5.8.2

EE25BTECH11004 - Aditya Appana

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Question

10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Solution

Let the number of girls in the class be g, and the number of boys be b. Let the vector representing this data be

$$\mathbf{x} = \begin{pmatrix} g \\ b \end{pmatrix} \tag{1}$$

Since the total number of students in the class is 10, $g + b = 10$ which can be expressed as:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \mathbf{x} = 10 \tag{2}$$

Since there are 4 more girls than boys, $b + 4 = g$, which can be expressed as:

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \mathbf{x} = -4 \tag{3}$$

Organising these two equations into the form $\mathbf{Ax} = \mathbf{b}$:

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} \tag{4}$$

Normalising $\mathbf{A}$:

$$\sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathbf{x} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} \tag{5}$$

$$\tag{6}$$

Let $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ be $\mathbf{M}$. $\mathbf{M}$ is orthogonal, therefore $\mathbf{M^T M} = \mathbf{I}$.
Multiplying by $\mathbf{M^T}$ on both the sides:

$$\sqrt{2}\mathbf{x} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 10 \\ -4 \end{pmatrix} \tag{7}$$

$$\mathbf{x} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 10 \\ -4 \end{pmatrix} \tag{8}$$

Solving we get:

$$\mathbf{x} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \tag{9}$$

$$g = 7 \tag{10}$$

$$b = 3 \tag{11}$$

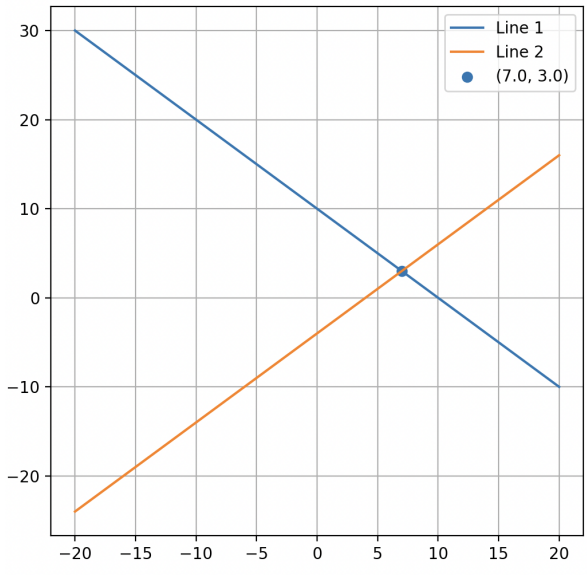

Figure 1: Plot