9.4.24

EE25BTECH11012-BEERAM MADHURI

Question:
A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was $750Rs$. We would like to find out the number of toys produced on that day.

Solution:
Let number of toys produced per day = x
cost of each toy= $55 - x$
Total Cost of toys = $x(55 - x)$

On a particular day cost = 750

$$(55 - x)x = 750 \tag{0.1}$$

$$y = x^2 - 55x + 750 = 0 \tag{0.2}$$

which can be expressed as the conic

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.3}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{55}{2} \\ -\frac{1}{2} \end{pmatrix}, f = 750 \tag{0.4}$$

find roots of (0.3), we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{0.5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.6}$$

The values of k are given by:

$$k_i = \frac{1}{1}\left(\frac{55}{2} \pm \sqrt{\left(\frac{55}{2}\right)^2 - 750} \right) \tag{0.7}$$

$$k_1 = 25, \quad k_2 = 30. \tag{0.8}$$

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} 25 \\ 0 \end{pmatrix}, \begin{pmatrix} 30 \\ 0 \end{pmatrix} \tag{0.9}$$

$\therefore$ no. of toys produced that day can be either 25 or 30.

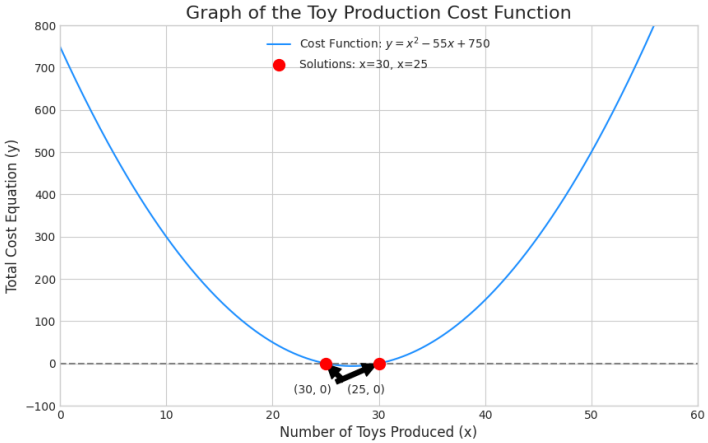

Fig. 0.1: points of intersection of parabola with X axis.