University of Texas at Dallas
CS 6364
Artificial Intelligence
Fall 2020

Instructor: Dr. Sanda Harabagiu

Preparation for the Final Exam

Problem 1. Propositional logic.

A. Translate in CNF the following two sentences:
i) Rain $\Leftrightarrow$ (Wet $\vee$ Umbrella)
ii) (Free $\Rightarrow$ Happy) $\Leftrightarrow$ (Money $\vee$ Gold)

Solution:
i) Rain $\Leftrightarrow$ (Wet $\vee$ Umbrella)

Step 1: Eliminate $\Leftrightarrow$ by replacing a$\Leftrightarrow$ b with (a $\Rightarrow$ b) $\wedge$ (b$\Rightarrow$ a)
(Rain $\Rightarrow$ (Wet $\vee$ Umbrella)) $\wedge$ ((Wet $\vee$ Umbrella) $\Rightarrow$ Rain)

Step 2: Eliminate $\Rightarrow$ by replacing (a $\Rightarrow$ b) with $\neg$a $\vee$ b
($\neg$Rain $\vee$ (Wet $\vee$ Umbrella)) $\wedge$ ($\neg$(Wet $\vee$ Umbrella) $\vee$ Rain)

Step 3: Move $\neg$ inwards:
($\neg$Rain $\vee$ Wet $\vee$ Umbrella) $\wedge$ (($\neg$Wet $\wedge$ $\neg$Umbrella) $\vee$ Rain)

Step 4: Distribute $\vee$ over $\wedge$ by replacing (a $\wedge$ b) $\vee$ c with (a $\vee$ c) $\wedge$ (b $\vee$ c)
($\neg$Rain $\vee$ Wet $\vee$ Umbrella) $\wedge$ ($\neg$Wet $\vee$ Rain) $\wedge$ ($\neg$Umbrella $\vee$ Rain) $\Leftarrow$This is in **CNF**

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ii) (Free $\Rightarrow$ Happy) $\Leftrightarrow$ (Money $\vee$ Gold)

Step 1: Eliminate $\Leftrightarrow$ by replacing a$\Leftrightarrow$b with (a $\Rightarrow$b) $\wedge$ (b$\Rightarrow$a)
((Free $\Rightarrow$ Happy) $\Rightarrow$ (Money $\vee$ Gold)) $\wedge$ ((Money $\vee$ Gold) $\Rightarrow$ (Free $\Rightarrow$ Happy))

Step 2: Eliminate $\Rightarrow$ by replacing (a $\Rightarrow$ b) with $\neg$a $\vee$ b
($\neg$ ($\neg$Free $\vee$ Happy) $\vee$ (Money $\vee$ Gold)) $\wedge$ ($\neg$ (Money $\vee$ Gold) $\vee$ ($\neg$Free $\vee$ Happy))

Step 3: Move $\neg$ inwards:
((Free $\wedge$ $\neg$Happy) $\vee$ (Money $\vee$ Gold)) $\wedge$ (($\neg$Money ^ $\neg$Gold) $\vee$ ($\neg$Free $\vee$ Happy))

Step 4: Distribute $\vee$ over $\wedge$ by replacing $(a \wedge b) \vee c$ with $(a \vee c) \wedge (b \vee c)$
(Free $\vee$ Money $\vee$ Gold) $\wedge$ ($\neg$Happy $\vee$ Money $\vee$ Gold) $\wedge$ ($\neg$Money $\vee$ $\neg$Free $\vee$ Happy) $\wedge$ ($\neg$Gold $\vee$ $\neg$Free $\vee$ Happy) $\Leftarrow$This is in **CNF**

B. If the two sentences in CNF are used to form a Knowledge Base, use resolution to prove Q: Wet $\wedge$ Money $\wedge$ Happy

Solution:

The KB is:
S1: $\neg$Rain $\vee$ Wet $\vee$ Umbrella
S2: $\neg$Wet $\vee$ Rain
S3: $\neg$Umbrella $\vee$ Rain
S4: Free $\vee$ Money $\vee$ Gold
S5: $\neg$Happy $\vee$ Money $\vee$ Gold
S6: $\neg$Money $\vee$ $\neg$Free $\vee$ Happy
S7: $\neg$Gold $\vee$ $\neg$Free $\vee$ Happy

The negated query is : $\neg$Q: $\neg$(Wet $\wedge$ Money $\wedge$ Happy)= $\neg$Wet $\vee$ $\neg$Money $\vee$ $\neg$Happy

From S4 and S7, after applying resolution we obtain:
R1: Money $\vee$ Happy
From S1 and S3, after applying resolution we obtain:
R2: Wet
From $\neg$Q and R1, after applying resolution, we obtain:
R3: $\neg$Wet
From R2 and R3, after applying resolution, we obtain NIL
Therefore, we were able to prove Q from the KB.

