Chapter 16
Application of derivatives

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G : Comprehension Based Questions

PASSAGE-1

If a continuous function f defined on the real line R, assumes positive and negative values in R then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuos function f on R is positive at some point and its minimum value is negative then the eequation $f(x) = 0$ has a root in R.

Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

1) The line $y = x$ meets curve $y = ke^x$ for $k \leq 0$ at $(2007 - 4marks)$
   a) no point
   b) one point
   c) two points
   d) more than two points
2) The positive value of k for which $ke^x - x = 0$ has only one root is $(2007 - 4marks)$
   a) $\frac{1}{e}$
   b) 1
   c) e
   d) $\log_e 2$
3) For $k > 0$, the set of all values of k for which equation $ke^x - x = 0$ has two distinct roots is $(2007 - 4marks)$
   a) $\left(0, \frac{1}{e}\right)$
   b) $\left(\frac{1}{e}, 1\right)$
   c) $\left(\frac{1}{e}, \infty\right)$
   d) $(0, 1)$

PASSAGE-2

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\, dt$ for all $x \in (1, \infty)$.

4) Consider the statements:
   P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2\left(1 + x^2\right)$
   Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$
   Then (2012)
   a) both P and Q are true
   b) P is true and Q is false
   c) P is false and Q is true
   d) both P and Q are false
5) Which of the following is true? (2012)
   a) g is increasing on $(1, \infty)$
   b) g is decreasing on $(1, \infty)$
   c) g is increasing in (1,2) and decreasing on $(2, \infty)$
   d) g is decreasing in (1,2) and increasing on $(2, \infty)$

PASSAGE-3

Let $f(x) : [0, 1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable , $f(0) = f(1) = 0$ and satisfies

$$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]. \quad (1)$$

6) Which of the following is true for interval
   $$0 < x < 1$$
   ? $(JEEAdv.2013)$
   a) $0 < f(x) < \infty$
   b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
   c) $-\frac{1}{4} < f(x) < 1$
   d) $-\infty < f(x) < 0$
7) If the function $e^{-x} f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = \frac{1}{4}$, which of the following is true?
   $(JEEAdv.2013)$

1) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$

2) $f'(x) > f(x)$,$0 < x < \frac{1}{4}$

3) $f'(x) < f(x)$,$0 < x < \frac{1}{4}$

4) $f'(x) < f(x)$,$\frac{3}{4} < x < 1$

I:Integer Value Correct Type

1) The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \left\{x | x^2 + 20 \le 9x\right\}$ is (2009)

2) Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim_{x\to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2$. Then the value of $p(2)$ is (2009)

3) Let f be a real-valued differentiable function on $\mathbf{R}(the set of all real numbers)$ such that $f(1) = 1$. If the y-intercept of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then find the value of $f(-3)$ (2010)