8.4.7

EE25BTECH11004 - Aditya Appana

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Question

Let $\mathbf{O}$ be the vertex and $\mathbf{Q}$ be any point on the parabola $x^2 = 8y$. If the point $\mathbf{P}$ divides the line segment $\mathbf{OQ}$ internally in the ratio (1:3), then the locus of $\mathbf{P}$ is:

A) $y^2 = 2x$ B) $x^2 = 2y$ C) $x^2 = y$ D) $y^2 = x$

Solution

The equation of conic with directrix $\mathbf{n}^T\mathbf{x} = c$ and focus at $\mathbf{F}$, and eccentricity e is

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0$$

where:

$$\mathbf{V} = \|\mathbf{n}\|^2\mathbf{I} - e^2\mathbf{n}\mathbf{n^T}$$
$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2\mathbf{F}$$
$$f = \|\mathbf{n}\|^2\mathbf{F} - c^2e^2$$

The directrix of the given parabola is $y = -2$, which expressed in the form $\mathbf{n^T x} = c$ is

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \mathbf{x} = -2$$

The focus $\mathbf{F} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$. This is a parabola, therefore $e = 1$.

Therefore:

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\mathbf{u} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$
$$f = 0$$

The parabola can be represented as

$$\mathbf{x}^{\mathbf{T}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 0 \\ 8 \end{pmatrix} \mathbf{x} = 0 \tag{1}$$

Since the point **P** divides **OQ** internally in the ratio 1:3,

$$\mathbf{P} = \frac{\mathbf{x}}{4} \tag{2}$$

Substituting **P** in (1),

$$\mathbf{4P}^{\mathbf{T}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{P} - \begin{pmatrix} 0 \\ 8 \end{pmatrix} \mathbf{P} = 0 \tag{3}$$

$$\mathbf{P}^{\mathbf{T}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{P} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \mathbf{P} = 0 \tag{4}$$

Expanding this equation, we get the locus of **P** as $x^2 = 2y$.

The correct option is **B**.

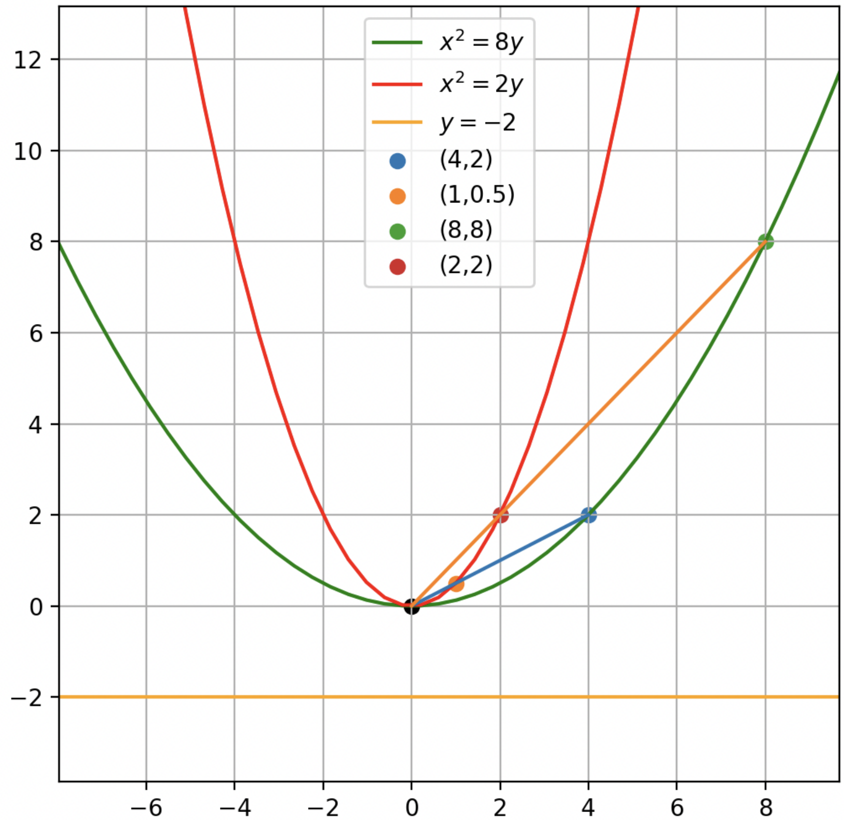

Figure 1: Plot