## EE24BTECH11063 - Y.Harsha Vardhan Reddy

## **Ouestion:**

Prove that the curves  $y^2 = 4x$  and  $x^2 = 4y$  divide the area of the square bounded by sides x = 0, x = 4, y = 4, and y = 0 into three equal parts.

**Solution:** The parameters of the conics are

Variable	Description
$V_1, u_1, f_1$	Parameters of Parabola-1
$V_2, u_2, f_2$	Parameters of Parabola-2
$P_{1}, P_{2}$	Points of intersection
$A_1, A_2, A_{overlap}$	Areas swept by the parabolas

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ u_1 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \ f_1 = 0$$
 (0.1)

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ u_2 = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, \ f_2 = 0$$
 (0.2)

Area of square is 16 sq.units Intersection points are,

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.3}$$

Let,  $A_1$ =Area under  $y^2 = 4x$ ,  $A_2$ =Area under  $x^2 = 4y$ 

$$A_1 = \int_0^4 \left(2\sqrt{x}\right) dx = \frac{32}{3} \tag{0.4}$$

$$A_2 = \int_0^4 \left(\frac{x^2}{4}\right) dx = \frac{16}{3} \tag{0.5}$$

$$A_{overlap} = A_1 - A_2 = \frac{16}{3} \tag{0.6}$$

By removing overlapped portion from  $A_1$  we get  $\frac{16}{3}$ Therefore, the curves  $y^2 = 4x$  and  $x^2 = 4y$  divides the area of square into 3 equal parts.

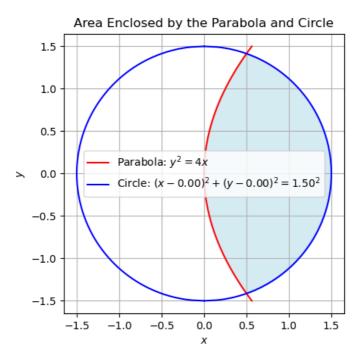


Fig. 0.1