Problem 2. Inference in First-Order logic.

There are three girls: Mary, Ann and Julie.
They want to pick 6 different flowers.
Each of them will pick two flowers.
The flowers are: roses, lilies, carnations, calla, daisies and orchids.
Mary does not like carnations.
Mary, the girl that picked orchid and the girl that picked the calla rode the same bus.
Roses go with carnations, thus they were picked by the same girl.
What flowers did each of them pick?

Solution:
Step 1: Translate in FOL

"They want to pick 6 different flowers."
"The flowers are: roses, lilies, carnations, calla, daisies and orchids."

P1: flower(Rose)
P2: flower(Lily)
P3: flower(Carnation)
P4: flower(Calla)
P5: flower(Daisy)
P6: flower(Orchid)

"There are three girls: Mary, Ann and Julie."

P7: girl(Mary)
P8: girl(Ann)
P9: girl(Julie)

"They want to pick 6 different flowers.
Each of them will pick two flowers."

P10: $\forall$ x,y,z $\exists$ a,b,c,d,e,f girl(x) $\wedge$ girl(y) $\wedge$ girl(z) $\wedge$ flower(a) $\wedge$ flower(b) $\wedge$ flower(c) $\wedge$ flower(d) $\wedge$ flower(e) $\wedge$ flower(f) $\Rightarrow$ pick(x,a) $\wedge$ pick(x,b) $\wedge$ pick(y,c) $\wedge$ pick(y,d) $\wedge$ pick(z,e) $\wedge$ pick(z,f)

"Mary does not like carnations."

P11: $\neg$like(Mary, Carnation)

"A girl that does not like a flower will not pick it"

P12: $\forall$ x, a like(x,a) $\wedge$ girl(x) $\wedge$ flower(a) $\Rightarrow$ pick(x,a)

"Roses go with carnations, thus they were picked by the same girl."

P13: $\exists$ x girl(x) $\Rightarrow$ pick(x,Rose) $\wedge$ pick(x, Carnation)

"Mary, the girl that picked orchid and the girl that picked the cala rode the same bus."

P14: $\neg$pick(Mary, Orchid)

P15: $\neg$pick(Mary, Cala)

"What flowers did each of them pick?"

$\neg$Q: (pick(Mary, f1) $\wedge$ pick(Mary,f2) $\wedge$ pick(Ann,f3) $\wedge$ pick(Ann,f4) $\wedge$ pick(Julie,f5) $\wedge$ pick(Julie,f6) $\vee$ $\neg$(pick(Mary,f1) $\wedge$ pick(Mary,f2) $\wedge$ pick(Ann,f3) $\wedge$ pick(Ann,f4) $\wedge$ pick(Julie,f5) $\wedge$ pick(Julie,f6))

Note that P10 and Q are the most complex sentences.

<u>Step 2:</u> Transform in CNF

P1: flower(Rose)
P2: flower(Lily)
P3: flower(Carnation)
P4: flower(Cala)
P5: flower(Daisy)
P6: flower(Orchid)
P7: girl(Mary)
P8: girl(Ann)
P9: girl(Julie)

P10: $\forall$ x,y,z $\exists$ a,b,c,d,e,f girl(x) $\wedge$ girl(y) $\wedge$ girl(z) $\wedge$ flower(a) $\wedge$ flower(b) $\wedge$ flower(c) $\wedge$ flower(d) $\wedge$ flower(e) $\wedge$ flower(f) $\Rightarrow$ pick(x,a) $\wedge$ pick(x,b) $\wedge$ pick(y,c) $\wedge$ pick(y,d) $\wedge$ pick(z,e) $\wedge$ pick(z,f)

P10 looks like this:
x1 $\wedge$ x2 $\wedge$ x3 $\wedge$ x4 $\wedge$ x5 $\wedge$ x6 $\wedge$ x7 $\wedge$ x8 $\wedge$ x9 $\Rightarrow$ y1 $\wedge$ y2 $\wedge$ y3 $\wedge$ y4 $\wedge$ y5 $\wedge$ y6

- we can translate it in CNF similarly as we would translate:

S: (x1 $\wedge$ x2) $\Rightarrow$ (y1 $\wedge$ y2)

this is equivalent to:
S: $\neg$(x1 $\wedge$ x2) $\vee$ (y1 $\wedge$ y2)

which becomes:
S: $\neg$x1 $\vee$ $\neg$x2 $\vee$ (y1 $\wedge$ y2)

which becomes:
S: ($\neg$x1 $\vee$ $\neg$x2 $\vee$ y1) $\wedge$ ($\neg$x1 $\vee$ $\neg$x2 $\vee$ y2)

or S: S1 $\wedge$ S2
with S1 = $\neg$x1 $\vee$ $\neg$x2 $\vee$ y1

S2 = (¬x1 ∨ ¬x2 ∨ y2

- therefore, we can generalize and see that P10 will be transformed in 6 different sentences:

