10.4.1

EE25BTECH11025 - Ganachari Vishwambhar

Question:
Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.
Solution:
Let the point of contact of tangent and conic be $\mathbf{q}$ and also point of intersection of normal and conic be $\mathbf{q}$.
Given

$$\mathbf{q} = \begin{pmatrix} 2 \\ k \end{pmatrix}; k > 0 \tag{1}$$

Let the equation of given ellipse in quadratic form be:

$$\mathbf{x}^\top V\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

$$\tag{3}$$

where,

$$V = \begin{pmatrix} \frac{16}{145} & 0 \\ 0 & \frac{9}{145} \end{pmatrix} \tag{4}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} f = -1 \tag{5}$$

Since $\mathbf{q}$ lies on the ellipse:

$$\mathbf{q}^\top V\mathbf{q} + f = 0 \tag{6}$$

$$\mathbf{q} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{7}$$

The tangent equation can be given by

$$(V\mathbf{q} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{q} + f = 0 \tag{8}$$

The normal equation can be given by

$$(V\mathbf{q} + \mathbf{u})^\top R(\mathbf{x} - \mathbf{q}) = 0 \tag{9}$$

$$R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{10}$$

After substituting values we get tangent equation in normal form as:

$$\begin{pmatrix} \frac{32}{145} \\ \frac{27}{145} \end{pmatrix}^\top \mathbf{x} - 1 = 0 \tag{11}$$

After substituting values we get normal equation in normal form as:

$$\begin{pmatrix} \frac{-27}{145} \\ \frac{32}{145} \end{pmatrix}^{\top} \mathbf{x} - \frac{42}{145} = 0 \tag{12}$$

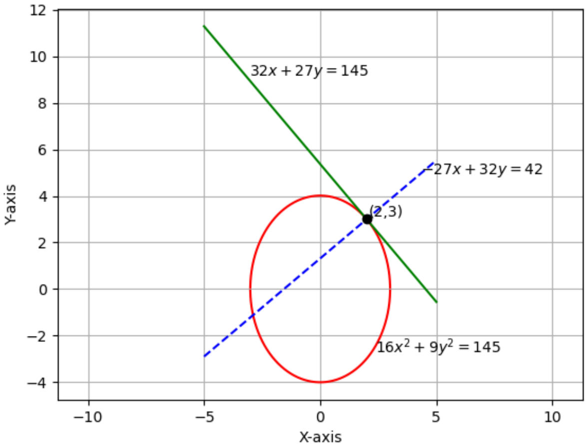

Fig. 1: Plot of the ellipse, tangent and normal