

12.66

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:

The matrix

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix}$$

is:

- 1) orthogonal 2) symmetric 3) anti-symmetric 4) unitary

(PH 2014)

Solution: Given matrix:

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix} \quad (4.1)$$

Check 1: Symmetric ($A = A^T$)

The transpose of A is:

$$A^T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix} \quad (4.2)$$

Since $A^T \neq A$ (the off-diagonal elements are different: $1+i \neq 1-i$),
 A is NOT symmetric.

Check 2: Anti-symmetric ($A = -A^T$)

For anti-symmetric:

$$-A^T = -\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix} \quad (4.3)$$

$$= \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & -1+i \\ -1-i & 1 \end{pmatrix} \quad (4.4)$$

Since $A \neq -A^T$,

A is NOT anti-symmetric.

Check 3: Orthogonal ($AA^T = I$)

Note: For real matrices, orthogonal means $AA^T = I$. However, A contains complex entries, so we need to check if it satisfies the real orthogonal property.

$$AA^T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix} \quad (4.5)$$

$$= \frac{1}{3} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix} \begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix} \quad (4.6)$$

Computing the (1, 1) entry:

$$(AA^T)_{11} = \frac{1}{3} [1 \cdot 1 + (1+i)(1+i)] = \frac{1}{3} [1 + 1 + 2i + i^2] = \frac{1}{3} [1 + 2i] \neq 1 \quad (4.7)$$

Since this is complex (not real), $AA^T \neq I$,

A is NOT orthogonal.

Check 4: Unitary ($AA^T = I$)

For a unitary matrix, we need $A\overline{A^T} = I$, where $\overline{A^T} = \overline{A^T}$ is the conjugate transpose.

The conjugate transpose is:

$$\overline{A^T} = \overline{A^T} = \overline{\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix}} \quad (4.8)$$

$$= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix} \quad (4.9)$$

Notice that $\overline{A^T} = A$, but let's verify unitarity:

$$A\overline{A^T} = \frac{1}{3} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix} \quad (4.10)$$

$$A\overline{A^T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \quad (4.11)$$

A is UNITARY.

Option 4: The matrix A is unitary