P10(i): ¬x1 ∨ ¬x2 ∨ ¬x3 ∨ ¬x4 ∨ ¬x5 ∨ ¬x6 ∨ ¬x7 ∨ ¬x8 ∨ ¬x9 ∨ y(i)

i.e.:

………

p10(1): ∀ x,y,z ∃ a,b,c,d,e ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(a) ∨ ¬ flower(b) ∨ ¬ flower(c) ∨ ¬ flower(d) ∨ ¬ flower(e) ∨ ¬ flower(f) ∨ pick(x,a)

p10(2): ∀ x,y,z ∃ a,b,c,d,e ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(a) ∨ ¬ flower(b) ∨ ¬ flower(c) ∨ ¬ flower(d) ∨ ¬ flower(e) ∨ ¬ flower(f) ∨ pick(x,b)

p10(3): ∀ x,y,z ∃ a,b,c,d,e ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(a) ∨ ¬ flower(b) ∨ ¬ flower(c) ∨ ¬ flower(d) ∨ ¬ flower(e) ∨ ¬ flower(f) ∨ pick(y,c)

p10(4): ∀ x,y,z ∃ a,b,c,d,e ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(a) ∨ ¬ flower(b) ∨ ¬ flower(c) ∨ ¬ flower(d) ∨ ¬ flower(e) ∨ ¬ flower(f) ∨ pick(y,d)

p10(5): ∀ x,y,z ∃ a,b,c,d,e ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(a) ∨ ¬ flower(b) ∨ ¬ flower(c) ∨ ¬ flower(d) ∨ ¬ flower(e) ∨ ¬ flower(f) ∨ pick(z,e)

p10(6): ∀ x,y,z ∃ a,b,c,d,e ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(a) ∨ ¬ flower(b) ∨ ¬ flower(c) ∨ ¬ flower(d) ∨ ¬ flower(e) ∨ ¬ flower(f) ∨ pick(z,f)

………..

Now, when eliminating the existential quantifiers in each sentence P10(i), we generate each time six Skolem functions, two for each variable of the predicate girl. In this way we are sure that (1) the flowers are different and (2) no two girls will pick the same flower. We obtain:

………..

p10(1): ∀ x,y,z ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(f1(x)) ∨ ¬ flower(f2(x)) ∨ ¬flower(f3(y)) ∨ ¬ flower(f4(y)) ∨ ¬ flower(f5(z)) ∨ ¬ flower(f6(z)) ∨ pick(x,f1(x))

p10(2): ∀ x,y,z ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(f1(x)) ∨ ¬ flower(f2(x)) ∨ ¬flower(f3(y)) ∨ ¬ flower(f4(y)) ∨ ¬ flower(f5(z)) ∨ ¬ flower(f6(z)) ∨ pick(x,f2(x))

p10(3): ∀ x,y,z ¬ girl(x) ∨ ¬ girl(y) ∨ ¬ girl(z) ∨ ¬ flower(f1(x)) ∨ ¬ flower(f2(x)) ∨ ¬flower(f3(y)) ∨ ¬ flower(f4(y)) ∨ ¬ flower(f5(z)) ∨ ¬ flower(f6(z)) ∨ pick(y,f3(y)) p10(4): $\forall$ x,y,z $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(y,f4(y))

p10(5): $\forall$ x,y,z $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(z,f5(z))

p10(6): $\forall$ x,y,z $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(z,f6(z))

………..

Finally, we can eliminate the universal quantifiers, and obtain the six sentences in CNF

p10(1): $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(x,f1(x))

p10(2): $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(x,f2(x))

p10(3): $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(y,f3(y))

p10(4): $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(y,f4(y))

p10(5): $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(z,f5(z))

p10(6): $\neg$ girl(x) $\vee \neg$ girl(y) $\vee \neg$ girl(z) $\vee \neg$ flower(f1(x)) $\vee \neg$ flower(f2(x)) $\vee$ $\neg$flower(f3(y)) $\vee \neg$ flower(f4(y)) $\vee \neg$ flower(f5(z)) $\vee \neg$ flower(f6(z)) $\vee$ pick(z,f6(z))

P11: $\neg$like(Mary, Carnation)

P12: $\neg$ like(x,a) $\vee$ $\neg$ girl(x) $\vee$ $\neg$ flower(a) $\vee$ pick(x,a)

Because P13: $\exists$ x girl(x) $\Rightarrow$ pick(x,Rose) $\wedge$ pick(x, Carnation)

- When eliminating the existential quantifier in P13, x may be substituted by any constant. The constants for predicate girl are given by P7, P8 and P9. Thus we have three choices:

C1: Mary/x
C2: Ann/x
C3: Julie/x
But let us also remember:
P14: $\neg$pick(Mary, Orchid)

P15: $\neg$pick(Mary, Cala)

If we pick C1, we will have to separate the variables because of P14 and P15, therefore C1 is not a good choice. We can select between choices C2 and C3. Let us try C2. But we shall remember that a different solution could be obtained when C3 is selected.

