8.2.20

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Question

Find the equation of the conic, that satisfies the given conditions: Vertex (0,0) passing through (2,3) and axis is along X axis.

Solution

The general conic is

$$g(\vec{x}) = \vec{x}^{\top}\vec{V}\vec{x} + 2\vec{u}^{\top}\vec{x} + f = 0 \tag{1}$$

For axis along the x-axis,

$$\vec{V} = \begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} D \\ 0 \end{pmatrix} \tag{2}$$

Since the vertex is at the origin,

$$\nabla g(\vec{0}) = 2\vec{u} = \vec{0} \implies \vec{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

If $\det(\vec{V}) = AC \neq 0$, then the conic has a center at the origin, which corresponds to an ellipse or hyperbola.
But the problem specifies a single vertex at the origin, not a center, so this case is invalid.

Solution

$$\therefore \det(\vec{V}) = 0 \quad \implies \quad \text{The conic is a parabola.} \tag{4}$$

$$g(\vec{x}) = \vec{x}^\top \vec{V}\vec{x} + 2\vec{u}^\top \vec{x} + f = 0 \tag{5}$$

For a parabola with axis along the x-axis, vertex at origin, focus $\vec{F} = \begin{pmatrix} p \\ 0 \end{pmatrix}$ and directrix $\vec{n}^\top \vec{x} = c$, we have $\vec{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ c = -p$.

$$\vec{V} = \|\vec{n}\|^2 \vec{I} - \vec{n}\vec{n}^\top = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

$$\vec{u} = c\vec{n} - \vec{F} = (-p)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} p \\ 0 \end{pmatrix} = \begin{pmatrix} -2p \\ 0 \end{pmatrix} \tag{7}$$

Solution

$$f = \|\vec{F}\|^2 - c^2 = p^2 - (-p)^2 = 0 \tag{8}$$

Thus, the parabola equation becomes:

$$\vec{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \vec{x} + 2 \begin{pmatrix} -2p & 0 \end{pmatrix} \vec{x} + 0 = 0 \tag{9}$$

$$y^2 - 4px = 0 \tag{10}$$

Since $(2, 3)$ lies on the parabola:

$$3^2 - 4p(2) = 0 \tag{11}$$

$$9 - 8p = 0 \tag{12}$$

$$p = \frac{9}{8} \tag{13}$$

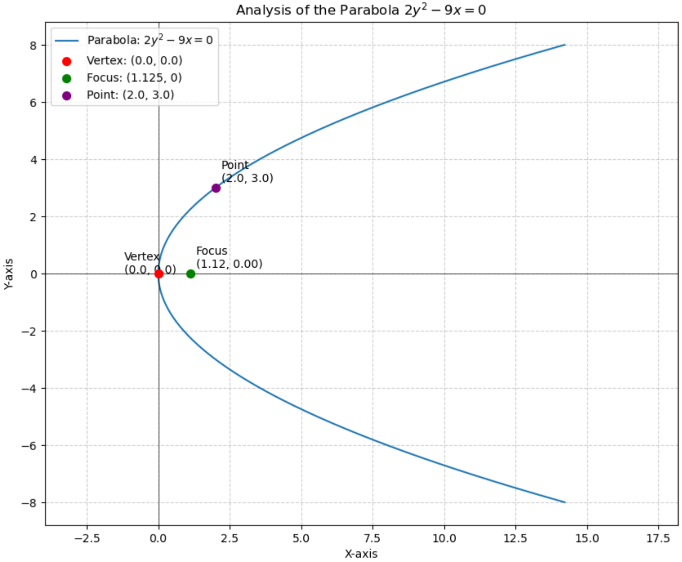
Therefore,

Solution

Therefore,the equation of the required parabola is

$$y^2 = \frac{9}{2}x \tag{14}$$

Plot



Analysis of the Parabola $2y^2 - 9x = 0$
Parabola: $2y^2 - 9x = 0$
Vertex: (0.0, 0.0)
Focus: (1.125, 0)
Point: (2.0, 3.0)
Point
(2.0, 3.0)
Vertex
(0.0, 0.0)
Focus
(1.12, 0.00)
Y-axis
X-axis
8
6
4
2
0
−2
−4
−6
−8
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5