Problem 12.557

ee25btech11023-Venkata Sai

October 10, 2025

Problem

Let $\mathbf{A} = \begin{pmatrix} 5 & -3 \\ 6 & -4 \end{pmatrix}$. Then the trace of $\mathbf{A}^{1000}$ equals

Given

Given

$$\mathbf{A} = \begin{pmatrix} 5 & -3 \\ 6 & -4 \end{pmatrix} \tag{3.1}$$

$$\tag{3.2}$$

To find eigen values

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{3.3}$$

$$\left| \begin{pmatrix} 5 & -3 \\ 6 & -4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \tag{3.4}$$

$$\left| \begin{pmatrix} 5 & -3 \\ 6 & -4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0 \tag{3.5}$$

$$\left| \begin{pmatrix} 5 - \lambda & -3 \\ 6 & -4 - \lambda \end{pmatrix} \right| = 0 \tag{3.6}$$

$$(5 - \lambda)(-4 - \lambda) + 3(6) = 0 \tag{3.7}$$

Finding eigen values

$$\lambda^2 + 4\lambda - 5\lambda - 20 + 18 = 0 \tag{3.8}$$
$$\lambda^2 - \lambda - 2 = 0 \tag{3.9}$$
$$(\lambda - 2)(\lambda + 1) = 0 \tag{3.10}$$
$$\lambda_1 = 2 \ \text{(and)} \ \lambda_2 = -1 \tag{3.11}$$

For a given matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{PDP}^{-1} \tag{3.12}$$
$$\mathbf{A}^2 = \left(\mathbf{PDP}^{-1}\right)^2 \tag{3.13}$$
$$= \mathbf{PDP}^{-1}\mathbf{PDP}^{-1} \tag{3.14}$$
$$= \mathbf{PDIDP}^{-1} \tag{3.15}$$
$$= \mathbf{PD}^2\mathbf{P}^{-1} \tag{3.16}$$

where

Formula

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.17}$$

$$\begin{aligned} \mathbf{A}^k &= \mathbf{P}\mathbf{D}^k\mathbf{P}^{-1} && (3.18) \\ \text{trace}\left(\mathbf{A}^k\right) &= \text{trace}\left(\mathbf{P}\mathbf{D}^k\mathbf{P}^{-1}\right) && (3.19) \\ &= \text{trace}\left(\left(\mathbf{P}\mathbf{D}^k\right)\mathbf{P}^{-1}\right) && (3.20) \end{aligned}$$

Since trace(**AB**)=trace(**BA**)

$$\begin{aligned} \text{trace}\left(\mathbf{A}^k\right) &= \text{trace}\left(\left(\mathbf{P}\mathbf{D}^k\right)\mathbf{P}^{-1}\right) && (3.21) \\ &= \text{trace}\left(\mathbf{P}^{-1}\left(\mathbf{P}\mathbf{D}^k\right)\right) && (3.22) \\ \text{trace}\left(\mathbf{A}^k\right) &= \text{trace}\left(\mathbf{I}\mathbf{D}^k\right) = \text{trace}\left(\mathbf{D}^k\right) && (3.23) \end{aligned}$$

Conclusion

$$\operatorname{trace}\left(\mathbf{A}^{1000}\right) = \operatorname{trace}\left(\mathbf{D}^{1000}\right) \tag{3.24}$$

$$= \operatorname{trace}\begin{pmatrix} 2^{1000} & 0 \\ 0 & (-1)^{1000} \end{pmatrix} \tag{3.25}$$

$$= 2^{1000} + 1 \tag{3.26}$$

C code

```c
void get_matrix_A(double* data) {
    data[0] = 5.0;
    data[1] = -3.0;
    data[2] = 6.0;
    data[3] = -4.0;
}
```

Python code for Solving

```
import ctypes
import numpy as np

def get_trace_of_power_matrix():

    lib = ctypes.CDLL('./code.so')

    double_array_4 = ctypes.c_double * 4
    lib.get_matrix_A.argtypes = [ctypes.POINTER(ctypes.c_double)]

    out_data_c = double_array_4()
    lib.get_matrix_A(out_data_c)

    A = np.array(list(out_data_c)).reshape((2, 2))

    eigenvalues = np.linalg.eigvals(A)
```

Python code for Solving

```
lambda_1, lambda_2 = eigenvalues
    trace_A_1000 = lambda_1**1000 + lambda_2**1000
    return A, (lambda_1, lambda_2), trace_A_1000

if __name__ == '__main__':
    matrix_A, eigs, final_trace = get_trace_of_power_matrix()
    l1, l2 = eigs
    print(f\ntrace(A^1000) = ({l1.real:.0f})^1000 + ({l2.real:.0f
        })^1000)
```