MA
2018

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Q.1 to Q.5 carry 1 mark each

1) Let X and Y be metric spaces, and let $f : X \to Y$ be a continuous map. For any subset S of X, which one of the following statements is true?
   a) If S is open, then $f(S)$ is open
   b) If S is connected, then $f(S)$ is connected
   c) If S is closed, then $f(S)$ is closed
   d) If S is bounded, then $f(S)$ is bounded

2) The general solution of the differential equation
$$xy' = y + \sqrt{x^2 + y^2} \quad \text{for } x > 0$$
is given by (with an arbitrary positive constant k)
   a) $ky^2 = x + \sqrt{x^2 + y^2}$
   b) $kx^2 = x + \sqrt{x^2 + y^2}$
   c) $kx^2 = y + \sqrt{x^2 + y^2}$
   d) $ky^2 = y + \sqrt{x^2 + y^2}$

3) Let $p_n(x)$ be the polynomial solution of the differential equation $\frac{d}{dx}\left[(1 - x^2)y'\right] + n(n+1)y = 0$ with $p_n(1) = 1$ for $n = 1, 2, 3, \ldots$. If $\frac{d}{dx}[p_{n+2}(x) - p_n(x)] = \alpha_n p_{n+1}(x)$, then α_n is
   a) $2n$
   b) $2n + 1$
   c) $2n + 2$
   d) $2n + 3$

4) In the permutation group S_6, the number of elements of order 8 is
   a) 0
   b) 1
   c) 2
   d) 4

5) Let R be a commutative ring with 1 (unity) which is not a field. Let $I \subset R$ be a proper ideal such that every element of R not in I is invertible in R. Then the number of maximal ideals of R is
   a) 1
   b) 2
   c) 3
   d) infinite

6) Let $f : \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function. The order of convergence of the secant method for finding the root of the equation $f(x) = 0$ is
   a) $\frac{1+\sqrt{5}}{2}$
   b) $\frac{2}{1+\sqrt{5}}$
   c) $\frac{1+\sqrt{5}}{3}$
   d) $\frac{3}{1+\sqrt{5}}$

7) The Cauchy problem $u_{xx} + yu_y = x$ with $u(x, 1) = 2x$, when solved using its characteristic equations with an independent variable t, is found to admit a solution in the form $x = \frac{3}{2}e^t - \frac{1}{2}e^{-t}, \quad y = e^t, \quad u = f(s,t)$. Then $f(s,t) =$

a) $\frac{3}{2}se^{t} + \frac{1}{2}se^{-t}$

b) $\frac{1}{2}se^{t} - \frac{3}{2}se^{-t}$

c) $\frac{3}{2}se^{t} - \frac{1}{2}se^{-t}$

d) $\frac{3}{2}se^{t} - \frac{1}{2}se^{-t}$

8) An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is

a) $\frac{2}{7}$ b) $\frac{3}{8}$ c) $\frac{1}{2}$ d) $\frac{5}{7}$

9) For a linear programming problem, which one of the following statements is **FALSE**?
   a) If a constraint is an equality, then the corresponding dual variable is unrestricted in sign
   b) Both primal and its dual can be infeasible
   c) If primal is unbounded, then its dual is infeasible
   d) Even if both primal and dual are feasible, the optimal values of the primal and the dual can differ

10) Let $A = \begin{pmatrix} a & 2f & 0 \\ 2f & b & 3f \\ 0 & 3f & c \end{pmatrix}$, where a, b, c, f are real numbers and $f \neq 0$. The geometric multiplicity of the largest eigenvalue of A equals ________.

11) Consider the subspaces $W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = x_2 + 2x_3\}$ $W_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 3x_2 + 2x_3\}$ of $\mathbb{R}^3$. Then the dimension of $W_1 + W_2$ equals ________.

12) Let V be the real vector space of all polynomials of degree less than or equal to 2 with real coefficients. Let $T : V \to V$ be the linear transformation given by $T(p) = 2p + p'$ for $p \in V$, where p' is the derivative of p. Then the number of nonzero entries in the Jordan canonical form of a matrix of T equals ________.

13) Let $I = [2, 3]$, J be the set of all rational numbers in the interval $[4, 6]$, K be the Cantor (ternary) set, and let $L = \{x \in I : x \in K\}$. Then the Lebesgue measure of the set $I \cup J \cup L$ equals ________.