We obtain:

P13(1): $\neg$girl(Ann) $\vee$ pick(Ann, Rose)
P13(2): $\neg$girl(Ann) $\vee$ pick(Ann, Carnation)

And:

P14: $\neg$pick(Mary, Orchid)

P15: $\neg$pick(Mary, Cala)

Now we turn to the query.
Remember that the negated query is:
$\neg$Q: (pick(Mary, f1) $\wedge$ pick(Mary,f2) $\wedge$ pick(Ann,f3) $\wedge$ pick(Ann,f4) $\wedge$ pick(Julie,f5) $\wedge$ pick(Julie,f6) $\vee$ $\neg$(pick(Mary,f1) $\wedge$ pick(Mary,f2) $\wedge$ pick(Ann,f3) $\wedge$ pick(Ann,f4) $\wedge$ pick(Julie,f5) $\wedge$ pick(Julie,f6))

- The negated query has the form:

$\neg$Q: (a1 $\wedge$ a2 $\wedge$ a3 $\wedge$ a4 $\wedge$ a5 $\wedge$ a6 $\wedge$ a7 $\wedge$ a8 $\wedge$ a9 $\wedge$ a10 $\wedge$ a11 $\wedge$ a12) $\vee$ $\neg$(a1 $\wedge$ a2 $\wedge$ a3 $\wedge$ a4 $\wedge$ a5 $\wedge$ a6 $\wedge$ a7 $\wedge$ a8 $\wedge$ a9 $\wedge$ a10 $\wedge$ a11 $\wedge$ a12)

We can translate it in CNF similarly as we would translate

Q': (a1 $\wedge$ a2) $\vee$ $\neg$(a1 $\wedge$ a2)
Q' becomes (a1 $\wedge$ a2) $\vee$ $\neg$a1 $\vee$ $\neg$a2

In this case, in CNF, we have:
Q'(1): a1 $\vee$ $\neg$a1 $\vee$ $\neg$a2 and
Q'(2): a2 $\vee$ $\neg$a1 $\vee$ $\neg$a2

Therefore ¬Q will be transformed in 6 different queries Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 where Q_i = a_i v ∨ ¬a1 ∨ ¬a2 ∨ ¬a3 ∨ ¬a4 ∨ ¬a5 ∨ ¬a6 ∨ ¬a7 ∨ ¬a8 ∨ ¬a9 ∨ ¬a10 ∨ ¬a11 ∨ ¬a12

In this way we obtain the following six queries from ¬Q:
Q1: pick(Mary,f1) ∨ ¬pick(Mary,f1) ∨ ¬pick(Mary,f2) ∨ ¬pick(Ann,f3) ∨ ¬pick(Ann,f4) ∨ ¬pick(Julie,f5) ∨ ¬pick(Julie,f6)

Q2: pick(Mary,f2) ∨ ¬pick(Mary,f1) ∨ ¬pick(Mary,f2) ∨ ¬pick(Ann,f3) ∨ ¬pick(Ann,f4) ∨ ¬pick(Julie,f5) ∨ ¬pick(Julie,f6)

Q3: pick(Ann,f3) ∨ ¬pick(Mary,f1) ∨ ¬pick(Mary,f2) ∨ ¬pick(Ann,f3) ∨ ¬pick(Ann,f4) ∨ ¬pick(Julie,f5) ∨ ¬pick(Julie,f6)

Q4: pick(Ann,f4) ∨ ¬pick(Mary,f1) ∨ ¬pick(Mary,f2) ∨ ¬pick(Ann,f3) ∨ ¬pick(Ann,f4) ∨ ¬pick(Julie,f5) ∨ ¬pick(Julie,f6)

Q5: pick(Julie,f5) ∨ ¬pick(Mary,f1) ∨ ¬pick(Mary,f2) ∨ ¬pick(Ann,f3) ∨ ¬pick(Ann,f4) ∨ ¬pick(Julie,f5) ∨ ¬pick(Julie,f6)

Q6: pick(Julie,f6) ∨ ¬pick(Mary,f1) ∨ ¬pick(Mary,f2) ∨ ¬pick(Ann,f3) ∨ ¬pick(Ann,f4) ∨ ¬pick(Julie,f5) ∨ ¬pick(Julie,f6)

