University of Texas at Dallas
CS 6364
Artificial Intelligence
Fall 2020

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Bayesian Networks Problems
Set 1

Problem 1

We believe that the relation between the Boolean variables A, B, C can be approximated by:
A → B and B → C. We also have:

P(A) = 0.2 P(B|A=0) = 0.7
P(B|A=1) = 0.8 P(C|B=0) = 0.5 P(C|B=1) = 0.6

A) Produce a graphical representation of the Bayesian Network

Solution:

A	P(A)
T	0.2

B	P(C)
T	0.6
F	0.5

A	P(B)
T	0.8
F	0.7

B) We believe that the relation between the Boolean variables A,B,C can be approximated by: A and B are unrelated, but C can be determined from A and B. We also have:
P(A) = 0.2 P(B) = 0.01
P(C|A=1,B=1) = 0.9 P(C|A=1,B=0) = 0.5
P(C|A=0,B=1) = 0.8 P(C|A=0,B=0) = 0

Produce a graphical representation as a Bayesian network.

Solution:

$$\begin{tikzcd} A \arrow[dr] & & B \arrow[dl] \\ & C & \end{tikzcd}$$

A	P(A)
T	0.2

B	P(B)
T	0.01

A	B	P(C)
T	T	0.9
T	F	0.5
F	T	0.8
F	F	0

C) Compute the probability that A=1, B=1, C=0 in both networks

Solution:

Net 1:

$$\begin{tikzcd} A \arrow[d] \\ B \arrow[d] \\ C \end{tikzcd}$$

A	P(A)
T	0.2

B	P(C)
T	0.6
F	0.5

A	P(B)
T	0.8
F	0.7

$$P(A=1,B=1,C=0) = 0.2 \times 0.8 \times (1-0.6)$$

Net 2:

$$\begin{tikzcd} A \arrow[dr] & & B \arrow[dl] \\ & C & \end{tikzcd}$$

A	P(A)
T	0.2

B	P(B)
T	0.01

A	B	P(C)
T	T	0.9
T	F	0.5
F	T	0.8
F	F	0

$$P(A=1,B=1,C=0) = 0.2 \times 0.01 \times (1-0.9)$$

D) Compute the probability that C=0 in both networks.

Solution:

Net 1: P(C=0) = ?   C has B as father ==> P(B) = ?

A	P(B)	P(W)
T	0.6	0.72
F	0.7	0.8

==> $P(B=1) = 0.8 \times 0.2 + 0.7 \times 0.8 = 0.72$
$P(B=0) = 1 - 0.72 = 0.28$

Now we can compute the probabilities of the possible worlds that affect C:

B	P(C)	P(W)
T	0.6	0.72
F	0.5	0.28

$\Rightarrow P(C=1) = 0.6 \times 0.72 + 0.5 \times 0.28 = 0.716$
$\Rightarrow P(C=0) = 1 - 0.716 = 0.284$

Net 2:

A	P(A)
T	0.2

B	P(B)
T	0.01

A	B	P(C)
T	T	0.9
T	F	0.5
F	T	0.8
F	F	0

$P(C=1) = 0.9 \times 0.002 + 0.198 \times 0.5 + 0.008 \times 0.8 + 0.792 \times 0 = 0.1072$
$\Rightarrow P(C=0) = 0.8928$

E) Compute P(C=0|A=1) in the first network

Solution:

$$\begin{tikzcd} A \arrow[d] \\ B \arrow[d] \\ C \end{tikzcd}$$

A	P(A)
T	0.2

B	P(B)
T	0.6
F	0.5

A	P(B)
T	0.8
F	0.7

First compute P(B=1|A=1) because B is the parent of C

A	P(B)	P(B)
T	0.8	1

$\Rightarrow$ P(B=1|A=1) = 0.8 $\times$ 0.1 = 0.8
P(B=0|A=1) = 0.2

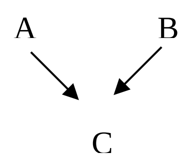
Then we can compute P(C=0|A=1) by considering the possible worlds:

B	P(C)	P(W)
T	0.6	0.8
F	0.5	0.2

P(C=1|A=1) = 0.6 $\times$ 0.8 + 0.5 $\times$ 0.2 = 0.74
P(C=0|A=1) = 1 - 0.74 = 0.26

F) Use network 2 to determine the probability that C=0 if it is known that A=1

Solution:



A	P(A)
T	0.2

B	P(B)
T	0.01

A	B	P(C)
T	T	0.9
T	F	0.5
F	T	0.8
F	F	0

Then we compute:

A	B	P(C)	P(W)
T	T	0.9	0.01
T	F	0.5	1 - 0.01 = 0.99

P(C=1|A=1) = 0.9 $\times$ 0.01 + 0.5 $\times$ 0.99 = 0.504
Then P(C=0|A=1) = 1 - 0.504 = 0.496
