7.4.19

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October 2025

Question

Let **C** be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of the chords of the circle **C** that subtend an angle of $\frac{2\pi}{3}$ at its centre is

1. $x^2 + y^2 = \frac{3}{2}$
2. $x^2 + y^2 = 1$
3. $x^2 + y^2 = \frac{27}{4}$
4. $x^2 + y^2 = \frac{9}{4}$

finding the locus:

Given radius $= 3$ units

$$\mathbf{C} = \text{center} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

Let $\mathbf{A}$ and $\mathbf{B}$ be the end points of chord

$$\|\mathbf{A}\| = \|\mathbf{B}\| = 3 \tag{2}$$

$$\mathbf{A}^\top \mathbf{A} = 9 \tag{3}$$

$$\mathbf{B}^\top \mathbf{B} = 9 \tag{4}$$

$$\mathbf{A}^\top \mathbf{B} = \|\mathbf{A}\|\|\mathbf{B}\| \cos\theta = -\frac{9}{2} \tag{5}$$

Let P be the midpoint of chords then,

$$\mathbf{P} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{6}$$

$$\|\mathbf{P}\| = \frac{1}{2}\|\mathbf{A}+\mathbf{B}\| \tag{7}$$

$$\mathbf{P}^\top\mathbf{P} = \frac{1}{4}(\mathbf{A}+\mathbf{B})^\top(\mathbf{A}+\mathbf{B}) \tag{8}$$

$$\mathbf{P}^\top\mathbf{P} = \frac{1}{4}(\mathbf{A}^\top\mathbf{A}+\mathbf{A}^\top\mathbf{B}+\mathbf{B}^\top\mathbf{A}+\mathbf{B}^\top\mathbf{B}) \tag{9}$$

$$\tag{10}$$

Substituting the values:

$$= \frac{1}{4}\left(9 - \frac{9}{2} - \frac{9}{2} + 9\right) = \frac{9}{4} \tag{11}$$

Hence, $\mathbf{P}^\top \mathbf{P} = 9/4$
option D is correct.

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Arc

# --- 1. Setup the figure and axis ---
fig, ax = plt.subplots(figsize=(8, 8))
ax.set_aspect('equal', adjustable='box')
ax.grid(True, linestyle='--', alpha=0.6)
ax.axhline(0, color='black', linewidth=0.5)
ax.axvline(0, color='black', linewidth=0.5)
```

python Code

```
# --- 2. Define and plot the original circle (C) ---
r_C = 3
theta = np.linspace(0, 2 * np.pi, 200)
x_C = r_C * np.cos(theta)
y_C = r_C * np.sin(theta)
ax.plot(x_C, y_C, 'b-', label=r'Original Circle C: $x^2 + y^2 = 9
    $')
```

python Code

```
# --- 3. Define and plot the locus of midpoints ---
# The distance of the midpoint from the center is r_C * cos((2pi
    /3)/2) = 3 * cos(pi/3) = 1.5
r_locus = 1.5
x_locus = r_locus * np.cos(theta)
y_locus = r_locus * np.sin(theta)
ax.plot(x_locus, y_locus, 'r--', label=r'Locus of Midpoints: $x^2
    + y^2 = \frac{9}{4}$')

# --- 4. Draw an example chord and radii to illustrate ---
# Angle of the radius to the chord midpoint
angle_midpoint = np.pi / 4
```

python Code

```
# Center
O = (0, 0)
# Midpoint P
P = (r_locus * np.cos(angle_midpoint), r_locus * np.sin(
    angle_midpoint))
# Endpoints of the chord (A and B)
angle_subtended_half = np.pi / 3 # Half of 2pi/3
A = (r_C * np.cos(angle_midpoint + angle_subtended_half), r_C *
    np.sin(angle_midpoint + angle_subtended_half))
B = (r_C * np.cos(angle_midpoint - angle_subtended_half), r_C *
    np.sin(angle_midpoint - angle_subtended_half))
```

python Code

