

Student Name: Kapil Gautam

Student NetID: KXG180032

University of Texas at Dallas
Department of Computer Science
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FINAL EXAM

Fall 2020

Instructor: Dr. Sanda Harabagiu

Instructions: Do not communicate with anyone in any shape or form. This is an independent exam. Do not delete any problem formulation, just attach your answer in the space provided. If the problem is deleted and you send only the answer, you shall receive ZERO points.

Copy and paste the Final Exam into a Word document, enter your answers (either by typing in Word, or by inserting a VERY CLEAR picture of your hand-written solution) and transform the file of the exam into a PDF format. If we cannot clearly read the picture, you will get ZERO for that answer! If you create an enormous file for your final exam (i.e. larger than 5 Mbytes) you will receive ZERO for your entire exam. Please follow the instructions from the attached **instructions_submission_EXAM.pdf** file to make sure your final pdf file is of reasonable size.

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Make sure that you insert EACH answer immediately after EACH question. Failure to do so will result in ZERO points for the entire exam! Submit the PDF file with the name **Final_Exam_netID.pdf**, where netID is your unique netid provided by UTD. If you submit your exam in any other format your will receive ZERO points.

The Final exam shall be submitted in eLearning before the deadline. No late submissions shall be graded! Any cheating attempt will determine the ENTIRE grade of the final exam to become ZERO.

Write your answers immediately after the problem statements.

Problem 1 Uncertainty (10 points)

Given the joint probability table:

	<i>rain</i>		<i>¬rain</i>	
	<i>sprinkle</i>	<i>¬sprinkle</i>	<i>sprinkle</i>	<i>¬sprinkle</i>
<i>Grasswet</i>	0.1	0.02	0.023	0.007
<i>¬Grasswet</i>	0.09	0.06	0.14	0.56

Compute:

1. $P(\neg \text{Grasswet})$ (2points)
2. $P(\text{rain} | \text{Grasswet})$ (3points)
3. $P(\text{Grasswet} | \text{rain} \vee \text{sprinkle})$ (5points)

Answers:

$$1. P(\neg \text{Grasswet}) = 0.09 + 0.06 + 0.14 + 0.56 = 0.85$$

$$2. P(\text{rain} | \text{Grasswet}) = \frac{P(\text{Grasswet}, \text{rain})}{P(\text{Grasswet})} = \frac{0.1 + 0.02}{0.1 + 0.02 + 0.023 + 0.007} = \frac{0.12}{0.15} = 0.8$$

$$3. P(\text{Grasswet} | \text{rain} \vee \text{sprinkle}) = \frac{P(\text{Grasswet}, \text{rain} \vee \text{sprinkle})}{P(\text{rain} \vee \text{sprinkle})}$$

$$\begin{aligned} \text{From the table given, } P(\text{Grasswet}, \text{rain} \vee \text{sprinkle}) &= P(\text{Grasswet}, \text{rain}) + P(\text{Grasswet}, \text{sprinkle}) \\ &= (0.1 + 0.02) + (0.1 + 0.023) \\ &= 0.243 \end{aligned}$$

$$\begin{aligned} P(\text{rain} \vee \text{sprinkle}) &= P(\text{rain}) + P(\text{sprinkle}) \\ &= (0.1 + 0.02 + 0.09 + 0.06) + (0.1 + 0.09 + 0.023 + 0.14) \\ &= 0.27 + 0.353 \\ &= 0.623 \end{aligned}$$

$$P(\text{Grasswet} | \text{rain} \vee \text{sprinkle}) = \frac{P(\text{Grasswet}, \text{rain} \vee \text{sprinkle})}{P(\text{rain} \vee \text{sprinkle})} = \frac{0.243}{0.623} = 0.39$$

Problem 2 First Order Logic Representations (30 points)

(a) **(15 points)** Represent in First-Order Logic (FOL) the following sentences, using the semantics of the predicates provided for each sentence:

1. *Every artist is inspired by a painting.*

Predicates: *artist(x)*- x is artist;

inspire(x,y): x is inspired by y;

painting(z): z is a painting.

