12.765

EE25BTECH11023 - Venkata Sai

Question:

Let $\mathbf{v_1} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\mathbf{v_2} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ be two vectors. The value of the coefficient α in the expression $\mathbf{v_1} = \alpha \mathbf{v_2} + \mathbf{e}$, which minimizes the length of the error vector $\mathbf{e}$, is

Solution:

Given expression

$$\mathbf{v_1} = \alpha \mathbf{v_2} + \mathbf{e} \tag{1}$$

where $\mathbf{e}$ is the error vector

For any linear system $\mathbf{Ax} = \mathbf{B}$, the least squares solution formula is given by

$$\left(\mathbf{A}^\top \mathbf{A}\right) \mathbf{x} = \mathbf{A}^\top \mathbf{B} \tag{2}$$

$$\mathbf{x} = \left(\mathbf{A}^\top \mathbf{A}\right)^{-1} \mathbf{A}^\top \mathbf{B} \tag{3}$$

On writing the given expression as a linear system

$$\mathbf{v_2} \alpha = \mathbf{v_1} \tag{4}$$

where α being an 1×1 vector

$$\mathbf{A} = \mathbf{v_2}, \mathbf{B} = \mathbf{v_1} \tag{5}$$

$$\alpha = \left(\mathbf{v_2}^\top \mathbf{v_2}\right)^{-1} \mathbf{v_2}^\top \mathbf{v_1} \tag{6}$$

$$= \left(\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}^\top \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}\right)^{-1} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}^\top \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \tag{7}$$

$$= \left(\begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}\right)^{-1} \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \tag{8}$$

$$= (4 + 1 + 9)^{-1} (2 + 2 + 0) \tag{9}$$

$$= \frac{1}{14} (4) \tag{10}$$

$$= \frac{2}{7} \tag{11}$$

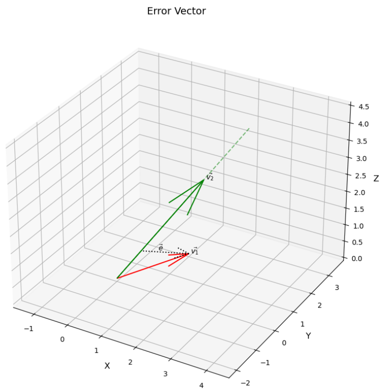
Error Vector
$\vec{v_2}$
$\vec{e}$
$\vec{v_1}$
X
Y
Z
−1
0
1
2
3
4
−2
−1
0
1
2
3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

Fig. 0.1