Chapter 16
Application of derivatives

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G : Comprehension Based Questions

PASSAGE-1

If a continuous function f defined on the real line R, assumes positive and negative values in R then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuos function f on R is positive at some point and its minimum value is negative then the eequation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

1) The line $y = x$ meets curve $y = ke^x$ for $k \leq 0$ at $(2007 - 4marks)$

a) no point
b) one point
c) two points
d) more than two points

2) The positive value of k for which $ke^x - x = 0$ has only one root is $(2007 - 4marks)$

a) $\frac{1}{e}$
b) 1
c) e
d) $\log_e 2$

3) For $k > 0$, the set of all values of k for which equation $ke^x - x = 0$ has two distinct roots is $(2007 - 4marks)$

a) $\left(0, \frac{1}{e}\right)$
b) $\left(\frac{1}{e}, 1\right)$
c) $\left(\frac{1}{e}, \infty\right)$
d) $(0, 1)$

PASSAGE-2

Let

$$f(x) = (1-x)^2 \sin^2 x + x^2 forallx \in \mathbb{IR} \tag{1}$$

and let

$$g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\, dt forallx \in (1, \infty). \tag{2}$$

4) Consider the statements:
P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2\left(1 + x^2\right)$
Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$
Then (2012)

a) both P and Q are true
b) P is true and Q is false
c) P is false and Q is true
d) both P and Q are false

5) Which of the following is true? (2012)
a) g is increasing on $(1, \infty)$
b) g is decreasing on $(1, \infty)$
c) g is increasing in (1,2) and decreasing on $(2, \infty)$
d) g is decreasing in (1,2) and increasing on $(2, \infty)$

PASSAGE-3

Let $f(x) : [0, 1] \rightarrow \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable , $f(0) = f(1) = 0$ and satisfies

$$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]. \tag{3}$$

6) Which of the following is true for interval

$$0 < x < 1$$

? $(JEEAdv.2013)$

a) $0 < f(x) < \infty$
b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
c) $-\frac{1}{4} < f(x) < 1$
d) $-\infty < f(x) < 0$

7) If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = \frac{1}{4}$, which of the following is true?

$(JEEAdv.2013)$

1) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$
2) $f'(x) > f(x)$,$0 < x < \frac{1}{4}$
3) $f'(x) < f(x)$,$0 < x < \frac{1}{4}$
4) $f'(x) < f(x)$,$\frac{3}{4} < x < 1$

I:Integer Value Correct Type

1) The maximum value of the function
$f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set
$A = \left\{x | x^2 + 20 \leq 9x\right\}$ is

(2009)

2) Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim_{x \to 0} \left(1 + \frac{p(x)}{x^2}\right) = 2$.
Then the value of $p(2)$ is (2009)

3) Let f be a real-valued differentiable function on $\mathbf{R}(the set of all real numbers)$ such that $f(1) = 1$. If the y-intercept of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then find the value of $f(-3)$

(2010)