5.8.17

EE25BTECH11019 - Darji Vivek M.

Question:
A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes $\frac{5}{6}$. Find the fraction.

Solution:

Matrix Method:
Let the numerator be n and the denominator be d. From the conditions we get

$$\frac{n+2}{d+2} = \frac{9}{11} \implies 11(n+2) = 9(d+2) \implies 11n - 9d = -4, \tag{1}$$
$$\frac{n+3}{d+3} = \frac{5}{6} \implies 6(n+3) = 5(d+3) \implies 6n - 5d = -3. \tag{2}$$

In matrix form:

$$\begin{pmatrix} 11 & -9 \\ 6 & -5 \end{pmatrix} \begin{pmatrix} n \\ d \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}. \tag{3}$$

Solve by row-reduction (augmented matrix). In the same `\augvec` format you provided:

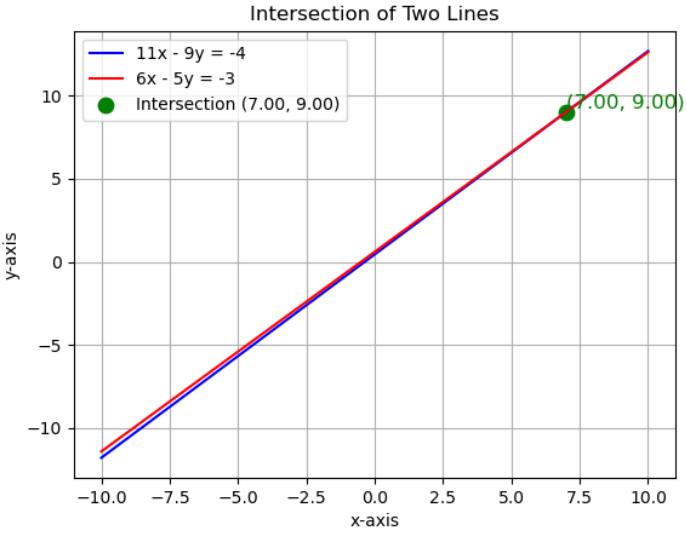
$$\left(\begin{array}{cc|c} 11 & -9 & -4 \\ 6 & -5 & -3 \end{array}\right) \xrightarrow{R_1 \leftarrow \frac{1}{11}R_1,\ R_2 \leftarrow R_2 - 6R_1} \left(\begin{array}{cc|c} 1 & -\frac{9}{11} & -\frac{4}{11} \\ 0 & -\frac{1}{11} & -\frac{9}{11} \end{array}\right) \xrightarrow{R_2 \leftarrow -11R_2,\ R_1 \leftarrow R_1 + \frac{9}{11}R_2} \left(\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 9 \end{array}\right)$$

Thus

$$n = 7, \qquad d = 9. \tag{4}$$

Therefore the fraction is $\boxed{\frac{7}{9}}$.

Check: $\frac{7+2}{9+2} = \frac{9}{11}$, $\frac{7+3}{9+3} = \frac{10}{12} = \frac{5}{6}$.


Intersection of Two Lines
11x - 9y = -4
6x - 5y = -3
Intersection (7.00, 9.00)
(7.00, 9.00)
y-axis
x-axis
10
5
0
−5
−10
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0