8.4.28

EE25BTECH11025 - Ganachari Vishwambhar

Question:
The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.
Solution:
Let:
The shorter side of the rectangle be x
The longer side of the rectangle be y
Then the diagonal of the rectangle will be $\sqrt{x^2 + y^2}$
Given:

$$\sqrt{x^2 + y^2} = x + 60 \tag{1}$$

$$y^2 - 120x - 3600 = 0 \tag{2}$$

$$-x + y = 30 \tag{3}$$

Writing equation (3) in conic/quadratic form:

$$\mathbf{x}^\top A\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + c = 0 \tag{4}$$

$$\tag{5}$$

where,

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

$$\mathbf{u} = \begin{pmatrix} -60 \\ 0 \end{pmatrix} \tag{7}$$

$$c = -3600 \tag{8}$$

Writing equation(4) in parametric form:

$$\mathbf{x} = \mathbf{p} + t\mathbf{m} \tag{9}$$

where,

$$\mathbf{p} = \begin{pmatrix} 0 \\ 30 \end{pmatrix} \tag{10}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{11}$$

Substituting (9) in (4), we get:

$$pt^2 + qt + r = 0 \tag{12}$$

$$\tag{13}$$

where,

$$p = \mathbf{m}^\top A\mathbf{m} \tag{14}$$

$$q = 2\left(\mathbf{p}^\top A\mathbf{m} + \mathbf{u}^\top \mathbf{m}\right) \tag{15}$$

$$r = \mathbf{p}^\top A\mathbf{p} + 2\mathbf{u}^\top \mathbf{p} \tag{16}$$

By using Sridharacharya's formula,

$$t = \frac{1}{\mathbf{m}^\top A\mathbf{m}}\left(-\mathbf{m}^\top (A\mathbf{p}+\mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (A\mathbf{p}+\mathbf{u})\right]^2 - (\mathbf{m}^\top A\mathbf{m})(\mathbf{p}^\top A\mathbf{p} + 2\mathbf{u}^\top\mathbf{p})}\right) \tag{17}$$

Substituting (17) in (9) we get:

$$\mathbf{x} = \mathbf{p} + \frac{1}{\mathbf{m}^\top A\mathbf{m}}\left(-\mathbf{m}^\top (A\mathbf{p}+\mathbf{u}) + \sqrt{\left[\mathbf{m}^\top (A\mathbf{p}+\mathbf{u})\right]^2 - (\mathbf{m}^\top A\mathbf{m})(\mathbf{p}^\top A\mathbf{p} + 2\mathbf{u}^\top\mathbf{p})}\right)\mathbf{m} \tag{18}$$

$$\mathbf{x} = \mathbf{p} + \frac{1}{\mathbf{m}^\top A\mathbf{m}}\left(-\mathbf{m}^\top (A\mathbf{p}+\mathbf{u}) - \sqrt{\left[\mathbf{m}^\top (A\mathbf{p}+\mathbf{u})\right]^2 - (\mathbf{m}^\top A\mathbf{m})(\mathbf{p}^\top A\mathbf{p} + 2\mathbf{u}^\top\mathbf{p})}\right)\mathbf{m} \tag{19}$$

After substituting values in equation (18) and (19), we get:

$$\mathbf{p}_1 = \begin{pmatrix} 90 \\ 120 \end{pmatrix} \tag{20}$$

$$\mathbf{p}_2 = \begin{pmatrix} -30 \\ 0 \end{pmatrix} \tag{21}$$

Since the side of the rectangle cannot be negative. The correct vector is $\mathbf{p}_1$. Therefore,

$$x = 90 \tag{22}$$

$$y = 120 \tag{23}$$

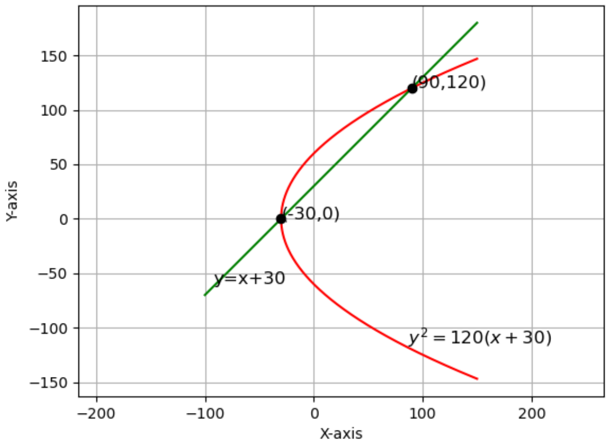

Fig. 1: Plot of the parabola and the line