# Current flow ,Heat generation in a Resistor

EE16B014
G.HARSHA KANAKA ESWAR

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**Abstract**

In this Report , we are going to discuss the current flow in resistor and the heat generated at various parts of the resistor surface.we will take an example model,and see the plots of current magnitude and direction using quiver plot ,along with that we will also see the surface 3d-plot for voltage.

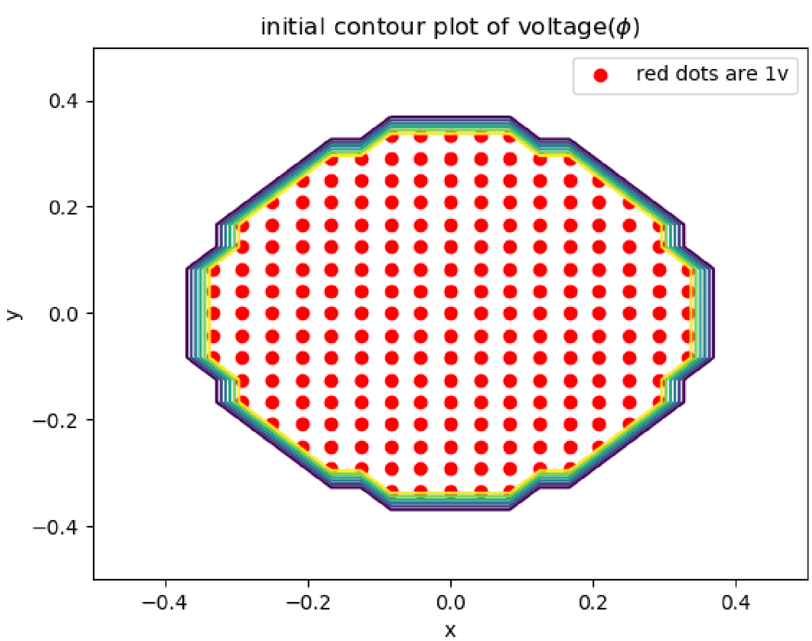
## 1 Resistor Model:

We are going to take a 1cm by 1cm copper plate where there is a circular region of radius 0.35 is held at a constant one volt and one end of the plate is grounded.Since current is directly proportional to current density we are going to consider current density for our calculations.
We will define the copper plate voltage($\phi$) with *phi* as a zeros matrix of size Nx$\times$Ny where value of *phi* is one's in the center of copper plate so we find the points within the circular region of radius 0.35 from the center and make it one's as shown below:

```
from numpy import *
from pylab import *
from matplotlib.pyplot import *
from scipy import *
import mpl_toolkits.mplot3d.axes3d as p3

y=linspace(0.5,-0.5,25)
x=linspace(-0.5,0.5,25)
i=arange(0,25,1)

Nx=int(sys.argv[1])
```

```
Ny=int(sys.argv[2])
phi=zeros((Nx,Ny))
Niter=1500
k1=arange(0,1500,1)
k2=arange(500,1500,1)
Y,X=meshgrid(y,x)
ii=where(Y*Y+X*X<=0.35*0.35)
phi[ii]=1.0
print( phi)
scatter(x[ii[0]],y[ii[1]],color='r',label="red dots are 1v")
contour(x,y,phi)
title("initial contour plot of voltage ")
xlabel('x')
ylabel('y')
legend(loc="upper right")
show()
```

We will first take a look at the voltage contour plot of the plate:




As we can see we made the circular region as one volt and the remaining others as zero.

## 2 Solving the phi from Laplace's equation:

We have the Laplace's equation given as:

$$\nabla^2\phi = 0$$

We can convert the above equation into Cartesian coordinates as follows:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

If we have the values of $\phi$ , clearly we can write:

$$\left.\frac{\partial\phi}{\partial x}\right|_{(x_i,y_i)} = \frac{\phi(x_{i+\frac{1}{2}}, y_j) - \phi(x_{i-\frac{1}{2}}, y_j)}{\Delta x}$$

and

$$\left.\frac{\partial^2\phi}{\partial x^2}\right|_{(x_i,y_i)} = \frac{\phi(x_{i+1}, y_j) - 2\phi(x_i, y_j) + \phi(x_{i-1}, y_j)}{(\Delta x)^2}$$

