University of Texas at Dallas
CS 6364
Artificial Intelligence
Fall 2020

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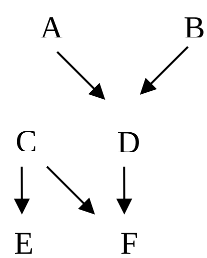
Bayesian Networks Problems
Set 2

Problem 1.

A) Given the following Bayesian Net:



With the CPTs:

At node A:

A	P(A)
T	0.4
F	0.6

At node B:

B	P(B)
T	0.8
F	0.2

At node C:

C	P(C)
T	0.3
F	0.7

At node D:

A	B	P(D)
T	T	1
T	F	0.5
F	T	0.9
F	F	0

At node E:

C	P(E)
T	0.9
F	0.7

At node F:

C	D	P(F)
T	T	0.4
T	F	0.3
F	T	0.2
F	F	0.1

Compute the probability of each node in the network.

Solution *Method 1 (Possible Worlds)*
P(A)=0.4 P(B)=0.8 P(C)=0.3

At node D:

A	B	P(D)	P(W)
T	T	1	0.4 x 0.8 = 0.32
T	F	0.5	0.4 x (1 - 0.8) = 0.08
F	T	0.9	(1 - 0.4) x 0.8 = 0.48
F	F	0	(1 - 0.4) x (1 - 0.8) = 0.12

We compute: P(D) = 1 x 0.32 + 0.5 x 0.08 + 0.9 x 0.48 + 0 x 0.12 = 0.792

At node E:

C	P(E)	P(W)
T	0.9	0.3
F	0.7	(1-0.3)=0.7

P(E) = 0.9 x 0.3 + 0.7 x 0.7 = 0.76

At node F:

C	D	P(F)	P(W)
T	T	0.4	0.3 x 0.792 = 0.2376
T	F	0.3	0.3 x (1 - 0.792) = 0.0624
F	T	0.2	(1 - 0.3) x 0.792 = 0.5544
F	F	0.1	(1 - 0.3) x (1 - 0.792) = 0.1456

P(F) = 0.4 x 0.2376 + 0.3 x 0.0624 + 0.2 x 0.5544 + 0.1 x 0.1456 = 0.2392

Method 2 (Variable Elimination)
Build initial factors:

A	g(A)
1	0.4
0	0.6

P(A)=0.4

B	g(B)
1	0.8
0	0.2

P(B)=0.8

C	g(C)
0	0.3
1	0.7

P(C)=0.3

A	B	D	g(D)
1	1	1	1
1	1	0	0
1	0	1	0.5
1	0	0	0.5
0	1	1	0.9
0	1	0	0.1
0	0	1	0
0	0	0	1

To compute P(D) **we need to compute:**
h1 = g(D) x g(A) x g(B)

A	B	D	h1
1	1	1	1 x 0.4 x 0.8 = 0.32
1	1	0	0 x 0.4 x 0.8 = 0
1	0	1	0.5 x 0.4 x 0.2 = 0.04
1	0	0	0.5 x 0.4 x 0.2 = 0.04
0	1	1	0.9 x 0.6 x 0.8 = 0.402
0	1	0	0.1 x 0.6 x 0.8 = 0.048
0	0	1	0 x 0.6 x 0.2 = 0
0	0	0	1 x 0.6 x 0.2 = 0.12

Then eliminate A => h2

B	D	h2
1	1	0.32 + 0.402 = 0.722
1	0	0 + 0.048 = 0.048
0	1	0.04 + 0 = 0.04
0	0	0.04 + 0.12 = 0.16

Then eliminate B => h3

D	h3
1	0.722 + 0.04 = 0.762
0	0.048 + 0.16 = 0.204

P(D) = alpha <0.762, 0.204> = <0.792, 0.208>
P(D) = 0.792

Then we still need to compute P(E) and P(F). Let us first compute P(E). To build the factors for E and C we have:

