12.690

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:
A real 2×2 non-singular matrix $\mathbf{A} = \begin{pmatrix} x & -3.0 \\ 3.0 & 4.0 \end{pmatrix}$ where x is a real positive number, has repeated eigenvalues. The value of x is ____________. (EC 2021)
Solution:
According to the Cayley-Hamilton theorem:

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{0.1}$$

$$\det\begin{pmatrix} x-\lambda & -3.0 \\ 3.0 & 4.0-\lambda \end{pmatrix} = 0 \tag{0.2}$$

$$(x-\lambda)(4-\lambda) - (-3)(3) = 0 \tag{0.3}$$

$$\lambda^2 - (x+4)\lambda + (4x+9) = 0 \tag{0.4}$$

For the matrix to have repeated eigenvalues, the discriminant of this quadratic characteristic equation must be zero.

$$\Delta = (-(x+4))^2 - 4(1)(4x+9) = 0 \tag{0.5}$$

$$(x+4)^2 - 4(4x+9) = 0 \tag{0.6}$$

$$x^2 - 8x - 20 = 0 \tag{0.7}$$

$$(x-10)(x+2) = 0 \tag{0.8}$$

This gives two possible solutions: $x = 10$ or $x = -2$. Since the problem states that x is a real positive number, we have:

$$x = 10$$