So From the above equations we can derive the value of $\phi$ at a corresponding point from its neighboring points like this:

$$\phi_{i,j} = \frac{\phi_{i+1,,j} + \phi_{i-1,,j} + \phi_{i,,j+1} + \phi_{i,,j-1}}{4}$$

So at every point we just have to update the value of *phi* from the average of its neighboring points potentials and lets keep updating the $\phi$ for 1500 no of iterations.

```
for k in arange(1,1501,1):
        oldphi=phi.copy()
        phi[1:Nx-1,1:Ny-1]=0.25*(phi[1:Nx-1,0:Ny-2]+phi[1:Nx-1,2:Ny]+ \
             phi[0:Nx-2,1:Ny-1]+phi[2:Nx,1:Ny-1])
        phi[ii]=1.0
        phi[0:-1,0]=phi[0:-1,1]
        phi[0:-1,-1]=phi[0:-1,-2]
        phi[0,0:-1]=phi[1,0:-1]
        errors[k-1]=(abs(phi-oldphi)).max()
```

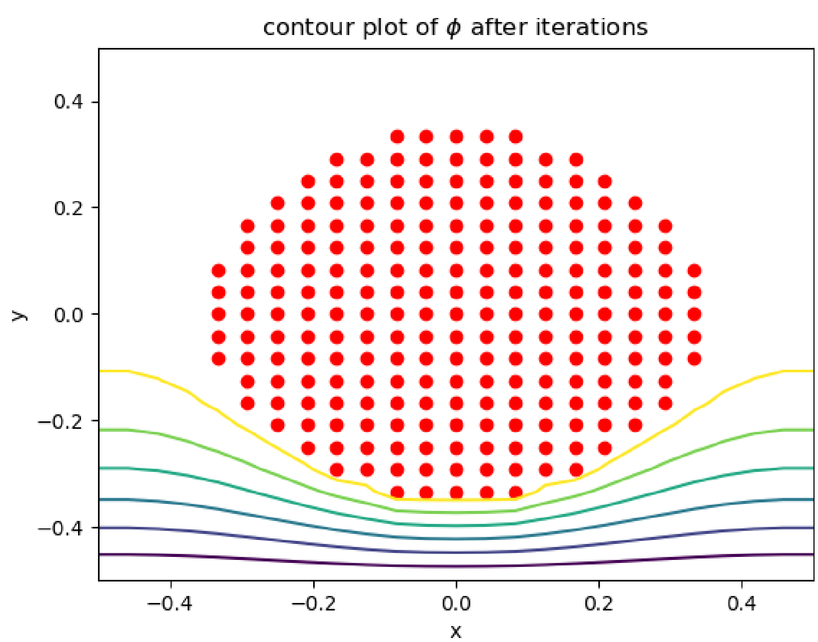
Here we have small exception about boundary conditions that where the electrode is present we give the same value of the electrode potential.At the other boundaries the current is tangential by the condition that charge can't leap out of the plate so ,it means the Field is tangential,i.e Gradient of $\phi$ in normal direction should be zero(because of no field in that direction which means the value of $\phi$ should remain the same in normal direction.

So at the boundary points we equate them with the adjacent points potential values and at electrode we kept constant zero potential in each iteration.

Now lets plot the contour of $\phi$ once again:

```
scatter(x[ii[0]],y[ii[1]],color='r')
contour(x,y,phi)
xlabel('x')
ylabel('y')
title("contour plot of $\phi$ after iterations")
show()
```

Here is the plot:




Along with the update of $\phi$ we have also calculated the error in $\phi$ which is defined as the maximum difference between elements in old and updated $\phi$ ,and at every iteration the value is stored in an array.

```
errors[k-1]=(abs(phi-oldphi)).max()
```

