12.34

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:

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Let $\mathbf{A} = 10\mathbf{I}_3$ where $\mathbf{I}_3$ is the 3×3 identity matrix. Find the nullity of $5\mathbf{A}\left(\mathbf{I}_3 + \mathbf{A} + \mathbf{A}^2\right)$. **Solution:**

$$\mathbf{A}^2 = 100\mathbf{I}_3 \tag{0.1}$$

$$\mathbf{M} = 5\mathbf{A}\left(\mathbf{I}_3 + \mathbf{A} + \mathbf{A}^2\right) = 5\left(10\mathbf{I}_3\right)\left(\mathbf{I}_3 + 10\mathbf{A} + 100\mathbf{A}\right) = 5550\mathbf{I}_3 \tag{0.2}$$

Solve $\mathbf{M}\mathbf{x} = 0$

$$5550\mathbf{I}_3\mathbf{x} = 0 \implies 5550\mathbf{x} = 0 \tag{0.3}$$

Since 5550 is a nonzero scalar, the only vector satisfying this is the zero vector:

$$\mathbf{x} = 0 \tag{0.4}$$

The nullspace contains only the zero vector, so its dimension is 0. Therefore

$$\textit{nullity}\left(5\mathbf{A}\left(\mathbf{I}_3 + \mathbf{A} + \mathbf{A}^2\right)\right) = 0 \tag{0.5}$$

Listing: nullity.c

```c
#include <stdio.h>

int find_nullity(double k) {
    double scalar = 5 * k * (1 + k + k * k);
    int n = 3; // 3x3 matrix
    int nullity = 0;

    if (scalar == 0)
        nullity = n;
    else
        nullity = 0;

    return nullity;
}
```