10.7.69

EE25BTECH11020 - Darsh Pankaj Gajare

October 11, 2025

Question:
Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point $(4, 5)$ with a pair of radii form a quadrilateral with area______.
Solution:

$$\text{Conic:} \quad \mathbf{x}^\top V\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{0.1}$$

$$V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}, \quad f = -11 \tag{0.2}$$

Matrix equation of a line through P:

$$\mathbf{x} = \mathbf{P} + t\mathbf{m}, \quad \mathbf{P} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \ \mathbf{m} = \begin{pmatrix} 1 \\ k \end{pmatrix} \tag{0.3}$$

Substitute into the conic:

$$\left(\mathbf{P} + t\mathbf{m}\right)^\top V \left(\mathbf{P} + t\mathbf{m}\right) + 2\mathbf{u}^\top \left(\mathbf{P} + t\mathbf{m}\right) + f = 0 \tag{0.4}$$

$$\left(k^2+1\right)t^2+(8k+4)\,t+4=0 \tag{0.5}$$

Tangency from P$\Rightarrow$ double root in t

$$(8k+4)^2-4\cdot\left(k^2+1\right)\cdot 4=0 \tag{0.6}$$

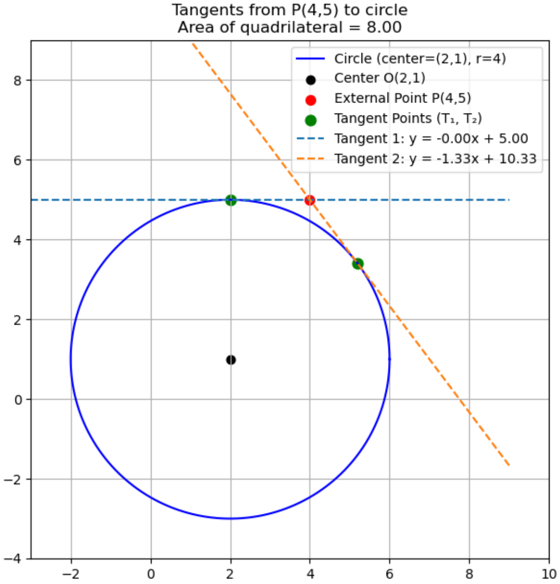
$$k=0,-\frac{4}{3} \tag{0.7}$$

$$\text{For each } k: \quad t=-\frac{8k+4}{2(k^2+1)} \tag{0.8}$$

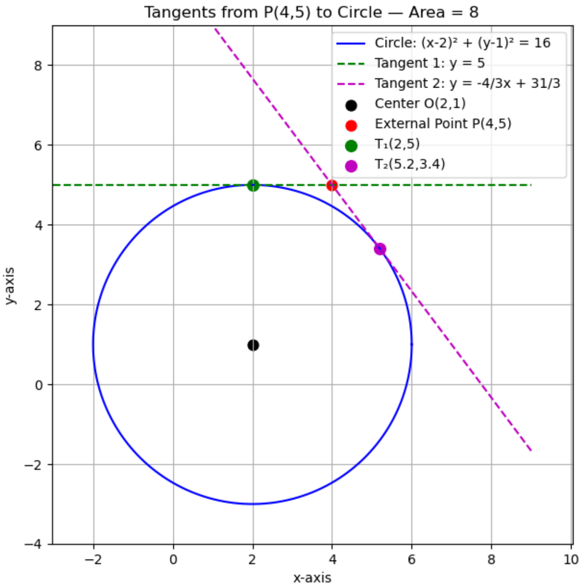
$$\text{Thus contact points } A=\mathbf{P}+t\mathbf{m} \text{ are } A_1=\begin{pmatrix}2\\5\end{pmatrix}, \quad A_2=\begin{pmatrix}\frac{26}{5}\\\frac{17}{5}\end{pmatrix} \tag{0.9}$$

$$C = -\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.10}$$

$$area\left(A_2CA_1P\right) = \frac{1}{2}\left(\|(\mathbf{A_1} - \mathbf{C}) \times (\mathbf{P} - \mathbf{C})\| + \|(\mathbf{A_2} - \mathbf{C}) \times (\mathbf{P} - \mathbf{C})\|\right) = 8 \tag{0.11}$$


Tangents from P(4,5) to circle
Area of quadrilateral = 8.00
Circle (center=(2,1), r=4)
Center O(2,1)
External Point P(4,5)
Tangent Points (T₁, T₂)
Tangent 1: y = -0.00x + 5.00
Tangent 2: y = -1.33x + 10.33

Plot using C libraries:


Tangents from P(4,5) to Circle — Area = 8
Circle: $(x-2)^2 + (y-1)^2 = 16$
Tangent 1: $y = 5$
Tangent 2: $y = -4/3x + 31/3$
Center O(2,1)
External Point P(4,5)
$T_1(2,5)$
$T_2(5.2,3.4)$
y-axis
x-axis
8
6
4
2
0
−2
−4
−2
0
2
4
6
8
10

Plot using Python: