8.4.7

Aditya Appana - EE25BTECH11004

October 4,2025

Question

Let **O** be the vertex and **Q** be any point on the parabola $x^2 = 8y$. If the point **P** divides the line segment **OQ** internally in the ratio (1:3), then the locus of **P** is:

1. $y^2 = 2x$
2. $x^2 = 2y$
3. $x^2 = y$
4. $y^2 = x$

Solution

The equation of conic with directrix $\mathbf{n}^T\mathbf{x} = c$ and focus at $\mathbf{F}$, and eccentricity e is

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0$$

where:

$$\mathbf{V} = ||\mathbf{n}||^2\mathbf{I} - e^2\mathbf{n}\mathbf{n^T}$$
$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F}$$
$$f = ||\mathbf{n}||^2\mathbf{F} - c^2e^2$$

The directrix of the given parabola is $y = -2$, which expressed in the form $\mathbf{n^T x} = c$ is

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \mathbf{x} = -2$$

The focus $\mathbf{F} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$. Since it is a parabola, $e = 1$.

Solution

Therefore:

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$u = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$f = 0$$

The parabola can be represented as

$$\mathbf{x}^{\mathbf{T}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 0 \\ 8 \end{pmatrix} \mathbf{x} = 0 \tag{1}$$

Since the point **P** divides **OQ** internally in the ratio 1:3,

$$\mathbf{P} = \frac{\mathbf{x}}{4} \tag{2}$$

Solution

Substituting $\mathbf{P}$ in (1),

$$\mathbf{4P^T}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\mathbf{P}-\begin{pmatrix}0\\ 8\end{pmatrix}\mathbf{P}=0 \tag{3}$$

$$\mathbf{P^T}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\mathbf{P}-\begin{pmatrix}0\\ 2\end{pmatrix}\mathbf{P}=0 \tag{4}$$

Expanding this equation, we get the locus of $\mathbf{P}$ as $x^2 = 2y$.

The correct option is **B**.

Python Code

```
import matplotlib.pyplot as plt
import numpy as np
fig = plt.figure(figsize = (6,6))
ax = fig.add_subplot(111)

X = np.linspace(-10,10,50)
p1 = X*X/8
p2 = X*X/2
l = np.zeros(50) - 2

ax.plot(X,p1, label = '$x^2 = 8y$', color = 'green')
ax.plot(X,p2, label = '$x^2 = 2y$', color = 'red')
ax.plot(X,l, label = '$y=-2$', color = 'orange')
```

Python Code

```
ax.scatter(4,2, label='(4,2)')
ax.scatter(1,0.5,label='(1,0.5)')
ax.scatter(8,8, label='(8,8)')
ax.scatter(2,2,label='(2,2)')
ax.scatter(0,0, color = 'black')
set1 = np.array([[4,2],
                 [1,0.5],
                 [0,0]])
set2 = np.array([[8,8],
                 [2,2],
                 [0,0]])
ax.plot(set1[:,0],set1[:,1])
ax.plot(set2[:,0],set2[:,1])
ax.scatter(0,2, label = 'Focus')
ax.grid(True)
ax.legend()
plt.show()
```

Figure

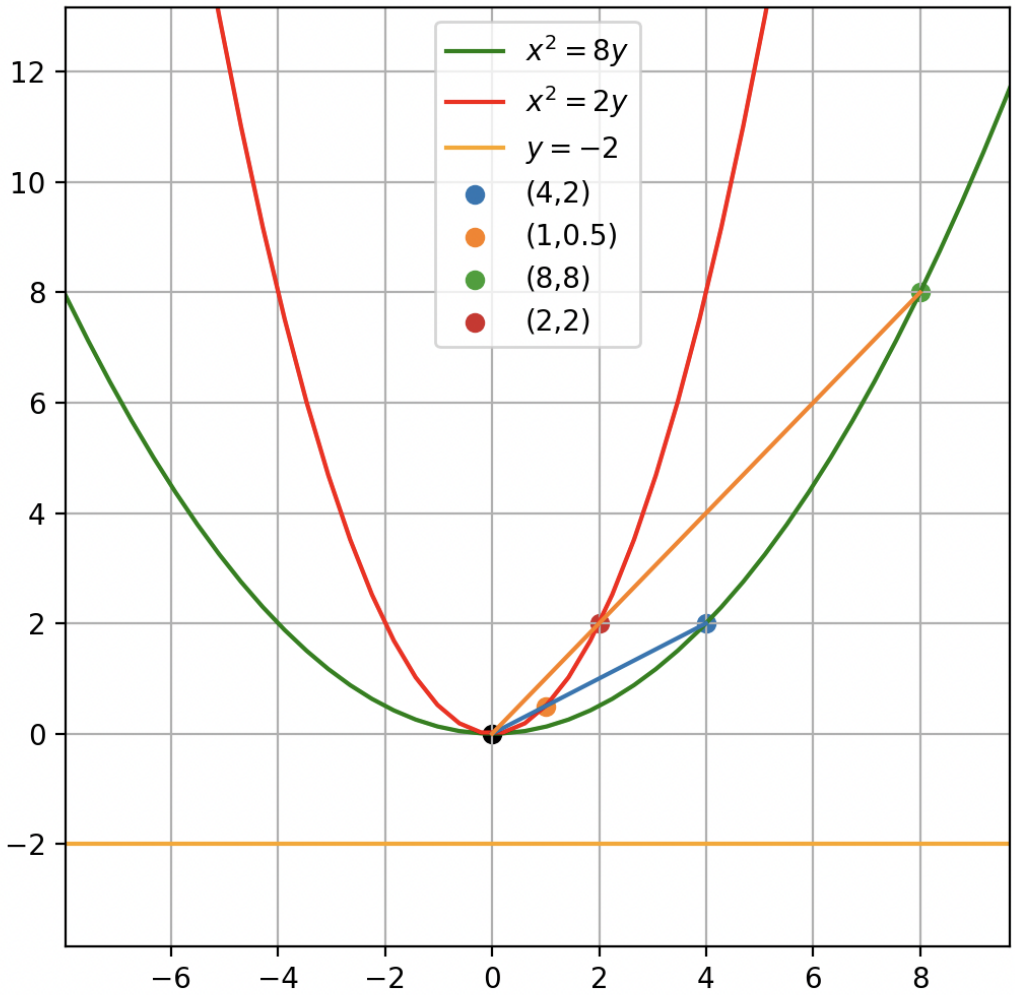

Figure: Plot