5.4.26

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Question

Using elementary transformations find the inverse of the given matrix

$$\begin{pmatrix} 1 & -3 & 2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

Theoretical Solution

Given

$$\mathbf{A} = \begin{pmatrix} 1 & -3 & 2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix} \tag{1}$$

To find the inverse, $\mathbf{A}^{-1}$, we augment the identity matrix $\mathbf{I}$ to $\mathbf{A}$ and apply row operations to this augmented matrix.

Theoretical Solution

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ -3 & 0 & -5 & 0 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_2 \to R_2 + 3R_1} \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right) \quad (2)$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \quad (3)$$

Theoretical Solution

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \xleftrightarrow{R_2 \to \frac{1}{9} R_2} \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \quad (4)$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \xleftarrow{R_1 \to R_1 - 3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \quad (5)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \xleftarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & \frac{25}{9} & \frac{-5}{3} & \frac{1}{9} & 1 \end{array}\right) \tag{6}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & \frac{25}{9} & \frac{-5}{3} & \frac{1}{9} & 1 \end{array}\right) \xleftarrow{R_3 \to \frac{9}{25} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \tag{7}$$

Theoretical Solution

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \xleftarrow{R_2 \longrightarrow R_2 + \frac{11}{9} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & 0 & \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \tag{8}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & 0 & \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \xleftarrow{R_1 \longrightarrow R_1 + \frac{5}{3} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \frac{-2}{5} & \frac{-3}{5} \\ 0 & 1 & 0 & \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \tag{9}$$

Theoretical Solution

Therefore,

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix}$$

C Code - Finding Inverse

```c
#include <stdio.h>

void inverse(double (*matrix)[3]) {
    double I[3][3] = {
        {1,0,0},
        {0,1,0},
        {0,0,1}
    };

    double pivot = matrix[0][0];
    for(int i = 0; i < 3; i++) {
        matrix[0][i] /= pivot;
        I[0][i] /= pivot;
    }
```

C Code - Finding Inverse

```c

    double factor = matrix[1][0];
    for(int i = 0; i < 3; i++) {
        matrix[1][i] -= factor * matrix[0][i];
        I[1][i] -= factor * I[0][i];
    }

    factor = matrix[2][0];
    for(int i = 0; i < 3; i++) {
        matrix[2][i] -= factor * matrix[0][i];
        I[2][i] -= factor * I[0][i];
    }

     pivot = matrix[1][1];
    for(int i = 0; i < 3; i++) {
        matrix[1][i] /= pivot;
        I[1][i] /= pivot;
    }
```

C Code - Finding Inverse

```c
    factor = matrix[0][1];
    for(int i = 0; i < 3; i++) {
        matrix[0][i] -= factor * matrix[1][i];
        I[0][i] -= factor * I[1][i];
    }

    factor = matrix[2][1];
    for(int i = 0; i < 3; i++) {
        matrix[2][i] -= factor * matrix[1][i];
        I[2][i] -= factor * I[1][i];
    }


    pivot = matrix[2][2];
    for(int i = 0; i < 3; i++) {
        matrix[2][i] /= pivot;
        I[2][i] /= pivot;
    }
```

C Code - Finding Inverse

```
    factor = matrix[0][2];
    for(int i = 0; i < 3; i++) {
        matrix[0][i] -= factor * matrix[2][i];
        I[0][i] -= factor * I[2][i];
    }

    factor = matrix[1][2];
    for(int i = 0; i < 3; i++) {
        matrix[1][i] -= factor * matrix[2][i];
        I[1][i] -= factor * I[2][i];
    }

    for (int i = 0; i < 3; i++) {
    for (int j = 0; j < 3; j++) {
        matrix[i][j] = I[i][j];
    }
}

}
```

Python Code - Using Shared Object

```python
import numpy as np
import ctypes

c_lib = ctypes.CDLL("./code.so")

c_lib.inverse.argtypes = [ctypes.POINTER((ctypes.c_double * 3))]

A = np.array([
    [1.0, -3.0, 2.0],
    [-3.0, 0.0, -5.0],
    [2.0, 5.0, 0.0]
], dtype=np.float64)
```

Python Code - Using Shared Object

```
B = A.ctypes.data_as(ctypes.POINTER((ctypes.c_double * 3)))

c_lib.inverse(B)

np.set_printoptions(precision=2)

print(A)
```

Python Code

```
import numpy as np
import numpy.linalg as LA

A = np.array([
    [1.0, -3.0, 2.0],
    [-3.0, 0.0, -5.0],
    [2.0, 5.0, 0.0]
])

A_inv = LA.inv(A)

print(A_inv)
```