5.2.61

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Question: Solve the system:

x - y + 2z = 1

2z - 3z = 1

3x - 2y + 4z = 2

Solution

Variable
x
y
z

TABLE 0: Variables Used

$$\begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \mathbf{X} = 1 \tag{0.1}$$

$$\begin{pmatrix} 0 & 2 & -3 \end{pmatrix} \mathbf{X} = 1 \tag{0.2}$$

$$\begin{pmatrix} 3 & -2 & 4 \end{pmatrix} \mathbf{X} = 2 \tag{0.3}$$

This system of equations can be solved using an augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & -3 & 1 \\ 3 & -2 & 4 & 2 \end{array}\right) \xrightarrow{R_3 \to R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & -3 & 1 \\ 0 & 1 & -2 & -1 \end{array}\right) \tag{0.4}$$

$$\xrightarrow{R_3 \to R_3 - \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \tag{0.5}$$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \tag{0.6}$$

$$\xrightarrow{R_3 \to -2R_3} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{0.7}$$

$$\xrightarrow{R_2 \to R_2 + \frac{3}{2}R_3} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{0.8}$$

$$\xrightarrow{R_1 \to R_1 - 2R_3} \left(\begin{array}{ccc|c} 1 & -1 & 0 & -5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{0.9}$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{0.10}$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} \tag{0.11}$$

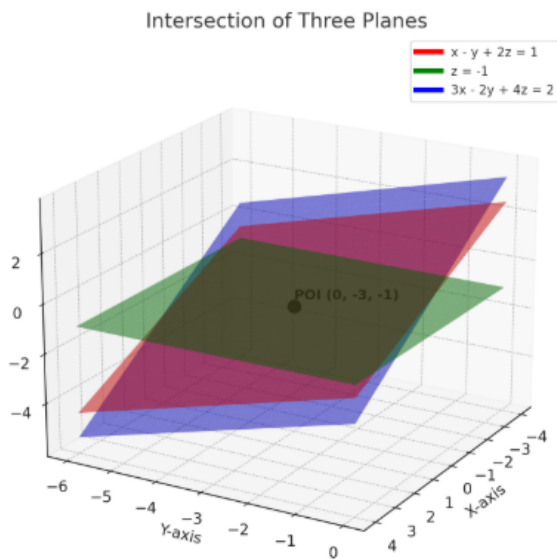$$x = 0, \quad y = 5, \quad z = 3 \tag{0.12}$$

Refer to Figure

Fig. 0.1