10.3.11

EE25BTECH11012-BEERAM MADHURI

Question:
Find the normal at the point $(1, 1)$ on the curve

$$2y + x^2 = 3 \tag{0.1}$$

Solution:

$$2y + x^2 = 3 \tag{0.2}$$

$$2y + x^2 - 3 = 0 \tag{0.3}$$

Which can be expressed as the conic:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.4}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, f = -3 \tag{0.5}$$

let

$$\mathbf{p}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{m} \text{ is normal vector} \tag{0.6}$$

$$\mathbf{m}^\top(\mathbf{V}\mathbf{p} + \mathbf{u}) = 0 \tag{0.7}$$

substitung the value :

$$\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} \tag{0.8}$$

$$\mathbf{V}\mathbf{p} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.9}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.10}$$

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}\begin{pmatrix} 1 + 0 \\ 0 + 1 \end{pmatrix} = 0 \tag{0.11}$$

$$m_1 = -m_2 \tag{0.12}$$

$$\therefore \mathbf{m} = \begin{pmatrix} -m \\ m \end{pmatrix} \tag{0.13}$$

equation of normal is

$$\mathbf{m}^\top(\mathbf{x} - \mathbf{p}) = 0 \tag{0.14}$$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x - 1 \\ y - 1 \end{pmatrix} = 0 \tag{0.15}$$

$$y = x \tag{0.16}$$

Hence equation of normal to $2y + x^2 - 3 = 0$ at $(1, 1)$ is $y = x$.

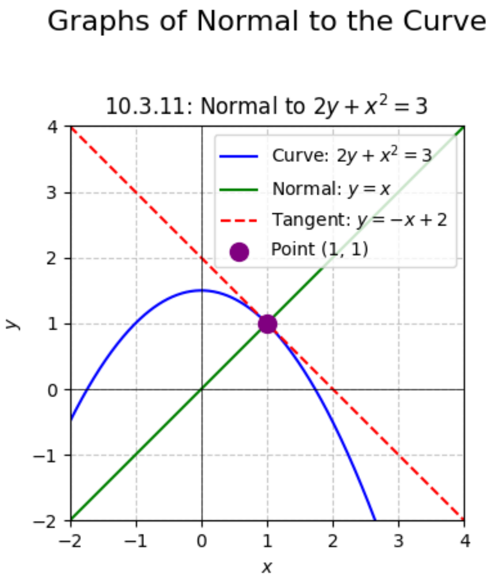

Fig. 0.1: 10.3.11