2. *All musicians in an orchestra follow the instructions of their conductor.* Predicates: *musician(x)*- x is musician;

inOrchestra(x,y): x is in orchestra y;

conductor(z,y): z is conductor of orchestra y;

follows(x,y): x follows the instructions of y;

3. *In each concert, every musician plays some instrument and sings some song and no one gets bored.*

Predicates: *concert(c)* – c is a concert;

musician(m) – m is a musician;

musicianInConcert(m,c) – m is a musician participating in concert c;

playsInstrument(m,i) – m plays instrument i;

singsSong(m,s) – m sings song s;

getBored(m) – m is getting bored;

Answers:

-
- (a) 1. **(5 points)**

Every artist is inspired by a painting.

Predicates: *artist(x)*- x is artist;

inspire(x,y): x is inspired by y;

painting(z): z is a painting.

First Order Logic:

$$\forall x [\text{artist}(x) \Rightarrow \exists z [\text{painting}(z) \wedge \text{inspired}(x,z)]]$$

- (a) 2. **(5 points)**

All musicians in an orchestra follow the instructions of their conductor.

Predicates: *musician(x)*- x is musician;

inOrchestra(x,y): x is in orchestra y;

conductor(z,y): z is conductor of orchestra y;

follows(x,y): x follows the instructions of y;

First Order Logic:

$$\forall x [\text{musician}(x) \Rightarrow \exists y \exists z [\text{inOrchestra}(x,y) \wedge \text{conductor}(z,y) \wedge \text{follows}(x,z)]]$$

(a) 3. (5 points)

In each concert, every musician plays some instrument and sings some song and no one gets bored.

Predicates: $concert(c)$ – c is a concert;

$musician(m)$ – m is a musician;

$musicianInConcert(m,c)$ – m is a musician participating in concert c;

$playsInstrument(m,i)$ – m plays instrument i;

$singsSong(m,s)$ – m sings song s;

$getBored(m)$ – m is getting bored;

First Order Logic:

$\forall c \forall m [concert(c) \wedge musician(m) \wedge musicianInConcert(m,c) \Rightarrow \exists i \exists s [playsInstrument(m,i) \wedge singsSong(m,s) \wedge \neg getBored(m)]]$

(b) Generate the Conjunctive Normal Form (CNF) for the FOL expressions obtained in

(a). Specify at each step of the conversion in CNF what you are doing!

Answers:

1. (5 points)

First Order Logic:

$\forall x [artist(x) \Rightarrow \exists z [painting(z) \wedge inspired(x,z)]]$

STEP 1: Eliminate Implications

$\forall x [\neg artist(x) \vee \exists z [painting(z) \wedge inspired(x,z)]]$

STEP 2: Eliminate \exists quantifiers. z is in scope of quantized variable x

Skolem function $z = f(x)$

STEP 3: Skolemize

$\forall x [\neg artist(x) \vee [painting(f(x)) \wedge inspired(x,f(x))]]$

STEP 4: Eliminate \forall quantifiers

$\neg artist(x) \vee [painting(f(x)) \wedge inspired(x, f(x))]$

STEP 5: Distribute \wedge over \vee

$(\neg artist(x) \vee painting(f(x))) \wedge (\neg artist(x) \vee inspired(x, f(x)))$

CNF obtained: $(\neg artist(x) \vee painting(f(x))) \wedge (\neg artist(x) \vee inspired(x, f(x)))$

2. (5 points)

First Order Logic:

$$\forall x [\text{musician}(x) \Rightarrow \exists y \exists z [\text{inOchestra}(x,y) \wedge \text{conductor}(z,y) \wedge \text{follows}(x,z)]]$$

STEP 1: Eliminate Implications

$$\forall x [\neg \text{musician}(x) \vee \exists y \exists z [\text{inOchestra}(x,y) \wedge \text{conductor}(z,y) \wedge \text{follows}(x,z)]]$$

STEP 2: Eliminate \exists quantifiers. y and z are in scope of quantized variable x

Skolem function $y = f(x)$, $z = g(x)$

STEP 3: Skolemize

$$\forall x [\neg \text{musician}(x) \vee [\text{inOchestra}(x,f(x)) \wedge \text{conductor}(g(x),f(x)) \wedge \text{follows}(x,g(x))]]$$

