The University of Texas at Dallas
CS 6364
Artificial Intelligence
Fall 2020
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<u>Homework 4: 100 points</u>
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Submit only in eLearning

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<u>PROBLEM 1:</u> Qualifying Uncertainty (**20 points**)

Problem 13.21 from the Textbook at page 509. (It should start with (Adapted from Pearl (1988)). Suppose you are a witness to a nightime….

- **(a) 10 points**
- **(b) 10 points**

<u>Solution 1:</u>

From the problem,

1. All taxis are blue or green.
2. Taxi is blue sweared under oath.
3. Discrimination between blue and green is 75% reliable.

a.) Let us take two random variables B and LB, where B means taxi is blue and LB means that the taxi look blue. Now, discrimination between blue and green is 75% reliable.

So, $P(LB \mid B) = 0.75$ and

$P(\neg LB \mid \neg B) = 0.75$

To evaluate: $P (B \mid LB) = P(LB \mid B) * P(B) \sim 0.75 * P(B)$ and

$P(\neg B \mid LB) \sim P(LB \mid \neg B) * P(\neg B) \sim 0.25* (1-P(B))$

From the above two equation, it seems that the probability of blue taxi is required to make a judgement. For example, if we knew that all the taxis are blue, then $P(B) = 1$, so $P(B|LB) = 1$

b.) Let two random variables be AC and OW, being actual color of taxi and color observed by witness respectively.

P(AC = Green | OW = Green) = 0.75

P(AC = Green | OW = Blue) = 0.25

P(AC = Blue | OW = Green) = 0.25

P(AC = Blue | OW = Blue) = 0.75

Since '9 out of 10 taxis are green':

P(AC = Green) = 0.9

P(AC = Blue) = 0.1

For simplicity in the further equation let us denote Green by G and Blue by B.

To evaluate: P(AC=B | OW=B)

=> P(AC=B | OW=B) = P(AC=B, OW=B) / p(OW = B)

Now P(AC=B, OW=B) = P(AC=B | OW=B) * P(OW=B) = P(OW=B | AC=B) * (AC=B)

Using the above rule, we get

$$P(AC=B \mid OW=B) = \frac{P(OW=B \mid AC=B) * P(AC=B)}{P(OW = B, AC= G) + P(OW = B; AC = B)}$$

$$= \frac{P(OW=B \mid AC=B) * P(AC=B)}{p(OW = B \mid AC= G)*P(AC=G) + P(OW = B; AC = B)*P(AC=B)}$$

Substituting the known values, we get:

=> P(AC=B | OW=B) = (0.75 * 0.1) / (0.25*0.9 + 0.75*0.1) = 0.25

Probability is less than half, so we can conclude that the witness is in fact wrong.

PROBLEM 2: Naïve Bayesian Reasoning (**25 points**)

Consider a traveler that wants to climb the Everest. He gets to Nepal in summer and also finds an experienced guide. Use Naive Bayesian reasoning to decide if the traveler will climb to 1000 ft from the top of the Everest based (**15 points**) on the following information:

1. 10% of all climbers get to 1000 ft from the top of the Everest.

2. Among all travelers who get to 1000 ft from the top of the Everest, 90% went to Nepal in summer and 80% used an experienced guide.

3. 50% of climbers that cannot get to 1000 ft from the top of the Everest went to Nepal in summer and 30% were able to find an experienced guide.

Explain your conclusion. (**10 points)**

Solution 2:

1. 10% of all climbers get to 1000 ft from the top of the Everest.
2. Out of 10% total, 90% went to Nepal in summer and 80% used an experienced guide.
3. 50% of climbers that cannot get to 1000 ft from the top of the Everest went to Nepal in summer and 30% were able to find an experienced guide.

Let us take few random variables,
CL represents all climbers, NP represents Nepal and EV represents Everest.

$$\text{So, } P(CL \mid NP, EV) = \frac{P(NP \mid CL) * P(EV \mid CL) * P(CL)}{P(NP, EV)}$$

$$= (0.9 * 0.8 * 0.1) / P(NP, EV)$$

$$= 0.072 / P(NP,EV) \qquad \text{-> Eqn 1}$$

Now, we know that $P(CL \mid NP, EV) + P(\neg CL \mid NP, EV) = 1$

$$\text{So, } P(\neg CL \mid NP, EV) = \frac{P(NP,EL \mid \neg CL) * P(\neg CL)}{P(NP,EV)}$$

