

12.546

EE25BTECH11012-BEERAM MADHURI

Question:

Consider the following two statements

P: $\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$ has infinitely many LU factorizations, where **L** is lower triangular with each diagonal entry 1 and **U** is upper triangular.

Q: $\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$ has no LU factorization, where **L** is lower triangular with each diagonal entry 1 and **U** is upper triangular.

Then which one of the following options is correct?

(MA 2018)

- a) P is TRUE and Q is FALSE
- b) Both P and Q are TRUE
- c) P is FALSE and Q is TRUE
- d) Both P and Q are FALSE

Solution:

statement	given matrix
P	$\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$
Q	$\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$

TABLE 0: Variables used

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \quad (0.1)$$

$$U = \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} \quad (0.2)$$

$$LU = \begin{pmatrix} u_{11} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} \end{pmatrix} \quad (0.3)$$

Statement P:

$$LU = \begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix} \quad (0.4)$$

$$u_{11} = 0, u_{12} = 5 \quad (0.5)$$

$$l_{21}u_{12} + u_{22} = 7 \quad (0.6)$$

has infinite solutions.

\therefore there are infinitely many pairs (l_{21}, u_{22})

Statement P is true

Statement Q:

$$LU = \begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix} \quad (0.7)$$

$$u_{11} = 0 \quad (0.8)$$

$$l_2 1 u_1 1 = 0 \quad (0.9)$$

$$\text{but } l_2 1 u_1 1 = 2 \quad (0.10)$$

\therefore no pairs(L,U) exists.

\therefore Statement Q is true.

Option b is correct.