12.661

EE25BTECH11023 - Venkata Sai

Question:

Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & a \\ b & 0 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} 3 & 1 \\ 3 & 2 \end{pmatrix}$.If $\mathbf{AX} = \mathbf{Y}$.Then $a + b$ equals

Solution:

Given

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & a \\ b & 0 \end{pmatrix} \text{ and } \mathbf{Y} = \begin{pmatrix} 3 & 1 \\ 3 & 2 \end{pmatrix} \tag{1}$$

$$\mathbf{AX} = \mathbf{Y} \tag{2}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{Y} \tag{3}$$

Augmented matrix of $\begin{pmatrix} \mathbf{A} \mid \mathbf{I} \end{pmatrix}$ is given by

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & 1 \end{array}\right) \xrightarrow{R_2 \to \frac{2}{3}R_2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & \frac{2}{3}(-3) & \frac{2}{3}(-2) & \frac{2}{3}(1) \end{array}\right) \tag{4}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -\frac{4}{3} & \frac{2}{3} \end{array}\right) \xrightarrow{R_1 \to R_1 + R_2} \left(\begin{array}{cc|cc} 1 & 0 & 1 - \frac{4}{3} & \frac{2}{3} \\ 0 & -2 & -\frac{4}{3} & \frac{2}{3} \end{array}\right) \xrightarrow{R_2 \to -\frac{1}{2}R_2} \left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \end{array}\right) \tag{5}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \tag{6}$$

$$\mathbf{X} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3}(3) + \frac{2}{3}(3) & -\frac{1}{3}(1) + \frac{2}{3}(2) \\ \frac{2}{3}(3) - \frac{1}{3}(3) & \frac{2}{3}(1) - \frac{1}{3}(2) \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} 1 & a \\ b & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \tag{8}$$

Hence $a = 1, b = 1 \implies a + b = 1 + 1 = 2$