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EE25BTECH11018 - Darisy Sreetej

Question:
Point $\mathbf{P}(0, 2)$ is the point of intersection of the y-axis and the perpendicular bisector of the line segment joining the points $\mathbf{A}(-1, 1)$ and $\mathbf{B}(3, 3)$.
True or **False**
Solution:

TABLE I

A	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

Let the equation of perpendicular bisector be

$$\mathbf{n}^\top \mathbf{x} = C \tag{1}$$

Let $\mathbf{R}$ be the midpoint of the line segment $\mathbf{AB}$

$$\mathbf{R} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{\begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix}}{2} \tag{2}$$

$$\mathbf{R} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{3}$$

The direction vector of $\mathbf{AB}$ is

$$\mathbf{n} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{4}$$

As it passes through the midpoint $\mathbf{R}$,

$$\begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = C \tag{5}$$

$$C = 8 \tag{6}$$

Therefore , the equation of the perpendicular bisector is

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix}^\top \mathbf{x} = 8 \tag{7}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}^\top \mathbf{x} = 4 \tag{8}$$

Let $\mathbf{P}$ be the point of intersection of y-axis with the perpendicular bisector
Intersection with y-axis ($x = 0$) ,

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = 4 \tag{9}$$

$$y = 4 \tag{10}$$

Thus,

$$P = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{11}$$

The point of intersection is $P(0, 4)$
Therefore the Statement is **False**

Perpendicular Bisector of AB and Intersection with Y-axis