Here the errors are actually varying exponential wit no of iterations in the form:

$$y = Ae^{Bx}$$

Take logarithm on both sides to get a linear variance of $logy$ and $x$:

$$logy = logA + Bx$$

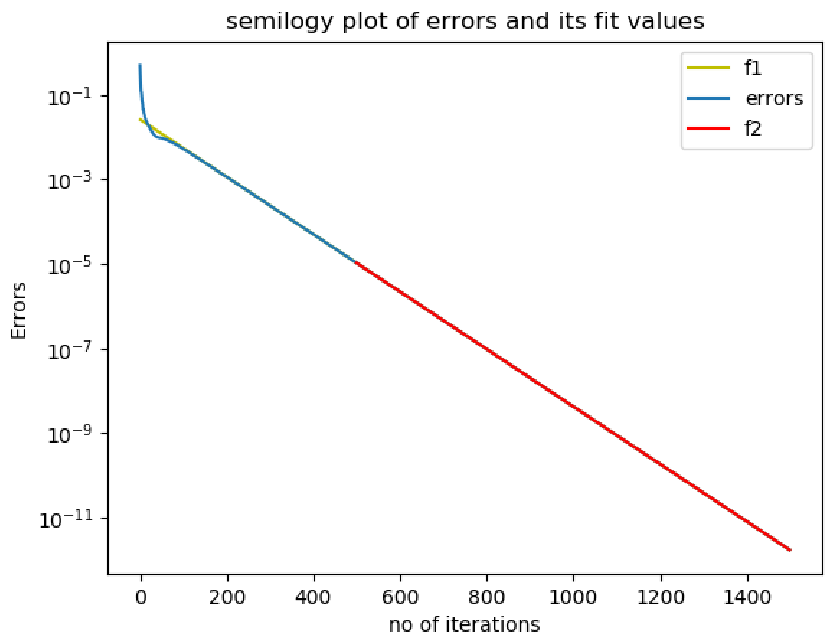
Now we shall use least square approach for solving $logA, B$ starting from first iteration as one fit and starting from $500^{th}$as the other fit and lets see the best fit among them:

```
y1=log(errors)
A=zeros((Niter,2))
A[:,0]=1
A[:,1]=k1
cl=lstsq(A,y1)[0]
a=exp(cl[0])
b=cl[1]

y2=log(errors[500:1500])
B=zeros((1000,2))
B[:,0]=1
B[:,1]=range(500,1500)
cl1=lstsq(B,y2)[0]

c1=a*e**(k1[1:]*b)
c2=e**(cl1[0])*e**(cl1[1]*k2)
```

Here we got fit values in c1,c2 lets plot them:

```
ylabel('Errors')
xlabel('no of iterations')
semilogy(range(1,1500),c1,'y',label="f1")
semilogy(k1,errors,label="errors")
semilogy(k2,c2,'r',label="f2")
legend(loc="upper right")
title('semilogy plot of errors and its fit values')
show()
```

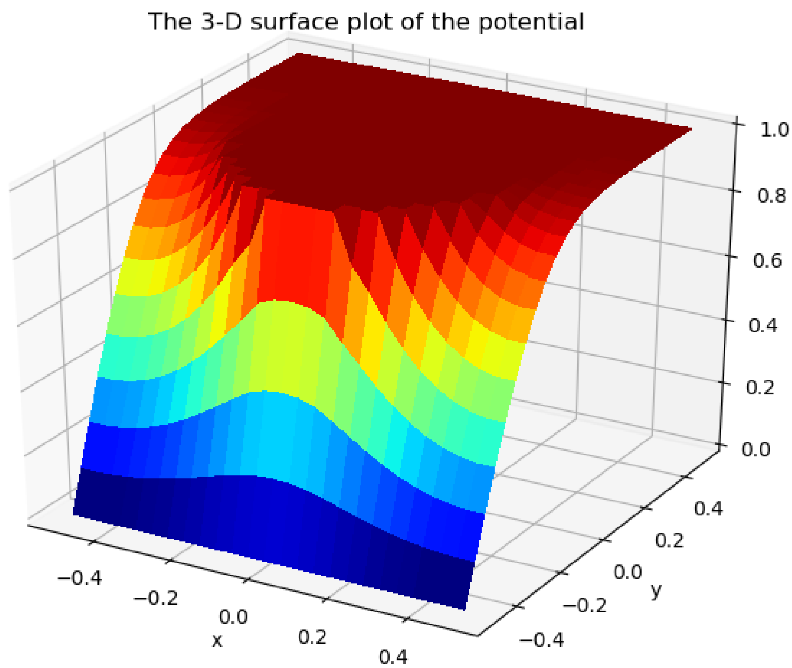
Plot of errors and its fits in semilogy scale:




Among these fit2 seems to be more accurate because the errors are not exactly exponentially varying for the less no of iterations so the fit1 is slightly away from the errors where as the fit2 is exactly coinciding with the errors.Thus fit2 is better approximation .

