

5.13.38

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Question:

Let \mathbf{X} and \mathbf{Y} be two arbitrary, 3×3 , non-zero, skew-symmetric matrices and \mathbf{Z} be an arbitrary 3×3 , non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

- 1) $\mathbf{Y}^3\mathbf{Z}^4 - \mathbf{Z}^4\mathbf{Y}^3$ 2) $\mathbf{X}^{44} + \mathbf{Y}^{44}$ 3) $\mathbf{X}^4\mathbf{Z}^3 - \mathbf{Z}^3\mathbf{X}^4$ 4) $\mathbf{X}^{23} + \mathbf{Y}^{23}$

Solution:

Given,

$$\begin{aligned}\mathbf{X}^\top &= -\mathbf{X} \quad (\text{Skew-Symmetric}) \\ \mathbf{Y}^\top &= -\mathbf{Y} \quad (\text{Skew-Symmetric}) \\ \mathbf{Z}^\top &= \mathbf{Z} \quad (\text{Symmetric})\end{aligned}\tag{4.1}$$

Checking all options:

a) $\mathbf{Y}^3\mathbf{Z}^4 - \mathbf{Z}^4\mathbf{Y}^3$

Let

$$\mathbf{A} = \mathbf{Y}^3\mathbf{Z}^4 - \mathbf{Z}^4\mathbf{Y}^3\tag{4.2}$$

$$\mathbf{A}^\top = (\mathbf{Y}^3\mathbf{Z}^4 - \mathbf{Z}^4\mathbf{Y}^3)^\top\tag{4.3}$$

$$\mathbf{A}^\top = (\mathbf{Y}^3\mathbf{Z}^4)^\top - (\mathbf{Z}^4\mathbf{Y}^3)^\top\tag{4.4}$$

$$\mathbf{A}^\top = (\mathbf{Z}^\top)^4(\mathbf{Y}^\top)^3 - (\mathbf{Y}^\top)^3(\mathbf{Z}^\top)^4\tag{4.5}$$

$$\mathbf{A}^\top = (\mathbf{Z}^\top)^4(\mathbf{Y}^\top)^3 - (\mathbf{Y}^\top)^3(\mathbf{Z}^\top)^4\tag{4.6}$$

Now, substitute the given properties ($\mathbf{Z}^\top = \mathbf{Z}$ and $\mathbf{Y}^\top = -\mathbf{Y}$):

$$\mathbf{A}^\top = (\mathbf{Z})^4(-\mathbf{Y})^3 - (-\mathbf{Y})^3(\mathbf{Z})^4\tag{4.7}$$

$$\text{Since } (-\mathbf{Y})^3 = (-1)^3\mathbf{Y}^3 = -\mathbf{Y}^3, \text{ we get:}\tag{4.8}$$

$$\mathbf{A}^\top = \mathbf{Z}^4(-\mathbf{Y}^3) - (-\mathbf{Y}^3)\mathbf{Z}^4\tag{4.9}$$

$$\mathbf{A}^\top = -\mathbf{Z}^4\mathbf{Y}^3 + \mathbf{Y}^3\mathbf{Z}^4\tag{4.10}$$

$$\mathbf{A}^\top = \mathbf{Y}^3\mathbf{Z}^4 - \mathbf{Z}^4\mathbf{Y}^3\tag{4.11}$$

$$= \mathbf{A}\tag{4.12}$$

Hence, \mathbf{A} is Symmetric Matrix.

b) $\mathbf{X}^{44} + \mathbf{Y}^{44}$

Let

$$B = \mathbf{X}^{44} + \mathbf{Y}^{44} \quad (4.13)$$

$$B^\top = (\mathbf{X}^{44} + \mathbf{Y}^{44})^\top \quad (4.14)$$

$$B^\top = (\mathbf{X}^{44})^\top + (\mathbf{Y}^{44})^\top \quad (4.15)$$

$$B^\top = (\mathbf{X}^\top)^{44} + (\mathbf{Y}^\top)^{44} \quad (4.16)$$

Now, substitute the given properties ($\mathbf{X}^\top = -\mathbf{X}$ and $\mathbf{Y}^\top = -\mathbf{Y}$):

$$B^\top = (-\mathbf{X})^{44} + (-\mathbf{Y})^{44} \quad (4.17)$$

$$(-1)^{44} = 1 \quad (4.18)$$

$$\therefore (-\mathbf{X})^{44} = \mathbf{X}^{44} \text{ and } (-\mathbf{Y})^{44} = \mathbf{Y}^{44} \quad (4.19)$$

$$B^\top = \mathbf{X}^{44} + \mathbf{Y}^{44} \quad (4.20)$$

$$B^\top = B \quad (4.21)$$

Hence, B is Symmetric Matrix.