<u>Step 3:</u> <u>Perform Resolution</u>
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R1: By using the following substitutions **SUBST1**: {x/Mary; y/Ann; z/Julie;
f1(x)/Lily ; f2(x)/Daisy; f3(y)/Rose ; f4(y)/Carnation ;
f5(z)/Orchid; f6(z)/Cala} and applying a chain of resolutions:
-in P1 & P10(3) ⇒P10(3)_1 & P2 ⇒P10(3)_2 & P3 ⇒P10(3)_3 & P4 ⇒P10(3)_5 & P5 ⇒P10(3)_6 & P6⇒P10(3)_7 & P7⇒P10(3)_8 & P8⇒P10(3)_9 & P9⇒

pick(Ann, Rose)

R2: By using SUBST1 and applying a chain of resolutions:
P1 & P10(4) ⇒P10(4)_1 & P2⇒P10(4)_2 & P3⇒P10(4)_3 & P4⇒P10(4)_4 & P5⇒P10(4)_5 & P6⇒P10(4)_6 & P7⇒P10(4)_7 & P8⇒P10(4)_8 & P9⇒

pick(Ann, Carnation)

R3: By using SUBST1 and applying a chain of resolutions
P1 & & P10(5) ⇒P10(5)_1 & P2 ⇒P10(5)_2 & P3⇒P10(5)_3 & P4⇒P10(5)_4 & P5⇒P10(5)_5 & P6⇒P10(5)_6 & P7⇒P10(5)_7 & P8⇒P10(5)_8 & P9 ⇒

pick(Julie, Orchid)

R4: By using SUBST1 and applying a chain of resolutions:
P1 & & P10(6) ⇒P10(6)_1 & P2⇒P10(6)_2 & P3⇒P10(6)_3 & P4 ⇒P10(6)_4 & P5 ⇒P10(6)_5 & P6⇒P10(6)_6 & P7⇒P10(6)_7 & P8⇒P10(6)_8 & P9 ⇒

pick(Julie, Calla)

R5: By using SUBST1 and applying a chain of resolutions:
P1 & P10(1) ⇒P10(10)_1 P2 ⇒P10(10)_2 & P3 ⇒P10(10)_3 & P4⇒P10(10)_4 & P5 ⇒P10(10)_5 & P6⇒P10(10)_6 & P7⇒P10(10)_7 & P8 ⇒P10(10)_8 & P9 ⇒

pick(Mary, Lily)

R6: By using SUBST1 and applying a chain of resolutions:
P1 & P10(2) ⇒P10(2)_1 P2⇒P10(2)_2 & P3⇒P10(2)_3 & P4 ⇒P10(2)_4 & P5⇒P10(2)_5 & P6⇒P10(2)_6 & P7⇒P10(2)_7 & P8⇒P10(2)_8 & P9⇒

pick(Mary, Daisy)

R(Q1): By using **SUBST2**: {f1/Lily; f2/Daisy; f3/Rose; f4/Carnation; f5/Orchid; f6/Cala} and applying a chain of resolutions:
Q1 & R1⇒Q1_1 & R2⇒Q1_2 & R3⇒Q1_3 & R4⇒Q1_4 & R5⇒Q1_5 & R6 ⇒

pick(Mary, Lily)

R(Q2): By using SUBST2 and applying a chain of resolutions:
Q2 & R1⇒Q2_1 & R2⇒Q2_2 & R3⇒Q2_3 & R4⇒Q2_4 & R5⇒Q2_5 & R6 ⇒

pick(Mary, Daisy)

R(Q3): By using SUBST2 and applying a chain of resolutions:
Q3 & R1 ⇒Q3_1 & R2⇒Q2_2 & R3⇒Q2_3 & R4⇒Q2_4 & R5⇒Q2_5 & R6 ⇒

pick(Ann, Rose)

R(Q4): By using SUBST2 and applying a chain of resolutions:
Q4 & R1⇒Q4_1 & R2⇒Q4_2 & R3 ⇒Q4_3 & R4 ⇒Q4_4 & R5⇒Q4_5 & R6 ⇒

pick(Ann, Carnation)

R(Q5): By using SUBST2 and applying a chain of resolutions:
Q5 & R1 ⇒Q5_1 & R2 ⇒Q5_2 & R3⇒Q5_3 & R4⇒Q5_4 & R5⇒Q5_5 & R6 ⇒

pick(Julie, Orchid)

R(Q6): By using SUBST2 and applying a chain of resolutions:
Q6 & R1 ⇒Q6_1 & R2⇒Q6_2 & R3⇒Q6_3 & R4⇒Q6_4 & R5⇒Q6_5 & R6 ⇒

pick(Julie, Calla)

From R(Q1) to R(Q6) we have the answers:

