5.9.13

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question: A shopkeeper has 3 varieties of pens A, B and C. Meenu purchased 1 pen of each variety for a total of Rs 21. Jeevan purchased 4 pens of A variety, 3 pens of B variety and 2 pens of C variety for Rs 60. While Shikha purchased 6 pens of A variety, 2 pens of B variety and 3 pens of C variety for Rs 70. Using matrix method, find the cost of each variety of pen.

Solution:

Variable	Value
a	cost of pen A
b	cost of pen B
c	cost of pen C

TABLE 0: Variables Used

Let unit cost matrix X be

$$X = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{0.1}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{pmatrix} X = \begin{pmatrix} 21 \\ 60 \\ 70 \end{pmatrix} \tag{0.2}$$

Solving it using a Augmented Matrix

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 21 \\ 4 & 3 & 2 & 60 \\ 6 & 2 & 3 & 70 \end{array}\right) \xrightarrow{R_2 \to R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 21 \\ 0 & -1 & -2 & -24 \\ 6 & 2 & 3 & 70 \end{array}\right) \tag{0.3}$$

$$\xrightarrow{R_3 \to R_3 - 6R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 21 \\ 0 & -1 & -2 & -24 \\ 0 & -4 & -3 & -56 \end{array}\right) \tag{0.4}$$

$$\xrightarrow{R_2 \to -1 \cdot R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 21 \\ 0 & 1 & 2 & 24 \\ 0 & -4 & -3 & -56 \end{array}\right) \tag{0.5}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 24 \\ 0 & -4 & -3 & -56 \end{array}\right) \tag{0.6}$$

$$\xrightarrow{R_3 \to R_3 + 4R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & 5 & 40 \end{array}\right) \tag{0.7}$$

$$\xrightarrow{R_3 \to \frac{1}{5}R_3} \left(\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & 1 & 8 \end{array}\right) \tag{0.8}$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & 1 & 8 \end{array}\right) \tag{0.9}$$

$$\xrightarrow{R_2 \to R_2 - 2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 8 \end{array}\right) \tag{0.10}$$

$$\mathbf{X} = \begin{pmatrix} 5 \\ 8 \\ 8 \end{pmatrix} \tag{0.11}$$

Therefore,
cost of pen A = Rs 5
cost of pen B = Rs 8
cost of pen C = Rs 8