5.4.26

EE25BTECH11031 - Sai Sreevallabh

Question:

Using elementary transformations find the inverse of the given matrix

$$\begin{pmatrix} 1 & -3 & 2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

Solution:

Given

$$\mathbf{A} = \begin{pmatrix} 1 & -3 & 2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix} \tag{0.1}$$

To find the inverse, $\mathbf{A}^{-1}$, we augment the identity matrix $\mathbf{I}$ to $\mathbf{A}$ and apply row operations to this augmented matrix.

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ -3 & 0 & -5 & 0 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_2 \to R_2 + 3R_1} \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right) \tag{0.2}$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \tag{0.3}$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 9 & -11 & 3 & 1 & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \xleftrightarrow{R_2 \to \frac{1}{9} R_2} \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \tag{0.4}$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \xleftarrow{R_1 \to R_1 - 3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \tag{0.5}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & -1 & 4 & -2 & 0 & 1 \end{array}\right) \xleftarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & \frac{25}{9} & \frac{-5}{3} & \frac{1}{9} & 1 \end{array}\right) \tag{0.6}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & \frac{25}{9} & \frac{-5}{3} & \frac{1}{9} & 1 \end{array}\right) \xleftarrow{R_3 \longrightarrow \frac{9}{25} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \tag{0.7}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{-11}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \xleftarrow{R_2 \longrightarrow R_2 + \frac{11}{9} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & 0 & \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \tag{0.8}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{3} & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & 0 & \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \xleftarrow{R_1 \longrightarrow R_1 + \frac{5}{3} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \frac{-2}{5} & \frac{-3}{5} \\ 0 & 1 & 0 & \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ 0 & 0 & 1 & \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{array}\right) \tag{0.9}$$

Therefore,

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix}$$