5.2.37

EE25BTECH11034 - Kishora Karthik

Question:
Solve the following system of linear equations.

$$\frac{x}{2} + \frac{y}{3} = -1$$

$$x - \frac{y}{3} = 0$$

Solution:
The equation of line L_1 is,

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{3} \end{pmatrix} \mathbf{x} = -1 \tag{1}$$

The equation of line L_2 is,

$$\begin{pmatrix} 1 & -\frac{1}{3} \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

On putting the equations in a matrix, we will get

$$\implies \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ 1 & -\frac{1}{3} \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{3}$$

So the augmented matrix is,

$$\left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & -1 \\ 1 & -\frac{1}{3} & 0 \end{array} \right) \tag{4}$$

$$R_2 \to R_2 - 2R_1 \implies \left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & -1 \\ 0 & -1 & 2 \end{array} \right) \tag{5}$$

$$R_2 \to -R_2 \implies \left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & -1 \\ 0 & 1 & -2 \end{array} \right) \tag{6}$$

$$R_1 \to R_1 - \frac{1}{3}R_2 \implies \left(\begin{array}{cc|c} \frac{1}{2} & 0 & -\frac{1}{3} \\ 0 & 1 & -2 \end{array} \right) \tag{7}$$

$$R_1 \to 2R_1 \implies \left(\begin{array}{cc|c} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & -2 \end{array} \right) \tag{8}$$

$$\implies \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \equiv \begin{pmatrix} -\frac{2}{3} \\ -2 \end{pmatrix} \tag{9}$$

Therefore the two lines will intersect at $\begin{pmatrix} -\frac{2}{3} \\ -2 \end{pmatrix}$.

Graph using C intersection function