Matgeo Presentation - Problem 2.9.20

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Question

X and **Y** are two points with position vectors $3\mathbf{a}+\mathbf{b}$ and $\mathbf{a}$-$3\mathbf{b}$, respectively. Write the position vector of a point Z which divides the line segment **XY** in the ratio 2:1 externally.

solution

$$\mathbf{X} = 3\mathbf{a} + \mathbf{b} \tag{0.1}$$

$$\mathbf{Y} = \mathbf{a} - 3\mathbf{b} \tag{0.2}$$

Now, the matrix form for $\mathbf{Y}$ and $\mathbf{X}$ is:

$$\begin{pmatrix} \mathbf{Y} & \mathbf{X} \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix} \tag{0.3}$$

Using the section formula, the point $\mathbf{Z}$ dividing $\mathbf{Y} - \mathbf{X}$ in ratio $2:1$ externally is:

$$\mathbf{Z} = \frac{2\mathbf{Y} - \mathbf{X}}{2-1} \tag{0.4}$$

$$\mathbf{Z} = \begin{pmatrix} \mathbf{Y} & \mathbf{X} \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{0.5}$$

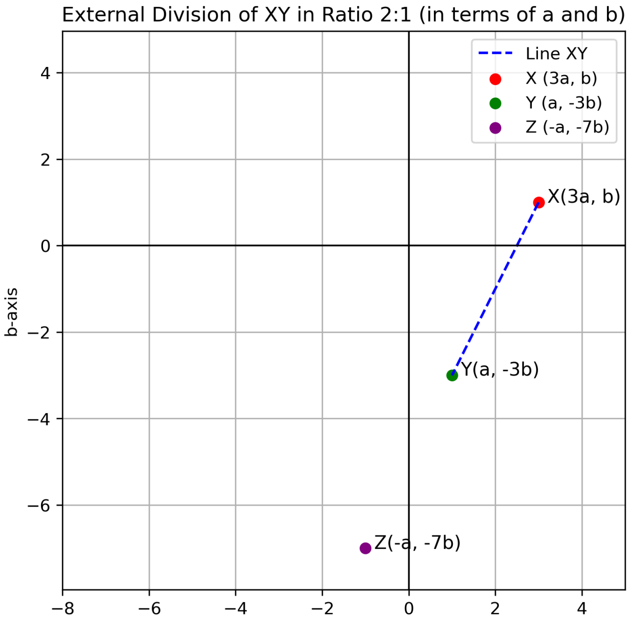
$$\mathbf{Z} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{0.6}$$

solution

$$\mathbf{Z} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{0.7}$$

$$\mathbf{Z} = -\mathbf{a} - 7\mathbf{b} \tag{0.8}$$

Plot


External Division of XY in Ratio 2:1 (in terms of a and b)
Line XY
X (3a, b)
Y (a, -3b)
Z (-a, -7b)
X(3a, b)
Y(a, -3b)
Z(-a, -7b)
b-axis
4
2
0
−2
−4
−6
−8
−6
−4
−2
0
2
4

C Code: Code.c

```
#include <stdio.h>

int main() {
    FILE *fp;
    float a, b;
    float X1, X2, Y1, Y2, Z1, Z2;
    int k = 2; // Ratio 2:1 (external division)

    // Input a and b
    printf("Enter values of a and b: ");
    scanf("%f %f", &a, &b);

    // Position vectors
    X1 = 3 * a;
    X2 = b;
    Y1 = a;
    Y2 = -3 * b;

    // External division formula: Z = (kY - X) / (k - 1)
    Z1 = (k * Y1 - X1) / (k - 1);
    Z2 = (k * Y2 - X2) / (k - 1);

    // Open file to write output
    fp = fopen("solution.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
```

C Code: Code.c

```c
    // Write result to file
    fprintf(fp, "External␣Division␣of␣XY␣in␣ratio␣2:1\n");
    fprintf(fp, "Given:\n");
    fprintf(fp, "X␣=␣(3a,␣b)\nY␣=␣(a,␣-3b)\n");
    fprintf(fp, "Computed␣position␣vector␣of␣Z:\n");
    fprintf(fp, "Z␣=␣(%.2fa,␣%.2fb)\n", Z1/a, Z2/b); // symbolic form
    fprintf(fp, "Or␣numerically:␣(%.2f,␣%.2f)\n", Z1, Z2);

    fclose(fp);

    printf("␣Output␣written␣to␣'solution.dat'\n");
    return 0;
}
```

Python: plot.py

```
  import numpy as np
import matplotlib.pyplot as plt

# Symbolic vectors (set a = 1, b = 1 for proportional plotting)
a, b = 1, 1

# Given position vectors
X = np.array([3*a, b]) # (3a, b)
Y = np.array([a, -3*b]) # (a, -3b)
k = 2 # ratio 2:1 externally

# External division formula: Z = (kY - X) / (k - 1)
Z = (k*Y - X) / (k - 1) # should give (-a, -7b)

# Plot
plt.figure(figsize=(6,6))
plt.plot([X[0], Y[0]], [X[1], Y[1]], 'b--', label='Line XY')
plt.scatter(*X, color='red', label='X (3a, b)')
plt.scatter(*Y, color='green', label='Y (a, -3b)')
plt.scatter(*Z, color='purple', label='Z (-a, -7b)')

# Annotate points
plt.text(X[0]+0.2, X[1], 'X(3a, b)', fontsize=11)
plt.text(Y[0]+0.2, Y[1], 'Y(a, -3b)', fontsize=11)
plt.text(Z[0]+0.2, Z[1], 'Z(-a, -7b)', fontsize=11)

# Axes styling
plt.axhline(0, color='black', linewidth=1)
plt.axvline(0, color='black', linewidth=1)
plt.grid(True)
plt.axis('equal')
plt.xlim(-8, 5)
plt.ylim(-8, 5)
```

Python: plot.py

```
plt.xlabel('a-axis')
plt.ylabel('b-axis')
plt.title('External Division of XY in Ratio 2:1 (in terms of a and b)')
plt.legend()

# Save figure
plt.savefig("external_division.png", dpi=300, bbox_inches='tight')
plt.show()

print(" Figure saved as 'external_division.png'")
```