4.13.23

EE25BTECH11018 - Darisy Sreetej

Question:
Let a, b, c, d be non-zero numbers. If the point of intersection of the lines $4a\mathbf{x} + 2a\mathbf{y} + c = 0$ and $5b\mathbf{x} + 2b\mathbf{y} + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

1) $3bc - 2ad = 0$
2) $2bc - 3ad = 0$
3) $3bc + 2ad = 0$
4) $2bc + 3ad = 0$

Solution:
The two lines are

$$4a\mathbf{x} + 2a\mathbf{y} + c = 0, \tag{1}$$
$$5b\mathbf{x} + 2b\mathbf{y} + d = 0 \tag{2}$$

According to the condition,the intersection point is equidistant from the axes and lies in the fourth quadrant, so its coordinates satisfy $y = -x$

$$\mathbf{x} + \mathbf{y} = 0 \tag{3}$$

This equation can be expressed in terms of matrices

$$\begin{pmatrix} 4a \\ 2a \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = -c \tag{4}$$
$$\begin{pmatrix} 5b \\ 2b \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = -d \tag{5}$$
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = 0 \tag{6}$$

They can be represented as,

$$\begin{pmatrix} 4a & 5b \\ 2a & 2b \\ 1 & 1 \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} -c \\ -d \\ 0 \end{pmatrix} \tag{7}$$

Using augmented matrix,

$$\left(\begin{array}{cc|c} 4a & 2a & -c \\ 5b & 2b & -d \\ 1 & 1 & 0 \end{array}\right) \tag{8}$$

$R_3 = R_3 - 4aR_1$
$R_2 = R_2 - 5bR_1$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3b & -d \\ 0 & -2a & -c \end{array}\right) \tag{9}$$

$R_3 = R_3 - \frac{2a}{3b}R_2$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3b & -d \\ 0 & 0 & -c + \frac{2ad}{3b} \end{array}\right) \tag{10}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3b & -d \\ 0 & 0 & \frac{2ad-3bc}{3b} \end{array}\right) \tag{11}$$

The last row of the matrix represents the equation

$$\mathbf{y}0 + \mathbf{x}0 = \frac{2ad - 3bc}{3b} \tag{12}$$

For the system to be consistent(i.e., to have a solution),the right-hand side must be zero.

$$\frac{2ad - 3bc}{3b} = 0 \tag{13}$$

$$2ad - 3bc = 0 \tag{14}$$

Therefore,

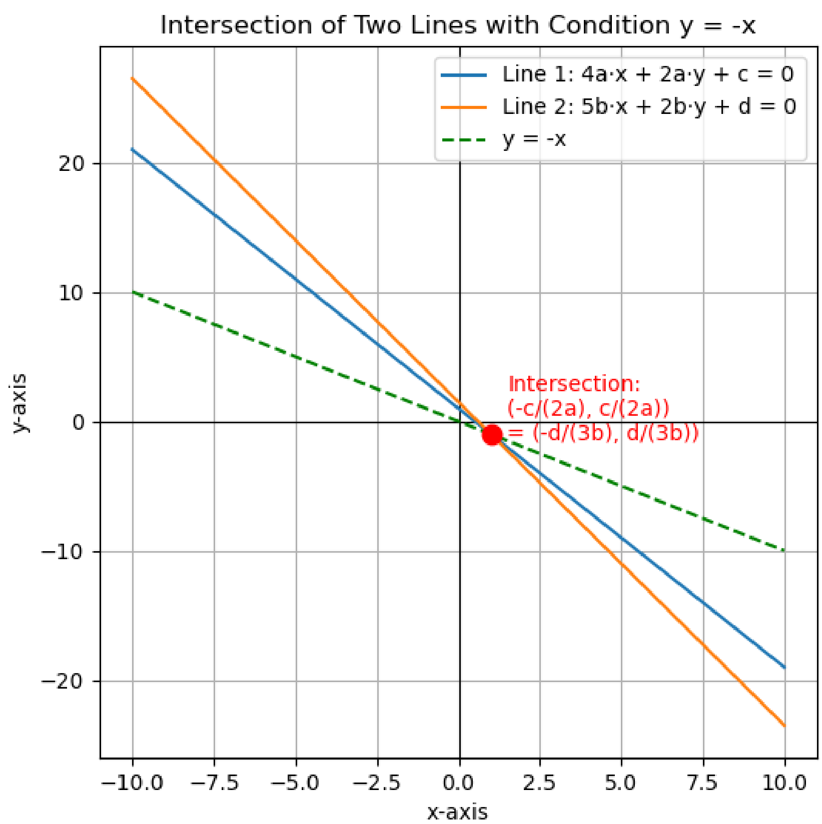
$$3bc = 2ad \tag{15}$$

Therefore, option(a) is correct
For the point of intersection , solve the equations from (11),
The point of intersection is

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \frac{-d}{3b} \\ \frac{d}{3b} \end{pmatrix} \tag{16}$$

Also ,

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \frac{-c}{2a} \\ \frac{c}{2a} \end{pmatrix} \quad (from(15)) \tag{17}$$



Intersection of Two Lines with Condition y = -x
Line 1: 4a·x + 2a·y + c = 0
Line 2: 5b·x + 2b·y + d = 0
y = -x
Intersection:
(-c/(2a), c/(2a))
= (-d/(3b), d/(3b))
y-axis
x-axis
20
10
0
−10
−20
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0