5.8.16

EE25BTECH11018 - Darisy Sreetej

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Question

The taxi charges in a city consist of a fixed charge together with the charges for the distance covered. For a distance of 10 km, the charge paid is Rs.105 and for a distance of 15 km, the charge paid is Rs.155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km ?

Solution

Let us solve the given question theoretically and then verify the solution computationally.
Let x = fixed charge , y = charge per km
Then total fare = $x + y \times distance$
According to the question, The equation of lines given

$$\begin{pmatrix} 1 & 10 \end{pmatrix} \mathbf{x} = 105 \tag{1}$$

$$\begin{pmatrix} 1 & 15 \end{pmatrix} \mathbf{x} = 115 \tag{2}$$

where , $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

From the question ,

$$\begin{pmatrix} 1 & 25 \end{pmatrix} \mathbf{x} = c \tag{3}$$

where , c = total fare the person should pay for travelling 25 km

$$\therefore \begin{pmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 25 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 105 \\ 155 \\ c \end{pmatrix} \tag{4}$$

Using augmented matrix,

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 1 & 25 & c \end{array}\right) \tag{5}$$

Upon doing row reduction,

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 1 & 25 & c \end{array}\right) \xleftrightarrow{R_3 = R_3 - R_1} \left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 0 & 15 & c-105 \end{array}\right) \tag{6}$$

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 0 & 15 & c-105 \end{array}\right) \xleftrightarrow{R_2 = R_2 - R_1} \left(\begin{array}{cc|c} 1 & 10 & 105 \\ 0 & 5 & 50 \\ 0 & 15 & c-105 \end{array}\right) \tag{7}$$

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 0 & 5 & 50 \\ 0 & 15 & c-105 \end{array}\right) \xleftrightarrow{R_3 = R_3 - 3\times R_2} \left(\begin{array}{cc|c} 1 & 10 & 105 \\ 0 & 5 & 50 \\ 0 & 0 & c-255 \end{array}\right) \tag{8}$$

From (0.8),

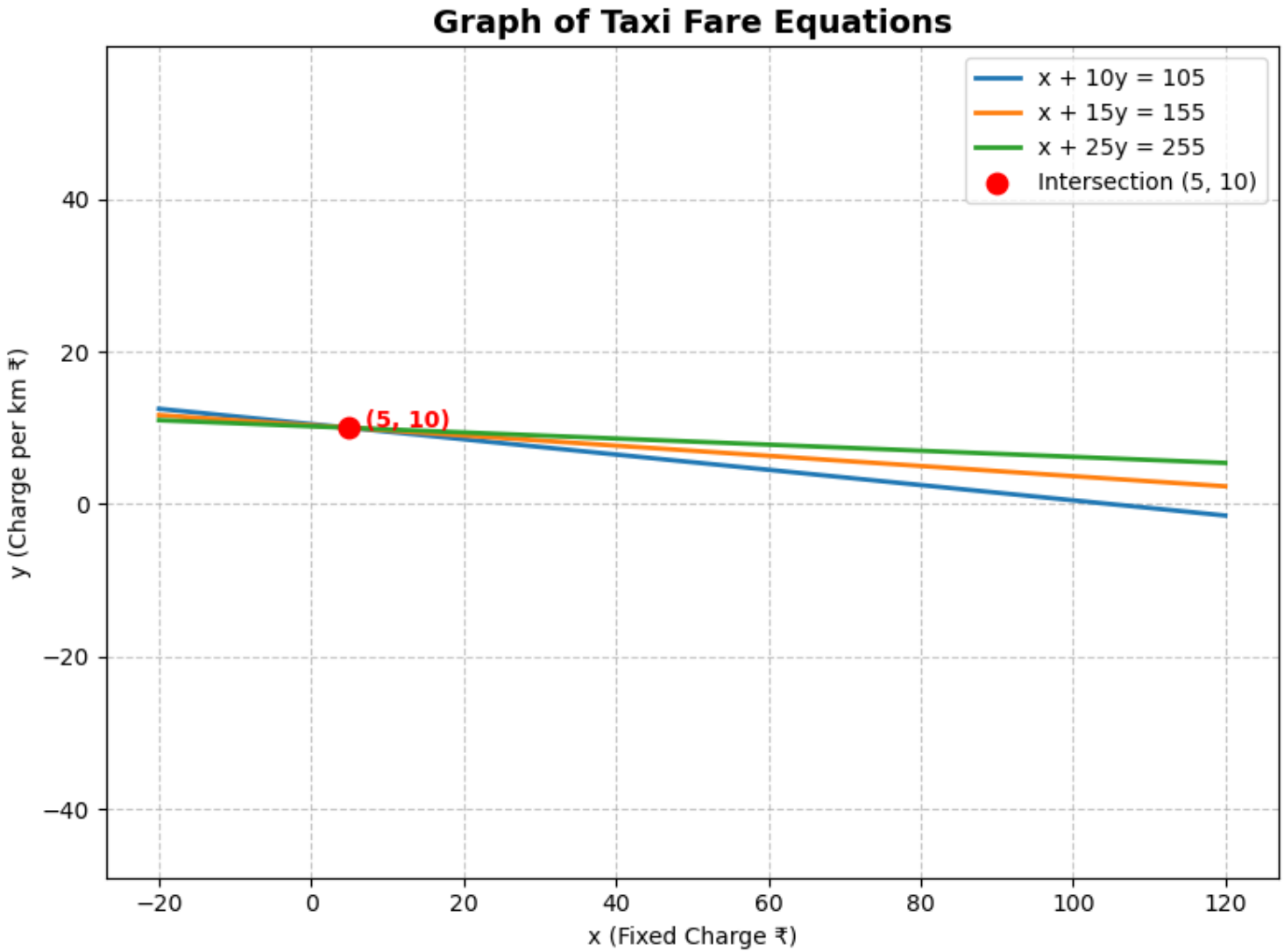
$$\begin{aligned} 0 &= c - 255 \\ c &= 255 \end{aligned} \tag{9}$$

$$\begin{aligned} 5y &= 50 \\ y &= 10 \\ x + 10y &= 105 \\ x &= 5 \end{aligned} \tag{10}$$

$$\implies \mathbf{x} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \tag{11}$$

