4.2.22

EE25BTECH11019 – Darji Vivek M.

Question

Solve for the system of linear equations:

$$\sqrt{2}x + \sqrt{3}y = 0$$
$$\sqrt{3}x - \sqrt{8}y = 0$$

Theoretical Solution

According to the question,
The equation of lines given,

$$\begin{pmatrix}\sqrt{2} & \sqrt{3}\end{pmatrix}\mathbf{x} = 0 \quad \begin{pmatrix}\sqrt{3} & -\sqrt{8}\end{pmatrix}\mathbf{x} = 0 \tag{1}$$

$$\therefore \begin{pmatrix}\sqrt{2} & \sqrt{3} \\ \sqrt{3} & -\sqrt{8}\end{pmatrix}\mathbf{x} = \begin{pmatrix}0 \\ 0\end{pmatrix} \tag{2}$$

Theoretical Solution

Forming an augmented matrix,

$$\left(\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 0 \\ \sqrt{3} & -\sqrt{8} & 0 \end{array}\right)$$

Upon doing row reduction,

$$\left(\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 0 \\ \sqrt{3} & -\sqrt{8} & 0 \end{array}\right) \xrightarrow{R_2 \leftarrow R_2 - \frac{\sqrt{3}}{\sqrt{2}} R_1} \left(\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 0 \\ 0 & -\sqrt{8} - \frac{3}{\sqrt{2}} & 0 \end{array}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 + 0 \times R_2} \left(\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 0 \\ 0 & -\sqrt{8} - \frac{3}{\sqrt{2}} & 0 \end{array}\right)$$

$$\implies \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

C Code: parallel_funcs.c

```c
#include <math.h>
// Function to find the intersection of two lines
// Line 1: a1*x + b1*y = c1
// Line 2: a2*x + b2*y = c2
// result[0] = x, result[1] = y
void intersection(double a1, double b1, double c1,
                  double a2, double b2, double c2,
                  double *result)
{
    double det = a1*b2 - a2*b1;
    if (fabs(det) < 1e-9) {
        // Lines are parallel or coincident
        result[0] = NAN;
        result[1] = NAN;
        return;
    }
    result[0] = (c1*b2 - c2*b1) / det;
    result[1] = (a1*c2 - a2*c1) / det;
}
```

Python: Plotting Lines

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the C library
lib = ctypes.CDLL('./10.so')

# Define argument and return types
lib.intersection.argtypes = [ctypes.c_double, ctypes.c_double,
    ctypes.c_double,
                             ctypes.c_double, ctypes.c_double,
                                 ctypes.c_double,
                             ctypes.POINTER(ctypes.c_double)]

# Prepare variables
a1, b1, c1 = np.sqrt(2), np.sqrt(3), 0
a2, b2, c2 = np.sqrt(3), -np.sqrt(8), 0

# Result array
res = (ctypes.c_double * 2)()
lib.intersection(a1, b1, c1, a2, b2, c2, res)
```

Python: Plotting Lines

```
x_inter, y_inter = res[0], res[1]
print(f"Intersection point: ({x_inter:.2f}, {y_inter:.2f})")

# Generate line points
x = np.linspace(-5, 5, 100)
y1 = (-a1 * x + c1) / b1
y2 = (-a2 * x + c2) / b2

# Plot lines
plt.plot(x, y1, label=r'$\sqrt{2}x + \sqrt{3}y = 0$')
plt.plot(x, y2, label=r'$\sqrt{3}x - \sqrt{8}y = 0$')
plt.scatter(x_inter, y_inter, color='red', label='Intersection
    Point (0,0)')
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Intersection of Two Lines')
plt.legend()
plt.grid(True)
plt.show()
```

Pyhton plot

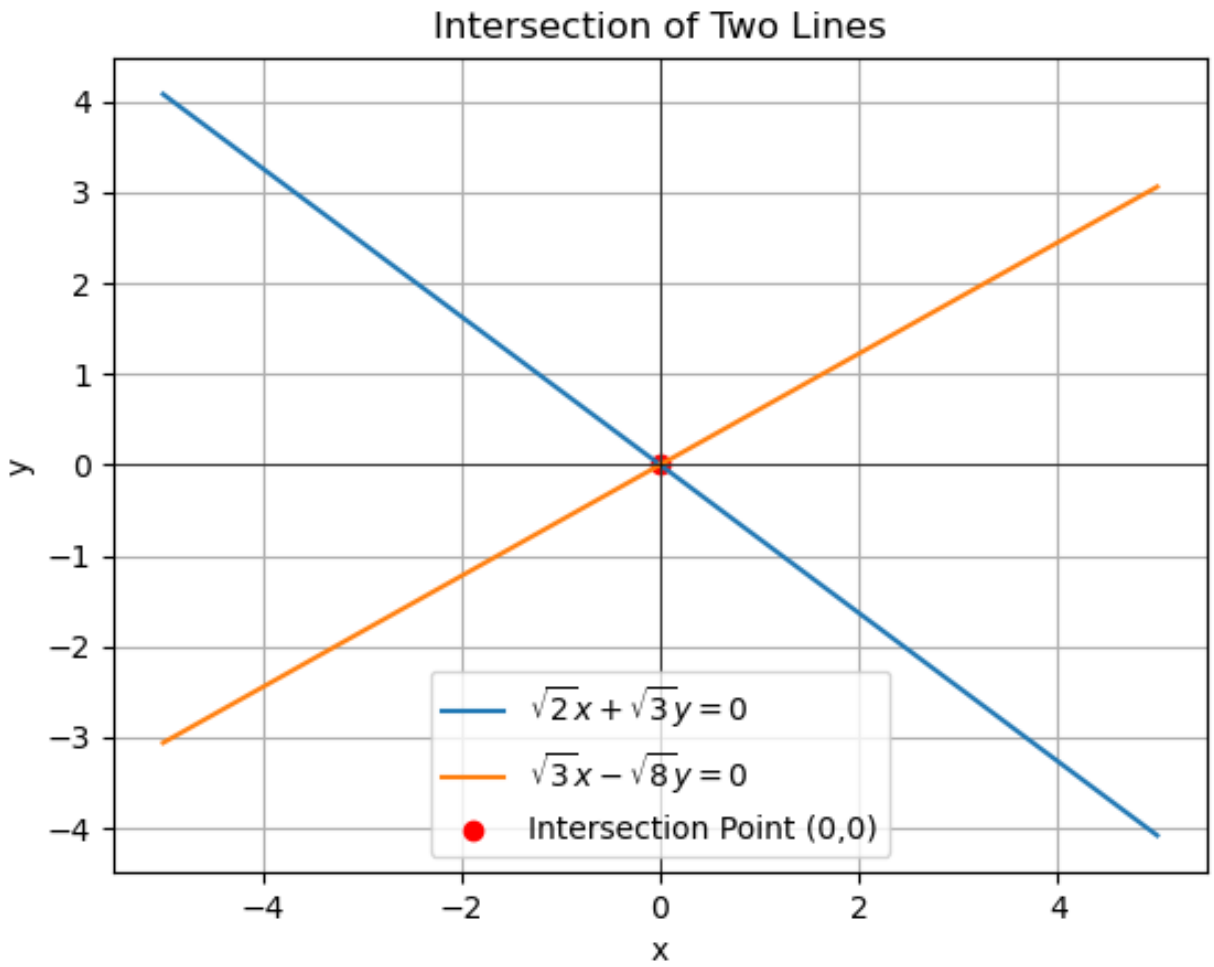

Figure: parallel lines