4.13.24

EE25BTECH11019 - Darji Vivek M.

Question:
The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0, 0)$, $(0, 41)$ and $(41, 0)$ is:

1) 820
2) 780
3) 901
4) 861

Solution:

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 \\ 41 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 41 \\ 0 \end{pmatrix} \tag{1}$$

Step 1: Area using determinant.

$$A = \frac{1}{2} \left\| \begin{matrix} 0 & 41 \\ 41 & 0 \end{matrix} \right\| \tag{2}$$

$$A = \frac{1}{2} \left| 0 - (41)(41) \right| \tag{3}$$

$$A = \frac{1681}{2} \tag{4}$$

Step 2: Boundary lattice points. For a line joining integer points (x_1, y_1) and (x_2, y_2), the number of lattice points on it is given by

$$N = \gcd(|x_2 - x_1|, |y_2 - y_1|) + 1 \tag{5}$$

For each side:

$$B_1 = \gcd(|0 - 0|, |41 - 0|) + 1 = 41 + 1 = 42 \tag{6}$$

$$B_2 = \gcd(|41 - 0|, |0 - 0|) + 1 = 41 + 1 = 42 \tag{7}$$

$$B_3 = \gcd(|41 - 0|, |0 - 41|) + 1 = 41 + 1 = 42 \tag{8}$$

Since each vertex is counted twice,

$$B = 42 + 42 + 42 - 3 = 123 \tag{9}$$

Step 3: Apply Pick's theorem.

$$A = I + \frac{B}{2} - 1 \tag{10}$$

Rearranging for I,

$$I = A - \frac{B}{2} + 1 \tag{11}$$

Substitute $A = \frac{1681}{2}$ and $B = 123$:

$$I = \frac{1681}{2} - \frac{123}{2} + 1 \tag{12}$$

$$I = \frac{1558}{2} + 1 \tag{13}$$

$$I = 780 \tag{14}$$

The number of integer lattice points lying strictly inside the triangle is $\boxed{780}$

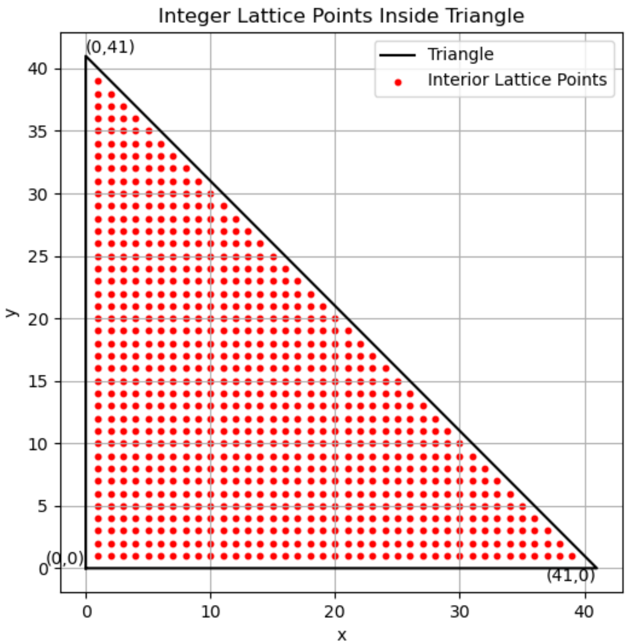

Fig. 4.1: plot