12.378

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:
The eigenvalues of the matrix

$$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}$$

are

(a) (3, 6) (b) (1, -2) (c) (5, 4) (d) (1, 6)

Solution
According to the Cayley-Hamilton theorem, $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$.

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \det\begin{pmatrix} 6-\lambda & 1 \\ -2 & 3-\lambda \end{pmatrix} \tag{0.1}$$

$$= (6-\lambda)(3-\lambda) - (1)(-2) \tag{0.2}$$

$$= 18 - 6\lambda - 3\lambda + \lambda^2 + 2 \tag{0.3}$$

$$= \lambda^2 - 9\lambda + 20 = 0 \tag{0.4}$$

$$= (\lambda - 5)(\lambda - 4) = 0 \tag{0.5}$$

Thus, the eigenvalues are $\lambda_1 = 5$ and $\lambda_2 = 4$. This corresponds to option (c).