12.690

Shriyansh Chawda-EE25BTECH11052 October 11, 2025

Question

A real 2×2 non-singular matrix $\mathbf{A} = \begin{pmatrix} x & -3.0 \\ 3.0 & 4.0 \end{pmatrix}$ where x is a real positive number, has repeated eigenvalues. The value of x is _____. (EC 2021)

Solution

According to the Cayley-Hamilton theorem:

$$\det(\mathbf{A} - \lambda \mathbf{I}) = 0 \tag{1}$$

$$\det \begin{pmatrix} x - \lambda & -3.0 \\ 3.0 & 4.0 - \lambda \end{pmatrix} = 0 \tag{2}$$

$$(x - \lambda)(4 - \lambda) - (-3)(3) = 0 \tag{3}$$

$$\lambda^2 - (x + 4)\lambda + (4x + 9) = 0 \tag{4}$$

For the matrix to have repeated eigenvalues, the discriminant of this quadratic characteristic equation must be zero.

Solution

$$\Delta = (-(x+4))^2 - 4(1)(4x+9) = 0 \tag{5}$$
$$(x+4)^2 - 4(4x+9) = 0 \tag{6}$$
$$x^2 - 8x - 20 = 0 \tag{7}$$
$$(x-10)(x+2) = 0 \tag{8}$$

This gives two possible solutions: $x = 10$ or $x = -2$. Since the problem states that x is a real positive number, we have:

$$x = 10$$