

5.8.16

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Question: The taxi charges in a city consist of a fixed charge together with the charges for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a distance of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km ?

Solution:

Let us solve the given question theoretically and then verify the solution computationally.

Let x = fixed charge , y = charge per km

Then total fare = $x + y \times \text{distance}$

According to the question, The equation of lines given

$$\begin{pmatrix} 1 & 10 \end{pmatrix} \mathbf{x} = 105 \quad (0.1)$$

$$\begin{pmatrix} 1 & 15 \end{pmatrix} \mathbf{x} = 115 \quad (0.2)$$

where , $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

From the question ,

$$\begin{pmatrix} 1 & 25 \end{pmatrix} \mathbf{x} = c \quad (0.3)$$

where , c = total fare the person should pay for travelling 25 km

$$\therefore \begin{pmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 25 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 105 \\ 155 \\ c \end{pmatrix} \quad (0.4)$$

Using augmented matrix,

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 1 & 25 & c \end{array} \right) \quad (0.5)$$

Upon doing row reduction,

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 1 & 25 & c \end{array} \right) \xleftrightarrow{R_3=R_3-R_1} \left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 0 & 15 & c-105 \end{array} \right) \quad (0.6)$$

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 1 & 15 & 155 \\ 0 & 15 & c-105 \end{array} \right) \xleftrightarrow{R_2=R_2-R_1} \left(\begin{array}{cc|c} 1 & 10 & 105 \\ 0 & 5 & 50 \\ 0 & 15 & c-105 \end{array} \right) \quad (0.7)$$

$$\left(\begin{array}{cc|c} 1 & 10 & 105 \\ 0 & 5 & 50 \\ 0 & 15 & c - 105 \end{array} \right) \xleftrightarrow{R_3 = R_3 - 3 \times R_2} \left(\begin{array}{cc|c} 1 & 10 & 105 \\ 0 & 5 & 50 \\ 0 & 0 & c - 255 \end{array} \right) \quad (0.8)$$

From (0.8),

$$\begin{aligned} 0 &= c - 255 \\ c &= 255 \end{aligned} \quad (0.9)$$

$$\begin{aligned} 5y &= 50 \\ y &= 10 \\ x + 10y &= 105 \\ x &= 5 \end{aligned} \quad (0.10)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \quad (0.11)$$

Thus , fixed charge = ₹5

charge per km = ₹10

The total fare the person should pay for travelling 25 km = ₹255

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.

