2.9.10

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question: Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $\|\mathbf{a} + \mathbf{b}\| = \|\mathbf{b}\|$. Prove that $\mathbf{a} + 2\mathbf{b}$ is perpendicular to $\mathbf{a}$.

Solution:

Variable	Value
$\mathbf{a}$	vector $\mathbf{a}$
$\mathbf{b}$	vector $\mathbf{b}$

TABLE 0: Variables Used

$$(\mathbf{a} + \mathbf{b})^T(\mathbf{a} + \mathbf{b}) = \mathbf{b}^T\mathbf{b}. \tag{0.1}$$

$$(\mathbf{a} + \mathbf{b})^T(\mathbf{a} + \mathbf{b}) = \mathbf{a}^{\mathbf{T}}\mathbf{a} + \mathbf{a}^{\mathbf{T}}\mathbf{b} + \mathbf{b}^{\mathbf{T}}\mathbf{a} + \mathbf{b}^{\mathbf{T}}\mathbf{b}. \tag{0.2}$$

Since dot product is symmetric

$$\mathbf{a}^{\mathbf{T}}\mathbf{b} = \mathbf{b}^{\mathbf{T}}\mathbf{a} \tag{0.3}$$

$$\mathbf{a}^{\mathbf{T}}\mathbf{a} + 2\,\mathbf{a}^{\mathbf{T}}\mathbf{b} + \mathbf{b}^{\mathbf{T}}\mathbf{b} = \mathbf{b}^{\mathbf{T}}\mathbf{b}. \tag{0.4}$$

$$\mathbf{a}^{\mathbf{T}}\mathbf{a} + 2\,\mathbf{a}^{\mathbf{T}}\mathbf{b} = 0. \tag{0.5}$$

We want to show $(\mathbf{a} + 2\mathbf{b})$ is perpendicular to $\mathbf{a}$.

$$\text{To prove: } \mathbf{a}^{\mathbf{T}}(\mathbf{a} + 2\mathbf{b}) = 0 \tag{0.6}$$

$$\mathbf{a}^{\mathbf{T}}(\mathbf{a} + 2\mathbf{b}) = \mathbf{a}^{\mathbf{T}}\mathbf{a} + 2\,\mathbf{a}^{\mathbf{T}}\mathbf{b} \tag{0.7}$$

By eq (0.5) and (0.7)

$$\mathbf{a}^{\mathbf{T}}(\mathbf{a} + 2\mathbf{b}) = 0 \tag{0.8}$$

Hence proved

Refer to Figure

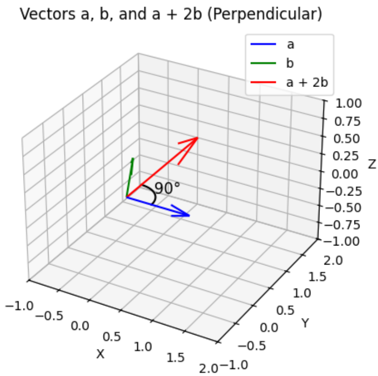

Fig. 0.1