

UNIVERSITY OF RUHUNA DEPARTMENT OF MATHEMATICS

BACHELOR OF SCIENCE (GENERAL) DEGREE (LEVEL III) MATHEMATICS

MAT 327β : Introduction to Financial Mathematics

Tutorial No: 05 Semester II, 2023

Submit answer sheets on or before: 25/10/2023

1. The following table shows the information of a government bond traded in the secondary market:

Type of Bond	Government bond
Issue Date	June 15, 2005
Date of Purchase	June 15, 2009
Maturity Date	June 15, 2020
Coupon Rate	4.2% payable semiannually
Yield Rate	4.0% convertible semiannually

Assume that the redemption value of the bond is the same as the face value, which is \$100. Find the purchase price of the bond immediately after its 8th coupon payment on June 15, 2009.

- 2. Assume that, Jayani has decided to invest in bond X, an n-year bond with semiannual coupons and the following characteristics:
 - Par value is \$1,000.
 - The ratio of the semiannual coupon rate to the desired semi-annual yield rate, $\left(\frac{r}{i}\right)$ is 1.03125.
 - The present value of the redemption value is 381.50.

Given $\nu^n = 0.5889$, what is the price of bond X?

- 3. Nimali buys a 10-year \$1,000 par value 6% bond with semiannual coupons. The price assumes a nominal yield of 6%, compounded semiannually. As Nimali receives each coupon payment, she immediately puts the money into an account earning interest at an annual effective rate of i. At the end of 10 years, immediately after Nimali receives the final coupon payment and the redemption value of the bond, Nimali has earned an annual effective yield of 7% on her investment in the bond. Calculate i.
- **4.** A 10-year \$100 par value bond pays 8% coupons semiannually. The bond is priced at \$118.20 to yield an annual nominal rate of 6% convertible semiannually. Calculate the redemption value of the bond.
- 5. An investor borrows an amount at an annual effective interest rate of 5% and will repay all interest and principal in a lump sum at the end of 10 years. She uses the amount borrowed to purchase a \$1,000 par value 10-year bond with 8% semiannual coupons bought to yield 6% convertible semiannually. All coupon payments are reinvested at a nominal rate of 4% convertible semiannually. Calculate the net gain to the investor at the end of 10 years after the loan is repaid.

- 6. Danushka purchases a \$1,000 par value 10-year bond with 9% semiannual coupons for \$925. He is able to reinvest his coupon payments at a nominal rate of 7% convertible semiannually. Calculate his nominal annual yield rate convertible semiannually over the 10-year period. Calculate the number of years bond was held.
- 7. Maya purchased a 10-year par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually at a price of \$1,021.50. The bond can be called at par value \$1,100 on any coupon date starting at the end of year five. What is the minimum yield that Maya could receive, expressed as a nominal annual rate of interest convertible semiannually?
- 8. Consider a \$1,000 face value 15-year bond with coupon rate of 4% convertible semiannually. The bond is callable and the first call date is the date immediately after the 15^{th} coupon payment. Assume that the issuer will only call the bond at a date immediately after the n^{th} coupon ($15 \le n \le 30$) and the call price (i.e., redemption value) is

$$c = \begin{cases} 1,000, & \text{if } 15 \le n \le 20, \\ 1,000 + 10(n - 20), & \text{if } 20 < n \le 30. \end{cases}$$

Find the price of the bond if the investor wants to achieve a yield of at least 5% compounded semiannually.

- **9.** A premium bond is purchased to yield 5% convertible semiannually. The amount of premium amortized in the third payment is \$4.10. Find the amount of premium amortized in the 8th payment.
- 10. Sachin purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually at a price of \$1,722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The price guarantees that Sachin will receive a nominal annual rate of interest convertible semiannually of at least 6%. Calculate X.
- 11. A \$1,000 par bond with 8% semiannual coupons has payment dates of May 31 and November 30. The bond matures on November 30, 2010. On May 31, 2007 the coupon payment of 40 is paid. The bond is sold 70 days later on the settlement date of August 9. The bond is sold to yield 7.4% convertible semiannually. Find the price plus accrued interest, the accrued interest and the price.

Supplementary Examples

- 12. A 10-year \$1,000 par bond with 6% semiannual coupons is priced to yield 5.6% convertible semiannually. How much of the premium is amortized in the 8th period?
- 13. A five year \$1,000 bond has a 6% coupon rate convertible semiannually. It is callable in two years. An investor wishes to buy the bond to yield 5.5% convertible semiannually. Find the purchase price of the bond.

2 Continued.

- 14. Susantha purchased a 10-year par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually at a price of \$41,021.50. The bond can be called at par value X on any coupon date starting at the end of year five. The price guarantees that Susantha will receive a nominal annual rate of interest convertible semiannually of at least 6%. Calculate X.
- 15. A bond with par value of \$1,000 has payment dates of April 15 and October 15. The nominal coupon rate convertible semiannually is 7%. The bond matures on October 15, 2009. On April 15, 2007 a coupon payment of 35 was made. The bond is sold 80 days later on the settlement date of July 4, 2007 to yield 6% convertible semiannually. There are 183 days between the coupon payment dates of April 15, 2007 and October 15, 2007. Find the price-plus accrued, the accrued interest and the price.

3

Last Page