4.7.26

EE25BTECH11034 - Kishora Karthik

Question:
Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of X-axis.

Solution:
Let p be the length of perpendicular from the origin and θ be the angle made by the line with the positive X-axis.
Given, $p = 4$ and $\theta = 120°$. Let the angle made by the perpendicular with the positive X-axis be α.

$$\alpha = 90° - (180° - \theta) \tag{1}$$

$$\alpha = 90° - (180° - 120°) \tag{2}$$

$$\alpha = 30° \tag{3}$$

$p\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$ is a point on the line as well as the normal vector. Hence the equation of the line is,

$$p\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}^{\top}\left(\mathbf{x} - p\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}\right) = 0 \tag{4}$$

$$\implies p\begin{pmatrix}\cos\alpha & \sin\alpha\end{pmatrix}\left(\mathbf{x} - p\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}\right) = 0 \tag{5}$$

$$\implies p\begin{pmatrix}\cos\alpha & \sin\alpha\end{pmatrix}\mathbf{x} = p^2(\cos^2\alpha + \sin^2\alpha) \tag{6}$$

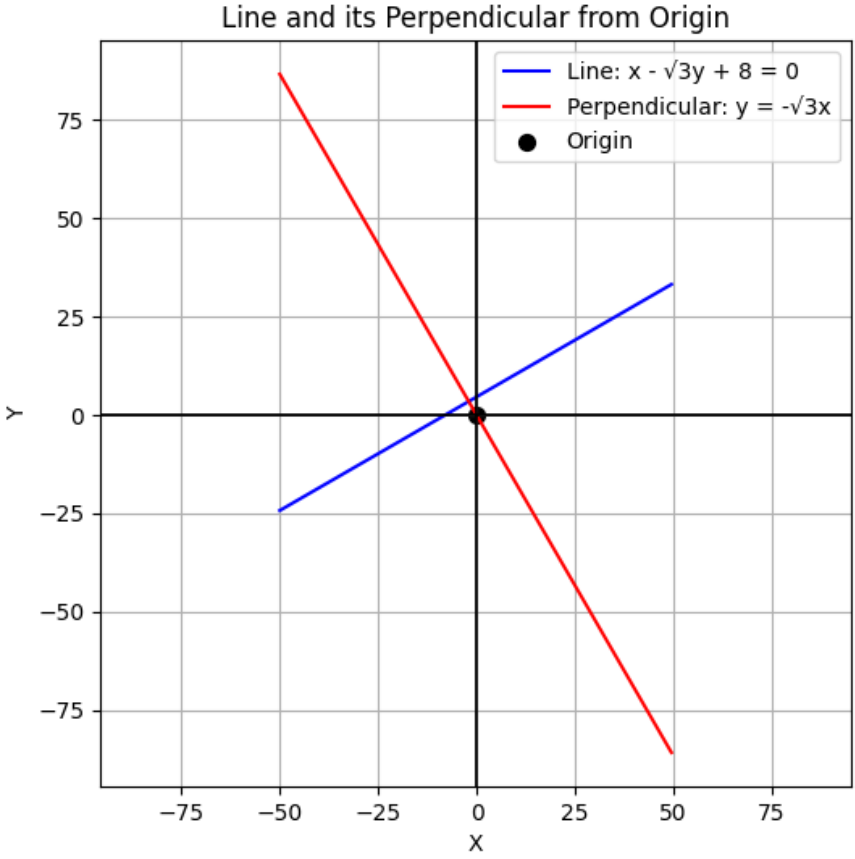
$$\implies \begin{pmatrix}\cos\alpha & \sin\alpha\end{pmatrix}\mathbf{x} = p \tag{7}$$

So the equation of the straight line is,

$$\begin{pmatrix}\cos 30° & \sin 30°\end{pmatrix}\mathbf{x} = 4 \tag{8}$$

$$\begin{pmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\mathbf{x} = 4 \tag{9}$$

$\therefore$ The equation of the required line is $\begin{pmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\mathbf{x} = 4$ or $\sqrt{3}x + y = 8$.


Line and its Perpendicular from Origin
Line: x - √3y + 8 = 0
Perpendicular: y = -√3x
Origin
75
50
25
0
−25
−50
−75
Y
−75
−50
−25
0
25
50
75
X