Problem 12.245

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Problem

Which one of the following matrices has the same eigenvalues as that of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

1. $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$
3. $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$
4. $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

Equation

Let the given matrix be

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \tag{3.1}$$

Characteristic equation of Matrix is given by

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{3.2}$$

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{3.3}$$

$$\left| \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0 \tag{3.4}$$

$$\left| \begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} \right| = 0 \tag{3.5}$$

$$(a-\lambda)(d-\lambda) - bc = 0 \tag{3.6}$$

$$\lambda^2 - a\lambda - d\lambda + ad - bc = 0 \tag{3.7}$$

Verification

$$\lambda^2 - (a+d)\,\lambda + ad - bc = 0 \tag{3.8}$$

$$\lambda^2 - (tr\mathbf{A})\,\lambda + \det \mathbf{A} = 0 \tag{3.9}$$

where λ is the eigen value and tr$\mathbf{A}$ is the trace of $\mathbf{A}$

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \tag{3.10}$$

$$tr\mathbf{A} = 1 + 3 = 4, \det \mathbf{A} = 3 - 8 = -5 \tag{3.11}$$

Option (1)

$$\mathbf{V} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \tag{3.12}$$

$$tr\mathbf{V} = 3 + 2 = 5, \det \mathbf{V} = 6 - 4 = 2 \tag{3.13}$$

Not equal to the given matrix $\mathbf{A}$. Hence the eigen values are not same

Verification

Option (2)

$$\mathbf{V} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \tag{3.14}$$

$$tr\mathbf{V} = 1 + 3 = 4, \det \mathbf{V} = 3 - 8 = -5 \tag{3.15}$$

Equal to the given matrix $\mathbf{A}$. Hence the eigen values are same

Option (3)

$$\mathbf{V} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \tag{3.16}$$

$$tr\mathbf{V} = 4 + 3 = 7, \det \mathbf{V} = 12 - 2 = 10 \tag{3.17}$$

Not equal to the given matrix $\mathbf{A}$. Hence the eigen values are not same

Option (4)

$$\mathbf{V} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \tag{3.18}$$

$$tr\mathbf{V} = 2 + 3 = 5, \det \mathbf{V} = 6 - 4 = 2 \tag{3.19}$$

Conclusion

Not equal to the given matrix **A**. Hence the eigen values are not same

Hence option (2) is the correct answer

C Code

```
void get_matrices(double* data) {
    // Given Matrix
    data[0] = 1.0; data[1] = 2.0;
    data[2] = 4.0; data[3] = 3.0;
    // Option 1
    data[4] = 3.0; data[5] = 4.0;
    data[6] = 1.0; data[7] = 2.0;
    // Option 2
    data[8] = 1.0; data[9] = 4.0;
    data[10] = 2.0; data[11] = 3.0;
    // Option 3
    data[12] = 4.0; data[13] = 2.0;
    data[14] = 1.0; data[15] = 3.0;
    // Option 4
    data[16] = 2.0; data[17] = 4.0;
    data[18] = 1.0; data[19] = 3.0;
}
```

Python Code for Solving

```
import ctypes
import numpy as np

lib = ctypes.CDLL('./code.so')
double_array_20 = ctypes.c_double * 20
lib.get_matrices.argtypes = [ctypes.POINTER(ctypes.c_double)]

out_data_c = double_array_20()
lib.get_matrices(out_data_c)
data = np.array(list(out_data_c))
options = data.reshape(5, 2, 2)

given = options[0]

trace = np.trace(given)
det = np.linalg.det(given)

match_index = -1
```

Python Code for Solving

```
for i in range(1,4):
    if np.isclose(np.trace(options[i]),trace) and np.isclose(np.
        linalg.det(options[i]),det):
        print(Option,i,is the correct answer )
        break
```