7.4.25

EE25BTECH11018 - Darisy Sreetej

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Question

The locus of the centre of a circle, which touches the circle is $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation:

1. $x^2 - 6x - 10y + 14 = 0$
2. $x^2 - 10x - 6y + 14 = 0$
3. $y^2 - 6x - 10y + 14 = 0$
4. $y^2 - 10x - 6y + 14 = 0$

Solution

Given circle equation is ,

$$x^2 + y^2 - 6x - 6y + 14 = 0$$

can be represented as

$$\|\mathbf{x}\|^2 + 2\begin{pmatrix} -3 \\ -3 \end{pmatrix}^\top \mathbf{x} + 14 = 0 \tag{1}$$

The centre of circle is $\mathbf{c}_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and radius $r = 2$

$$\left(\because f = 14, \quad \mathbf{u} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}\right)$$

Let the centre of the moving circle be $\mathbf{c} = \begin{pmatrix} h \\ k \end{pmatrix}$

As the circle touches $X-$axis , Distance of a point from x-axis is given by

$$R = |\mathbf{n}^\top \mathbf{c}| \tag{2}$$

where $\mathbf{n}$ is the unit vector normal to x-axis

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

Distance between their centers equal to sum of their radius

$$\left\| \mathbf{c} - \mathbf{c_1} \right\| = R \pm r \tag{4}$$

$$\left\| \mathbf{c} - \mathbf{c_1} \right\| = \left| \mathbf{n}^\top \mathbf{c} \right| \pm r \tag{5}$$

$$\left\| \mathbf{c} - \mathbf{c_1} \right\|^2 = \left(\left| \mathbf{n}^\top \mathbf{c} \right| \pm r \right)^2 \tag{6}$$

$$\left(\mathbf{c} - \mathbf{c_1} \right) \left(\mathbf{c} - \mathbf{c_1} \right)^\top = \left(\left| \mathbf{n}^\top \mathbf{c} \right| \pm r \right)^2 \tag{7}$$

$$\mathbf{c}^\top \mathbf{c} + \mathbf{c_1} \mathbf{c_1}^\top - \mathbf{c_1}^\top \mathbf{c} - \mathbf{c}^\top \mathbf{c_1} = \left(\left| \mathbf{n}^\top \mathbf{c} \right| \right)^2 \pm 2r \left| \mathbf{n}^\top \mathbf{c} \right| + r^2 \tag{8}$$

$$\mathbf{c}^\top \mathbf{c} + \mathbf{c_1} \mathbf{c_1}^\top - \mathbf{c_1}^\top \mathbf{c} - \mathbf{c}^\top \mathbf{c_1} = \left(\mathbf{n}^\top \mathbf{c} \right)^\top \left(\mathbf{n}^\top \mathbf{c} \right) \pm 2r \left| \mathbf{n}^\top \mathbf{c} \right| + r^2 \tag{9}$$

$$\mathbf{c}^\top \mathbf{c} + \left\| \mathbf{c_1} \right\|^2 - 2\mathbf{c_1}^\top \mathbf{c} = \left(\mathbf{c}^\top \mathbf{n} \mathbf{n}^\top \mathbf{c} \right) \pm 2r \left| \mathbf{n}^\top \mathbf{c} \right| + r^2 \tag{10}$$

$$\mathbf{c}^\top\mathbf{c} + 18 = \left(\mathbf{c}^\top\mathbf{n}\mathbf{n}^\top\mathbf{c}\right) + 2\mathbf{n}^\top\mathbf{c} \pm 2r\left|\mathbf{n}^\top\mathbf{c}\right| + r^2 + 2\mathbf{c_1}^\top\mathbf{c} \tag{11}$$

$$\mathbf{c}^\top\mathbf{c} + 14 = \left(\mathbf{c}^\top\mathbf{n}\mathbf{n}^\top\mathbf{c}\right) + 2\mathbf{n}^\top\mathbf{c} \pm 4\left|\mathbf{n}^\top\mathbf{c}\right| + 2\mathbf{c_1}^\top\mathbf{c} \quad \text{(Since r=2)} \tag{12}$$

