12.754

EE25BTECH11012-BEERAM MADHURI

Question:
Let $\mathbf{Q} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ be a 2×2 matrix. Which one of the following statements is **TRUE**?

a) $\mathbf{Q}$ is equal to its transpose.
b) $\mathbf{Q}$ is equal to its inverse.
c) $\mathbf{Q}$ is full rank.
d) $\mathbf{Q}$ has linearly dependent columns.

Solution:
a)

$$\mathbf{Q}^\top = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{Q} \neq \mathbf{Q}^\top \tag{0.2}$$

b)

$$Q = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \tag{0.3}$$

$$\text{If } \mathbf{Q} = \mathbf{Q}^{-}1 \text{ then } \mathbf{Q}^2 = I \tag{0.4}$$

$$\mathbf{Q}^2 = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \tag{0.5}$$

$$= \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix} \neq \mathbf{I} \tag{0.6}$$

c)

$$\mathbf{Q} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \tag{0.7}$$

Using Row reduction:-

$$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \xrightarrow{R_2 - 2(R_1)} \begin{pmatrix} 1 & -2 \\ 0 & -3 \end{pmatrix} \tag{0.8}$$

$$rank = 2 \tag{0.9}$$

$\therefore$ $\mathbf{Q}$ is a full rank Matrix.

d) Columns of $\mathbf{Q}$ are linearly dependent if

$$\mathbf{c_1} = \lambda \mathbf{c_2} \quad (\lambda \neq 0) \tag{0.10}$$

where $\mathbf{c_1}$ = first column of $\mathbf{Q}$
$\mathbf{c_2}$ = second column of $\mathbf{Q}$.

$$\mathbf{c_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{c_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \tag{0.11}$$

$$\mathbf{c_1} \neq \lambda \mathbf{c_2} \text{ for any } \lambda \neq 0 \tag{0.12}$$

$\therefore$ columns of $\mathbf{Q}$ are linearly independent.

$\therefore$ Option C is correct.