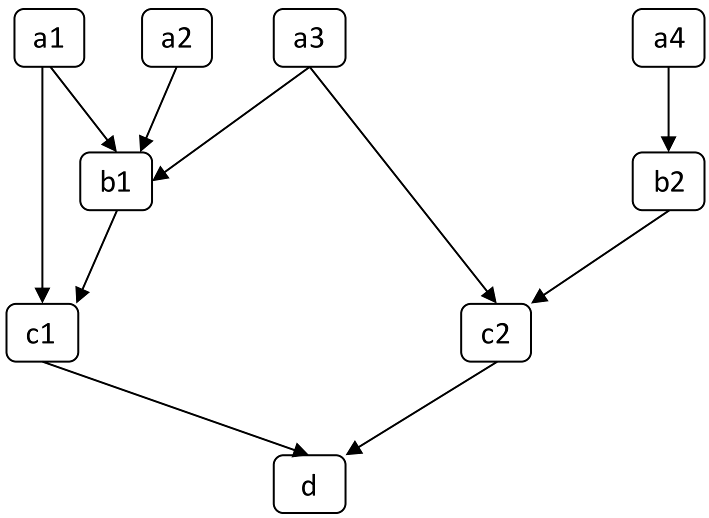
Solution 1:
Mary picks a lily and a daisy.
Ann picks a rose and a carnation.
Julie picks an orchid and a calla.

We could also have proved that:

Solution 2:
Mary picks a lily and a daisy.
Ann picks a calla and a orchid.
Julie picks a rose and a carnation.

<u>Problem 3.</u> Bayesian Networks.

Given the following Bayesian network:



at node a_1:

Prob(a_1)
0.6

at node a_2:

Prob(a_2)
0.8

at node a_3:

Prob(a_3)
0.4

at node a_4:

Prob(a_4)
0.3

at node b_1:

a_1	a_2	a_3	Prob(b_1)
0	0	0	0.5
0	0	1	0.2
0	1	0	0.7
0	1	1	0.9
1	0	0	0.4
1	0	1	0.1
1	1	0	0.3
1	1	1	0.0

at node b_2:

a_4	Prob(b_2)
0	0.4
1	0.3

at node c_1:

a_1	b_1	Prob(c_1)
0	0	0.3
0	1	0.1
1	0	0.5
1	1	0.8

at node c_2:

a_3	b_2	Prob(c_2)
0	0	0.2
0	1	0.6
1	0	0.3
1	1	0.1

at node d:

c_1	c_2	Prob(d)
0	0	0.4
0	1	0.3
1	0	0.6
1	1	0.2

1. Compute the probability that $b_1 = 1$, $c_2 = 0$ and $d = 1$.
2. Compute the probability of b_1 given that d happened.
3. If $P(a_1 = 1) = x$, what is the value of x such that it is more likely that d happened rather than it did not happen.

Solution:

1. $P(b_1 = 1, c_2 = 0, d = 1) = P((b_1 = 1) \times P(c_2 = 0) \times P(d)$

   -Where:
   $P(b_1) = 0.5 \times (1 - 0.6) \times (1 - 0.8) \times (1 - 0.4) + 0.2 \times (1 - 0.6) \times (1 - 0.8) \times 0.4$
   $+0.7 \times (1 - 0.6) \times 0.8 \times (1 - 0.4) + 0.9 \times (1 - 0.6) \times 0.8 \times (1 - 0.4) +$
   $+0.4 \times 0.6 \times (1 - 0.8) \times (1 - 0.4) + 0.1 \times 0.6 \times (1 - 0.8) \times 0.4$
   $+ 0.3 \times 0.6 \times 0.8 \times (1 - 0.4) =$
   $= 0.024 + 0.0064 + 0.1344 + 0.1152 + 0.0288 + 0.0048 + 0.0864 = 0.4$

   Now we know $P(b_1) = 0.4$

   To compute $P(c_2 = 0)$ we need to compute $P(c_2)$ which in turn requires that we compute $P(b_2)$

   $P(b_2) = 0.4 \times (1 - 0.3) + 0.3 \times 0.3 = 0.4 \times 0.7 + 0.09 = 0.28 + 0.09 = 0.37$

   We now have:
   $P(\neg b_2) = 0.63$
   $P(c_2) = (1 - 0.4) \times 0.63 \times 0.2 + (1 - 0.4) \times 0.37 \times 0.6 + 0.4 \times 0.63 \times 0.3$
   $+ 0.4 \times 0.37 \times 0.1 = 0.0756 + 0.1332 + 0.0756 + 0.0148 = 0.2992$
   We now have:
   $P(\neg c_2) = 0.7008$

