

4.2.22

EE25BTECH11019 - Darji Vivek M.

Question:

Show that the two lines

$$a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0$$

with $b_1b_2 \neq 0$ are parallel iff $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

Solution:

Matrix form:

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = - \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}. \quad (1)$$

Augmented matrix:

$$\left(\begin{array}{cc|c} a_1 & b_1 & -c_1 \\ a_2 & b_2 & -c_2 \end{array} \right). \quad (2)$$

Perform row-reduction:

$$R_2 \rightarrow R_2 - \frac{a_2}{a_1}R_1 \implies \left(\begin{array}{cc|c} a_1 & b_1 & -c_1 \\ 0 & \frac{a_1b_2 - a_2b_1}{a_1} & -c_2 + \frac{a_2c_1}{a_1} \end{array} \right). \quad (3)$$

For two lines to be parallel, the coefficient matrix must have rank 1 and the augmented matrix must have rank 2.

Thus,

$$a_1b_2 - a_2b_1 = 0. \quad (4)$$

\therefore The two lines are parallel if

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}, \quad \text{where } b_1b_2 \neq 0. \quad (5)$$

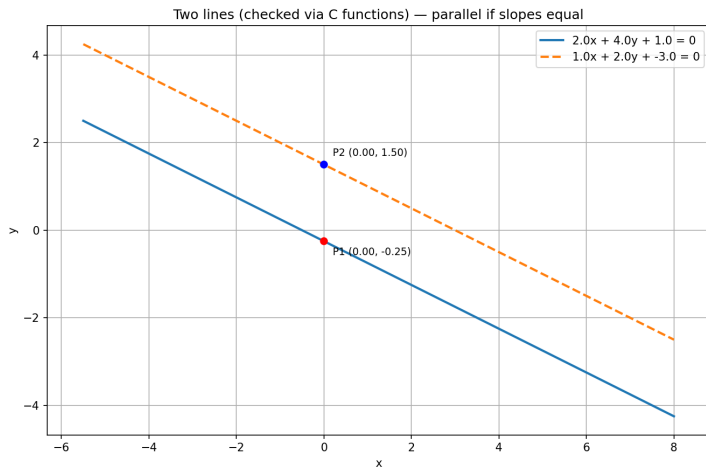


Fig. 0.1: plot