9-9.5-8

EE24BTECH11063 - Y.Harsha Vardhan Reddy

Question:
Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides $x = 0$, $x = 4$, $y = 4$, and $y = 0$ into three equal parts.
Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola-1
V_2, u_2, f_2	Parameters of Parabola-2
P_1, P_2	Points of intersection
$A_1, A_2, A_{overlap}$	Areas swept by the parabolas

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} , \ u_1 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} , \ f_1 = 0 \tag{0.1}$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \ u_2 = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} , \ f_2 = 0 \tag{0.2}$$

Area of square is 16 sq.units Intersection points are,

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} , \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.3}$$

Let, A_1=Area under $y^2 = 4x$, A_2=Area under $x^2 = 4y$

$$A_1 = \int_0^4 \left(2\sqrt{x}\right) dx = \frac{32}{3} \tag{0.4}$$

$$A_2 = \int_0^4 \left(\frac{x^2}{4}\right) dx = \frac{16}{3} \tag{0.5}$$

$$A_{overlap} = A_1 - A_2 = \frac{16}{3} \tag{0.6}$$

By removing overlapped portion from A_1 we get $\frac{16}{3}$
Therefore, the curves $y^2 = 4x$ and $x^2 = 4y$ divides the area of square into 3 equal parts.

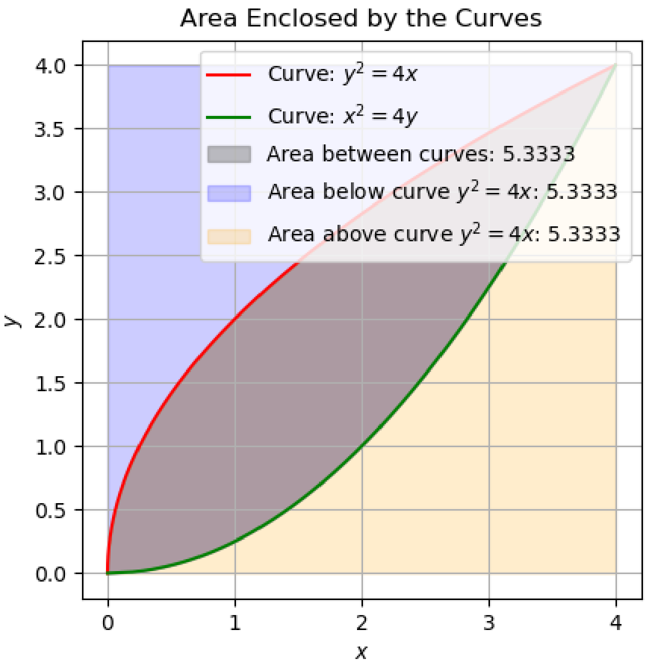
Area Enclosed by the Curves
Curve: $y^2 = 4x$
Curve: $x^2 = 4y$
Area between curves: 5.3333
Area below curve $y^2 = 4x$: 5.3333
Area above curve $y^2 = 4x$: 5.3333
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
4
x

Fig. 0.1