Case 1 : (External Tangency)

$$\mathbf{c}^\top\mathbf{c} + 14 = \left(\mathbf{c}^\top\mathbf{n}\mathbf{n}^\top\mathbf{c}\right) + 2\mathbf{n}^\top\mathbf{c} + 4\left|\mathbf{n}^\top\mathbf{c}\right| + 2\mathbf{c_1}^\top\mathbf{c} \tag{13}$$

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + 14 = \begin{pmatrix} x & 0 \end{pmatrix}\begin{pmatrix} x \\ 0 \end{pmatrix} + 4\left(\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\right) + 2\left(\begin{pmatrix} 3 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\right) \tag{14}$$

$$x^2 + y^2 + 14 = x^2 + 4x + 6x + 6y \tag{15}$$

$$y^2 - 10x - 6y + 14 = 0 \tag{16}$$

Case 2 : (Internal Tangency)

$$\mathbf{c}^\top\mathbf{c} + 14 = \left(\mathbf{c}^\top\mathbf{n}\mathbf{n}^\top\mathbf{c}\right) + 2\mathbf{n}^\top\mathbf{c} - 4|\mathbf{n}^\top\mathbf{c}| + 2\mathbf{c_1}^\top\mathbf{c} \tag{17}$$

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + 14 = \begin{pmatrix} x & 0 \end{pmatrix}\begin{pmatrix} x \\ 0 \end{pmatrix} - 4\left(\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\right) + 2\left(\begin{pmatrix} 3 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\right) \tag{18}$$

$$x^2 + y^2 + 14 = x^2 - 4x + 6x + 6y \tag{19}$$

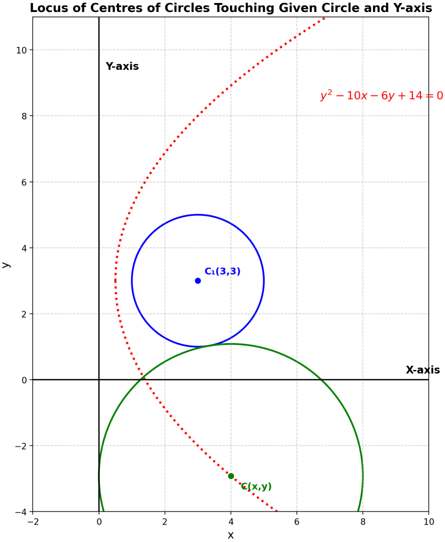
$$y^2 - 2x - 6y + 14 = 0 \tag{20}$$

Therefore , the locus of the centre of a circle is

$$y^2 - 10x - 6y + 14 = 0 \quad \text{(from the options)}$$

Conclusion

Hence, option (4) is correct.


Locus of Centres of Circles Touching Given Circle and Y-axis
Y-axis
$y^2 - 10x - 6y + 14 = 0$
C₁(3,3)
X-axis
C(x,y)
x
y
10
8
6
4
2
0
−2
−4
−2
0
2
4
6
8
10

C code

```c
#include <stdio.h>

void get_circle_params(double* out_data) {
    // Given circle: x^2 + y^2 - 6x - 6y + 14 = 0
    // Centre: (3,3), radius = 2
    out_data[0] = 3.0; // x-coordinate
    out_data[1] = 3.0; // y-coordinate
    out_data[2] = 2.0; // radius
}
```

Python + C code

```
import ctypes
import sympy

def find_locus_equation():
    lib = ctypes.CDLL('./code.so')

    double_array_3 = ctypes.c_double * 3
    lib.get_circle_params.argtypes = [ctypes.POINTER(ctypes.
        c_double)]
    out_data_c = double_array_3()

    lib.get_circle_params(out_data_c)
    c1_x, c1_y, r1 = list(out_data_c) # c1_x=3.0, c1_y=3.0, r1
        =2.0

    # Symbols: h,k for centre of variable circle
    h, k = sympy.symbols('h k', real=True)
```