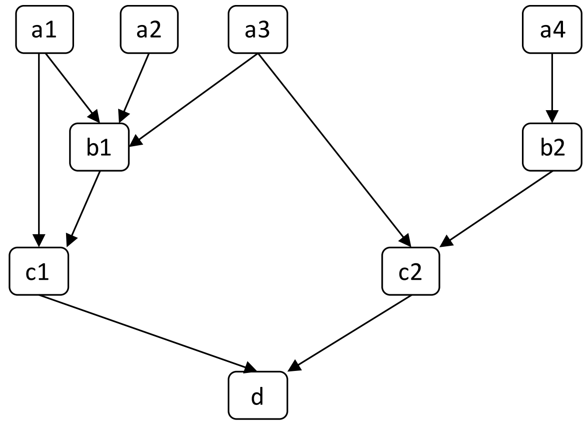
   To compute $P(d = 1)$ we need to also compute $P(c_1)$
   $P(c_1) = (1 - 0.6) \times (1 - 0.4) \times 0.3 + (1 - 0.6) \times 0.4 \times 0.1 + 0.6 \times 0.6 \times 0.5$
   $+ 0.4 \times 0.6 \times 0.8 = 0.256 + 0.016 + 0.18 = 0.46$

   We now have:
   $P(\neg c_1) = 0.54$

   Finally, we can compute
   $P(d = 1) = P(\neg c_1) \times P(\neg c_2) \times 0.4 + P(\neg c_1) \times P(c_2) \times 0.3 + P(c_1) \times P(\neg c_2) \times 0.6$
   $+ P(c_1) \times P(c_2) \times 0.2$
   $= 0.54 \times 0.7008 \times 0.4 + 0.54 \times 0.2992 \times 0.3 + 0.46 \times 0.7008 \times 0.6$
   $+ 0.46 \times 0.2992 \times 0.2 = 0.1514 + 0.0485 + 0.1934 + 0.0275$
   $= 0.4208$

   We have now everything that will enable us to compute:
   $P(b_1 = 1, c_2 = 0, d = 1) = P((b_1 = 1) \times P(c_2 = 0) \times P(d) = 0.4 \times 0.7008 \times 0.4208$
   $= 0.1241$

2. $P(b_1|d) = P(d|b_1) \times P(b_1)/P(d)$



We already know that $P(b_1) = 0.4$ and $P(d) = 0.4208$

Now we need to compute $P(c_1|b_1)$ because d depends on c_1 and on c_2. We also know $P(c_2) = 0.2992$

First:

$$P(c_1|b_1) = P(c_1|b_1, a_1)P(a_1) + P(c_1|b_1, \neg a_1)P(\neg a_1) = 0.8 * 0.6 + 0.1 * 0.4$$
$$= 0.48 + 0.04 = 0.52$$

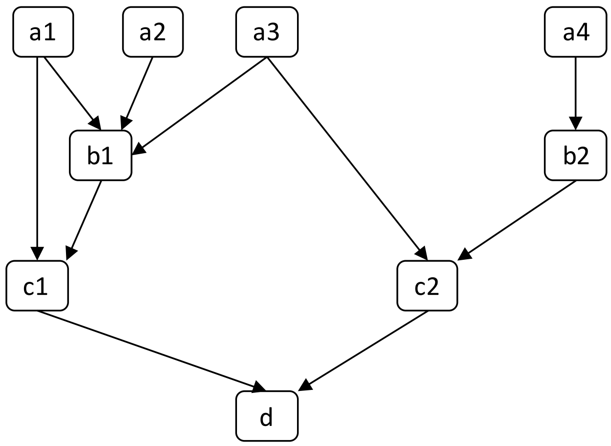
Then:

$$P(d|b_1) = P(d|c_1, c_2)P(c_1|b_1)P(c_2) + P(d|\neg c_1, c_2)P(\neg c_1|b_1)P(c_2)$$
$$+ P(d|c_1, \neg c_2)P(c_1|b_1)P(\neg c_2) + P(d|\neg c_1, \neg c_2)P(\neg c_1|b_1)P(\neg c_2)$$
$$= 0.2 * 0.52 * 0.2992 + 0.3 * 0.48 * 0.2992 + 0.6 * 0.52 * 0.7008$$
$$+ 0.4 * 0.48 * 0.7008 = 0.0311 + 0.0431 + 0.2186 + 0.1346$$
$$= 0.4274$$

Now:

$$P(b_1|d) = 0.4274 \times \frac{0.4}{0.4208} = 0.4063$$

3. If $P(a_1 = 1) = x$, if we compute $P(d = 1)$, we need to compute before that $P(c_1)$ as well as $P(b_1)$ before that!!!



