

# 12.245

EE25BTECH11023 - Venkata Sai

## Question:

Which one of the following matrices has the same eigenvalues as that of  $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

1)  $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$

2)  $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

3)  $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$

4)  $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

## Solution:

Let the given matrix be

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \quad (1)$$

Characteristic equation of Matrix is given by

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (2)$$

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \quad (3)$$

$$\left| \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0 \quad (4)$$

$$\left| \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} \right| = 0 \quad (5)$$

$$(a - \lambda)(d - \lambda) - bc = 0 \quad (6)$$

$$\lambda^2 - a\lambda - d\lambda + ad - bc = 0 \quad (7)$$

$$\lambda^2 - (a + d)\lambda + ad - bc = 0 \quad (8)$$

$$\lambda^2 - (\text{tr}\mathbf{A})\lambda + \det\mathbf{A} = 0 \quad (9)$$

where  $\lambda$  is the eigen value and  $\text{tr}\mathbf{A}$  is the trace of  $\mathbf{A}$

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \quad (10)$$

$$\text{tr}\mathbf{A} = 1 + 3 = 4, \det\mathbf{A} = 3 - 8 = -5 \quad (11)$$

Option (1)

$$\mathbf{V} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \quad (12)$$

$$\text{tr}\mathbf{V} = 3 + 2 = 5, \det\mathbf{V} = 6 - 4 = 2 \quad (13)$$

Not equal to the given matrix **A**. Hence the eigen values are not same  
Option (2)

$$\mathbf{V} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \quad (14)$$

$$\text{tr}\mathbf{V} = 1 + 3 = 4, \det \mathbf{V} = 3 - 8 = -5 \quad (15)$$

Equal to the given matrix **A**. Hence the eigen values are same  
Option (3)

$$\mathbf{V} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \quad (16)$$

$$\text{tr}\mathbf{V} = 4 + 3 = 7, \det \mathbf{V} = 12 - 2 = 10 \quad (17)$$

Not equal to the given matrix **A**. Hence the eigen values are not same  
Option (4)

$$\mathbf{V} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \quad (18)$$

$$\text{tr}\mathbf{V} = 2 + 3 = 5, \det \mathbf{V} = 6 - 4 = 2 \quad (19)$$

Not equal to the given matrix **A**. Hence the eigen values are not same

Hence option (2) is the correct answer