2.10.47

EE25BTECH11018 - DARISY SREETEJ

Question: The value of a so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is

1) -3
2) 3
3) $\frac{1}{\sqrt{3}}$
4) $\sqrt{3}$

Solution: Let us consider,

$$\mathbf{p} = \hat{i} + a\hat{j} + \hat{k}$$
$$\mathbf{q} = \hat{j} + a\hat{k}$$
$$\mathbf{r} = a\hat{i} + \hat{k}$$

then, the Volume of the parallelopiped formed by $\mathbf{p}, \mathbf{q}, \mathbf{r}$ is ,

$$V = \mathbf{p} \cdot (\mathbf{q} \times \mathbf{r}) \tag{4.1}$$

$$V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} \tag{4.2}$$

$$V = a^3 - a + 1 \tag{4.3}$$

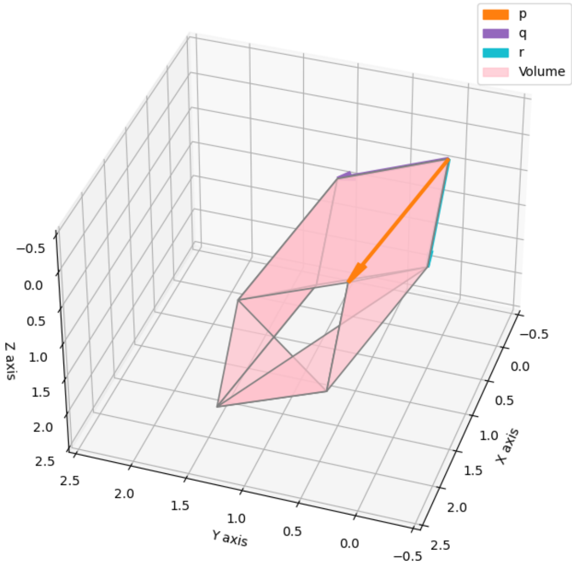
Now , consider

$$f(a) = a^3 - a + 1 \tag{4.4}$$
$$f'(a) = 3a^2 + 1 \tag{4.5}$$

$$\text{Set } f'(a) = 0 \Rightarrow a^2 = \frac{1}{\sqrt{3}} \Rightarrow a = \frac{1}{\sqrt{3}} or - \frac{1}{\sqrt{3}}$$

$$\text{Second derivative } f''(a) = 6a \tag{4.6}$$
$$\text{At } a = \frac{1}{\sqrt{3}}, f'' > 0 \Rightarrow minimum \tag{4.7}$$
$$\text{At } a = -\frac{1}{\sqrt{3}}, f'' < 0 \Rightarrow maximum \tag{4.8}$$

Therefore , $a = \frac{1}{\sqrt{3}}$ for which the Volume of the parallelopiped becomes minimum.



Parallelopiped with Vectors $\mathbf{p}, \mathbf{q}, \mathbf{r}$ for which $a = \frac{1}{\sqrt{3}}$(Volume is minimum)