The Laplace Transform

EE16B014
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Abstract

In this report we are going to discuss about how to use scipy.signal for solving 'Linear' time Invariant system's and see how to get the output if we know both input function and frequency response of the system.

1 Time Response from the Laplace transform:

Question1:
Lets take the function :

$$f(t) = cos(1.5t)e^{-0.5t}u_0(t)$$

Which has Laplace transform:

$$F(s) = \frac{s+0.5}{(s+0.5)^2+2.25}$$

Lets solve the equation:

$$\ddot{x} + 2.25x = f(t)$$

So we can write $X(s)$ as:

$$X(s) = \frac{s+0.5}{((s+0.5)^2+2.25)\times(s^2+2.25)}$$

So we have $X(s)$ lets get its time response using sp.impulse:

```
import scipy.signal as sp
t=linspace(0,50,1000)
X=sp.lti([1,0.5],polymul([1,1,2.5],[1,0,2.25]))
t1,x1=sp.impulse(X,None,t)
```

Question2:

Similar to the early case we have slight change in $f(t)$:

$$f(t) = cos(1.5t)e^{-0.05t}u_0(t)$$

Here there is only small change in $X(s)$

```
X=sp.lti([1,0.05],polymul([1,0.1,2.2525],[1,0,2.25]))
```

Question3:

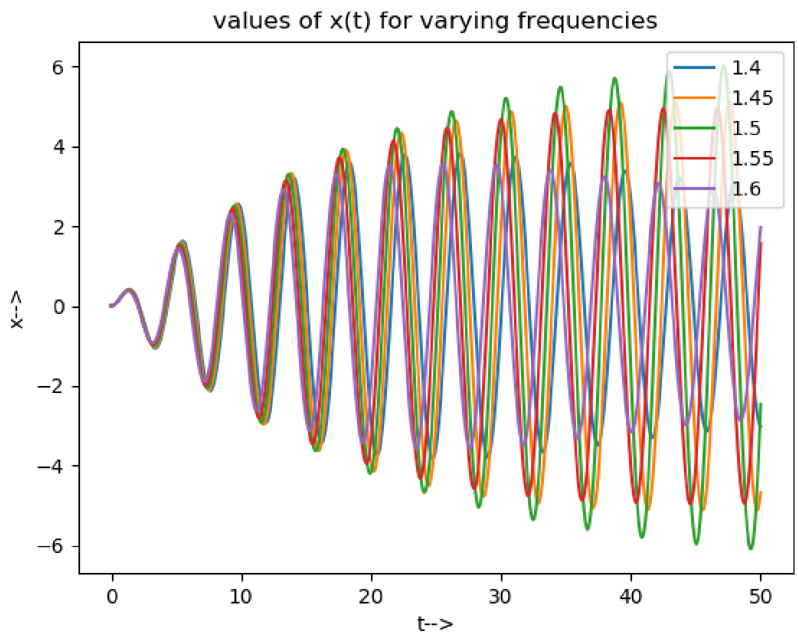
Here we have transfer function $\frac{X(s)}{F(s)} = H(s)$,where $H(s)$ is given as:

$$H(s) = \frac{1}{(s^2 + 2.25)}$$

Now will use a for loop and send the $f(t)$'s at variable frequencies and see the behavior of the output:

```
H=sp.lti([1],[1,0,2.25])
w=arange(1.4,1.6,0.05)
t=linspace(0,50,1000)
for k in w:
        f=cos(k*t)*exp(-0.05*t)
        t1,y,svec=sp.lsim(H,f,t)
        plot(t1,y,label=k)
```

Here is the output:



The maximum value of peak is occurring at frequency=1.5 which is the resonant frequency of the system.

