4.7.26

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Question

Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of X-axis.

Solution

Let p be the length of perpendicular from the origin and θ be the angle made by the line with the positive X-axis.
Given, $p = 4$ and $\theta = 120°$. Let the angle made by the perpendicular with the positive X-axis be α.

$$\alpha = 90° - (180° - \theta) \tag{1}$$

$$\alpha = 90° - (180° - 120°) \tag{2}$$

$$\alpha = 30° \tag{3}$$

$p \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$ is a point on the line as well as the normal vector. Hence the equation of the line is,

$$p \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}^{\top} \left(\mathbf{x} - p \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \right) = 0 \tag{4}$$

Solution

$$\implies p \begin{pmatrix} \cos\alpha & \sin\alpha \end{pmatrix} \left(\mathbf{x} - p \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \right) = 0 \tag{5}$$

$$\implies p \begin{pmatrix} \cos\alpha & \sin\alpha \end{pmatrix} \mathbf{x} = p^2(\cos^2\alpha + \sin^2\alpha) \tag{6}$$

$$\implies \begin{pmatrix} \cos\alpha & \sin\alpha \end{pmatrix} \mathbf{x} = p \tag{7}$$

So the equation of the straight line is,

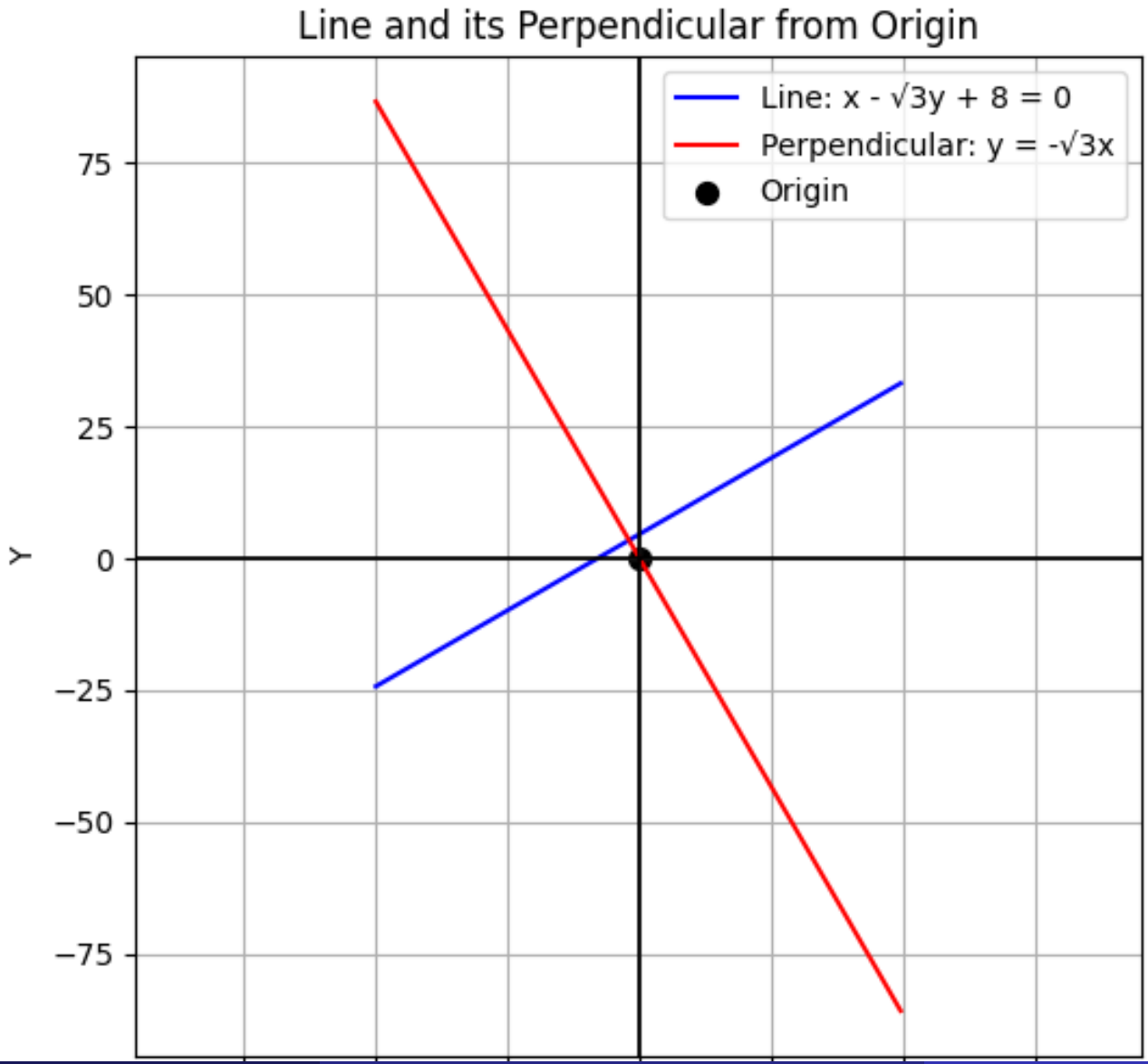
$$\begin{pmatrix} \cos 30^\circ & \sin 30^\circ \end{pmatrix} \mathbf{x} = 4 \tag{8}$$

Solution

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 4 \tag{9}$$

$\therefore$ The equation of the required line is $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 4$ or $\sqrt{3}x + y = 8$.

Plot


Line and its Perpendicular from Origin
Line: x - √3y + 8 = 0
Perpendicular: y = -√3x
Origin
75
50
25
0
−25
−50
−75
Y