Python + C code

```
# We take external tangency and assume h >= 0 => r = h
r = h

lhs = (h - c1_x)**2 + (k - c1_y)**2
rhs = (r + r1)**2

equation = sympy.Eq(lhs, rhs)
locus_expr = sympy.simplify(equation.lhs - equation.rhs)

# Rename symbols to conventional x,y for the returned
    equation
x, y = sympy.symbols('x y', real=True)
final_locus = sympy.simplify(locus_expr.subs({h: x, k: y}))

return sympy.Eq(final_locus, 0)
```

Python + C code

```
if __name__ == '__main__':
    eq = find_locus_equation()
    print("Derived locus equation:", eq)det_val = lib.determinant
        (Y)
```

Python code

```python
import sys
sys.path.insert(0, './')
import numpy as np
import matplotlib.pyplot as plt
from call import find_locus_equation

# Get the locus equation
locus_equation = find_locus_equation()
print(f"Locus equation: {locus_equation}")
fig, ax = plt.subplots(figsize=(9, 9))

# Given circle: x + y - 6x - 6y + 14 = 0
given_center = (3, 3)
given_radius = 2
given_circle = plt.Circle(given_center, given_radius, color='blue
    ', fill=False, linewidth=2)
ax.add_patch(given_circle)
```

Python code

```
    # Mark the moving circle centre
ax.plot(x_sample, y_sample, 'go')
ax.text(x_sample + 0.3, y_sample - 0.4, "C(x,y)", color='green',
    fontsize=11, fontweight='bold')

# Mark centre of given circle
ax.plot(*given_center, 'bo')
ax.text(given_center[0] + 0.2, given_center[1] + 0.2, "C(3,3)",
    color='blue', fontsize=11, fontweight='bold')

# Locus (Parabola):
x_vals = np.linspace(0.5, 9, 400)
sqrt_term = np.sqrt(np.maximum(0, 5 * (2 * x_vals - 1)))
y_plus = 3 + sqrt_term
y_minus = 3 - sqrt_term
ax.plot(x_vals, y_plus, 'r:', linewidth=2.5)
ax.plot(x_vals, y_minus, 'r:', linewidth=2.5)
```

Python code

```
# Write the parabola equation neatly near the curve
ax.text(6.7, 8.5, r"$y^2 - 10x - 6y + 14 = 0$", color='red',
    fontsize=13, fontweight='bold')

# Moving circle
x_sample = 4
y_sample = 3 - np.sqrt(5 * (2 * x_sample - 1))
r_sample = x_sample # because circle touches y-axis  r = x
moving_circle = plt.Circle((x_sample, y_sample), r_sample, color=
    'green', fill=False, linewidth=2)
ax.add_patch(moving_circle)

# Formatting , Axes setup
ax.axvline(0, color='black', linestyle='-', linewidth=1.5)
ax.axhline(0, color='black', linestyle='-', linewidth=1.5)
ax.text(9.3, 0.2, "X-axis", fontsize=12, fontweight='bold')
ax.text(0.2, 9.4, "Y-axis", fontsize=12, fontweight='bold')
ax.set_xlim(-2, 10)
```

Python code

```
ax.set_ylim(-4, 11)
ax.set_aspect('equal', 'box')
ax.set_xlabel("x", fontsize=13)
ax.set_ylabel("y", fontsize=13)
ax.set_title("Locus of Centres of Circles Touching Given Circle
    and Y-axis", fontsize=14, fontweight='bold')
ax.grid(True, linestyle='--', alpha=0.6)

# Save and show
plt.tight_layout()
plt.savefig('fig.png', dpi=200, bbox_inches='tight')
plt.show()
```