4.3.23

EE25BTECH11034 - Kishora Karthik

Question:
The line segment joining the points $\mathbf{A}(3,2)$ and $\mathbf{B}(5,1)$ is divided at the point $\mathbf{P}$ in the ratio $1:2$ which lies on $3x-18y+k=0$. Find the value of k.

Solution:
Given the points,

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{2}$$

and the line L_1,

$$L_1 : \begin{pmatrix} 3 & -18 \end{pmatrix} \mathbf{x} = -k \tag{3}$$

$$\implies \mathbf{n}^\top \mathbf{x} = 0 \tag{4}$$

Where,

$$\mathbf{n} = \begin{pmatrix} 3 \\ -18 \end{pmatrix} \tag{5}$$

Let the vector $\mathbf{P}$ be a point on the line $3x-18y+k=0$ wihch divides the line segment joining the points $\mathbf{A}$ and $\mathbf{B}$.
Section formula for a vector $\mathbf{P}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{B}$ in the ratio $m:1$ is given by

$$\mathbf{P} = \frac{m\mathbf{B} + \mathbf{A}}{m+1} \tag{6}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{m+1} \\ \frac{m}{m+1} \end{pmatrix} \tag{7}$$

Here, $m = 1/2$.

$$\implies \mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \tag{8}$$

Since $\mathbf{P}$ lies on line L_1,

$$\mathbf{n}^\top \mathbf{P} = 0 \tag{9}$$

$$\implies \begin{pmatrix} 3 & -18 \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \tag{10}$$

$$\implies \begin{pmatrix} 3 & -18 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \tag{11}$$

$$\implies \begin{pmatrix} 3 \cdot 3 + (-18) \cdot 2 & 3 \cdot 5 + (-18) \cdot 1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \tag{12}$$

$$\implies \begin{pmatrix} -27 & -3 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \tag{13}$$

$$\implies \left((-27) \cdot \tfrac{2}{3} + (-3) \cdot \tfrac{1}{3} \right) = -k \tag{14}$$

$$\implies k = 19 \tag{15}$$

$\therefore$ The value of k is 19 and the equation of the line is $3x - 18y + 19 = 0$.