Fitting Data To models

EE16B014
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Abstract

In this report we are going to discuss about fitting of data by taking example of Bessel function and approximate it to two function:1)$Acos(x_i)+Bsin(x_i)$ and 2)$A\frac{cos(x_i)}{\sqrt{x_i}}+B\frac{sin(x_i)}{\sqrt{x_i}}$ which are near functions to Bessel function,and we will evaluate the their coefficients by best fitting method and we will see the effect of noise in the fitting.

1 Bessel Function:

Bessel function is generally the a solution to the differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-\alpha^2)y=0$$

- The function is in such away that the output is same for both $+\alpha$ and $-\alpha$ and its a smooth function with α.
- The most important cases are when α is an integer or half-integer. Bessel functions for integer α are also known as cylinder functions or the cylindrical harmonics because they appear in the solution to Laplace's equation in cylindrical coordinates. Spherical Bessel functions with half-integer α are obtained when the Helmholtz equation is solved in spherical coordinates.

Lets take α as v and proceed.Now we have approximate form for the Bessel function as given below:

$$J_v(x)\approx\sqrt{\frac{2}{\pi x}}cos\left(x-\frac{v\pi}{2}-\frac{\pi}{4}\right)$$

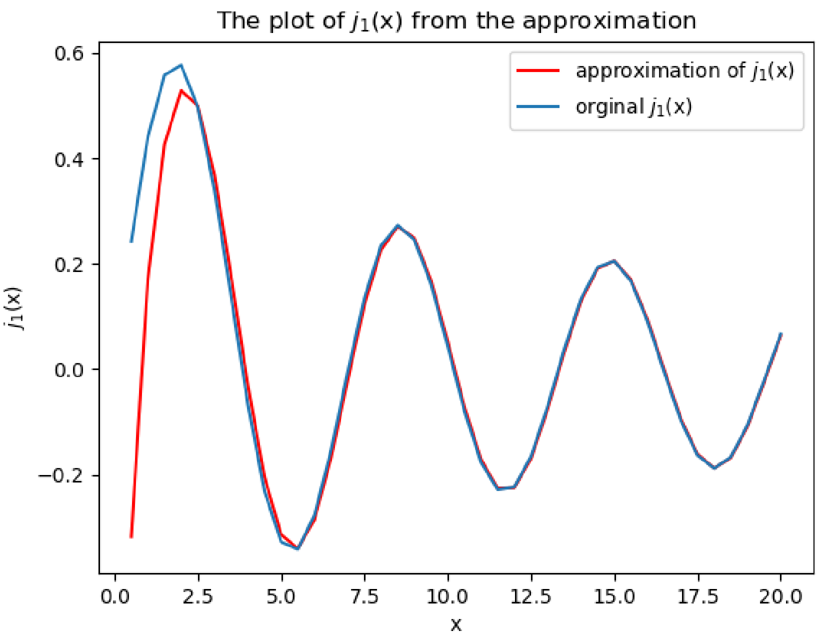
Now we first plot the approximation of Bessel function along with the original one.

```
from numpy import *
from pylab import *
from math import *
from scipy import *
from scipy.special import j1
x=linspace(0,20,41)
x=x[1:]

jk=((2/pi)**(1/2))*cos(x-(3*pi/4))/(x**(1/2.0))

plot(x,jk,"r",label="approximation of $j_1$(x)")
plot(x,j1(x),label="orginal $j_1$(x)")
legend(loc="upper right")
xlabel('x')
ylabel('$j_1$(x)')
title('The plot of $j_1$(x) from the approximation')
show()
```

Plot:



As we can see the approximation graph is slightly distorted from the original one which means our approximate function is not that accurate for less values of x.

Now lets define a function called *calcnu* Which can calculate the coefficients of A,B for our required functions when approximated as shown below:

$$A cos(x_i) + B sin(x_i) \approx J_1(x_i)$$
$$A\frac{cos(x_i)}{\sqrt{x_i}} + B\frac{sin(x_i)}{\sqrt{x_i}} \approx J_1(x_i)$$

Calcnu:

Here is the function which we have defined:

```
calcnu(y,xo,k,eps,model)
```

It takes the input arguments y,which defines the vector range for our approximation;$x_0$defines from which point of vector y to begin with,k is the Bessel function v value,eps

is the error to be added to the function ,model defines the approximation to be considered whether to take the first or the second one.

```
def calcnu(y,xo,k,eps,model):
        l=where(y>=xo)
        y1=y[l]
        l1=len(y1)
        A=zeros((l1,2))
        b=j1(y1)+eps*(rand(len(y1)))
        if model==0:
                A[:,0]=cos(y1)
                A[:,1]=sin(y1)
        if model==1:
                A[:,0]=cos(y1)/(y1**(0.5))
                A[:,1]=sin(y1)/(y1**(0.5))
        c=lstsq(A,b)[0]
        i1=arccos(c[0]/((c[0])**2+(c[1])**2)**(0.5))
        v1=-1*(-2*i1/pi+0.5)
        return v1
```