$$
\begin{aligned}
P(b_1) = {} & P(b_1|a_1,a_2,a_3)P(a_1)P(a_2)P(a_3) + P(b_1|a_1,a_2,\neg a_3)P(a_1)P(a_2)P(\neg a_3) \\
& + P(b_1|a_1,\neg a_2,a_3)P(a_1)P(\neg a_2)P(a_3) \\
& + P(b_1|a_1,\neg a_2,\neg a_3)P(a_1)P(\neg a_2)P(\neg a_3) \\
& + P(b_1|\neg a_1,a_2,a_3)P(\neg a_1)P(a_2)P(a_3) \\
& + P(b_1|\neg a_1,a_2,\neg a_3)P(\neg a_1)P(a_2)P(\neg a_3) \\
& + P(b_1|\neg a_1,\neg a_2,a_3)P(\neg a_1)P(\neg a_2)P(a_3) \\
& + P(b_1|\neg a_1,\neg a_2,\neg a_3)P(\neg a_1)P(\neg a_2)P(\neg a_3) \\
& = 0 + 0.3x(0.8)(0.6) + 0.1x(0.2)(0.4) + 0.4x(0.2)(0.6) \\
& + 0.9(1-x)(0.8)(0.4) + 0.7(1-x)(0.8)(0.6) + 0.2(1-x)(0.2)(0.4) \\
& + (0.5)(1-x)(0.2)(0.6) \\
& = 0.144x + 0.008x + 0.048x + 0.288(1-x) + 0.336(1-x) + 0.16(1-x) \\
& + 0.06(1-x) = 0.7 - 0.5x
\end{aligned}
$$

We now have:
$P(b_2) = 0.3 + 0.5x$

Now we can compute

$$
\begin{aligned}
P(c_1) = {} & P(c_1|a_1,b_1)P(a_1)P(b_1) + P(c_1|a_1,\neg b_1)P(a_1)P(\neg b_1) + P(c_1|\neg a_1,b_1)P(\neg a_1)P(b_1) \\
& + P(c_1|\neg a_1,\neg b_1)P(\neg a_1)P(\neg b_1) \\
& = 0.8x(0.7-0.5x) + 0.5x(0.3+0.5x) + 0.1(1-x)(0.7-0.5x) \\
& + 0.3(1-x)(0.3+0.5x) \\
& = 0.56x - 0.4x^2 + 0.15x + 0.25x^2 + 0.07 - 0.05x - 0.07x + 0.05x^2 + 0.09 \\
& + 0.15x - 0.09x + 0.15x^2 = 0.16 + 0.65x + 0.05x^2
\end{aligned}
$$

Similarly:
$P(\neg c_1) = 0.84 - 0.65x - 0.05x^2$

This allows us to compute $P(d=1)$:

$$
\begin{aligned}
P(d) = {} & P(d|c_1,c_2)P(c_1)P(c_2) + P(d|c_1,\neg c_2)P(c_1)P(\neg c_2) + P(d|\neg c_1,c_2)P(\neg c_1)P(c_2) \\
& + P(d|\neg c_1,\neg c_2)P(\neg c_1)P(\neg c_2) \\
& = 0.2(0.16+0.65x+0.05x^2)(0.2992) \\
& + 0.6(0.16+0.65x+0.05x^2)(0.7008) \\
& + 0.3(0.84-0.65x-0.05x^2)(0.2992) \\
& + 0.4(0.84-0.65x-0.05x^2)(0.7008) \\
& = 0.0359+0.0673+0.0754+.2355 \\
& + x(0.0389+0.2733-0.0583-0.1822) \\
& + x^2(0.0030+0.0210-0.0045-0.0140) = 0.4141+0.0717x+0.0055x^2 \\
& > 0.5
\end{aligned}
$$

If we desire that $P(d) > 0.5$ than we require that:

$$0.4141+0.0717x+0.0055x^2 > 0.5$$

This is equivalent with:

$0.0055x^2+0.0717x-0.5959 > 0$

This is a second-degree inequality. If we think of the quadratic equation:

$$f(x) = 0.0055x^2+0.0717x-0.5959 = 0$$

A second-degree polynomial also referred as a quadratic equation can be expressed as below:

$ax^2+bx+c=0$. In our case $a=0.0055$; $b=0.0717$; and $c=-0.5959$

to solve the equation we can use the quadratic formulas as shown below:

$$x_1 = [-b+(b^2-4ac)^{\frac{1}{2}}]/2a$$
$$x_2 = [-b-(b^2-4ac)^{\frac{1}{2}}]/2a$$

A quadratic equation has two solutions when $b^2-4ac > 0$, which is our case! We can compute $x_1 = -14.14$ and $x_2 = 1.104$

We also know that since $a = 0.0055 > 0$; $f(x) > 0$ only when:

$x > 1.104$ or $x < -14.14$

But because:

$0 < x < 1$, because $P(a_1=1) = x$; we find that <u>there are no values of x</u> which allow $P(d) > 0.5$