# 5.2.21

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October 4, 2025

# Question

Solve for the system of linear equations:

$$2x + 3y = 13$$
$$4x + 5y = 23$$

# Solution

Let us solve the given question theoretically and then verify the solution computationally.

According to the question,
The equation of lines given,

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}^\top \mathbf{x} = 13 \tag{1}$$

$$\begin{pmatrix} 4 \\ 5 \end{pmatrix}^\top \mathbf{x} = 23 \tag{2}$$

$$\therefore \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} 13 \\ 23 \end{pmatrix} \tag{3}$$

# Solution

Using augmented matrix,

$$\left(\begin{array}{cc|c} 2 & 3 & 13 \\ 4 & 5 & 23 \end{array}\right) \tag{4}$$

Upon doing row reduction,

$$\left(\begin{array}{cc|c} 2 & 3 & 13 \\ 4 & 5 & 23 \end{array}\right) \xleftrightarrow{R_1 = \frac{1}{2} \times R_1} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 4 & 5 & 23 \end{array}\right) \tag{5}$$

$$\left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 4 & 5 & 23 \end{array}\right) \xleftrightarrow{R_2 = R_2 - 4 \times R_1} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 0 & -1 & -3 \end{array}\right) \tag{6}$$

# Solution

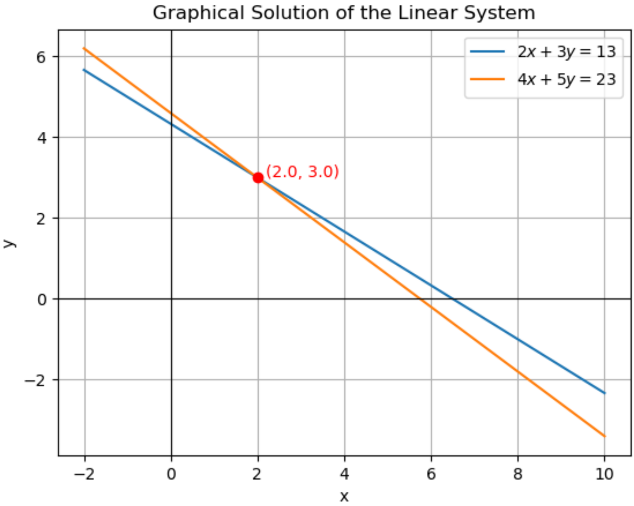
$$\left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 4 & 5 & 23 \end{array}\right) \xleftrightarrow{R_2 = -R_2} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 0 & 1 & 3 \end{array}\right) \tag{7}$$

$$\left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 4 & 5 & 23 \end{array}\right) \xleftrightarrow{R_1 = R_1 - \frac{3}{2} \times R_1} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right) \tag{8}$$

$$\implies \mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{9}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.

# Plot



Graphical Solution of the Linear System
$2x + 3y = 13$
$4x + 5y = 23$
(2.0, 3.0)
6
4
2
0
−2
y
−2
0
2
4
6
8
10
x

# C code

```c
#include <stdio.h>

void rref_solver(double aug[2][3], double solution[2]) {
    // Normalize first row (pivot = aug[0][0])
    double pivot = aug[0][0];
    for (int j = 0; j < 3; j++) {
        aug[0][j] /= pivot;
    }
    // Eliminate below pivot
    double factor = aug[1][0];
    for (int j = 0; j < 3; j++) {
        aug[1][j] -= factor * aug[0][j];
    }
```

# C Code

```
    //Normalize second row (pivot = aug[1][1])
    pivot = aug[1][1];
    for (int j = 0; j < 3; j++) {
        aug[1][j] /= pivot;
    }

    // Eliminate above pivot
    factor = aug[0][1];
    for (int j = 0; j < 3; j++) {
        aug[0][j] -= factor * aug[1][j];
    }

    // Extract solution
    solution[0] = aug[0][2]; // x
    solution[1] = aug[1][2]; // y
}
```

# Python + C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")

# Load the shared C library (adjust filename if needed)
lib = ctypes.CDLL("./line_solver.so")

# Define argument and return types
lib.rref_solver.argtypes = [ctypes.c_double * 6, ctypes.c_double
    * 2]

# Augmented matrix for system:
# 2x + 3y = 13
# 4x + 5y = 23
aug = (ctypes.c_double * 6)(2, 3, 13, 4, 5, 23) # Flattened 2x3
solution = (ctypes.c_double * 2)()
```

```python
# Call C function
lib.rref_solver(aug, solution)

# Convert result to numpy vector (ensure flat)
x_sol = np.array([solution[0], solution[1]], dtype=float).flatten
    ()
print("Solution vector from C:", x_sol)

# Plot lines
x_vals = np.linspace(-2, 10, 400)
y1 = (13 - 2*x_vals) / 3
y2 = (23 - 4*x_vals) / 5

plt.plot(x_vals, y1, label=r"$2x+3y=13$")
plt.plot(x_vals, y2, label=r"$4x+5y=23$")

# Plot solution point
plt.scatter(x_sol[0], x_sol[1], color="red", zorder=5)
```

# Python + C code

```
plt.text(float(x_sol[0]) + 0.2, float(x_sol[1]),
        f"({x_sol[0]:.1f}, {x_sol[1]:.1f})", color="red")

plt.xlabel("x")
plt.ylabel("y")
plt.title("Graphical Solution of the Linear System")
plt.axhline(0, color="black", linewidth=0.8)
plt.axvline(0, color="black", linewidth=0.8)
plt.legend()
plt.grid(True)

plt.show()
```

# Python code

```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")

# Coefficient matrix and RHS vector
A = np.array([[2, 3],
              [4, 5]], dtype=float)
b = np.array([13, 23], dtype=float)

# Solve system Ax = b
x = np.linalg.solve(A, b)
print("Solution vector for the system of equations:", x)

# Prepare x values for plotting
x_vals = np.linspace(-2, 10, 400)
```

```python
# Express y in terms of x for both equations
y1 = (13 - 2*x_vals) / 3 # from 2x + 3y = 13
y2 = (23 - 4*x_vals) / 5 # from 4x + 5y = 23

# Plot both lines
plt.plot(x_vals, y1, label=r"$2x + 3y = 13$")
plt.plot(x_vals, y2, label=r"$4x + 5y = 23$")

# Mark the solution point
plt.scatter(x[0], x[1], color="red", zorder=5)
plt.text(x[0] + 0.2, x[1], f"({x[0]:.1f}, {x[1]:.1f})", color="
    red")

# Formatting
plt.xlabel("x")
plt.ylabel("y")
plt.title("Graphical Solution of the Linear System")
plt.axhline(0, color='black', linewidth=0.8)
```

# Python code

```
plt.axvline(0, color='black', linewidth=0.8)
plt.legend()
plt.grid(True)

plt.show()
```