Question3:

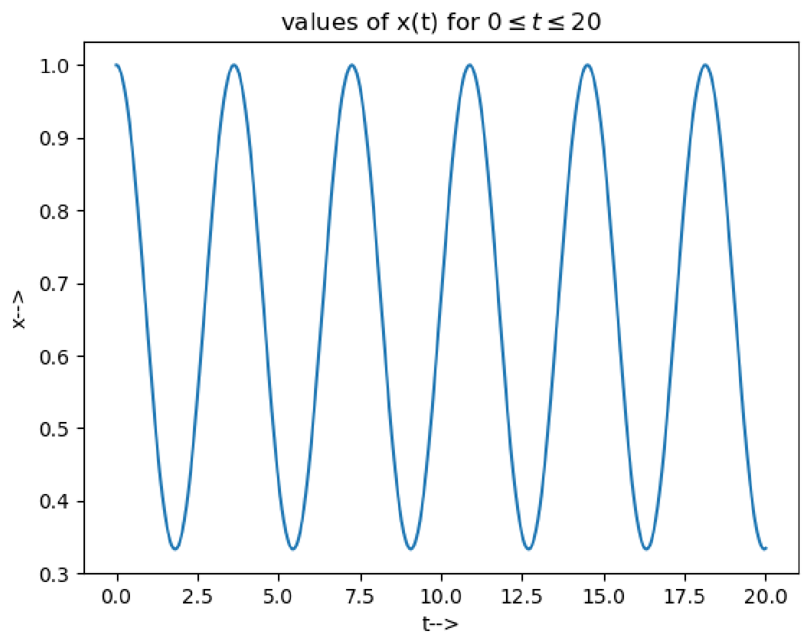
Now lets solve a coupled spring problem whose equations are given by:

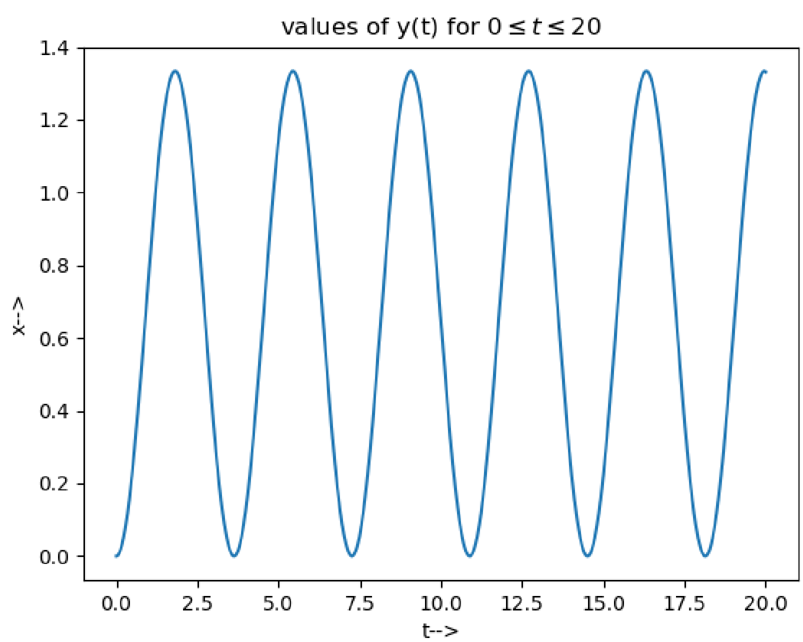
$$\ddot{x} + (x - y) = 0$$
$$\ddot{y} + 2(y - x) = 0$$

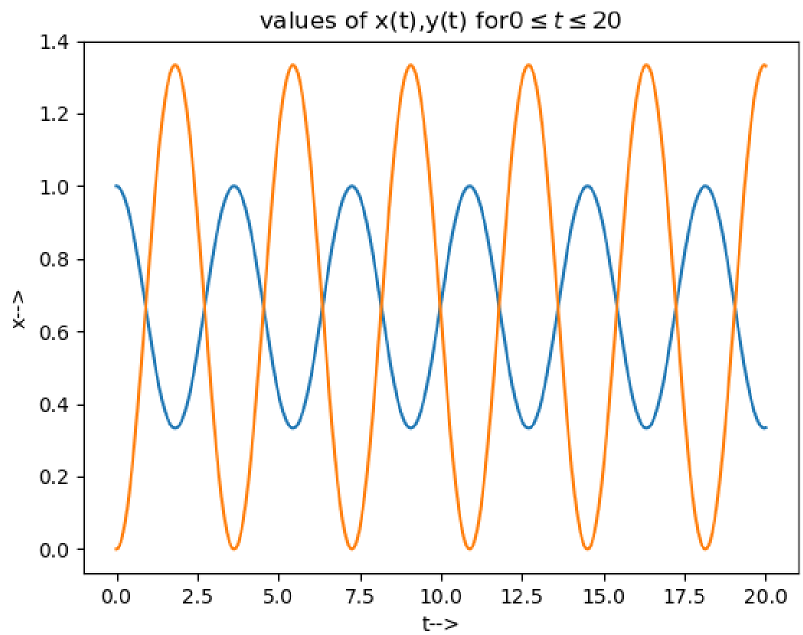
With initial conditions given as $x(0) = 1, \dot{x} = y(0) = \dot{y}(0) = 0$.

```
H=sp.lti([1,0,2,0],[1,0,3,0,0])
t=linspace(0,20,1000)
t1,x=sp.impulse(H,None,t)
H1=sp.lti([2],[1,0,3,0])
t2,y=sp.impulse(H1,None,t)
```