STEP 4: Eliminate \forall quantifiers

$$\neg \text{musician}(x) \vee [\text{inOchestra}(x,f(x)) \wedge \text{conductor}(g(x),f(x)) \wedge \text{follows}(x,g(x))]$$

STEP 5: Distribute \wedge over \vee

$$(\neg \text{musician}(x) \vee \text{inOchestra}(x,f(x))) \wedge (\neg \text{musician}(x) \vee \text{conductor}(g(x),f(x))) \wedge (\neg \text{musician}(x) \vee \text{follows}(x,g(x)))$$

CNF obtained: $(\neg \text{musician}(x) \vee \text{inOchestra}(x,f(x))) \wedge (\neg \text{musician}(x) \vee \text{conductor}(g(x),f(x))) \wedge (\neg \text{musician}(x) \vee \text{follows}(x,g(x)))$

3. (5 points)

First Order Logic:

$$\forall c \forall m [\text{concert}(c) \wedge \text{musician}(m) \wedge \text{musicianInConcert}(m,c) \Rightarrow \exists i \exists s [\text{playsInstrument}(m,i) \wedge \text{singsSong}(m,s) \wedge \neg \text{getBored}(m)]]$$

STEP 1: Eliminate Implications

$$\forall c \forall m [\neg (\text{concert}(c) \wedge \text{musician}(m) \wedge \text{musicianInConcert}(m,c)) \vee \exists i \exists s [\text{playsInstrument}(m,i) \wedge \text{singsSong}(m,s) \wedge \neg \text{getBored}(m)]]$$

STEP 2: Move \neg inwards

$$\forall c \forall m [\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c) \vee \exists i \exists s [\text{playsInstrument}(m,i) \wedge \text{singsSong}(m,s) \wedge \neg \text{getBored}(m)]]$$

STEP 3: Eliminate \exists quantifiers. i and s are in scope of quantized variable c and m

Skolem function $i = f(c,m)$, $s = g(c,m)$

STEP 4: Skolemize

$$\forall c \forall m [\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c) \vee [\text{playsInstrument}(m, f(c,m)) \wedge \text{singsSong}(m, g(c,m)) \wedge \neg \text{getBored}(m)]]$$

STEP 5: Eliminate \forall quantifiers

$(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee [\text{playsInstrument}(m, f(c,m)) \wedge \text{singsSong}(m, g(c,m)) \wedge \neg \text{getBored}(m)]$

STEP 6: Distribute \wedge over \vee

$(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee \text{playsInstrument}(m, f(c,m)) \wedge$
 $(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee \text{singsSong}(m, g(c,m)) \wedge$
 $(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee \neg \text{getBored}(m))$

CNF obtained:

$(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee \text{playsInstrument}(m, f(c,m)) \wedge$
 $(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee \text{singsSong}(m, g(c,m)) \wedge$
 $(\neg \text{concert}(c) \vee \neg \text{musician}(m) \vee \neg \text{musicianInConcert}(m,c)) \vee \neg \text{getBored}(m))$

Problem 3 (20 points) Inference in First Order Logic

You are given the following description:

"If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned"

1. Transform the text in First-Order Logic (FOL) **(4 points)**.
2. Convert each axiom in Conjunctive Normal Form (CNF) and produce a knowledge base (KB) containing all the clauses derived from the CNF. **(6 points)**.
3. Given the KB, can you prove that a unicorn is mythical? How about magical? Or horned? Use refutation to prove each of your answers and show the entire proof. **(10 points)**

Answer:

1. *"If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned."*

Transforming the propositions into FOL:

S1: $\text{Mythical} \Rightarrow \neg \text{Mortal}$

S2: $\neg \text{Mythical} \Rightarrow \text{Mortal} \wedge \text{Mammal}$

S3: $\neg \text{Mortal} \vee \text{Mammal} \Rightarrow \text{Horned}$

S4: $\text{Horned} \Rightarrow \text{Magical}$

2. Converting the axioms to CNF:

S1: $\text{Mythical} \Rightarrow \neg \text{Mortal}$

Step1: Remove the implication

C1: $\neg \text{Mythical} \vee \neg \text{Mortal}$

S2: $\neg \text{Mythical} \Rightarrow \text{Mortal} \wedge \text{Mammal}$

Step1: Remove the implication

S2: $\neg \neg \text{Mythical} \vee (\text{Mortal} \wedge \text{Mammal})$

Step2: Distribute \wedge over \vee

S2: $(\text{Mythical} \vee \text{Mortal}) \wedge (\text{Mythical} \vee \text{Mammal})$