C	E	g(E)
1	1	0.9
1	0	0.1
0	1	0.7
0	0	0.3

and

C	g(C)
1	0.3
0	0.7

Computing P(E) results from generating a new factor h4 = g(E) x g(C)

C	E	h4
1	1	0.9 x 0.3 = 0.27
1	0	0.1 x 0.3 = 0.03
0	1	0.7 x 0.7 = 0.49
0	0	0.3 x 0.7 = 0.21

Now we can eliminate C from h4=> h5, a new factor:

E	h5
1	0.27 + 0.49 = 0.76
0	0.03 + 0.21 = 0.24

P(E) = 0.76

Finally, we generate the factor g(F):

C	D	F	g(F)
1	1	1	0.4
1	1	0	0.6
1	0	1	0.3
1	0	0	0.7
0	1	1	0.2
0	1	0	0.8
0	0	1	0.1
0	0	0	0.9

Now we have two options:

Option1:
Since we already know the P(C) and P(D) we could eliminate them from g(F). Let us first use P(C)=0.3 and create a new factor h6 by eliminating C from g(F), while multiplying the values of g(F) with the corresponding values for P(C) or P(¬C):

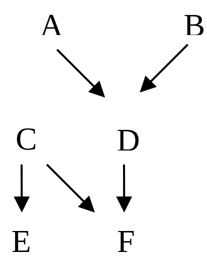
D	F	h6
1	1	0.4 x **0.3** + 0.2 x **0.7** = 0.12 + 0.14 = 0.26
1	0	0.6 x **0.3** + 0.8 x **0.7** = 0.18 + 0.56 = 0.74
0	1	0.3 x **0.3** + 0.1 x **0.7** = 0.09 + 0.07 = 0.16
0	0	0.7 x **0.3** + 0.9 x **0.7** = 0.21 + 0.63 = 0.84

Now we eliminate D from h6, while multiplying the values of h6 with the corresponding values of P(D)=> h7

F	h7
1	0.26 x **0.792** + 0.16 x **0.208** = 0.2392
0	0.74 x **0.792** + 0.84 x **0.208** = 0.7608

Option 2:
Compute the factors and eliminate variables from scratch. This entails that you will need to compute a new factor h8=g(C) x g(D) x g(F). From h8, by eliminating C, you will obtain h9. Then by eliminating D from h9, you should obtain h7.□

B) Compute the probability of F given B and C: P(F|BC)



Method 1: (Possible Worlds)

The parent of F is D $\Rightarrow$ need to compute the conditional probability of D given B and C.

A	B	P(D)	P(W)
1	1	1	0.4 x 1 = 0.4
1	0	0.5	0.4 x 0 = 0
0	1	0.9	(1 - 0.4) x 1 = 0.6
0	0	0	(1 - 0.4) x 0 = 0

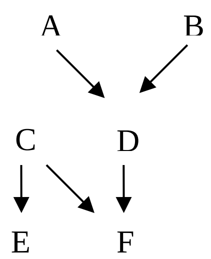
P(D) = 1 x 0.4 + 0.9 x 0.6 = 0.94

At node F:

C	D	P(F)	P(W)
1	1	0.4	1 x 0.94 = 0.94
1	0	0.3	1 x (1 - 0.94) = 0.06
0	1	0.2	0 x 0.94 = 0
0	0	0.1	0 x (1 - 0.94) = 0

P(F) = 0.4 x 0.94 + 0.3 x 0.06 = <u>0.394</u>

Method 2: (Variable Elimination) P(F|BC)



Since F depends on D and C, we first need to compute P(D|BC). Let us build initial factors for A and D when P(B)=1; P(C)=1:

A	g(A)
1	0.4
0	0.6

A	D	g1(D)
1	1	1
1	0	0
0	1	0.9
0	0	0.1

To compute P(D|BC) **we need to compute:**
 f1 = g1(D) x g(A)