c) $\mathbf{X}^4 \mathbf{Z}^3 - \mathbf{Z}^3 \mathbf{X}^4$

Let

$$C = (\mathbf{X}^4 \mathbf{Z}^3 - \mathbf{Z}^3 \mathbf{X}^4) \quad (4.22)$$

$$C^\top = (\mathbf{X}^4 \mathbf{Z}^3 - \mathbf{Z}^3 \mathbf{X}^4)^\top \quad (4.23)$$

$$C^\top = (\mathbf{X}^4 \mathbf{Z}^3)^\top - (\mathbf{Z}^3 \mathbf{X}^4)^\top \quad (4.24)$$

$$C^\top = (\mathbf{Z}^3)^\top (\mathbf{X}^4)^\top - (\mathbf{X}^4)^\top (\mathbf{Z}^3)^\top \quad (4.25)$$

$$C^\top = (\mathbf{Z}^\top)^3 (\mathbf{X}^\top)^4 - (\mathbf{X}^\top)^4 (\mathbf{Z}^\top)^3 \quad (4.26)$$

Now, substitute the given properties ($\mathbf{Z}^\top = \mathbf{Z}$ and $\mathbf{X}^\top = -\mathbf{X}$):

$$C^\top = (\mathbf{Z})^3 (-\mathbf{X})^4 - (-\mathbf{X})^4 (\mathbf{Z})^3 \quad (4.27)$$

$$(-\mathbf{X})^4 = \mathbf{X}^4 \quad (4.28)$$

$$\mathbf{C}^\top = \mathbf{Z}^3 \mathbf{X}^4 - \mathbf{X}^4 \mathbf{Z}^3 \quad (4.29)$$

$$C^\top = -(\mathbf{X}^4 \mathbf{Z}^3 - \mathbf{Z}^3 \mathbf{X}^4) \quad (4.30)$$

$$= -C \quad (4.31)$$

Hence, C is Skew Symmetric Matrix.

d) $\mathbf{X}^{23} + \mathbf{Y}^{23}$

Let

$$D = \mathbf{X}^{23} + \mathbf{Y}^{23} \quad (4.32)$$

$$D^\top = (\mathbf{X}^{23} + \mathbf{Y}^{23})^\top \quad (4.33)$$

$$D^\top = (\mathbf{X}^{23})^\top + (\mathbf{Y}^{23})^\top \quad (4.34)$$

$$D^\top = (\mathbf{X}^\top)^{23} + (\mathbf{Y}^\top)^{23} \quad (4.35)$$

Now, substitute the given properties ($\mathbf{X}^\top = -\mathbf{X}$ and $\mathbf{Y}^\top = -\mathbf{Y}$):

$$D^\top = (-\mathbf{X})^{23} + (-\mathbf{Y})^{23} \quad (4.36)$$

$$(-1)^{23} = -1 \quad (4.37)$$

$$(-\mathbf{X})^{23} = -\mathbf{X}^{23} \text{ and } (-\mathbf{Y})^{23} = -\mathbf{Y}^{23} \quad (4.38)$$

$$D^\top = -\mathbf{X}^{23} - \mathbf{Y}^{23} \quad (4.39)$$

$$D^\top = -(\mathbf{X}^{23} + \mathbf{Y}^{23}) \quad (4.40)$$

$$= -D \quad (4.41)$$

Hence, D is Skew Symmetric Matrix.

∴ Option 3. $\mathbf{X}^4\mathbf{Z}^3 - \mathbf{Z}^3\mathbf{X}^4$ and

Option 4. $\mathbf{X}^{23} + \mathbf{Y}^{23}$ are Skew Symmetric