Now, NP and EV are conditionally independent.
So, by Naïve Bayes,

$$P(\neg CL \mid NP, EV) = \frac{P(NP \mid \neg CL) * P(E \mid \neg CL) * P(\neg CL)}{P(N,E)}$$

$$= (0.5 * 0.3 * 0.9) / P(NP,EV)$$

$$= 0.135 / P(NP,EV) \qquad \text{-> Eqn 2}$$

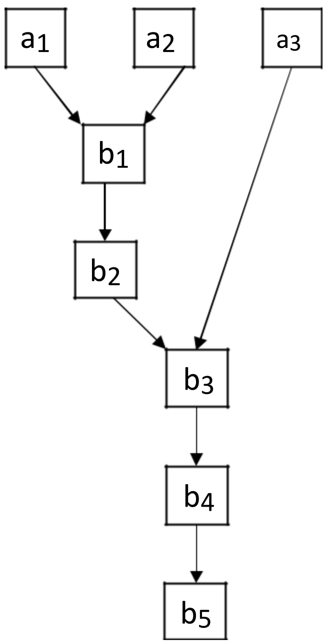
From Eq1, Eq2 and $P(CL \mid NP, EV) + P(\neg CL \mid NP, EV) = 1$
=> $0.072 / P(NP,EV) + 0.135 / P(NP,EV) = 1$
Solving it, we get
=> $P(N,E) = 0.207$
So, $P(CL \mid NP, EV) = 0.072 / P(NP,EV)$

$$= 0.072 / 0.207$$

$$= 0.3478$$

Given the conditions in the problem, the climber would not be able to climb to 1000 ft from the top of the Everest since the probability to climb is 0.1.

PROBLEM 3: Inference with Bayesian Networks (**55 points**)
Given the following Bayesian Network:



where:

at node a1 :

P(a1)
0.7

at node a2:

P(a2)
0.8

at node a3:

P(a3)
0.9

at node b1 :

a1	a2	P(b1)
0	0	0.2
0	1	0.6
1	0	0.7
1	1	0.9

at node b2:

b1	P(b2)
0	0.6
1	0.8

at node b3:

a3	b2	P(b3)
0	0	0
0	1	0.7
1	0	0.8
1	1	1

at node b4 :

b3	P(b4)
0	0.1
1	0.7

at node b5:

b4	P(b5)
0	0
1	1

You are asked to compute several probabilities by considering the above Bayesian network. In each case your answer can be a number or an expression that can be converted into a number by a pocket calculator.

A) (**5 points**) Compute the probability that: a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0

B) (**15 points**) Compute the probability that b5 = 1
C) (**10 points**) Compute the probability that b5 = 1 given that:
a1 = 1, a2 = 1, a3 = 1,
b1 = 0, b2 = 0, b3 = 0
D) (**5 points**) Compute the probability that b3=0 given that: b5=1
E) (**20 points**) The CPT in node a3 is changed to:

at node a3 :

P(a3)
x

where the value of x is unknown. What values of x would make it make it more likely that b5 happened than that b5 did not happen?

Solution 3:

A) Compute the probability that: a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0
=> P(a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0)
=> P(a1=1) * P(a2=1) * P(a3=1) * P(b1=0 | a1=1, a2=1) * P(b2=0 | b1=0) * P(b3=0 | b2=0, a3=1) * P(b4 = 0 | b3=0) * P(b5=0 | b4=0)
=> 0.7 * 0.8 * 0.9 * (1-0.9) * (1-0.6) * (1-0.8) * (1-0.1) * (1-0)
=> 0.0036288

B) Compute the probability that b5 = 1
=> P(b5=1)
=> P(b5=1 | b4=0) * P(b4=0) + P(b5=1 | b4=1) * P(b4=1)
=> 0 + 1*P(b4=1)
=> P(b4=1)
=> 0.7

C) Compute the probability that b5 = 1 given a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0
=> Node b5 is dependent on the node b4.
=> P(b5=1) = P(b4=1)
=> Also b4 is independent of other nodes if b3 is known. From the given problem b3=0
=> P(b5=1) = P(b4=1 | b3 =0)
=> 0.1

D) Compute the probability that b3=0 given that: b5=1
P(b3=0|b5=1) = P(b5=1|b3=0)*P(b3=0) / P(b5=1)
=> P(b5=1|b3=0) = 0.1
=> P(b3=0) = 0.0921
=> P(b5=1) = 0.64474
=> (0.1 * 0.0921) / 0.64474
=> 0.01428

E) CPT for node a3: P(a3) = x

Need to find x such that $P(b5=1) > P(b5=0)$

=> $P(b4=1 \mid b3=1) > P(b4=1 \mid b3=0)$

=> $P(b5=1)$ is possible only if b4=1. Now b4 is more probable if b3=1 from above.

=> Also, b3=1 is more likely when a3=1.

=> So a3=1 is more likely when $P(a3=1)$ => x=1

=> Therefore, x=1 is more probable for $P(b5=1) > P(b5=0)$