C2: $\text{Mythical} \vee \text{Mortal}$

C3: $\text{Mythical} \vee \text{Mammal}$

S3: $\neg \text{Mortal} \vee \text{Mammal} \Rightarrow \text{Horned}$

Step1: Remove the implication

S3: $\neg(\neg \text{Mortal} \vee \text{Mammal}) \vee \text{Horned}$

Step2: Move \neg inward

S3: $(\neg \neg \text{Mortal} \wedge \neg \text{Mammal}) \vee \text{Horned}$

S3: $(\text{Mortal} \wedge \neg \text{Mammal}) \vee \text{Horned}$

Step3: Distribute \wedge over \vee

S3: $(\text{Mortal} \wedge \neg \text{Mammal}) \vee \text{Horned}$

S3: $(\text{Mortal} \vee \text{Horned}) \wedge (\neg \text{Mammal} \vee \text{Horned})$

C4: $\text{Mortal} \vee \text{Horned}$

C5: $\neg \text{Mammal} \vee \text{Horned}$

S4: $\text{Horned} \Rightarrow \text{Magical}$

Step1: Remove the implication

C6: $\neg \text{Horned} \vee \text{Magical}$

We get the following knowledge base(KB) after conversion :

C1: $\neg \text{Mythical} \vee \neg \text{Mortal}$

C2: $\text{Mythical} \vee \text{Mortal}$

C3: $\text{Mythical} \vee \text{Mammal}$

C4: $\text{Mortal} \vee \text{Horned}$

C5: $\neg \text{Mammal} \vee \text{Horned}$

C6: $\neg \text{Horned} \vee \text{Magical}$

3. Derived the above KB, we need to prove 3 queries by refutation:

- a. Q1: Mythical
- b. Q2: Magical
- c. Q3: Horned

a.) Q1: Mythical

¬Q1: ¬Mythical

Knowledge Base:

C1: ¬Mythical \vee ¬Mortal

C2: Mythical \vee Mortal

C3: Mythical \vee Mammal

C4: Mortal \vee Horned

C5: ¬Mammal \vee Horned

C6: ¬Horned \vee Magical

With any combination, the query cannot be proved.

Thus, the unicorn is not mythical.

(If additional information is given like, a unicorn is not a mammal, then this query can be proven)

b.) Q2: Magical

¬Q2: ¬Magical

Knowledge Base:

C1: ¬Mythical \vee ¬Mortal

C2: Mythical \vee Mortal

C3: Mythical \vee Mammal

C4: Mortal \vee Horned

C5: ¬Mammal \vee Horned

C6: ¬Horned \vee Magical

Step1: Resolve C6 and ¬Q2

C6: ¬Horned \vee Magical

¬Q2: ¬Magical

Y1: ¬Horned

Step2: Resolve Y1 and C5

Y1: ¬Horned

C5: ¬Mammal \vee Horned

Y2: ¬Mammal

Step3: Resolve C4 and Y1

Y1: ¬Horned

C4: Mortal \vee Horned

Y3: Mortal

Step4: Resolve C3 and Y2

Y2: ¬Mammal

C3: Mythical \vee Mammal

Y4: Mythical

Step5: Resolve C1 and Y4

Y4: Mythical

C1: $\neg\text{Mythical} \vee \neg\text{Mortal}$

Y5: $\neg\text{Mortal}$

Step6: Resolve Y3 and Y5

Y3: Mortal

Y5: $\neg\text{Mortal}$

Y6: NIL

Thus, the unicorn is indeed magical.

c.) Q3: Horned

$\neg\text{Q3}$: $\neg\text{Horned}$

Knowledge Base:

