmatrix}$

4) If

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2 \tag{2}$$

   , then the ordered pair (A, B) is equal to: [JEE M 2018]

   a) $(-4, 3)$
   b) $(-4, 5)$
   c) $(4, 5)$
   d) $(-4, -5)$

5) If the system of linear equations

$$x + ky + 3z = 0 \tag{3}$$
$$3x + ky - 2z = 0 \tag{4}$$
$$2x + 4y - 3z = 0 \tag{5}$$

   has a non-zero solution (x, y, z),then $\frac{xz}{y^2}$ is equal to : [JEE M 2018]

a) 10 b) -30 c) 30 d) -10

6) The system of linear equations

$$x + y + z = 2 \tag{6}$$
$$2x + 3y + 2z = 5 \tag{7}$$
$$2x + 3y + \left(a^2 - 1\right) z = a + 1 \tag{8}$$

[JEE M2019-9 Jan(M)]

a) is consistent when $a = 4$
b) has a unique solution for $|a| = \sqrt{3}$
c) has infinitely many solutions for $a = 4$
d) is consistent when $|a| = \sqrt{3}$

7) If $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to: [JEE M 2019-9 Jan(M)]

a) $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ b) $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$

c) $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

8) If

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & n-1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 78 \\ 0 & 1 \end{pmatrix}. \tag{9}$$

then the inverse of $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ is [JEE M2019-9 April(M)]

a) $\begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & -12 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 13 & 1 \end{pmatrix}$

9) Let α and β be the roots of the equation

$$x^2 + x + 1 = 0$$

. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix} \tag{10}$$

is equal to : [JEE M 2019-9 April(M)]

a) $y(y^2 - 1)$ b) $y(y^2 - 3)$

c) y^3 d) $y^3 - 1$

Chapter 12
Differentiation

E : SUBJECTIVE PROBLEMS

1) Let f be a twice differentiable function such that $f''(x) = -f(x)$, and
$$f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2 \tag{11}$$
. Find $h(10)$ if $h(5) = 11$. (1982 – 3*Marks*)

2) If α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x), B(x)$ and $C(x)$ be polynomials of degree 3,4 and 5 respectively, then show that
$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} \tag{12}$$
is divisible by $f(x)$, where prime denotes the derivatives. (1984 – 4*Marks*)

3) If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that
$$\left(x^2 + 4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2 + 4\right) \tag{13}$$
(1989 – 2*Marks*)

4) Find $\frac{dy}{dx}$ at $x = -1$, when
$$(\sin y)^{\sin\left(\frac{\pi}{2}x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x\tan(\ln(x+2)) = 0 \tag{14}$$
(1991 – 4*Marks*)

5) If
$$y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + 1 \tag{15}$$
, prove that
$$\frac{y'}{y} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right) \tag{16}$$
(1998 – 8*Marks*)

H : Assertion & Reason Type Questions

1) Let $f(x) = 2 + \cos x$ for all real x.
STATEMENT - 1:For each real t, there exists a point c in $[t, t+\pi]$ such that $f'(c) = 0$ because
STATEMENT - 2: $f(t) = f(t + 2\pi)$ for each real t. (2007 – 3*Marks*)
   a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
   b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   c) Statement-1 is True, Statement-2 is False
   d) Statement-1 is False, Statement-2 is True

2) Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0.g'(0) = 0, g''(0) \neq 0$, and $f(x) = g(x)\sin x$
STATEMENT-1:
$\lim_{x\to 0}\left[g(x)\cot x - g(0)\csc x\right] = f''(0)$ and
STATEMENT-2: $f'(0) = g(0)$ (2008)
   a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
   b) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
   c) Statement-1 is True, Statement-2 is False
   d) Statement-1 is False, Statement-2 is True

I:Integer Value Correct Type

1) If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is (2009)
2) Let

$$f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right), where -\frac{\pi}{4} < \theta < \frac{\pi}{4} \tag{17}$$

. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

(2011)