12.122

EE25BTECH11004 - Aditya Appana

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Question

The S_2 operation on a molecule with the axis of rotation as the Z-axis, moves a nucleus at (x, y, z) to

A)
$$(-x, -y, z)$$

B)
$$(x, -y, -z)$$

A)
$$(-x, -y, z)$$
 B) $(x, -y, -z)$ C) $(-x, y, -z)$ D) $(-x, -y, -z)$

D)
$$(-x, -y, -z)$$

Solution

The rotation matrix for a rotation by an angle θ about the z-axis is:

$$R_z(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0\\ \sin \theta & \cos \theta & 0\\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Let the point be $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Therefore the rotated vector will be:

$$R_z(\theta)\mathbf{x} = \begin{pmatrix} \cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x\\ y\\ z \end{pmatrix}$$
 (2)

$$\begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \end{pmatrix} \tag{3}$$

It can be seen that a rotation about the Z-axis does not change the z-coordinate. Hence $option(\mathbf{A})$ is correct.

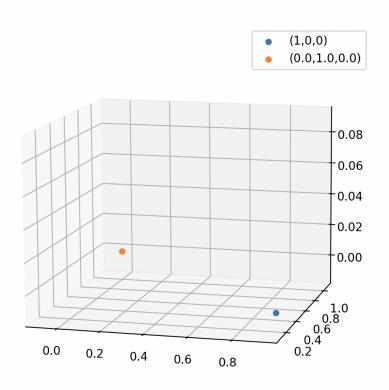


Figure 1: Plot