A	D	f1
1	1	1 x 0.4=0.4
1	0	0 x 0.4=0
0	1	0.9 x 0.6=0.54
0	0	0.1 x 0.6=0.06

Then eliminate A from f1=> f2

D	f2
1	0.4_0.54=0.94
0	0.06

We can see how we can generate the factor for F, when P(C)=1 and P(B)=1. We notice that F is influenced by both C and D, but P(C)=1, therefore we have the factor g1 for F as:

D	F	g1(F)
1	1	0.4
1	0	0.6
0	1	0.3
0	0	0.7

The new factor for F when P(C)=1 and P(B)=1 results from f2 x g1(F)=f3

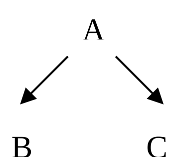
D	F	f3
1	1	0.4 x P(D) = 0.4 x 0.94 = 0.376
1	0	0.6 x P(D) = 0.6 x 0.94 = 0.564
0	1	0.3 x P(¬D) = 0.3 x 0.06 = 0.018
0	0	0.7 x P(¬D) = 0.7 x 0.06 = 0.042

Now we can reduce D from f3 ⇒ generate a new factor f4

F	f4
1	0.376+0.018=**0.394**
0	0.564+0.042=0.606

Problem 2

Given the Bayesian Network:



and the CPTs:

At node A:

A	P(A)
T	a1
F	(1-a1)

At node B:

A	P(B)
T	b1
F	b0

At node C:

A	P(C)
T	c1
F	c0

Compute P(A|B,¬C)

Solution:

Use Bayes' Rule:

$$P(A|B,\neg C) = \frac{P(B,\neg C|A)\times P(A)}{P(B,\neg C)}$$

a) Compute P(A) = a1

b) Compute P(B,¬C|A) = P(B|A) x P(¬C|A) = b1(1-c1)

c) Compute P(B, ¬C) or the normalization constant, *alpha*.

For computing *alpha* we need to compute also P(B,¬C|¬A) x P(¬A) ⇒

1) P(B,¬C|¬A) = P(B|¬A) x P(¬C|¬A) = b0 x (1 - c0)

2) Then P(B, ¬C|¬A) x P(¬A) = b0×(1-c0)×(1-a1)

$$alpha = \frac{1}{a1\times b1\times(1-c1)+b0\times(1-c0)\times(1-a1)}$$

Then we have:

$$P(A|B,\neg C) = \frac{a1\times b1\times(1-c1)}{a1\times b1\times(1-c1)+b0\times(1-c0)\times(1-a1)}$$

Problem 3

Given the following Bayesian network:

$$\begin{array}{c} A \\ \downarrow \\ B \end{array}$$

with the following CPTs:

At node A:

A	P(A)
T	a1
F	(1-a1)

At node B:

B	P(B)
T	b1
F	b0

Compute P(A|B).

Solution:

$$P(A\mid B) = \frac{P(B\mid A)\times P(A)}{P(B)}$$

Compute P(A) ⇒ P(A) = a1

P(B|A) ⇒ P(B|A) = b1

P(B) ⇒ P(B) = a1×b1 + (1-a1)×b0

because

A	P(B)	P(W)
T	b1	a1
F	b0	(1-a1)

then $$P(A \mid B) = \frac{a1 \times b1}{a1 \times b1 + (1 - a1) \times b0}$$

We could have computed P(A|B) by considering the normalization constant as well:

P(A|B) = alpha × P(B|A) × P(A) = alpha × a1 × b1

we also need to compute P(¬A|B) = alpha × P(B|¬A) × P(¬A)

But P(¬A) = 1 - a1 and P(B|¬A) = b0

⇒ alpha × (a1 × b1 + b0 × (1 - a1) = 1 *therefore* $$alpha = \frac{1}{a1 \times b1 + b0 \times (1 - a1)}$$
