# 4.11.36

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**Question**: Find the coordinates of the point where the line through the points $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane determined by the points $(1, 2, 3)$, $(4, 2, -3)$ and $(0, 4, 3)$.

**Solution**

| Variable | Value |
|---|---|
| **A** | $(3, -4, -5)$ |
| **B** | $(2, -3, 1)$ |
| **P** | $(1, 2, 3)$ |
| **Q** | $(4, 2, -3)$ |
| **R** | $(0, 4, 3)$ |

TABLE 0: Variables Used

Let eq of plane be

$$\mathbf{n^T x} = 1 \tag{0.1}$$

As $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ lie on the plane

$$\mathbf{n^T P} = 1 \tag{0.2}$$

$$\mathbf{n^T Q} = 1 \tag{0.3}$$

$$\mathbf{n^T R} = 1 \tag{0.4}$$

$$\begin{pmatrix} P^T \\ Q^T \\ R^T \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.5}$$

From eq (0.2), (0.3), (0.4)*and*(0.5)

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 2 & -3 & 1 \\ 0 & 4 & 3 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -6 & -15 & -3 \\ 0 & 4 & 3 & 1 \end{array}\right) \tag{0.6}$$

$$\xrightarrow{R_2 \to -\frac{1}{3}R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 4 & 3 & 1 \end{array}\right) \tag{0.7}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & -7 & -1 \end{array}\right) \tag{0.8}$$

$$\xrightarrow{R_3 \to -\frac{1}{7}R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \tag{0.9}$$

$$\xrightarrow{R_2 \to R_2 - 5R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 0 & \frac{2}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \tag{0.10}$$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \tag{0.11}$$

$$\xrightarrow{R_1 \to R_1 - 3R_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & \frac{4}{7} \\ 0 & 1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \tag{0.12}$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \tag{0.13}$$

$$\mathbf{n} = \begin{pmatrix} \frac{2}{7} \\ \frac{1}{7} \\ \frac{1}{7} \end{pmatrix} \tag{0.14}$$

hence eq of plane is

$$\begin{pmatrix} \frac{2}{7} & \frac{1}{7} & \frac{1}{7} \end{pmatrix} \mathbf{x} = 1 \tag{0.15}$$

let a point on line **AB** be

$$\mathbf{c} = k\mathbf{A} + (1-k)\mathbf{B} \tag{0.16}$$

$$\mathbf{n^T}(k\left(A\right) + (1-k)\left(B\right)) = 1 \tag{0.17}$$

$$\begin{pmatrix} \frac{2}{7} & \frac{1}{7} & \frac{1}{7} \end{pmatrix} \begin{pmatrix} 2+k \\ -3-k \\ 1-6k \end{pmatrix} = 1 \tag{0.18}$$

$$4 + 2k - 3 - k + 1 - 6k = 7 \tag{0.19}$$

$$2 - 5k = 7 \tag{0.20}$$

$$k = -1 \tag{0.21}$$

The point **c** is

$$\mathbf{c} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \tag{0.22}$$

Refer to Figure

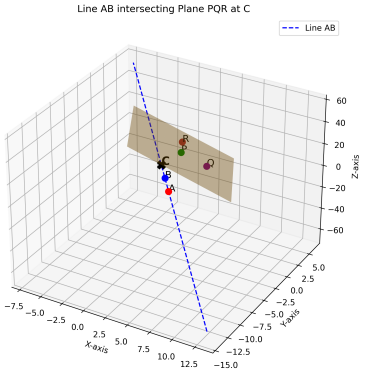


Fig. 0.1