5.2.61

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Question

Solve the system:

$x - y + 2z = 1$

$2z - 3z = 1$

$3x - 2y + 4z = 2$

Solution

Variable
x
y
z

Table: Variables Used

Solution

$$\begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \mathbf{X} = 1 \tag{1}$$

$$\begin{pmatrix} 0 & 2 & -3 \end{pmatrix} \mathbf{X} = 1 \tag{2}$$

$$\begin{pmatrix} 3 & -2 & 4 \end{pmatrix} \mathbf{X} = 2 \tag{3}$$

Solution

This system of equations can be solved using an augmented matrix and Gaussian elimination

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & -3 & 1 \\ 3 & -2 & 4 & 2 \end{array}\right) \xrightarrow{R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & -3 & 1 \\ 0 & 1 & -2 & -1 \end{array}\right) \tag{4}$$

$$\xrightarrow{R_3 - \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \tag{5}$$

solution

$$\xrightarrow{R_2 \to \frac{1}{2} R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \tag{6}$$

$$\xrightarrow{R_3 \to -2R_3} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{7}$$

$$\xrightarrow{R_2 \to R_2 + \frac{3}{2} R_3} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{8}$$

Solutions

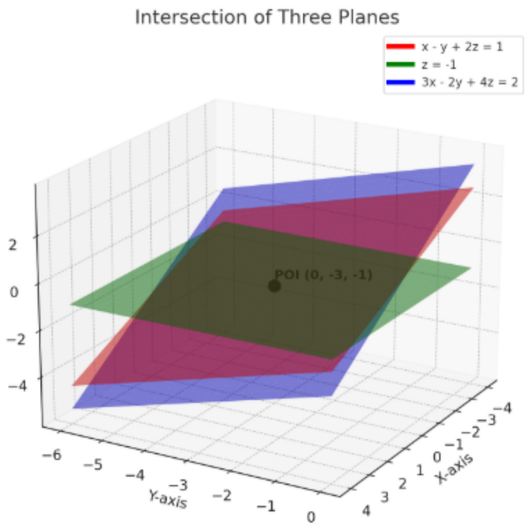
$$\xrightarrow{R_1 \rightarrow R_1 - 2R_3} \left(\begin{array}{ccc|c} 1 & -1 & 0 & -5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{9}$$

$$\xrightarrow{R_1 \rightarrow R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \tag{10}$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} \tag{11}$$

$$x = 0, \quad y = 5, \quad z = 3 \tag{12}$$

Refer to Figure



Python Code

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.lines import Line2D

# Intersection point (calculated from the system)
poi = np.array([0, -3, -1])

# Grid for plotting planes
x_range = np.linspace(-4, 4, 30)
y_range = np.linspace(-6, 0, 30)
X, Y = np.meshgrid(x_range, y_range)

# Plane 1: x - y + 2z = 1  ->  z = (1 - x + y) / 2
Z1 = (1 - X + Y) / 2

# Plane 2: z = -1
Z2 = -1 * np.ones_like(X)

# Plane 3: 3x - 2y + 4z = 2  -> z = (2 - 3x + 2y) / 4
```

Python Code

```
# Create plot
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

# Plot planes with distinct colors
ax.plot_surface(X, Y, Z1, alpha=0.5, color='red')
ax.plot_surface(X, Y, Z2, alpha=0.5, color='green')
ax.plot_surface(X, Y, Z3, alpha=0.5, color='blue')

# Mark and label the intersection point
ax.scatter([poi[0]], [poi[1]], [poi[2]], s=100, c='
    black', marker='o')
ax.text(poi[0], poi[1], poi[2]+0.3, f'POI (0, -3, -1)'
    ,
        color='black', fontsize=11, weight='bold')

# Labels and title
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
```

```python
ax.set_zlabel('Z-axis')
ax.set_title('Intersection of Three Planes')

# Add legend manually
legend_elements = [
    Line2D([0], [0], color='red', lw=4, label='x - y + 2z = 1'),
    Line2D([0], [0], color='green', lw=4, label='z = -1'),
    Line2D([0], [0], color='blue', lw=4, label='3x - 2 y + 4z = 2')
]
ax.legend(handles=legend_elements, loc='upper right')

# Adjust view angle
ax.view_init(elev=20, azim=30)

# Save the figure
plt.savefig( graph9.png , dpi=300, bbox_inches='tight')
```

C Code

```c
#include <stdio.h>

#define N 3  // number of equations

// Function to perform Gaussian elimination
void gaussian_elimination(double A[N][N+1], double x[N
    ]) {
    int i, j, k;

    // Forward elimination
    for (i = 0; i < N-1; i++) {
        for (k = i+1; k < N; k++) {
            double factor = A[k][i] / A[i][i];
            for (j = i; j <= N; j++) {
                A[k][j] -= factor * A[i][j];
            }
        }
    }
```

C Code

```c
    // Back-substitution
    for (i = N-1; i >= 0; i--) {
        x[i] = A[i][N];
        for (j = i+1; j < N; j++) {
            x[i] -= A[i][j] * x[j];
        }
        x[i] = x[i] / A[i][i];
    }
}

// Exposed function for ctypes
void solve_system(double *solution) {
    // Augmented matrix for given system:
    // x - y + 2z = 1
    // 0x + 2y - 3z = 1
```

C Code

```c
    // 3x - 2y + 4z = 2
    double A[N][N+1] = {
        {1, -1,  2, 1},
        {0,  2, -3, 1},
        {3, -2,  4, 2}
    };

    double x[N];
    gaussian_elimination(A, x);

    for (int i = 0; i < N; i++) {
        solution[i] = x[i];
    }
}
```

Python and C Code

```python
import ctypes

# Load the shared object
lib = ctypes.CDLL( ./code.so )

# Define return type and argument type of the exposed
    function
lib.solve_system.argtypes = [ctypes.POINTER(ctypes.
    c_double)]
lib.solve_system.restype = None

# Prepare solution array
solution = (ctypes.c_double * 3)()

# Call C function
lib.solve_system(solution)

# Print the result
print( Solution of system: )
```