```
# Plot the illustrative elements
ax.plot([A[0], B[0]], [A[1], B[1]], 'g-', label=r'Example Chord
    subtending $\frac{2\pi}{3}$') # Chord AB
ax.plot([O[0], A[0]], [O[1], A[1]], 'k:', alpha=0.8) # Radius OA
ax.plot([O[0], B[0]], [O[1], B[1]], 'k:', alpha=0.8) # Radius OB
ax.plot([O[0], P[0]], [O[1], P[1]], 'm-', alpha=0.8) # Line to
    midpoint OP
# Plot and label the points
ax.plot(O[0], O[1], 'ko')
ax.text(O[0] - 0.2, O[1] - 0.2, 'O', fontsize=12)
ax.plot(P[0], P[1], 'mo')
ax.text(P[0] + 0.1, P[1] + 0.1, 'P (midpoint)', fontsize=12,
    color='m')
```

python Code

```
# Add angle annotation
angle_rad = 2 * np.pi / 3
arc = Arc(0, 2, 2, angle=0,
          theta1=np.degrees(angle_midpoint - angle_subtended_half)
              ,
          theta2=np.degrees(angle_midpoint + angle_subtended_half)
              ,
          color='purple', lw=1.5, linestyle='-')
ax.add_patch(arc)
angle_text_pos = (0.8 * np.cos(angle_midpoint), 0.8 * np.sin(
    angle_midpoint))
ax.text(angle_text_pos[0], angle_text_pos[1], r'$\frac{2\pi}{3}$'
    , fontsize=14, color='purple', ha='center', va='center')
```

python Code

```
# --- 5. Finalize and show the plot ---
ax.set_title('Locus of the Midpoints of Chords', fontsize=16)
ax.set_xlabel('x-axis')
ax.set_ylabel('y-axis')
ax.legend()
plt.xlim(-3.5, 3.5)
plt.ylim(-3.5, 3.5)
plt.show()
```

C Code

```c
 #include <stdio.h>
#include <math.h>

// Define PI if it's not available in math.h (M_PI is common but
    not standard)
#ifndef M_PI
#define M_PI 3.14159265358979323846
#endif
```

C Code

```c
int main() {
    // --- Given Parameters ---
    // Radius of the main circle C
    double radius = 3.0;
    // Angle subtended by the chords at the center in radians
    double angle_rad = (2.0 * M_PI) / 3.0;
    // The locus of the midpoints is another circle. Its radius (
        let's call it r_locus)
    // is found using the formula: r_locus = radius * cos(angle /
         2).
```

C Code

```
// 1. Find half the angle
double half_angle = angle_rad / 2.0;
// 2. Calculate the radius of the locus circle
double locus_radius = radius * cos(half_angle);
// 3. The equation of the locus is x^2 + y^2 = (locus_radius)
    ^2.
// We need to find the value of the radius squared.
double locus_radius_squared = pow(locus_radius, 2);
```

C Code

```
    // --- Output ---
    printf("Problem: Find the locus of the midpoints of chords of
        the circle x^2 + y^2 = %.0f\n", pow(radius, 2));
    printf("where the chords subtend an angle of 2*pi/3 at the
        center.\n\n");

    printf("--- Calculation Results ---\n");
    printf("Radius of the locus circle: %.2f\n", locus_radius);
```

C Code

```
    printf("Radius of the locus circle squared: %.2f\n\n",
        locus_radius_squared);

    printf("The final equation of the locus is: x^2 + y^2 = %.2f\
        n", locus_radius_squared);
    printf("This corresponds to option (d): x^2 + y^2 = 9/4\n");

    return 0;
}
```

Python and C Code

```
from ctypes import c_double
import math

# Define M_PI if not available
M_PI = math.pi

# --- Given Parameters ---
# Radius of the main circle C
radius = c_double(3.0)
```

Python and C Code

```
# Angle subtended by the chords at the center in radians
angle_rad = c_double((2.0 * M_PI) / 3.0)

# --- Calculation ---
# 1. Find half the angle
half_angle = c_double(angle_rad.value / 2.0)

# 2. Calculate the radius of the locus circle
locus_radius = c_double(radius.value * math.cos(half_angle.value)
    )
```

Python and C Code

```
# 3. Radius squared
locus_radius_squared = c_double(math.pow(locus_radius.value, 2))

# --- Output ---
print(f"Problem: Find the locus of the midpoints of chords of the
    circle x^2 + y^2 = {math.pow(radius.value, 2):.0f}")
print("where the chords subtend an angle of 2*pi/3 at the center
    .\n")
```

Python and C Code

