10.3.21

EE25BTECH11020 - Darsh Pankaj Gajare

Question:
Find the point at which the line $y = x + 1$ is a tangent to the curve $y^2 = 4x$.
Solution: The given conic can be expressed as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \quad f = 0 \tag{2}$$

The given line is

$$\mathbf{n}^\top \mathbf{x} = C \tag{3}$$

where

$$\mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, C = 1 \tag{4}$$

The eigenvector corresponding to the zero eigenvalue is

$$\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} (\mathbf{u} + \kappa\mathbf{n})^\top \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa\mathbf{n} - \mathbf{u} \end{pmatrix} \tag{6}$$

$$\kappa = \frac{\mathbf{p_1}^\top \mathbf{u}}{\mathbf{p_1}^\top \mathbf{n}} = \frac{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 0 \end{pmatrix}}{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix}} = 2 \tag{7}$$

$$\begin{pmatrix} \left(\begin{pmatrix} -2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right)^\top \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 2\begin{pmatrix} -1 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} -4 & 2 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \tag{9}$$

Using augmented matrix,

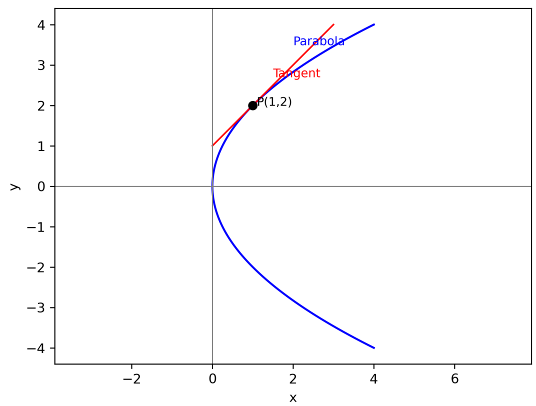
$$\left(\begin{array}{cc|c} -4 & 2 & 0 \\ 0 & 1 & 2 \end{array}\right) \tag{10}$$

$R_1 = \frac{R_1 - 2R_2}{-4}$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right) \tag{11}$$

$$\mathbf{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{12}$$

Plot using C libraries:



Plot using Python: