

# Assignment-2

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## CHAPTER 15 MATRICES AND DETERMINANTS SINGLE CORRECT TYPE

- 1) Let  $k$  be an integer such that triangle with vertices  $(k, -3k)$ ,  $(5, k)$  and  $(-k, 2)$  has area  $28sq.units$ . Then the orthocentre of the triangle is at the point : [JEE M 2017]

- a)  $(2, \frac{1}{2})$                       b)  $(2, -\frac{1}{2})$   
c)  $(1, \frac{3}{4})$                       d)  $(1, -\frac{3}{4})$

- 2) Let  $\omega$  be a complex number such that  $2\omega + 1 = z$  where  $z = \sqrt{-3}$ . If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$$

, then  $k$  is equal to: [JEE M 2017]

- a) 1                                      b)  $-z$   
c)  $z$                                       d)  $-1$

- 3) If  $A = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$ , then  $adj(3A^2 + 12A)$  is equal to: [JEE M 2017]

- a)  $\begin{pmatrix} 72 & -63 \\ -84 & 51 \end{pmatrix}$                       b)  $\begin{pmatrix} 72 & -84 \\ -63 & 51 \end{pmatrix}$   
c)  $\begin{pmatrix} 51 & 63 \\ 84 & 72 \end{pmatrix}$                       d)  $\begin{pmatrix} 51 & 84 \\ 63 & 72 \end{pmatrix}$

- 4) If

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$$

, then the ordered pair  $(A, B)$  is equal to: [JEE M 2018]

- a)  $(-4, 3)$    b)  $(-4, 5)$    c)  $(4, 5)$    d)  $(-4, -5)$

- 5) If the system of linear equations

$$\begin{aligned} x + ky + 3z &= 0 \\ 3x + ky - 2z &= 0 \\ 2x + 4y - 3z &= 0 \end{aligned}$$

has a non-zero solution  $(x, y, z)$ , then  $\frac{xz}{y^2}$  is equal to : [JEE M 2018]

- a) 10                      b)  $-30$                       c) 30                      d)  $-10$

- 6) The system of linear equations

$$\begin{aligned} x + y + z &= 2 \\ 2x + 3y + 2z &= 5 \\ 2x + 3y + (a^2 - 1)z &= a + 1 \end{aligned}$$

[JEE M2019-9 Jan(M)]

- a) is consistent when  $a = 4$   
b) has a unique solution for  $|a| = \sqrt{3}$   
c) has infinitely many solutions for  $a = 4$   
d) is consistent when  $|a| = \sqrt{3}$

- 7) If  $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ , then the matrix  $A^{-50}$  when  $\theta = \frac{\pi}{12}$ , is equal to: [JEE M 2019-9 Jan(M)]

- a)  $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$                       b)  $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$   
c)  $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$                       d)  $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

- 8) If  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & n-1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 78 \\ 0 & 1 \end{pmatrix}$ , then the inverse of  $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$  is [JEE M2019-9 April(M)]

$$\begin{array}{ll} \text{a) } \begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix} & \text{b) } \begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix} \\ \text{c) } \begin{pmatrix} 1 & -12 \\ 0 & 1 \end{pmatrix} & \text{d) } \begin{pmatrix} 1 & 0 \\ 13 & 1 \end{pmatrix} \end{array}$$

9) Let  $\alpha$  and  $\beta$  be the roots of the equation

$$x^2 + x + 1 = 0$$

. Then for  $y \neq 0$  in  $\mathbb{R}$ ,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

is equal to : [JEE M 2019-9 April(M)]

- a)  $y(y^2 - 1)$                       b)  $y(y^2 - 3)$   
c)  $y^3$                                 d)  $y^3 - 1$

## CHAPTER 12 DIFFERENTIATION

### E : SUBJECTIVE PROBLEMS

- 1) Let  $f$  be a twice differentiable function such that  $f''(x) = -f(x)$ , and

$$f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2$$

. Find  $h(10)$  if  $h(5) = 11$ . (1982-3 Marks)

- 2) If  $\alpha$  be a repeated root of a quadratic equation  $f(x) = 0$  and  $A(x)$ ,  $B(x)$  and  $C(x)$  be polynomials of degree 3, 4 and 5 respectively, then show that

$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

is divisible by  $f(x)$ , where prime denotes the derivatives. (1984-4 Marks)

- 3) If  $x = \sec \theta - \cos \theta$  and  $y = \sec^n \theta - \cos^n \theta$ , then show that

$$(x^2 + 4) \left( \frac{dy}{dx} \right)^2 = n^2 (y^2 + 4)$$

(1989-2 Marks)

- 4) Find  $\frac{dy}{dx}$  at  $x = -1$ , when

$$(\sin y)^{\sin(\frac{\pi}{2}x)} + \frac{\sqrt{3}}{2} \sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$$

(1991- 4 Marks)

5) If

$$y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + 1$$

, prove that

$$\frac{y'}{y} = \frac{1}{x} \left( \frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x} \right)$$

(1998- 8 Marks)

### H : ASSERTION & REASON TYPE QUESTIONS

- 1) Let  $f(x) = 2 + \cos x$  for all real  $x$ .

**STATEMENT - 1:** For each real  $t$ , there exists a point  $c$  in  $[t, t+\pi]$  such that  $f'(c) = 0$  because

**STATEMENT - 2:**  $f(t) = f(t + 2\pi)$  for each real  $t$ . (2007-3 Marks)

- a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1  
b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1  
c) Statement-1 is True, Statement-2 is False  
d) Statement-1 is False, Statement-2 is True
- 2) Let  $f$  and  $g$  be real valued functions defined on interval  $(-1, 1)$  such that  $g''(x)$  is continuous,  $g(0) \neq 0$ ,  $g'(0) = 0$ ,  $g''(0) \neq 0$ , and  $f(x) = g(x) \sin x$
- STATEMENT-1:**  
 $\lim_{x \rightarrow 0} [g(x) \cot x - g(0) \csc x] = f''(0)$  and  
**STATEMENT-2:**  $f'(0) = g(0)$  (2008)
- a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1  
b) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1  
c) Statement-1 is True, Statement-2 is False  
d) Statement-1 is False, Statement-2 is True

### I: INTEGER VALUE CORRECT TYPE

- 1) If the function  $f(x) = x^3 + e^{\frac{x}{2}}$  and  $g(x) = f^{-1}(x)$ , then the value of  $g'(1)$  is (2009)

2) Let

$$f(\theta) = \sin \left( \tan^{-1} \left( \frac{\sin \theta}{\sqrt{\cos 2\theta}} \right) \right), \text{ where } -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

. Then the value of  $\frac{d}{d(\tan \theta)}(f(\theta))$  is

(2011)