12.26

EE25BTECH11012-BEERAM MADHURI

Question:
Phani starts from point **P**, goes North for 3 km, and then East for 4 km to reach point **Q**. She then turns to face point **P** and goes 15 km in that direction. She then goes North for 6 km. How far is she from point **P**, and in which direction should she go to reach point **P**?

a) 8 km, East
b) 12 km, North
c) 6 km, East
d) 10 km, North

Solution:
Let point **P** be the origin:

$$\mathbf{P} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{0.1}$$

Moving from **P** to **Q**
First, move North by 3 km:

$$\mathbf{A} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \tag{0.2}$$

Then, move East by 4 km:

$$\mathbf{B} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \tag{0.3}$$

Position at point **Q** is:

$$\mathbf{Q} = \mathbf{P} + \mathbf{A} + \mathbf{B} \tag{0.4}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} \tag{0.5}$$

$$= \begin{bmatrix} 4 \\ 3 \end{bmatrix} \tag{0.6}$$

Move 15 km Toward **P** from **Q**
Direction vector from **Q** to **P**:

$$\mathbf{D} = \mathbf{P} - \mathbf{Q} \tag{0.7}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 \\ 3 \end{bmatrix} \tag{0.8}$$

$$= \begin{bmatrix} -4 \\ -3 \end{bmatrix} \tag{0.9}$$

$$|\mathbf{D}| = \sqrt{(-4)^2 + (-3)^2} \tag{0.10}$$
$$= \sqrt{16 + 9} \tag{0.11}$$
$$= \sqrt{25} = 5 \tag{0.12}$$

$$\hat{\mathbf{D}} = \frac{1}{5}\begin{bmatrix} -4 \\ -3 \end{bmatrix} \tag{0.13}$$

Now multiply by 15 km:

$$\mathbf{C} = 15 \cdot \hat{\mathbf{D}} \tag{0.14}$$
$$= 3 \cdot \begin{bmatrix} -4 \\ -3 \end{bmatrix} \tag{0.15}$$
$$= \begin{bmatrix} -12 \\ -9 \end{bmatrix} \tag{0.16}$$

New position :

$$\mathbf{R} = \mathbf{Q} + \mathbf{C} \tag{0.17}$$
$$= \begin{bmatrix} 4 \\ 3 \end{bmatrix} + \begin{bmatrix} -12 \\ -9 \end{bmatrix} \tag{0.18}$$
$$= \begin{bmatrix} -8 \\ -6 \end{bmatrix} \tag{0.19}$$

Moving North by 6 km

$$\mathbf{F} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} \tag{0.20}$$

Final position:

$$\mathbf{S} = \mathbf{R} + \mathbf{F} \tag{0.21}$$
$$= \begin{bmatrix} -8 \\ -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \end{bmatrix} \tag{0.22}$$
$$= \begin{bmatrix} -8 \\ 0 \end{bmatrix} \tag{0.23}$$

Distance and Direction from Final Position to **P**

$$\mathbf{P} - \mathbf{S} \tag{0.24}$$
$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -8 \\ 0 \end{bmatrix} \tag{0.25}$$
$$= \begin{bmatrix} 8 \\ 0 \end{bmatrix} \tag{0.26}$$

Distance:

$$||\mathbf{P} - \mathbf{S}|| = \sqrt{8^2 + 0^2} = 8 \text{ km} \tag{0.27}$$

Direction: Since the vector is along the positive x-axis, the direction is **East**.
∴ Option a is correct

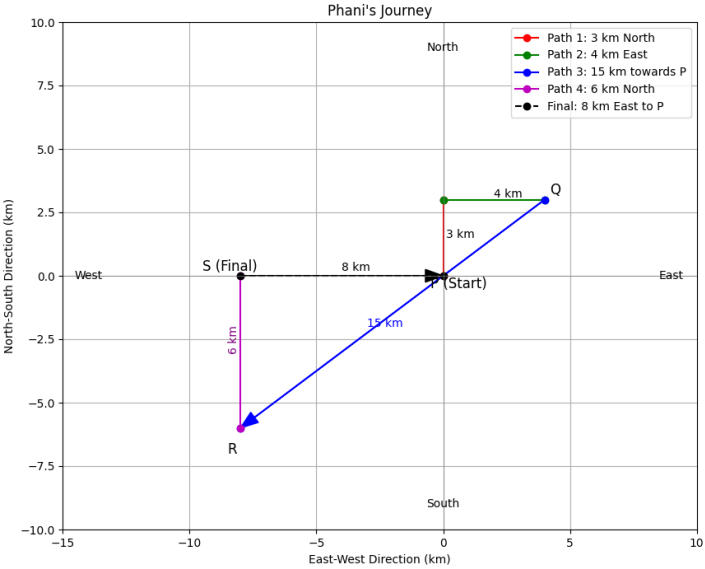

Fig. 0.1: 12.26