Dominating Set

Input A graph $G=(V,E)$ and int. k

Quest. Does G have a dominating set of size $\leq k$?

(A set $D \subseteq V$ is dominating set if $\forall v \in V$: $v \in D$ or v is adj to a vertex in D)

Theorem. Dominating set problem is NP-complete.

Proof. We show $VC \leq_p DS$

Given an instance $(G(V,E), k)$ of VC, construct G' by:

For each $(u,v) \in E$ add vertex uv to V' and add edges (u, uv), (v, uv) to E'

Claim. G has VC of size $m \iff$
G' — DS —— m

Let D be DS in G'. W.l.o.g assume D contains no vertex of form uv. ~~If it~~ did, replace uv by either u or v, and D remains a DS.

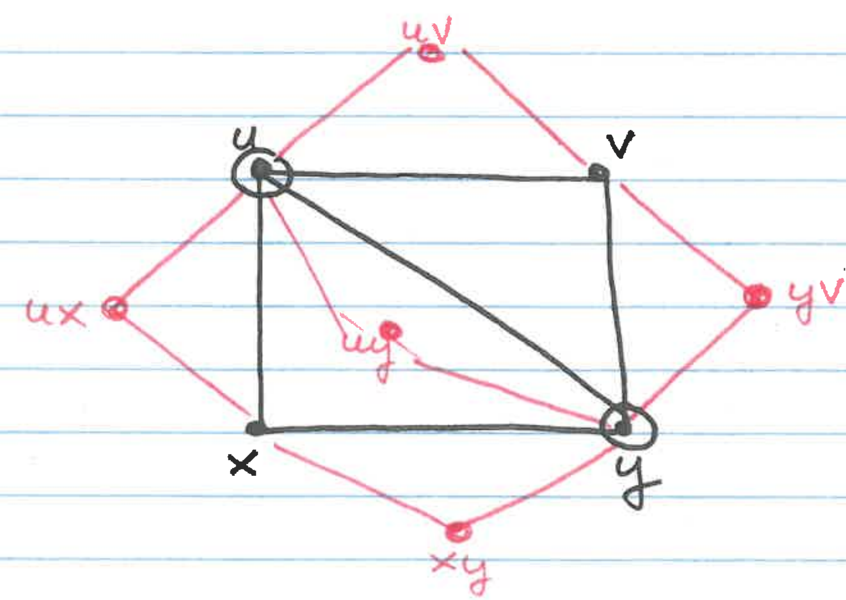
Thus assume $D \subseteq V$. Since D is a DS, a vertex of form uv must be adj. to some vertex in D which is either u or v

$\Longrightarrow \forall (u,v) \in E$: either $u \in D$ or $v \in D$

$\Longrightarrow$ D is also a VC in G

Conversely, if C is a VC in G, then every edge is covered by C

$\Longrightarrow$ new vertices of form uv are dominated by C $\Longrightarrow$ C is DS in G'.



Problem #3

1. Define the Subset Sum problem and describe the polynomial-time reduction from $3SAT$ to Subset Sum. 2 10

2. Based on the above reduction construct an instance of Subset Sum for the following $3SAT$ instance: 10

$$(\neg x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee x_4)$$

Construct for the Boolean assignment $x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1$ the corresponding solution of the instance of Subset Sum. 3

1. Input. A set S of integers and a target (integer) t

Question. Is there a subset $C \subseteq S$ s.t. $\sum_{x \in C} x = t$?

For reduction $3SAT \leq_p$ SubsetSum: see classnotes

2. subset Sum instance is constr. as follows:

	x_1	x_2	x_3	x_4	C_1	C_2	C_3	Integers
y_1	1	0	0	0	0	1	1	1000011
z_1	1	0	0	0	1	0	0	1000100
y_2	0	1	0	0	1	0	0	0100100
z_2	0	1	0	0	0	1	1	0100011
y_3	0	0	1	0	0	0	0	0010000
z_3	0	0	1	0	1	1	0	0010110
y_4	0	0	0	1	0	0	1	0001001
z_4	0	0	0	1	0	0	0	0001000
h_1	0	0	0	0	1	0	0	0000100
g_1	0	0	0	0	1	0	0	0000100
h_2	0	0	0	0	0	1	0	0000010
g_2	0	0	0	0	0	1	0	0000010
h_3	0	0	0	0	0	0	1	0000001
g_3	0	0	0	0	0	0	1	0000001
t	1	1	1	1	3	3	3	1111333

S

$x_1 = 0 \Longrightarrow z_1$
$x_2 = 1 \Longrightarrow y_2$
$x_3 = 0 \Longrightarrow z_3$
$x_4 = 1 \Longrightarrow y_4$

$\Longrightarrow$ SubsetSum sol. instance is

$$\boxed{\{z_1, y_2, z_3, y_4, h_2, g_2, h_3, g_3\}}$$