

7.4.19

EE25BTECH11012-BEERAM MADHURI

Question:

Let **C** be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle **C** that subtend an angle of $\frac{2\pi}{3}$ at its centre is

$$1) x^2 + y^2 = \frac{3}{2} \quad 2) x^2 + y^2 = 1 \quad 3) x^2 + y^2 = \frac{27}{4} \quad 4) x^2 + y^2 = \frac{9}{4}$$

Solution:

Given radius = 3 units

$$\mathbf{C} = \text{center} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4.1)$$

Let **A** and **B** be the end points of chord

$$\|\mathbf{A}\| = \|\mathbf{B}\| = 3 \quad (4.2)$$

$$\mathbf{A}^\top \mathbf{A} = 9 \quad (4.3)$$

$$\mathbf{B}^\top \mathbf{B} = 9 \quad (4.4)$$

$$\mathbf{A}^\top \mathbf{B} = \|\mathbf{A}\| \|\mathbf{B}\| \cos \theta = -\frac{9}{2} \quad (4.5)$$

Let **P** be the midpoint of chords then,

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (4.6)$$

$$\|\mathbf{P}\| = \frac{1}{2} \|\mathbf{A} + \mathbf{B}\| \quad (4.7)$$

$$\mathbf{P}^\top \mathbf{P} = \frac{1}{4} (\mathbf{A} + \mathbf{B})^\top (\mathbf{A} + \mathbf{B}) \quad (4.8)$$

$$\mathbf{P}^\top \mathbf{P} = \frac{1}{4} (\mathbf{A}^\top \mathbf{A} + \mathbf{A}^\top \mathbf{B} + \mathbf{B}^\top \mathbf{A} + \mathbf{B}^\top \mathbf{B}) \quad (4.9)$$

$$(4.10)$$

Substituting the values:

$$= \frac{1}{4} \left(9 - \frac{9}{2} - \frac{9}{2} + 9 \right) = \frac{9}{4} \quad (4.11)$$

Hence, $\mathbf{P}^\top \mathbf{P} = 9/4$
option D is correct.

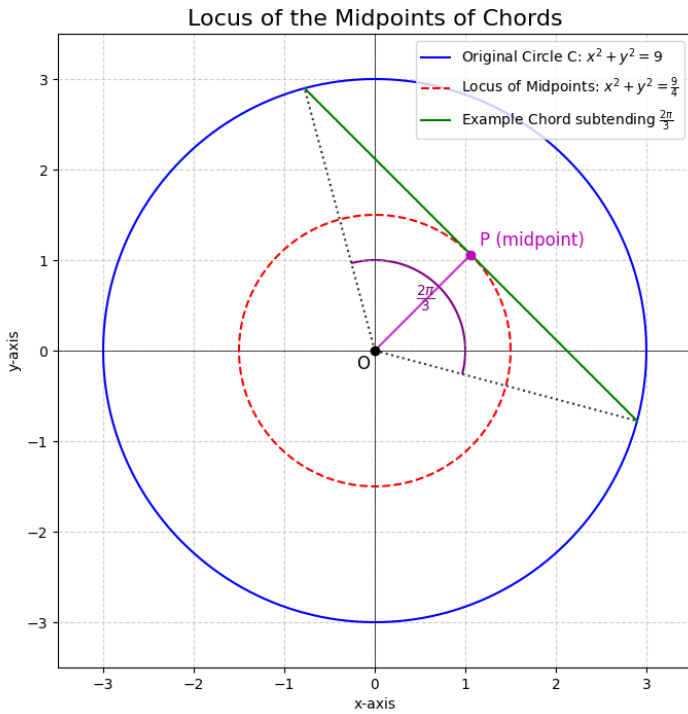


Fig. 4.1: 7.4.19