We have solved $X(s), Y(s)$ manually and got the $x(t)$ and $y(t)$.


values of x(t) for $0 \leq t \leq 20$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
x-->
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
t-->


values of y(t) for $0 \leq t \leq 20$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
x-->
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
t-->



The graph of $x(t), y(t)$ has a phase shift of $\frac{\pi}{2}$because $X(s), Y(s)$ are related by factor of $\frac{1}{s}$ which gives a phase shift of $\frac{\pi}{2}$.

Question4:

Here we Have a general R,L,C circuit whose transfer function is:

$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

Where the Values of R,L,C are given as $100\Omega, 1\mu H, 1.0\mu F$,So the equation is like:

$$H(s) = \frac{1}{10^{-12}s^2 + 10^{-4}s + 1}$$

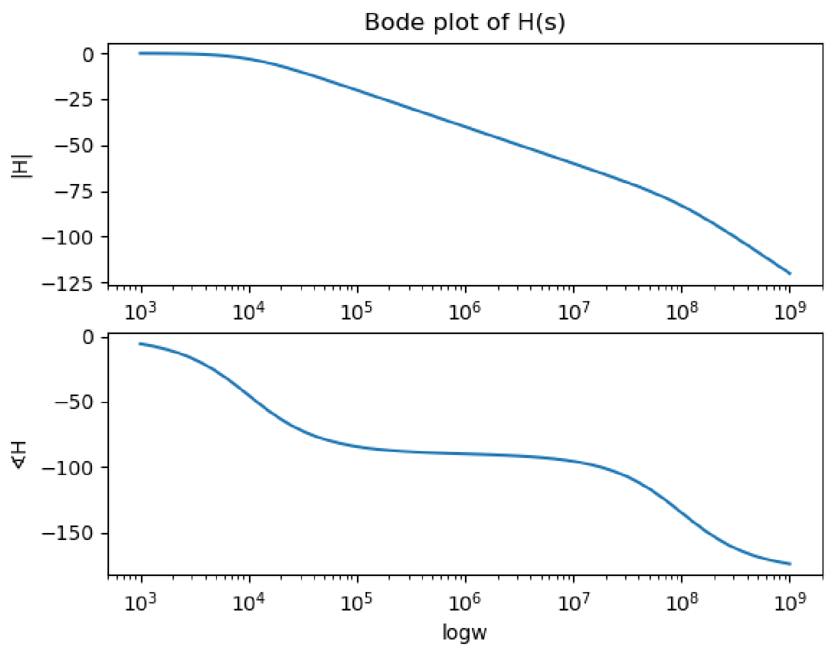
So now lets get the Bode plot of the $H(s)$:

```
H=sp.lti([1e12],[1,100e6,1e12])
w,S,phi=H.bode()
subplot(2,1,1)
semilogx(w,S)
ylabel('|H|')
```

```
title('Bode plot of H(s)')

subplot(2,1,2)
ylabel('$\sphericalangle$H')
xlabel('logw')
semilogx(w,phi)
show()
```

Bode PLOT:



Its nothing but a lowpass filter which can be interpreted through equation or from the plot.

