Problem 12.453

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1. Problem

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Problem

A 3×3 matrix $\mathbf{P}$ is such that, $\mathbf{P}^3 = \mathbf{P}$. Then the eigenvalues of $\mathbf{P}$ are

Formula

$$\mathbf{P} = \mathbf{QDQ}^{-1} \tag{3.1}$$

$$\mathbf{P}^2 = \left(\mathbf{QDQ}^{-1}\right)^2 \tag{3.2}$$

$$= \mathbf{QDQ}^{-1}\mathbf{QDQ}^{-1} \tag{3.3}$$

$$= \mathbf{QDIDQ}^{-1} \tag{3.4}$$

$$= \mathbf{QD}^2\mathbf{Q}^{-1} \tag{3.5}$$

where $\mathbf{D}$ is the Diagonal matrix containing eigen values

$$\mathbf{P}^k = \mathbf{QD}^k\mathbf{Q}^{-1} \tag{3.6}$$

$$\mathbf{P}^3 = \mathbf{QD}^3\mathbf{Q}^{-1} \tag{3.7}$$

Given

$$\mathbf{P}^3 = \mathbf{P} \tag{3.8}$$

$$\mathbf{P}^3 - \mathbf{P} = 0 \tag{3.9}$$

Conclusion

From (1) and (7)

$$\mathbf{Q}\mathbf{D}^3\mathbf{Q}^{-1} - \mathbf{Q}\mathbf{D}\mathbf{Q}^{-1} = 0 \tag{3.10}$$

$$\mathbf{Q}\left(\mathbf{D}^3 - \mathbf{D}\right)\mathbf{Q}^{-1} = 0 \tag{3.11}$$

$$\implies \left(\mathbf{D}^3 - \mathbf{D}\right) = 0 \tag{3.12}$$

$$\implies \left(\lambda^3 - \lambda\right) = 0 \tag{3.13}$$

where λ are the eigen values

$$\lambda\left(\lambda^2 - 1\right) = 0 \implies \lambda = 0 \text{ or } \lambda^2 - 1 = 0 \tag{3.14}$$

$$\lambda = 0 \text{ or } \lambda = \pm 1 \tag{3.15}$$