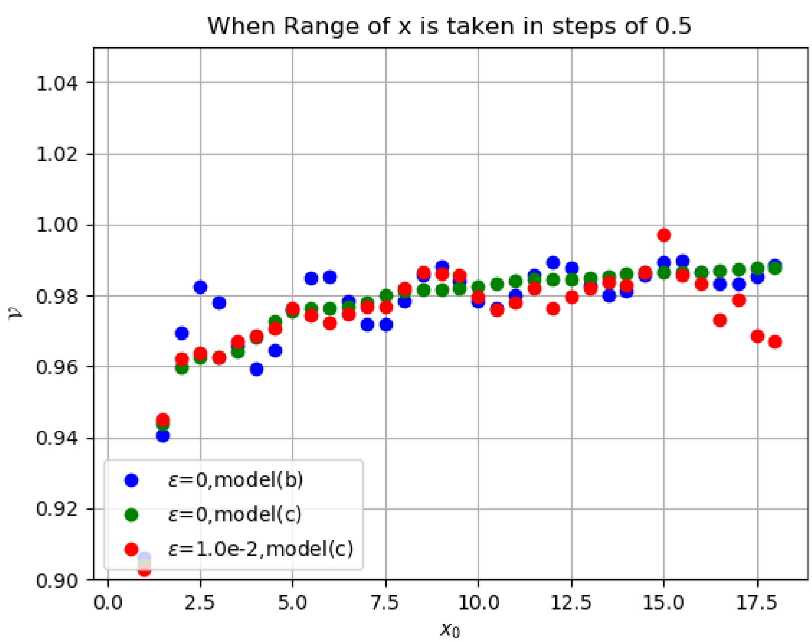
It returns the value of v (v is the subscript of$J_v(x)$),So that we can know that the approximation is very good if v value is very near to *one*.(Because we have considered the function to be $J_1(x)$.
The function is basically calculating coefficients then getting the angle ϕ when our function is written in the form $\sqrt{A^2+B^2}cos(x_i+\phi)$ where ϕ is nothing but $cos^{-1}\left(\frac{A}{\sqrt{A^2+B^2}}\right)$ so that we can get v value from the approximate Bessel function $\sqrt{\frac{2}{\pi x}}cos\left(x-\frac{v\pi}{2}-\frac{\pi}{4}\right)$.By equating $\phi=\frac{v\pi}{2}+\frac{\pi}{4}$.

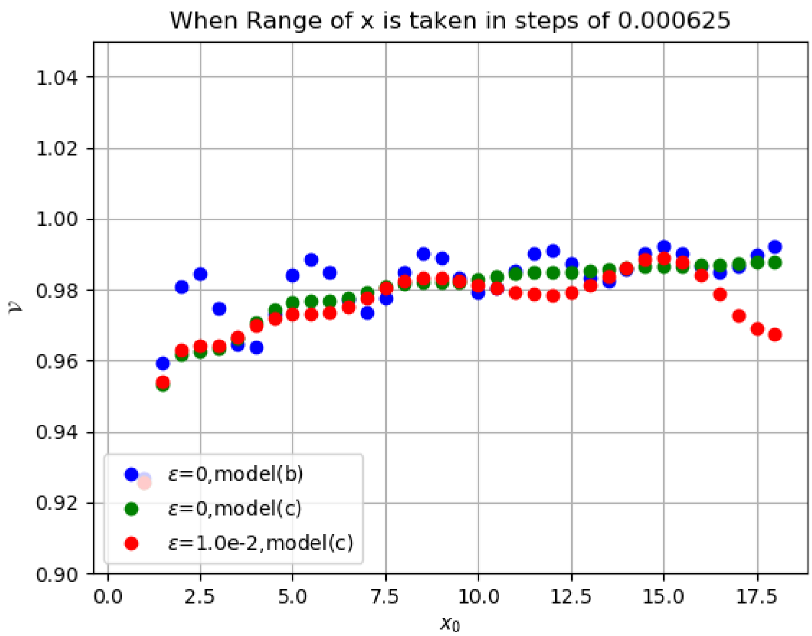
Calling the calcnu function for the v values and plotting:

```
y=arange(0.5,20.5,0.000625)
for xo in y3:
        nu=calcnu(y,xo,0,0,0)
        v1.append(nu)
for xo in y3:
        nu=calcnu(y,xo,0,0,1)
        v2.append(nu)
for xo in y3:
        nu=calcnu(y,xo,0,0.01,1)
        v3.append(nu)
```

```
#Plots
fig1=figure(2)
plot(y3,v1,'bo',label="$\epsilon$=0,model(b)")
plot(y3,v2,'go',label="$\epsilon$=0,model(c)")
plot(y3,v3,'ro',label="$\epsilon$=1.0e-2,model(c)")
title("Range of x in steps of 0.000625")
xlabel("$x_0$")
ylabel("$\mathcal{V}$")
grid()
axes=plt.gca()
axes.set_ylim([0.9,1.05])
legend(loc="lower left")
show()
```

Graph of v values for both approximate functions :





Conclusions:

- Looking at the Plots v vs x_0 we can say that model(c) is more accurate than model(b).
- By Varying the no of measurements by not changing the steps taken for *range of x* change from 0.5 to 0.000625we can say that the effect of noise in the graph for model(c) is considerably less.
- Moreover quality fit of error is very poor in the end values of x_o because of only less values are given for leastsq approximation.
- There is also a large deviation from "v" value "one" in the beginning,this is because the Bessel function is more near to the $\sqrt{\frac{2}{\pi x}} cos\left(x - \frac{v\pi}{2} - \frac{\pi}{4}\right)$ approximation from higher values of "x" so the approximate value is not near to one in the beginning.