4.11.12

EE25BTECH11034 - Kishora Karthik

Question:
Find the distance of the point $P(-1,-5,-10)$ from the point of intersection of the line $\mathbf{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda\,(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\mathbf{r}\cdot(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$.
Solution:
The line is given by $\mathbf{x} = \mathbf{h} + k\mathbf{m}$, where

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \tag{2}$$

The plane is of the form $\mathbf{n}^\top \mathbf{x} = c$ where $c = 5$ and,

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{3}$$

Given, $\mathbf{P} = \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$. To find the point of intersection, we substitute the vector equation of the line into the vector equation of the plane.

$$(\mathbf{a} + \lambda\mathbf{b}) \cdot \mathbf{n} = c \tag{4}$$

$$\mathbf{h}\cdot\mathbf{n} + k(\mathbf{m}\cdot\mathbf{n}) = c \tag{5}$$

$$\mathbf{h}\mathbf{n}^\top + k(\mathbf{m}\mathbf{n}^\top) = c \tag{6}$$

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} + k \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} = 5 \tag{7}$$

$$(2\cdot 1 + (-1)\cdot(-1) + (2)\cdot(1)) + k(3\cdot 1 + 4\cdot(-1) + 2\cdot(1)) = 5 \tag{8}$$

$$(5) + k(1) = 5 \tag{9}$$

$$k = 0 \tag{10}$$

The point of intersection is $\mathbf{x} = \mathbf{h} + 0(\mathbf{m}) = \mathbf{h}$.
Distance between two points is given by $\|\mathbf{x_2} - \mathbf{x_1}\|$.

$$d = \left\| \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right\| \tag{11}$$

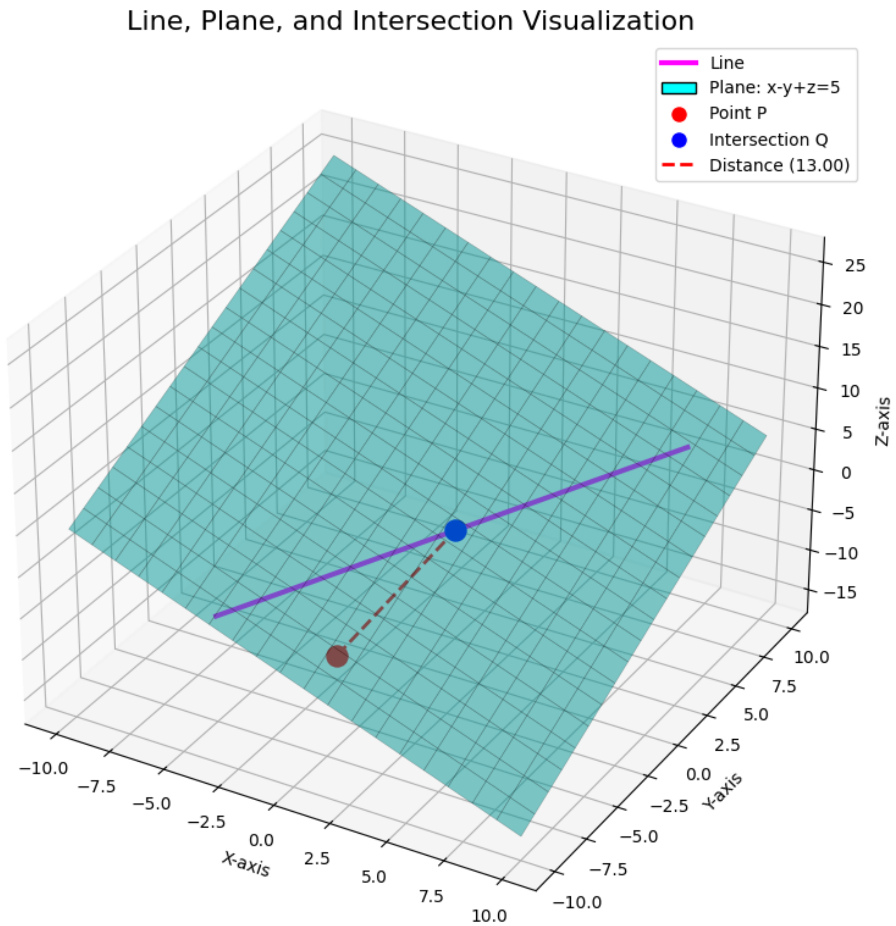
$$d = \left\| \begin{pmatrix} -3 \\ -4 \\ 12 \end{pmatrix} \right\| \tag{12}$$

$$d = \sqrt{(-3)^2 + (-4)^2 + (12)^2} \tag{13}$$

$$d = \sqrt{169} \tag{14}$$

$$d = 13 \tag{15}$$

$\therefore$ The distance between the points is 13 units.


Line, Plane, and Intersection Visualization
Line
Plane: x-y+z=5
Point P
Intersection Q
Distance (13.00)
X-axis
Y-axis
Z-axis
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
−15
−10
−5
0
5
10
15
20
25