# 12.130

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**Question**:

The linear operation L($\mathbf{x}$) is defined by the cross product L($\mathbf{x}$) = $\mathbf{b} \times \mathbf{x}$, where $\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ are three dimensional vectors. The $3 \times 3$ matrix $\mathbf{M}$ of this operation satisfies L($\mathbf{x}$) = $\mathbf{Mx}$. Then the eigenvalues of $\mathbf{M}$ are

(a) $0, +1, -1$
(b) $1, -1, 1$
(c) $i, -i, 1$
(d) $i, -i, 0$

**Solution:**

| **Point** | **Vector** |
|---|---|
| $\mathbf{b}$ | $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ |
| $\mathbf{x}$ | $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ |

TABLE 4: Variables used

Given,

$$L(x) = MX \tag{4.1}$$

$$L(x) = b \times X \tag{4.2}$$

Cross product can be written as skew symmetric matrix.

$$b \times X = \begin{pmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{pmatrix} X \tag{4.3}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} X \tag{4.4}$$

$$\therefore M = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \tag{4.5}$$

$$\tag{4.6}$$

finding eigenvalues :-

$$|M - \lambda I| = 0 \tag{4.7}$$

$$\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -1 & -\lambda \end{pmatrix} = 0 \tag{4.8}$$

$$-\lambda^3 - \lambda = 0 \tag{4.9}$$

$$-\lambda(\lambda^2 + 1) = 0 \tag{4.10}$$

$$\lambda_1 = 0 \tag{4.11}$$

$$\lambda_2 = i \tag{4.12}$$

$$\lambda_3 = -i \tag{4.13}$$

Hence Option d is correct.

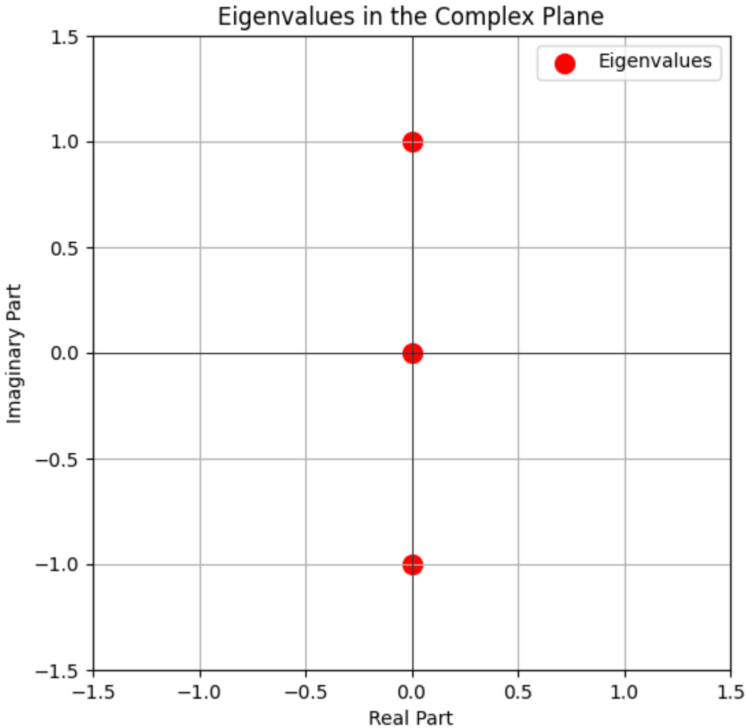
Eigenvalues in the Complex Plane
Eigenvalues
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Imaginary Part
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
Real Part

Fig. 4.1: 12.130