C1: $\neg\text{Mythical} \vee \neg\text{Mortal}$

C2: $\text{Mythical} \vee \text{Mortal}$

C3: $\text{Mythical} \vee \text{Mammal}$

C4: $\text{Mortal} \vee \text{Horned}$

C5: $\neg\text{Mammal} \vee \text{Horned}$

C6: $\neg\text{Horned} \vee \text{Magical}$

Step1: Resolve C5 and $\neg\text{Q3}$

C5: $\neg\text{Mammal} \vee \text{Horned}$

$\neg\text{Q3}$: $\neg\text{Horned}$

Z1: $\neg\text{Mammal}$

Step2: Resolve $\neg\text{Q3}$ and C4

$\neg\text{Q3}$: $\neg\text{Horned}$

C4: $\text{Mortal} \vee \text{Horned}$

Z2: Mortal

Step3: Resolve Z1 and C3

Z1: $\neg\text{Mammal}$

C3: $\text{Mythical} \vee \text{Mammal}$

Z3: Mythical

Step4: Resolve C1 and Z2 and Z3

C1: $\neg\text{Mythical} \vee \neg\text{Mortal}$

Z3: Mythical

Z2: Mortal

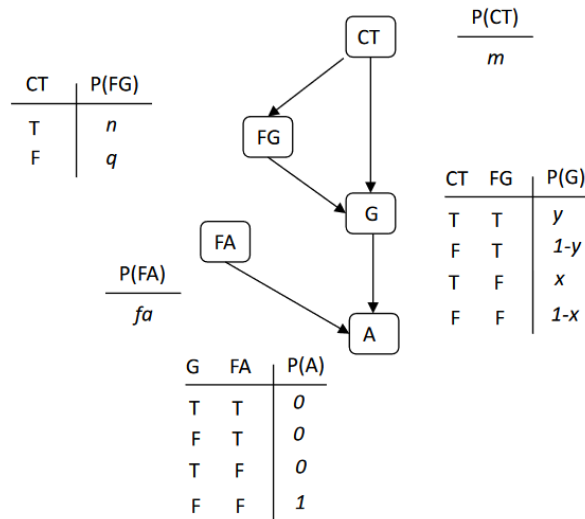
X4: NIL

Thus, the unicorn is indeed horned.

Problem 3 (20 points) Probabilistic Reasoning

In a nuclear power plant, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean random variables A (alarm sounds), FA (alarm is faulty), FG (gauge is faulty), G (gauge is reading) and CT (the core temperature is too high).

The following Bayesian network represents this domain:



- (5 points)** Compute the probability that $A=1$, $FA=0$, $FG=0$ and $CT=1$.
- (10 points)** Compute the probability that the core temperature is too high when the alarm sounds, and the alarm and the gauge work well, i.e. $P(CT|A, \neg FA, \neg FG)$.
- (5 points)** Decide whether A and FA have any effect on CT given G and explain why you reached that decision.

Answer:

- Compute the probability that $A=1$, $FA=0$, $FG=0$ and $CT=1$

$$\begin{aligned}
 P(A=1, FA=0, FG=0, CT=1) &= P(A=1) * P(FA=0) * P(FG=0) * P(CT=1) \\
 &\Rightarrow P(A=1 | FA=0, FG=0, CT=1) * P(FA=0) * P(FG=0 | CT=1) * P(CT=1) \\
 &\Rightarrow P(CT=1) = m \quad \text{---- 1}
 \end{aligned}$$

Since FG is dependent on CT, $P(FG=0 | CT=1)$

$$\Rightarrow P(FG=0 | CT=1) = 1-n \quad \text{---- 2}$$

Since G is dependent on CT and FG

$$\begin{aligned}
 &\Rightarrow P(G | FG=0, CT=1) = x \\
 &\Rightarrow P(FA=0) = 1-fa \quad \text{---- 3}
 \end{aligned}$$

Since A is dependent on FA and G

$$\Rightarrow P(A=1 | FA=0, FG=0, CT=1) = (1-x) * (1-fa) \quad \text{---- 4}$$

Combining 1,2,3 and 4 we get:

$$\Rightarrow P(A=1 \mid FA=0, FG=0, CT=1) * P(FA=0) * P(FG=0 \mid CT=1) * P(CT=1)$$

$$\Rightarrow (1-x) * (1-fa) * (1-n) * m$$

2. Compute the probability that the core temperature is too high when the alarm sounds, and the alarm and the gauge work well, i.e. $P(CT|A, \neg FA, \neg FG)$.

