10.3.2

Aditya Appana - EE25BTECH11004

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Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1,2).

Solution

To solve this question, we need to find the tangent at the given point to each of the curves. We can then prove that the curves touch each other at the point **if both tangent equations are the same**.
$y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ represented in the form

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0 \tag{0}$$

are:

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -2 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{1}$$

and

$$\mathbf{x^T} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -3 \\ 0 \end{pmatrix}^T \mathbf{x} + 1 = 0 \tag{2}$$

respectively.

Solution

Given the point of contact $\mathbf{q}$, the equation of tangent to (1) is:

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^T \mathbf{x} + \mathbf{u}^T \mathbf{q} + f = 0 \tag{3}$$

Therefore, the tangent to (1) at $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is:

$$\left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right)^T \mathbf{x} + \begin{pmatrix} -2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 0 \tag{4}$$

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix}^T \mathbf{x} - 2 = 0 \tag{5}$$

Solution

The tangent to (2) at $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is:

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \end{pmatrix} \right)^T \mathbf{x} + \begin{pmatrix} -3 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 1 = 0 \quad (6)$$

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix}^T \mathbf{x} - 2 = 0 \quad (7)$$

Both equations are the same, **hence proved**.

Python Code

```python
from sympy import symbols,solve
import numpy as np
import matplotlib.pyplot as plt
import math

x, y = symbols('x y')

eq1 = y**2 - 4*x
eq2 = x**2 + y**2 - 6*x + 1
solutions = solve([eq1, eq2], (x, y))

print(Solutions:)
for sol in solutions:
    print(fx = {sol[0]}, y = {sol[1]})
```

Python Code

```python
Y = np.linspace(-10,10,50)
p = Y*Y/4
l = Y - 1

fig = plt.figure(figsize = (6,6))
ax = fig.add_subplot(111)

ax.plot(p,Y, label='(1)')
ax.plot(l,Y, label = '$y=x+1$')
circle = plt.Circle((3, 0), math.sqrt(8), color='green', fill=
    False, linewidth=2, label = '(2)')
ax.add_patch(circle)
ax.scatter(1,2, label='(1,2)', color = 'red')

ax.grid(True)
plt.legend()
plt.show()
```
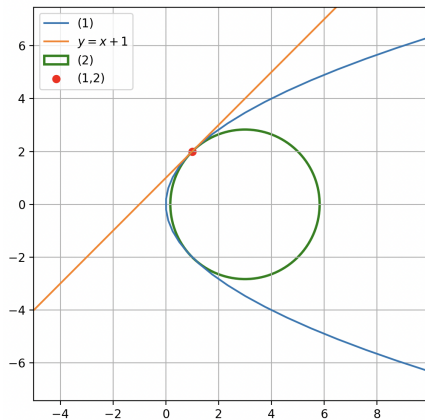
Figure

Figure: Plot