2.5.25

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question: If $\mathbf{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\mathbf{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$, then express $\mathbf{b}$ in the form $\mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2$, where $\mathbf{b}_1$ is parallel to $\mathbf{a}$ and $\mathbf{b}_2$ is perpendicular to $\mathbf{a}$.

Solution:

Variable	Value
$\mathbf{a}$	$2\hat{i} - \hat{j} - 2\hat{k}$
$\mathbf{b}$	$7\hat{i} + 2\hat{j} - 3\hat{k}$

TABLE 0: Variables Used

$$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \tag{0.1}$$

$$\mathbf{b} = \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} \tag{0.2}$$

Using the Gram-Schmidt approach
$\mathbf{b_1}$ is the projection of $\mathbf{b}$ on $\mathbf{a}$

$$\mathbf{b_1} = \frac{\mathbf{a^T b}}{\mathbf{a^T a}}\mathbf{a} \tag{0.3}$$

$$\mathbf{b_1} = \frac{18}{9}\mathbf{a} \tag{0.4}$$

$$\mathbf{b_1} = 2\mathbf{a} \tag{0.5}$$

$$\mathbf{b_1} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \tag{0.6}$$

$$\mathbf{b} = \mathbf{b_1} + \mathbf{b_2} \tag{0.7}$$

$$\mathbf{b}_2 = \mathbf{b} - \mathbf{b}_1 = \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \tag{0.8}$$

$$\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \tag{0.9}$$

Therefore,

$$\mathbf{b_1} = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix} \tag{0.10}$$

$$\mathbf{b_2} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \tag{0.11}$$

Refer to Figure

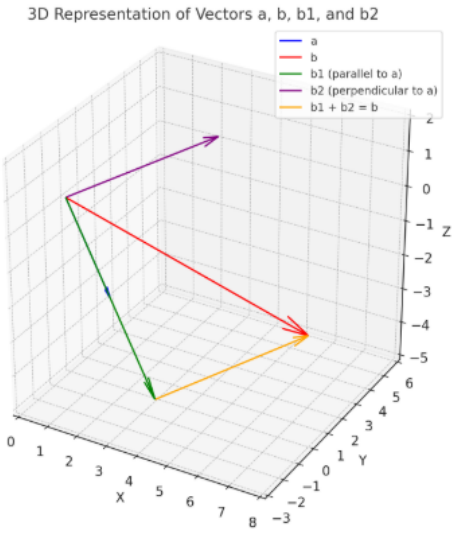

Fig. 0.1