

5.4.7

EE25BTECH11012-BEERAM MADHURI

Question:

Using elementary transformations, find the inverse of the following matrix

$$\begin{pmatrix} 2 & 2 \\ 4 & 3 \end{pmatrix}$$

Solution:

We know that

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \quad (0.1)$$

where \mathbf{I} is the 2×2 identity matrix

Now we get the augmented matrix

$$\begin{pmatrix} 2 & 2 & | & 1 & 0 \\ 4 & 3 & | & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - 2R_1} \begin{pmatrix} 2 & 2 & | & 1 & 0 \\ 0 & -1 & | & -2 & 1 \end{pmatrix} \quad (0.2)$$

$$\xrightarrow[\begin{matrix} R_1 \rightarrow \frac{R_1}{2} \\ R_2 \rightarrow -R_2 \end{matrix}]{\begin{pmatrix} 1 & 1 & | & \frac{1}{2} & 0 \\ 0 & 1 & | & 2 & -1 \end{pmatrix}} \xrightarrow{R_1 \rightarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & | & -\frac{3}{2} & 1 \\ 0 & 1 & | & 2 & -1 \end{pmatrix} \quad (0.3)$$

Therefore

$$\mathbf{A}^{-1} = \begin{pmatrix} -\frac{3}{2} & 1 \\ 2 & -1 \end{pmatrix} \quad (0.4)$$

This can be verified by $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$