

4.7.26

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Question

Solve the following system of linear equations.

$$\frac{x}{2} + \frac{y}{3} = -1$$

$$x - \frac{y}{3} = 0$$

Solution

The equation of line L_1 is,

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{3} \end{pmatrix} \mathbf{x} = -1 \quad (1)$$

The equation of line L_2 is,

$$\begin{pmatrix} 1 & -\frac{1}{3} \end{pmatrix} \mathbf{x} = 0 \quad (2)$$

On putting the equations in a matrix, we will get

$$\Rightarrow \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ 1 & -\frac{1}{3} \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (3)$$

So the augmented matrix is,

$$\left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & -1 \\ 1 & -\frac{1}{3} & 0 \end{array} \right) \quad (4)$$

$$R_2 \rightarrow R_2 - 2R_1 \implies \left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & -1 \\ 0 & -1 & 2 \end{array} \right) \quad (5)$$

$$R_2 \rightarrow -R_2 \implies \left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & -1 \\ 0 & 1 & -2 \end{array} \right) \quad (6)$$

$$R_1 \rightarrow R_1 - \frac{1}{3}R_2 \implies \left(\begin{array}{cc|c} \frac{1}{2} & 0 & -\frac{1}{3} \\ 0 & 1 & -2 \end{array} \right) \quad (7)$$

$$R_1 \rightarrow 2R_1 \implies \left(\begin{array}{cc|c} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & -2 \end{array} \right) \quad (8)$$

$$\implies \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \equiv \begin{pmatrix} -\frac{2}{3} \\ -2 \end{pmatrix} \quad (9)$$

Therefore the two lines will intersect at $\begin{pmatrix} -\frac{2}{3} \\ -2 \end{pmatrix}$.

