

5.3.37

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:

Draw the graphs of the following equations

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also, determine the co-ordinates of the vertices of the triangle formed by these lines and the X axis.

Solution:

The triangle is formed by the intersection of three lines:

$$L_1 : \begin{pmatrix} 3 \\ -4 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = -6 \quad (1)$$

$$L_2 : \begin{pmatrix} 3 \\ 1 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 9 \quad (2)$$

$$L_3 : \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad (3)$$

The vertices, which we will call **A**, **B**, and **C**, are the intersection points of these lines. We solve for them using Gaussian elimination (row reduction).

Vertex A: Intersection of L_1 and L_2

The system is: $3x - 4y = -6$ and $3x + y = 9$.

The augmented matrix is:

$$\left(\begin{array}{cc|c} 3 & -4 & -6 \\ 3 & 1 & 9 \end{array} \right) \quad (4)$$

Apply the row operation $R_2 \rightarrow R_2 - R_1$:

$$\left(\begin{array}{cc|c} 3 & -4 & -6 \\ 0 & 5 & 15 \end{array} \right) \quad (5)$$

From the second row, $5y = 15 \implies y = 3$. Substituting into the first row ($3x - 4y = -6$), we get:

$$3x - 4(3) = -6 \implies 3x - 12 = -6 \implies 3x = 6 \implies x = 2. \quad (6)$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (7)$$

Vertex B: Intersection of L_1 and L_3

The system is: $3x - 4y = -6$ and $y = 0$.

The augmented matrix is:

$$\left(\begin{array}{cc|c} 3 & -4 & -6 \\ 0 & 1 & 0 \end{array} \right) \quad (8)$$

This matrix is already in row-echelon form. From the second row, $y = 0$. Substituting into the first row ($3x - 4y = -6$), we get:

$$3x - 4(0) = -6 \implies 3x = -6 \implies x = -2. \quad (9)$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (10)$$

Vertex C : Intersection of L_2 and L_3

The system is: $3x + y = 9$ and $y = 0$.

The augmented matrix is:

$$\left(\begin{array}{cc|c} 3 & 1 & 9 \\ 0 & 1 & 0 \end{array} \right) \quad (11)$$

This matrix is in row-echelon form. From the second row, $y = 0$. Substituting into the first row ($3x + y = 9$), we get:

$$3x + 0 = 9 \implies 3x = 9 \implies x = 3. \quad (12)$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (13)$$

The coordinates of the vertices of the triangle are $(2, 3)$, $(-2, 0)$, and $(3, 0)$.

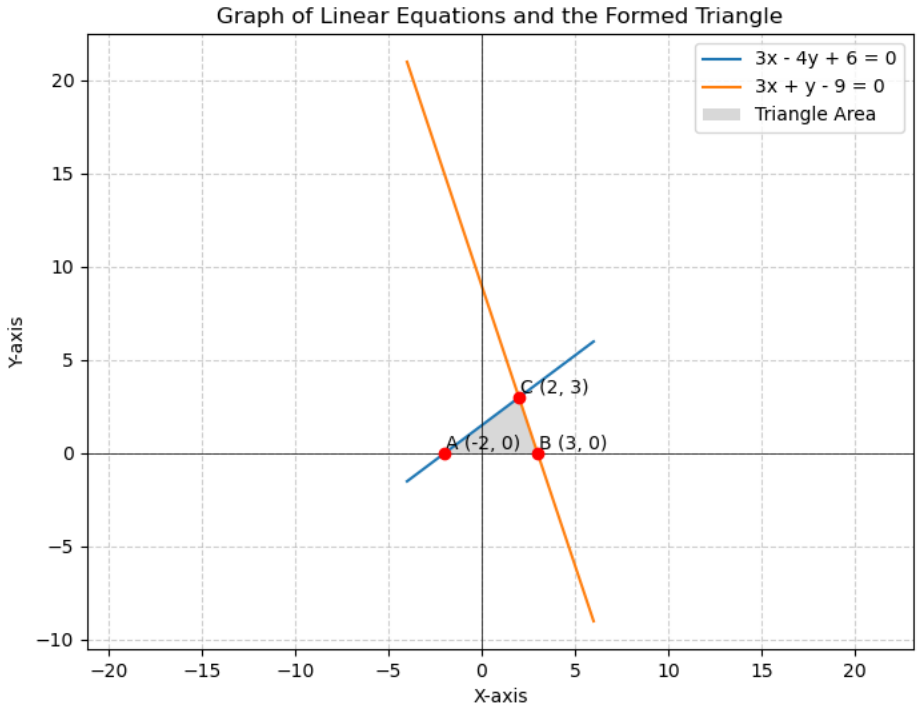
Plot of the Lines and Triangle:

Fig. 0: figure for 5.3.37