By Bayes Rule

$$\Rightarrow P(CT|A, \neg FA, \neg FG) = \frac{P(A, \neg FA, \neg FG | CT) P(CT)}{P(A, \neg FA, \neg FG)}$$

$$\Rightarrow P(A, \neg FA, \neg FG | CT) = P(A | CT) * P(\neg FA | CT) * P(\neg FG | CT)$$

\Rightarrow Let's compute $P(A | CT)$ first

$$P(CT=1) = m,$$

Since CT is known and FG depends on it

$$P(FG|CT=1) = n$$

Now G depends on CT and FG, and CT is given

Using Variable Elimination:

FG	G	g(D)	h1
1	1	y	$n*y$
1	0	$1-y$	$n*(1-y)$
0	1	x	$(1-n)*x$
0	0	$1-x$	$(1-n)*(1-x)$

Eliminating FG from the above table:

G	$h1 \Rightarrow h2$
1	$ny + (1-n)x$
0	$n(1-y) + (1-n)(1-x)$

$$\text{We get } P(G) = ny + (1-n)x,$$

$$P(\neg G) = n(1-y) + (1-n)(1-x)$$

$$P(FA=1) = fa,$$

$$P(FA=0) = 1-fa$$

Now A depends on FA and G

Also, Looking forward we see that the $P(A) = 1$ only when $FA=0$ and $G=0$, so we will only calculate that parts.

Using Variable Elimination:

FA	G	A	g(A)	h1
0	0	1	1	$(1-fa) * (n(1-y) + (1-n)(1-x))$
0	0	0	0	0

Eliminating and combining terms for A, we get

$$P(A|CT) = (1-fa) * (n*(1-y) + (1-n)*(1-x))$$

$$\Rightarrow \text{Now } P(\neg FA|CT)$$

Since FA is not depended on CT, so $P(\neg FA) = 1-fa$

$$\Rightarrow \text{Finally } P(\neg FG|CT) = 1-n$$

Overall, $P(A, \neg FA, \neg FG | CT) = P(A | CT) * P(\neg FA | CT) * P(\neg FG | CT)$
 $\Rightarrow (1-fa) * ((n)*(1-y) + (1-n)*(1-x)) * (1-fa) * (1-n)$

Since $P(m|s) = \alpha < P(s|m)P(m)$, $(P(s|\neg m)P(\neg m) >$

To calculate α , we also need to find $P(A, \neg FA, \neg FG | \neg CT) P(\neg CT)$
 $\Rightarrow P(\neg CT) = 1-m$
 $\Rightarrow P(A, \neg FA, \neg FG | \neg CT) = P(A | \neg CT) * P(\neg FA | \neg CT) * P(\neg FG | \neg CT)$

\Rightarrow Let's compute $P(A | \neg CT)$ first

$P(\neg CT) = 1-m$,

FG depends on CT, and $\neg CT$ is given, so $P(FG | \neg CT) = q$

Now G depends on CT and FG, and $\neg CT$ is given

Using Variable Elimination:

FG	G	g(D)	h1
1	1	1-y	$q*(1-y)$
1	0	y	$q*y$
0	1	1-x	$(1-q)*(1-x)$
0	0	x	$(1-q)*x$

Eliminating FG from the above table:

G	h1 \Rightarrow h2
1	$q(1-y) + (1-q)(1-x)$
0	$qy + (1-q)x$

We get $P(G) = q(1-y) + (1-q)(1-x)$,

$P(\neg G) = qy + (1-q)x$

$P(FA=1) = fa$,

$P(FA=0) = 1-fa$

Now A depends on FA and G

Also, Looking forward we see that the $P(A) = 1$ only when $FA=0$ and $G=0$, so we will only calculate that parts.

