4.6.8

EE25BTECH11018 - Darisy Sreetej

Question:
Find the equation of the plane passing through the points having position vectors $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}+\hat{k}$.Write the equation of the plane passing through a point $(2, 3, 7)$ and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.
Solution:

TABLE I

A	$\begin{pmatrix}1\\1\\-2\end{pmatrix}$
B	$\begin{pmatrix}2\\-1\\1\end{pmatrix}$
C	$\begin{pmatrix}1\\2\\1\end{pmatrix}$
P	$\begin{pmatrix}2\\3\\7\end{pmatrix}$

Let the equation of plane be

$$\mathbf{n}^\top \mathbf{x} = C_1 \tag{1}$$

A,B,C satisfies this equation,

$$\mathbf{n}^\top \mathbf{A} = C_1, \mathbf{n}^\top \mathbf{B} = C_1, \mathbf{n}^\top \mathbf{C} = C_1 \tag{2}$$

$$\begin{pmatrix}\mathbf{A}\\\mathbf{B}\\\mathbf{C}\end{pmatrix}^\top \mathbf{n} = \begin{pmatrix}C_1\\C_1\\C_1\end{pmatrix} \tag{3}$$

Using augmented matrix,

$$\left(\begin{array}{ccc|c}1 & 1 & -2 & C_1\\2 & -1 & 1 & C_1\\1 & 2 & 1 & C_1\end{array}\right) \tag{4}$$

$R_2 = R_2 - 2R_1$
$R_3 = R_3 - R_1$

$$\left(\begin{array}{ccc|c}1 & 1 & -2 & C_1\\0 & -3 & 5 & -C_1\\0 & 1 & 3 & 0\end{array}\right) \tag{5}$$

$R_2 \iff R_3$

$$\left(\begin{array}{ccc|c}1 & 1 & -2 & C_1\\0 & 1 & 3 & 0\\0 & -3 & 5 & -C_1\end{array}\right) \tag{6}$$

$R_3 = R_3 + 3R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & C_1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 14 & -C_1 \end{array}\right) \tag{7}$$

$$14z + C_1 = 0 \implies z = \frac{-C_1}{14} \tag{8}$$

$$y + 3z = 0 \implies y = \frac{3C_1}{14} \tag{9}$$

$$x + y - 2z = C_1 \implies x = \frac{9C_1}{17} \tag{10}$$

Let $C_1 = 14$

$$\mathbf{n} = \begin{pmatrix} 9 \\ 3 \\ -1 \end{pmatrix}, C_1 = 14 \tag{11}$$

Equation of the plane

$$\begin{pmatrix} 9 \\ 3 \\ -1 \end{pmatrix}^\top \mathbf{x} = 14 \tag{12}$$

For finding parallel plane passing through P ,

$$\mathbf{n}^\top \mathbf{x} = C_2 \tag{13}$$

$$\mathbf{n}^\top \mathbf{P} = C_2 \tag{14}$$

$$C_2 = \begin{pmatrix} 9 & 3 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} \tag{15}$$

$$C_2 = 20 \tag{16}$$

Equation of plane parallel to given plane passing through point P is

$$\begin{pmatrix} 9 \\ 3 \\ -1 \end{pmatrix}^\top \mathbf{x} = 20 \tag{17}$$

The 2 planes obtained are parallel since their normal vectors are the same
The normal vector of the planes $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 9 \\ 3 \\ -1 \end{pmatrix} \tag{18}$$

The distance between the planes is given by this formula

$$\text{Distance} = \frac{|d_1 - d_2|}{\|\mathbf{n}\|} \tag{19}$$

Where $d_1 = 14$ and $d_2 = 20$

$$\|\mathbf{n}\| = \left(\sqrt{(9)^2 + (3)^2 + (-1)^2}\right) = \sqrt{91} \tag{20}$$

Substituting these values in the distance formula, we get

$$\therefore \text{Distance} = \frac{|14 - 20|}{\sqrt{91}} \tag{21}$$

$$\text{Distance} = \frac{6}{\sqrt{91}} \tag{22}$$

Therefore, the distance between the planes is $\frac{6}{\sqrt{91}}$