Question5:

Now lets pass the below signal to the above lowpass filter and see the output;

$$v_i(t) = cos(10^3t)u(t) - cos(10^6t)u(t)$$

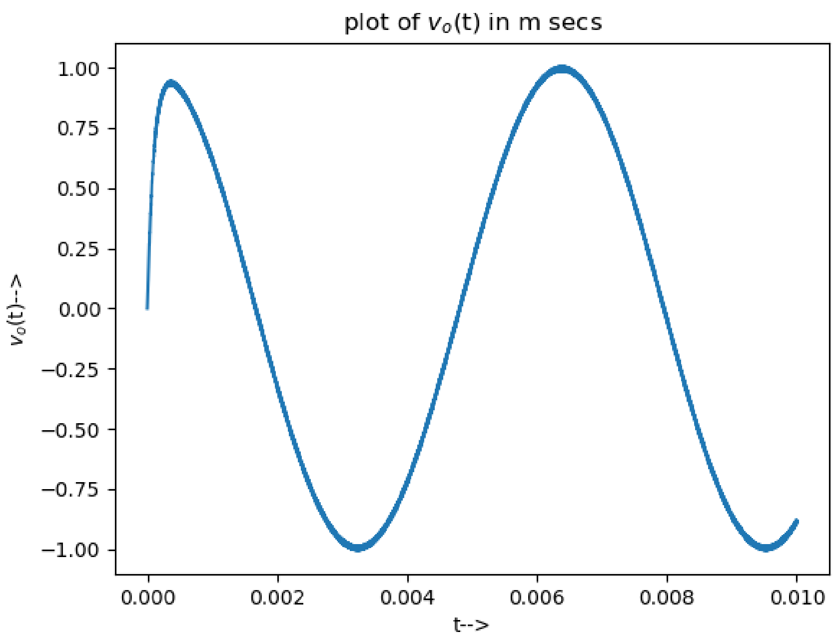
We will use sp.lsim and send the signal as the input for the above filter:

```
def v(t):
        return cos(1000*t)-cos((10**6)*t)
```

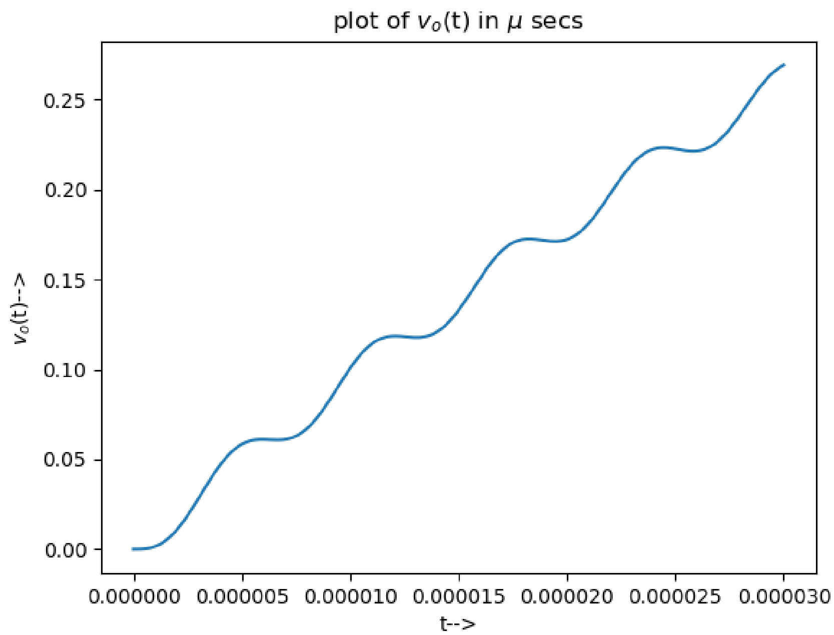
```
f1,y1,svec=sp.lsim(H,v(t1),t1)
```

We will plot the graph for two time intervals:

```
t2=linspace(0,1e-2,10000)
```




```
t1=linspace(0,30e-6,1000)
```



We see a nice sinusoidal wave in the plot one with amplitude one,whereas the second one is saying that it has some ripples in it,The reason is that the input is a combination of two sinusoids of amplitude, where the maximum value of input would be 2, But output has maximum amplitude one,i.e input has sinusoids of low,high frequency ,so the sinusoid with low frequency passed as it is with no change where as the second one is not passed by the low pass filter(where filter passed it with very low amplitude making it look like ripple on the first one).