Problem 12.141

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Problem

Let **A** be a 3×3 matrix. Suppose that the eigenvalues of **A** are -1, 0, 1 with respective eigenvectors $(1,-1,0)^\top, (1,1,-2)^\top$ and $(1,1,1)^\top$. Then 6**A** equals

1. $\begin{pmatrix} -1 & 5 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 5 & 3 \\ 5 & 1 & 3 \\ 3 & 3 & 3 \end{pmatrix}$
4. $\begin{pmatrix} -3 & 9 & 0 \\ 9 & -3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

Equation

For an invertible matrix $\mathbf{P}$

$$\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1} \tag{3.1}$$

Given eigen values are

$$\lambda_1 = -1, \lambda_2 = 0, \lambda_3 = 1 \tag{3.2}$$

Given eigen vectors are

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \mathbf{x_3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{3.3}$$

where

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.4}$$

$$\tag{3.5}$$

Augmented matrix

$$\mathbf{P} = \begin{pmatrix} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix} \tag{3.6}$$

$$|\mathbf{P}| = 1\,(1+2) - 1\,(-1+0) + 1\,(2+0) = 3+1+2 = 6 \neq 0 \tag{3.7}$$

$$\mathbf{P}\mathbf{P}^{-1} = \mathbf{I} \tag{3.8}$$

Augmented matrix of $\left(\mathbf{P} \mid \mathbf{I}\right)$ is given by

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \rightarrow \frac{1}{2}(R_1+R_2)} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \tag{3.9}$$

(3.10)

Operations

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_1 \to R_1 - R_2]{R_3 \to R_3 + 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right) \tag{3.11}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - \frac{1}{3} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right) \tag{3.12}$$

$$\mathbf{P}^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \tag{3.13}$$

$$6\mathbf{A} = 6\mathbf{PDP}^{-1} = (\mathbf{PD})\left(6\mathbf{P}^{-1}\right) \tag{3.14}$$

Conclusion

$$6\mathbf{A} = \left(\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \left(6 \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \right) \tag{3.15}$$

$$= \left(\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \begin{pmatrix} 3 & -3 & 0 \\ 1 & 1 & -2 \\ 2 & 2 & 2 \end{pmatrix} \tag{3.16}$$

$$= \begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & -3 & 0 \\ 1 & 1 & -2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} -3+2 & 3+2 & 0+2 \\ 3+2 & -3+2 & 0+2 \\ 0+2 & 0+2 & 0+2 \end{pmatrix} \tag{3.17}$$

$$= \begin{pmatrix} -1 & 5 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix} \tag{3.18}$$

C Code

```c
void get_eigen_data(double* out_data) {
    // Eigenvalues
    out_data[0] = -1.0;
    out_data[1] = 0.0;
    out_data[2] = 1.0;
    // Eigenvector 1,
    out_data[3] = 1.0;
    out_data[4] = -1.0;
    out_data[5] = 0.0;
    // Eigenvector 2
    out_data[6] = 1.0;
    out_data[7] = 1.0;
    out_data[8] = -2.0;
    // Eigenvector 3
    out_data[9] = 1.0;
    out_data[10] = 1.0;
    out_data[11] = 1.0;
}
```

Python Code for Solving

```python
import ctypes
import numpy as np

def calculate():
    lib = ctypes.CDLL('./code.so')
    double_array_12 = ctypes.c_double * 12
    lib.get_eigen_data.argtypes = [ctypes.POINTER(ctypes.c_double
        )]

    out_data_c = double_array_12()
    lib.get_eigen_data(out_data_c)
    raw_data = np.array(list(out_data_c))

    eigenvalues = raw_data[0:3]
    v1 = raw_data[3:6]
    v2 = raw_data[6:9]
    v3 = raw_data[9:12]
```

Python Code for Solving

```python
D = np.diag(eigenvalues)
    P = np.vstack([v1, v2, v3]).T
    P_inv = np.linalg.inv(P)
    A = P @ D @ P_inv
    result_6A = 6 * A

    return result_6A

if __name__ == '__main__':
    final_result = calculate()
    print(\n 6A =\n, final_result)
```