

4.11.12

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Question

Find the distance of the point $P(-1, -5, -10)$ from the point of intersection of the line $\mathbf{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

Solution

The line is given by $\mathbf{x} = \mathbf{h} + k\mathbf{m}$, where

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \quad (2)$$

The plane is of the form $\mathbf{n}^T \mathbf{x} = c$ where $c = 5$ and,

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (3)$$

Given, $\mathbf{P} = \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$.

Solution

To find the point of intersection, we substitute the vector equation of the line into the vector equation of the plane.

$$(\mathbf{a} + \lambda \mathbf{b}) \cdot \mathbf{n} = c \quad (4)$$

$$\mathbf{h} \cdot \mathbf{n} + k(\mathbf{m} \cdot \mathbf{n}) = c \quad (5)$$

$$\mathbf{h}\mathbf{n}^T + k(\mathbf{m}\mathbf{n}^T) = c \quad (6)$$

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} + k \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} = 5 \quad (7)$$

Solution

$$(2 \cdot 1 + (-1) \cdot (-1) + (2) \cdot (1)) + k(3 \cdot 1 + 4 \cdot (-1) + 2 \cdot (1)) = 5 \quad (8)$$

$$(5) + k(1) = 5 \quad (9)$$

$$k = 0 \quad (10)$$

The point of intersection is $\mathbf{x} = \mathbf{h} + 0(\mathbf{m}) = \mathbf{h}$.

Solution

Distance between two points is given by $\|\mathbf{x}_2 - \mathbf{x}_1\|$.

$$d = \left\| \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right\| \quad (11)$$

$$d = \left\| \begin{pmatrix} -3 \\ -4 \\ 12 \end{pmatrix} \right\| \quad (12)$$

$$d = \sqrt{(-3)^2 + (-4)^2 + (12)^2} \quad (13)$$

$$d = \sqrt{169} \quad (14)$$

$$d = 13 \quad (15)$$

\therefore The distance between the points is 13 units.

Line, Plane, and Intersection Visualization

