5.8.29

EE25BTECH11031 - Sai Sreevallabh

Question:

2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 women alone to finish the work, and also that taken by 1 man alone.

Solution:

Let the fraction of work done by a woman in a day be x and the fraction of work done by a man in a day be y, represented as

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.1}$$

Also, if a days are taken for a work to complete, the fraction of work completed in a single day is $\frac{1}{a}$.

Using the above, we can write the given data into two equations:

$$\begin{pmatrix} 2 & 5 \end{pmatrix} \mathbf{x} = \frac{1}{4} \tag{0.2}$$

$$\begin{pmatrix} 3 & 6 \end{pmatrix} \mathbf{x} = \frac{1}{3} \tag{0.3}$$

Converting into Reduced Row Echelon Form:

$$\begin{pmatrix} 2 & 5 & \Big| & \frac{1}{4} \\ 3 & 6 & \Big| & \frac{1}{3} \end{pmatrix} \xleftarrow{R_1 \to \frac{1}{2} R_1} \begin{pmatrix} 1 & \frac{5}{2} & \Big| & \frac{1}{4} \\ 3 & 6 & \Big| & \frac{1}{3} \end{pmatrix} \tag{0.4}$$

$$\begin{pmatrix} 1 & \frac{5}{2} & \Big| & \frac{1}{4} \\ 3 & 6 & \Big| & \frac{1}{3} \end{pmatrix} \xleftarrow{R_2 \to R_2 - 3R_1} \begin{pmatrix} 1 & \frac{5}{2} & \Big| & \frac{1}{4} \\ 0 & -\frac{3}{2} & \Big| & -\frac{1}{24} \end{pmatrix} \tag{0.5}$$

$$\begin{pmatrix} 1 & \frac{5}{2} & \Big| & \frac{1}{4} \\ 0 & -\frac{3}{2} & \Big| & -\frac{1}{24} \end{pmatrix} \xleftarrow{R_2 \to -\frac{2}{3} R_2} \begin{pmatrix} 1 & \frac{5}{2} & \Big| & \frac{1}{4} \\ 0 & 1 & \Big| & \frac{1}{36} \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} 1 & \frac{5}{2} & \Big| & \frac{1}{4} \\ 0 & 1 & \Big| & \frac{1}{36} \end{pmatrix} \xleftarrow{R_1 \to R_1 - \frac{5}{2} R_2} \begin{pmatrix} 1 & 0 & \Big| & \frac{1}{18} \\ 0 & 1 & \Big| & \frac{1}{36} \end{pmatrix} \tag{0.7}$$

We get

$$\mathbf{x} = \begin{pmatrix} \frac{1}{18} \\ \frac{1}{36} \end{pmatrix} \tag{0.8}$$

The number of days can be written as

$$\frac{1}{y} = 36 \quad \text{and} \quad \frac{1}{x} = 18 \tag{0.9}$$

$\therefore$. The time taken by one woman alone to finish the work is 18 days, and the time taken by one man alone to finish the work is 36 days.

(The below graph represents the solution to the equations (0.2) and (0.3))