9.5.6

EE25BTECH11043 - Nishid Khandagre

Question: Find the sum and product of the roots of the quadratic equation

$$2x^2 - 9x + 4 = 0$$

Solution: Given quadratic equation:

$$y = 2x^2 - 9x + 4 \tag{0.1}$$

Representing this equation as a conic section

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \ , \ \mathbf{V} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \ , \ \mathbf{u} = \begin{pmatrix} -9/2 \\ -1/2 \end{pmatrix} \ , \ f = 4 \tag{0.2}$$

We need to find intersection points with $y = 0$, that is, the X-axis.

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \ , \ \mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ , \ \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.3}$$

Substituting $\mathbf{x} = k\mathbf{m}$

$$k^2 \mathbf{m}^\top \mathbf{V} \mathbf{m} + 2k\mathbf{u}^\top \mathbf{m} + f = 0 \tag{0.4}$$

$$\implies k = \frac{1}{2\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left[-2\mathbf{u}^\top \mathbf{m} \ \pm \ \sqrt{(2\mathbf{u}^\top \mathbf{m})^2 - 4f\mathbf{m}^\top \mathbf{V} \mathbf{m}} \right] \tag{0.5}$$

$$\implies k = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left[-\mathbf{u}^\top \mathbf{m} \ \pm \ \sqrt{(\mathbf{u}^\top \mathbf{m})^2 - f\mathbf{m}^\top \mathbf{V} \mathbf{m}} \right] \tag{0.6}$$

$$\mathbf{u}^\top \mathbf{m} = \begin{pmatrix} -9/2 & -1/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -9/2 \tag{0.7}$$

$$\mathbf{m}^\top \mathbf{V} \mathbf{m} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 2 \tag{0.8}$$

$$k = \frac{1}{2} \left[-(-9/2) \ \pm \ \sqrt{(-9/2)^2 - 4 \cdot 2} \right] \tag{0.9}$$

$$k = \frac{1}{2} \left[\frac{9}{2} \ \pm \ \frac{7}{2} \right] \tag{0.10}$$

$$\implies k_1 = 4 \tag{0.11}$$

$$\implies k_2 = \frac{1}{2} \tag{0.12}$$

Substituting k into $\mathbf{x}$, we get the roots:

$$\mathbf{x} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \text{ OR } \mathbf{x} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} \tag{0.13}$$

This implies that the roots of $2x^2 - 9x + 4 = 0$ are 4 and $\frac{1}{2}$.

Now, calculate the sum and product of these roots:
Sum of the roots:

$$\text{Sum} = 4 + \frac{1}{2} = \frac{9}{2} \tag{0.14}$$

Product of the roots:
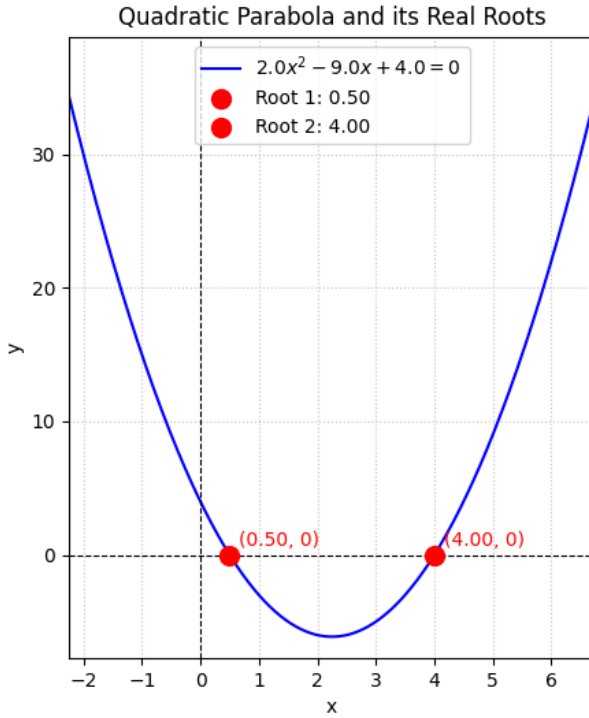
$$\text{Product} = 4 \times \frac{1}{2} = 2 \tag{0.15}$$

Fig. 0.1