2.10.59

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Question Two adjacent sides of a parallelogram ABCD are given by $\mathbf{B}-\mathbf{A} = \begin{pmatrix} 2 \\ 10 \\ 11 \end{pmatrix}$ and $\mathbf{D}-\mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$. The side $\mathbf{D}-\mathbf{A}$ is rotated by an acute angle α in the plane of the parallelogram so that $\mathbf{D}-\mathbf{A}$ becomes $(\mathbf{D}-\mathbf{A})^{|}$. If $(\mathbf{D}-\mathbf{A})^{|}$ makes a right angle with the side $\mathbf{B}-\mathbf{A}$ then the cosine of the angle α is given by

1) $\frac{8}{9}$
2) $\frac{\sqrt{17}}{9}$
3) $\frac{1}{9}$
4) $\frac{4\sqrt{5}}{9}$

Solution : Given details: ABCD is a parallelogram.

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 2 \\ 10 \\ 11 \end{pmatrix} \tag{1}$$

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \tag{2}$$

The side $(\mathbf{D}-\mathbf{A})^{|}$ is perpendicular to $\mathbf{B}-\mathbf{A}$.
Property: The cosine of the angle between vector 1 and vector 2 is given by $\dfrac{{n_1}^{\top} n_2}{\|n_1\| \, \|n_2\|}$.
Since $(\mathbf{D}-\mathbf{A})^{|}$ is perpendicular to $\mathbf{B}-\mathbf{A}$,

Let the angle between the vectors be θ.

$$\alpha + \theta = \frac{\pi}{2}$$

$$\cos\theta = \frac{\mathbf{B} - \mathbf{A}^{\top}\mathbf{D} - \mathbf{A}}{\|B - A\|\,\|D - A\|} \tag{3}$$

$$\cos\theta = \frac{\begin{pmatrix} 2 & 10 & 11 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}}{\sqrt{225}\,\sqrt{9}} \tag{4}$$

$$\cos\theta = \frac{40}{45} = \frac{8}{9}\left(\because \sin\theta = \sqrt{1 - \cos^2\theta}\right) \tag{5}$$

$$\sin\theta = \sqrt{1 - \frac{64}{81}} \tag{6}$$

$$\sin\theta = \frac{\sqrt{17}}{9} \tag{7}$$

Since $\cos\alpha = \sin\theta = \frac{\sqrt{17}}{9}$
Ans. option 2

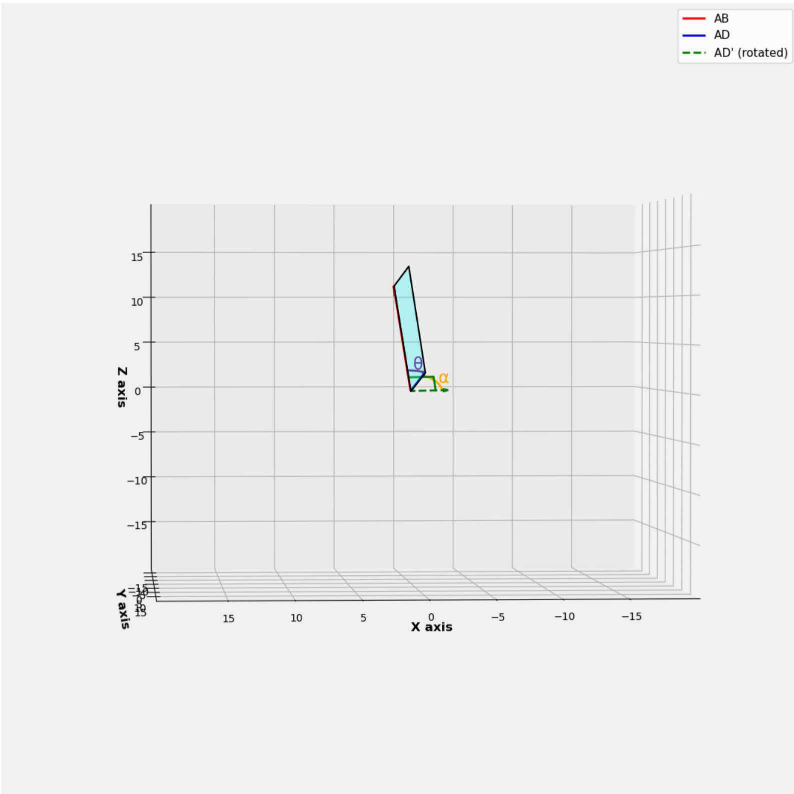

Fig. 4. Plot of the lines