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Question:
Solve for the system of linear equations:

$$2x + 3y = 13$$
$$4x + 5y = 23$$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

According to the question,
The equation of lines given,

$$\begin{pmatrix}2\\3\end{pmatrix}^{\top}\mathbf{x} = 13 \tag{0.1}$$

$$\begin{pmatrix}4\\5\end{pmatrix}^{\top}\mathbf{x} = 23 \tag{0.2}$$

$$\therefore \begin{pmatrix}2 & 4\\3 & 5\end{pmatrix}^{\top}\mathbf{x} = \begin{pmatrix}13\\23\end{pmatrix} \tag{0.3}$$

Using augmented matrix,

$$\left(\begin{array}{cc|c}2 & 3 & 13\\4 & 5 & 23\end{array}\right) \tag{0.4}$$

Upon doing row reduction,

$$\left(\begin{array}{cc|c}2 & 3 & 13\\4 & 5 & 23\end{array}\right) \xleftrightarrow{R_1=\frac{1}{2}\times R_1} \left(\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\4 & 5 & 23\end{array}\right) \tag{0.5}$$

$$\left(\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\4 & 5 & 23\end{array}\right) \xleftrightarrow{R_2=R_2-4\times R_1} \left(\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\0 & -1 & -3\end{array}\right) \tag{0.6}$$

$$\left(\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\4 & 5 & 23\end{array}\right) \xleftrightarrow{R_2=-R_2} \left(\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\0 & 1 & 3\end{array}\right) \tag{0.7}$$

$$\left(\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\4 & 5 & 23\end{array}\right) \xleftrightarrow{R_1=R_1-\frac{3}{2}\times R_1} \left(\begin{array}{cc|c}1 & 0 & 2\\0 & 1 & 3\end{array}\right) \tag{0.8}$$

$$\implies \mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{0.9}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.