

12.18

EE25BTECH11004 - Aditya Appana

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Question

X is 1 km northeast of **Y**. **Y** is 1 km southeast of **Z**. **W** is 1 km west of **Z**. **P** is 1 km south of **W**. **Q** is 1 km east of **P**. What is the distance between **X** and **Q** in km?

- A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2

Solution

Let **X** be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Every subsequent vector can be expressed as a translation by a unit vector, from the previous vector. The translation vector can be represented as:

$$\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (1)$$

X is 1km north-east of **Y**, so **Y** is 1km south-west of **X**. Therefore:

$$\mathbf{Y} - \mathbf{X} = \begin{pmatrix} \cos 225^\circ \\ \sin 225^\circ \end{pmatrix} \quad (2)$$

$$\mathbf{Y} - \mathbf{X} = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \quad (3)$$

Y is 1km south-east of **Z**, so **Z** is 1km north-west of **Y**. Therefore:

$$\mathbf{Z} - \mathbf{Y} = \begin{pmatrix} \cos 135^\circ \\ \sin 135^\circ \end{pmatrix} \quad (4)$$

$$\mathbf{Z} - \mathbf{Y} = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad (5)$$

W is 1km west of **Z**. Therefore:

$$\mathbf{W} - \mathbf{Z} = \begin{pmatrix} \cos 180^\circ \\ \sin 180^\circ \end{pmatrix} \quad (6)$$

$$\mathbf{W} - \mathbf{Z} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (7)$$

P is 1km south of **W**. Therefore:

$$\mathbf{P} - \mathbf{W} = \begin{pmatrix} \cos 270^\circ \\ \sin 270^\circ \end{pmatrix} \quad (8)$$

$$\mathbf{P} - \mathbf{W} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (9)$$

Q is 1km east of **P**. Therefore:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} \cos 0^\circ \\ \sin 0^\circ \end{pmatrix} \quad (10)$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (11)$$

Adding (3), (5), (7), (9), (11) together, we get $\mathbf{Q} - \mathbf{X} = \mathbf{Q} = \begin{pmatrix} -\sqrt{2} \\ -1 \end{pmatrix}$.

The distance between $\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -\sqrt{2} \\ -1 \end{pmatrix}$ is:

$$\|\mathbf{X} - \mathbf{Q}\| = \left\| \begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix} \right\| \quad (12)$$

$$\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3} \quad (13)$$

Therefore the correct option is **C**.

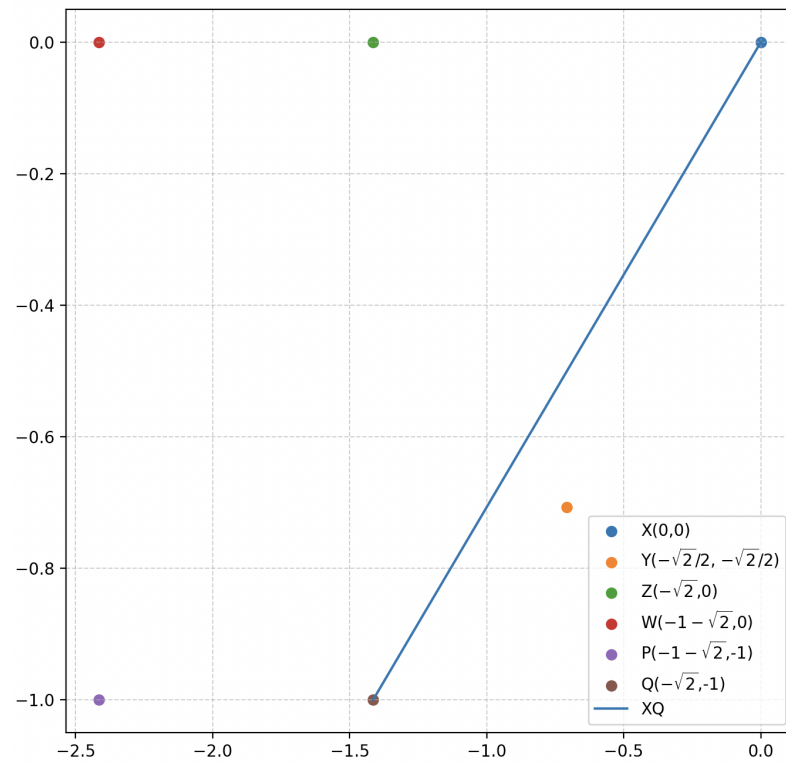


Figure 1: Plot