10.3.11

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Question

Find the normal at the point $(1, 1)$ on the curve

$$2y + x^2 = 3 \tag{1}$$

finding the Normal:

$$2y + x^2 = 3 \tag{2}$$

$$2y + x^2 - 3 = 0 \tag{3}$$

Which can be expressed as the conic:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, f = -3 \tag{5}$$

let

$$\mathbf{p}\begin{pmatrix}1\\1\end{pmatrix}, \mathbf{m} \text{ is normal vector} \tag{6}$$

$$\mathbf{m}^\top(\mathbf{Vp}+\mathbf{u}) = 0 \tag{7}$$

substitung the value :

$$\mathbf{m} = \begin{pmatrix}m_1\\m_2\end{pmatrix} \tag{8}$$

$$\mathbf{Vp} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \tag{9}$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \tag{10}$$

$$\begin{pmatrix}m_1\\m_2\end{pmatrix}\begin{pmatrix}1+0\\0+1\end{pmatrix} = 0 \tag{11}$$

$$m_1 = -m_2 \tag{12}$$

$$\therefore \mathbf{m} = \begin{pmatrix} -m \\ m \end{pmatrix} \tag{13}$$

equation of normal is

$$\mathbf{m}^\top (\mathbf{x} - \mathbf{p}) = 0 \tag{14}$$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x - 1 \\ y - 1 \end{pmatrix} = 0 \tag{15}$$

$$y = x \tag{16}$$

Hence equation of normal to $2y + x^2 - 3 = 0$ at $(1, 1)$ is $y = x$.

Python Code

```python
import numpy as np
import matplotlib.pyplot as plt

# Create the figure and axis
fig, ax2 = plt.subplots()
fig.suptitle('Graphs of Normal to the Curve', fontsize=16)
# Curve: 2y + x^2 = 3 => y = (3 - x^2)/2
# Point: (1, 1)
# Normal Line: y = x
# Tangent Line at (1,1): y = -x + 2
```

Python Code

```python
# Generate x-values for the curve
x_curve2 = np.linspace(-3, 4, 400)
y_curve2 = (3 - x_curve2**2) / 2

# Generate x-values for the lines
x_line2 = np.linspace(-2, 4, 100)
y_normal = x_line2
y_tangent2 = -x_line2 + 2
```

Python Code

```
# Plot the curve, normal, tangent, and the point
ax2.plot(x_curve2, y_curve2, label='Curve: $2y+x^2=3$', color='blue')
ax2.plot(x_line2, y_normal, label='Normal: $y=x$', color='green')
ax2.plot(x_line2, y_tangent2, label='Tangent: $y=-x+2$', color='red', linestyle='--')
ax2.scatter([1], [1], color='purple', s=100, zorder=5, label='Point (1, 1)')
```

```python
# Formatting for the plot
ax2.set_title('10.3.11: Normal to $2y+x^2=3$', fontsize=12)
ax2.set_xlabel('$x$')
ax2.set_ylabel('$y$')
ax2.axhline(0, color='black', linewidth=0.5)
ax2.axvline(0, color='black', linewidth=0.5)
ax2.grid(True, linestyle='--', alpha=0.7)
ax2.legend()
ax2.set_aspect('equal', adjustable='box')
ax2.set_xlim([-2, 4])
ax2.set_ylim([-2, 4])
```

Python Code

```
# Display the figure
plt.tight_layout(rect=[0, 0.03, 1, 0.95])
plt.show()
```

C Code

```c
#include <stdio.h>

int main() {
    // --- Given Problem Data ---
    // The point (x1, y1) on the curve 2y + x^2 = 3.
    double x1 = 1.0;
    double y1 = 1.0;
    // --- Calculations ---
    // 1. Find the slope of the tangent from the derivative dy/dx
         = -x.
    double m_tangent = -x1;
```

C Code

```c
// A horizontal tangent (slope=0) means the normal is a
    vertical line.
if (m_tangent == 0) {
    printf("The equation of the normal line is: x = %.2f\n",
        x1);
} else {
    // 2. The slope of the normal is the negative reciprocal.
    double m_normal = -1.0 / m_tangent;
```

C Code

```c
// 3. The y-intercept 'c' is calculated from y = mx + c.
double c = y1 - m_normal * x1;

// --- Print the Final Answer ---
// This logic helps format the equation cleanly (e.g.,
    printing "y = x"
// instead of "y = 1.00x + 0.00").
printf("The equation of the normal line is: ");
```

C Code

```
        if (m_normal == 1 && c == 0) {
            printf("y = x\n");
        } else if (m_normal == -1 && c == 0) {
            printf("y = -x\n");
        } else {
            printf("y = %.2fx", m_normal);
            if (c > 0) {printf(" + %.2f\n", c);
            } else if (c < 0) {printf(" - %.2f\n", -c);
            } else {printf("\n");
            }}}
    return 0;}
```

Python and C Code

```
from ctypes import c_double

def main():
    # --- Given Problem Data ---
    x1 = c_double(1.0)
    y1 = c_double(1.0)

    # --- Calculations ---
    m_tangent = c_double(-x1.value) # dy/dx = -x
```

Python and C Code

```
    if m_tangent.value == 0:
        print(f"The equation of the normal line is: x = {x1.value
            :.2f}")
    else:
        m_normal = c_double(-1.0 / m_tangent.value)
        c_intercept = c_double(y1.value - m_normal.value * x1.
            value)

        # --- Print the Final Answer ---
        print("The equation of the normal line is: ", end="")
```

Python and C Code

```python
if m_normal.value == 1 and c_intercept.value == 0:
    print("y = x")
elif m_normal.value == -1 and c_intercept.value == 0:
    print("y = -x")
else:
    print(f"y = {m_normal.value:.2f}x", end="")
    if c_intercept.value > 0:
        print(f" + {c_intercept.value:.2f}")
```

Python and C Code

```
            elif c_intercept.value < 0:
                print(f" - {abs(c_intercept.value):.2f}")
            else:
                print()

if __name__ == "__main__":
    main()
```

Graphs of Normal to the Curve

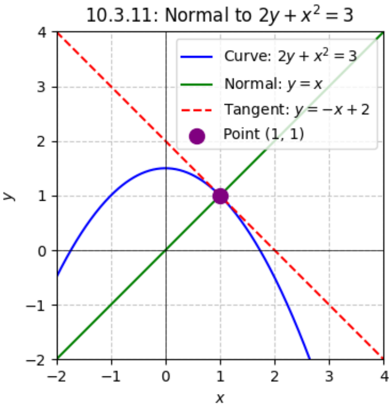

Figure: Plot