

4.13.9

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Question

Let the algebraic sum of the perpendicular distances from the points $(2,0)$, $(0,2)$ and $(1,1)$ to a variable straight line be zero; then the line passes through a fixed point whose coordinates are _____.

The normal form of a line is:

$$\mathbf{n}^T \mathbf{x} = c \quad (1)$$

The perpendicular distance of a point from a line is:

$$\frac{|\mathbf{n}^T \mathbf{x} - c|}{\|\mathbf{n}\|} \quad (2)$$

It is given that the algebraic sum of the perpendicular distances of three points (2,0), (0,2), and (1,1) to a line $\mathbf{n}^T \mathbf{x} = c$ is 0. Therefore:

$$\frac{\mathbf{n}^T \mathbf{x}_1 - c}{\|\mathbf{n}\|} + \frac{\mathbf{n}^T \mathbf{x}_2 - c}{\|\mathbf{n}\|} + \frac{\mathbf{n}^T \mathbf{x}_3 - c}{\|\mathbf{n}\|} = 0 \quad (3)$$

Solution

$$\frac{\mathbf{n}^T(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3) - 3c}{\|\mathbf{n}\|} = 0 \quad (4)$$

So, the required point is:

$$\frac{(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3)}{3} \quad (5)$$

Substituting the points:

$$\frac{\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{3} = \quad (6)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (7)$$

Therefore the line passes through the fixed point (1,1).

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

fig = plt.figure(figsize = (6,6))
ax = fig.add_subplot(111)
ax.set_aspect('equal', adjustable='box')
ax.set_title(Three Points)

vector = str(input(Input the vectors, and input X when you are
done))

counter = 0;
sum = np.zeros(2)
```

Python Code

```
while(vector != X):
    vector.strip()
    vector0, vector1 = vector.split( )
    vector0 = int(vector0)
    vector1= int(vector1)
    vectorA = np.array([vector0,vector1])
    sum += vectorA
    counter = counter + 1
    ax.scatter(vector0, vector1, label = f'({vector0}, {vector1})
    ')
    vector = str(input(Input the next vector))

commonpoint = sum/counter
print(fThe line passes through ( {commonpoint[0]}, {commonpoint
    [1]} ))
plt.legend()
ax.grid(True)
plt.show()
```