Thus , fixed charge =Rs.5
charge per km = Rs.10
The total fare the person should pay for travelling 25 km = Rs.255

Plot


Graph of Taxi Fare Equations
x + 10y = 105
x + 15y = 155
x + 25y = 255
Intersection (5, 10)
(5, 10)
y (Charge per km ₹)
x (Fixed Charge ₹)
40
20
0
−20
−40
−20
0
20
40
60
80
100
120

C code

```c
#include <stdio.h>

void solveByMatrix(float *x, float *y)
{
    float a[2][3] = {
        {1, 10, 105},
        {1, 15, 155}
    };

    float ratio;

    // Eliminate x from second row
    ratio = a[1][0] / a[0][0];
    for (int j = 0; j < 3; j++)
        a[1][j] = a[1][j] - ratio * a[0][j];
```

C Code

```
        // Solve
    *y = a[1][2] / a[1][1];
    *x = (a[0][2] - a[0][1] * (*y)) / a[0][0];
}
```

Python + C code

```python
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# --- Load shared library (.so) ---
lib = ctypes.CDLL("./taxi_matrix.so")

# Prepare C float variables
x = ctypes.c_float()
y = ctypes.c_float()

# Call the C function
lib.solveByMatrix(ctypes.byref(x), ctypes.byref(y))

# Extract computed values
x_val = x.value
y_val = y.value
```

Python + C code

```
print(f"Fixed charge (x) = {x_val:.2f}")
print(f"Charge per km (y) = {y_val:.2f}")

# --- Generate data for plotting ---
x_vals = np.linspace(-20, 120, 400)

# Line equations from the problem
y1 = (105 - x_vals) / 10
y2 = (155 - x_vals) / 15
y3 = (255 - x_vals) / 25 # third line for c = 255

# Intersection point (from C)
x_int, y_int = x_val, y_val

# --- Plot configuration ---
plt.figure(figsize=(9, 6))
plt.style.use('seaborn-v0_8-whitegrid')
```

Python + C code

```
        # Plot lines with clarity
plt.plot(x_vals, y1, color='royalblue', linewidth=2.8, label=r'$x
     + 10y = 105$')
plt.plot(x_vals, y2, color='darkorange', linewidth=2.8, linestyle
    ='--', label=r'$x + 15y = 155$')
plt.plot(x_vals, y3, color='green', linewidth=2.8, linestyle='-.'
    , label=r'$x + 25y = 255$')


# Intersection point
plt.scatter(x_int, y_int, color='red', s=100, edgecolors='black',
     zorder=5, label=f'Intersection ({x_int:.0f}, {y_int:.0f})')
plt.text(x_int + 5, y_int + 1, f'({x_int:.0f}, {y_int:.0f})',
    fontsize=12, color='red', fontweight='bold')
```

Python + C code

```
# --- Labels and aesthetics ---
plt.title("Graph of Taxi Fare Equations", fontsize=15, fontweight
    ='bold')
plt.xlabel("x (Fixed Charge )", fontsize=12)
plt.ylabel("y (Charge per km )", fontsize=12)

plt.grid(True, linestyle='--', linewidth=0.7, alpha=0.8)
plt.legend(fontsize=11, loc='upper right', frameon=True, shadow=
    True)
plt.xlim(-20, 120)
plt.ylim(-50, 50)
plt.tight_layout()

plt.show()
```

Python code

```
import numpy as np
import matplotlib.pyplot as plt

# Define y = (const - x)/coefficient equations
x_vals = np.linspace(-20, 120, 400)

# Line equations
y1 = (105 - x_vals) / 10
y2 = (155 - x_vals) / 15
y3 = (255 - x_vals) / 25 # c = 255

# Intersection point (x=5, y=10)
x_int, y_int = 5, 10

# Plot all lines
plt.figure(figsize=(8,6))
plt.plot(x_vals, y1, label="x + 10y = 105", linewidth=2)
```

Python code

```
plt.plot(x_vals, y2, label="x + 15y = 155", linewidth=2)
plt.plot(x_vals, y3, label="x + 25y = 255", linewidth=2)
# Mark intersection
plt.scatter(x_int, y_int, color='red', s=80, zorder=5, label="
    Intersection (5, 10)")
# Annotate intersection
plt.text(x_int + 2, y_int, "(5, 10)", color='red', fontsize=10,
    fontweight='bold')
# Labels and title
plt.title("Graph of Taxi Fare Equations", fontsize=14, fontweight
    ='bold')
plt.xlabel("x (Fixed Charge )")
plt.ylabel("y (Charge per km )")
plt.grid(True, linestyle="--", alpha=0.7)
plt.legend()
plt.axis("equal") # to maintain aspect ratio
plt.tight_layout()
plt.show()
```