```
print("--- Calculation Results ---")
print(f"Radius of the locus circle: {locus_radius.value:.2f}")
print(f"Radius of the locus circle squared: {locus_radius_squared
    .value:.2f}\n")
print(f"The final equation of the locus is: x^2 + y^2 = {
    locus_radius_squared.value:.2f}")
print("This corresponds to option (d): x^2 + y^2 = 9/4")
```

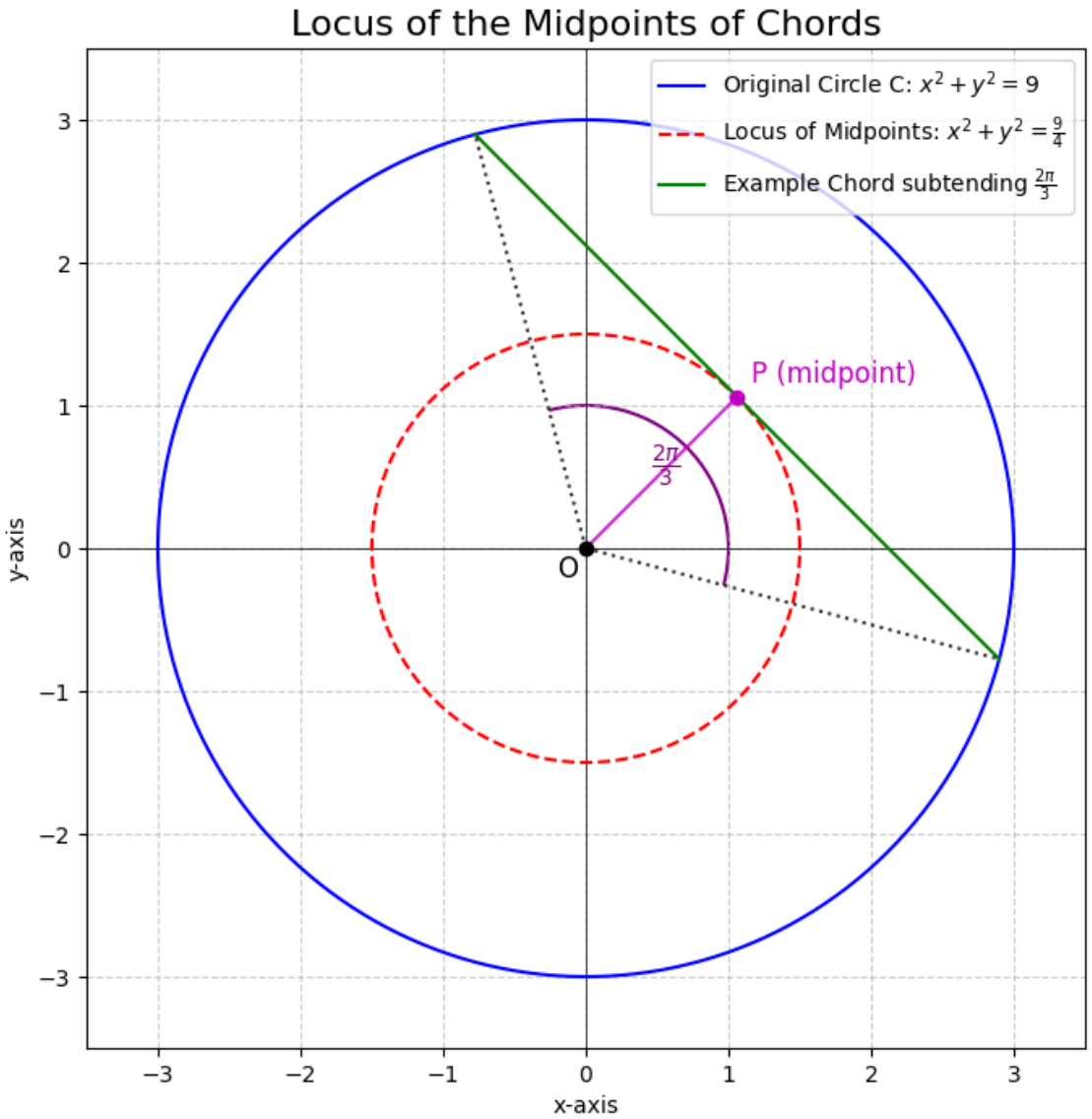
Locus of the Midpoints of Chords
Original Circle C: $x^2 + y^2 = 9$
Locus of Midpoints: $x^2 + y^2 = \frac{9}{4}$
Example Chord subtending $\frac{2\pi}{3}$
P (midpoint)
$\frac{2\pi}{3}$
O
y-axis
x-axis
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

Figure: Plot