Using Variable Elimination:

FA	G	A	g(A)	h1
0	0	1	1	$(1-fa) * (qy + (1-q)x)$
0	0	0	0	0

Eliminating and combining terms for A, we get

$P(A | \neg CT) = (1-fa) * (q*y + (1-q)*x)$

\Rightarrow Now $P(\neg FA | \neg CT) = P(\neg FA) = 1-fa$

\Rightarrow Finally $P(\neg FG | \neg CT) = 1-q$

Overall, $P(A, \neg FA, \neg FG | \neg CT) = P(A | \neg CT) * P(\neg FA | \neg CT) * P(\neg FG | \neg CT)$
 $\Rightarrow (1-fa) * (q*y + (1-q)*x) * (1-fa) * (1-q)$

Finally, putting in values we get

$$\Rightarrow P(CT | A, \neg FA, \neg FG) = \frac{P(A, \neg FA, \neg FG | CT) P(CT)}{P(A, \neg FA, \neg FG)}$$

$$\Rightarrow \frac{P(A, \neg FA, \neg FG | CT) P(CT)}{P(A, \neg FA, \neg FG | CT) P(CT) + P(A, \neg FA, \neg FG | \neg CT) P(\neg CT)}$$

$$\Rightarrow \frac{(1-fa) * (n*(1-y) + (1-n)*(1-x)) * (1-fa) * (1-n)*m}{(1-fa) * (n*(1-y) + (1-n)*(1-x)) * (1-fa) * (1-n)*m + (1-fa) * (q*y + (1-q)*x) * (1-fa) * (1-q)*(1-m)}$$

Taking (1-fa) common and removing from both sides

$$\Rightarrow \frac{(n*(1-y) + (1-n)*(1-x)) * (1-n)*m}{(n*(1-y) + (1-n)*(1-x)) * (1-n)*m + (q*y + (1-q)*x) * (1-q)*(1-m)}$$

3. Decide whether A and FA have any effect on CT given G and explain why you reached that decision.

FA is not dependent on anyone. However, A is dependent on G which in turn is dependent on CT.

$$P(CT | G) = \frac{P(G|CT)*P(CT)}{P(G)}$$

It only depends on P(G | CT)

Now P(G | CT) does not have A or FA dependency.

So, neither A, nor FA will affect CT given G.

Problem 4 (20 points) Inference in Propositional Logic

If your knowledge base (KB) contains the following Horn clauses:

1. $B \Rightarrow A$
2. $C \wedge D \wedge E \Rightarrow B$
3. $B \wedge F \Rightarrow C$
4. $F \wedge G \Rightarrow D$
5. $G \wedge H \Rightarrow E$
6. F
7. G
8. H

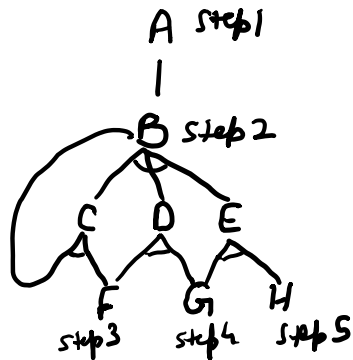
Use backward chaining to prove A.

i. Draw the AND=OR graph for this KB (5 points).

ii. Show each step of your proof using backward chaining (15 points).

Answer:

i.



AND-OR graph for this Knowledge base.

ii.

S1: $B \Rightarrow A$

S2: $C \wedge D \wedge E \Rightarrow B$

S3: $B \wedge F \Rightarrow C$

S4: $F \wedge G \Rightarrow D$

S5: $G \wedge H \Rightarrow E$

S6: F

S7: G

S8: H

Count	Step1	Step2
S1: 1	0	0
S2: 3	0	0
S3: 2	2	0
S4: 2	2	0
S5: 2	2	0
S6: 0	0	0
S7: 0	0	0
S8: 0	0	0

Agenda: A, B, C, D, E, F, G, H

Step1: We assume $A = T$, infer $B = T$
 $\text{count}(S2) = 0$

Step2: We know $B = T$, infer $C = T$, $D = T$, $E = T$
 $\text{count}(S3) = 0$, $\text{count}(S4) = 0$, $\text{count}(S5) = 0$

Step3: Because $\text{count}(S3) = 0$, infer $B = T$ and $F = T$.
 $F = T$ is a fact from the KB.

Step 4: Because $\text{count}(S4) = 0$, infer $F = T$, $G = T$.
 $F = T$ and $G = T$ are both facts from the KB.

Step 5: Because $\text{count}(S5) = 0$, infer $G = T$, $H = T$.
 $G = T$ and $H = T$ are both facts from the KB.

No more literals in Agenda

All literals have been inferred.

Query A is true.