### Current and surface plot of potential

Since we already got the potential lets just go ahead and plot the surface plot:




We can see clearly some circle having constant 1v potential, which is nothing but what we have done during the iterations making the center circular region of plate with radius 0.35 as 1v.We can also see that the values after the circle are almost same and near to one.
Okay Now lets derive $j_x$ from the following equations:

$$
\begin{aligned}
j_x &= -\frac{\partial \phi}{\partial x} \\
j_y &= -\frac{\partial \phi}{\partial y}
\end{aligned}
$$

Which can be written as :

$$
\begin{aligned}
J_{x,ij} &= \frac{1}{2}\left(\phi_{i,,j-1} - \phi_{i,,j+1}\right) \\
J_{x,ij} &= \frac{1}{2}\left(\phi_{i,,j-1} - \phi_{i,,j+1}\right)
\end{aligned}
$$

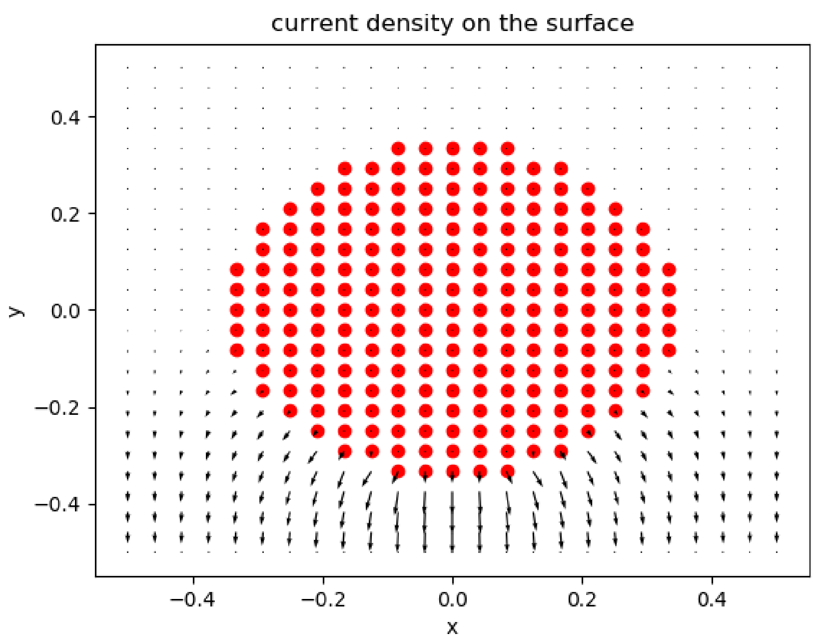
Now we can get $J_x, , J_y$:

```
jx=zeros((Nx,Ny))
jy=zeros((Nx,Ny))
jx[0:Nx,1:Ny-1]=0.5*-(phi[0:Nx,0:Ny-2]-phi[0:Nx,2:Ny])
jy[1:Nx-1,0:Ny]=0.5*-(phi[0:Nx-2,0:Ny]-phi[2:Nx,0:Ny])
```

Lets plot J using *quiver* plot as follows:

```
scatter(x[ii[0]],y[ii[1]],color='r')
quiver(y,x,jx[::-1,:],jy[::-1,:],scale=8)
title("current density")
show()
```

Here is the quiver plot of current density:




Here we can see the current is almost zero at the upper points that is because the potential difference is not very large which can be seen in the 3d surface potential plot.
And the current magnitude is increasing as we come down along the y axis its because the currents are summing up,as they can't escape outside. They are all adding up and approaching the ground electrode.Direction is just seen as from high to low potential.