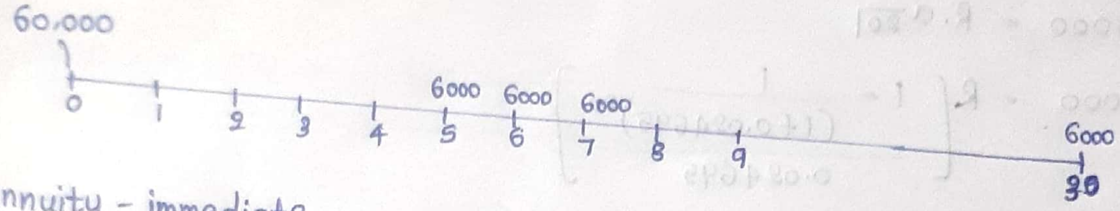
Introduction to Financial Mathematics - MAT 327β

Tutorial No. 03 - Answers.

(1)



Annuity - immediate

$$PV = R \cdot a_{\overline{n|}} V^5 \qquad R \kappa | a_{\overline{n|}}$$

$$= 6000\, a_{\overline{25|}} V^5$$

$$= 6000 \left[\frac{1 - \frac{1}{(1+0.06)^{25}}}{0.06} \right] \frac{1}{(1+0.06)^5}$$

$$= \$\ 57,314.8042$$

Annuity - due

$$PV = R \cdot \ddot{a}_{\overline{n|}} V^5$$

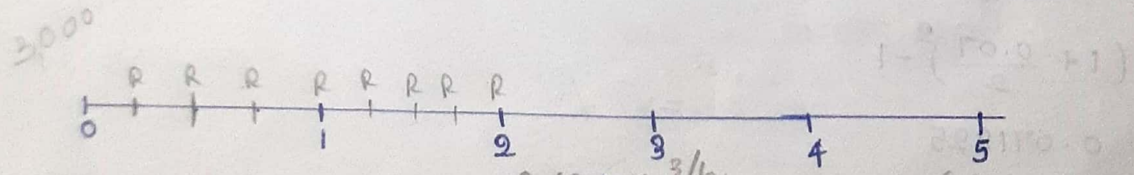
$$= 6000 \cdot \ddot{a}_{\overline{25|}} V^5$$

$$= 6000 \left[\frac{1 - \frac{1}{(1+0.06)^{25}}}{\left(\frac{0.06}{1.06}\right)} \right] \frac{1}{(1+0.06)^5}$$

$$= \$\ 60,753.569$$

The PV of the annuity-due is greater than the amount of debt. Therefore, the deal is feasible for Saman, when the Payment is an annuity-due.

(2)



$$\text{EAR Per quater} = \left(1 + \frac{0.10}{2}\right)^{1/2} - 1 = 3/6. \qquad \frac{i^{(2)}}{2} = 10\%$$

$$= 0.024695$$

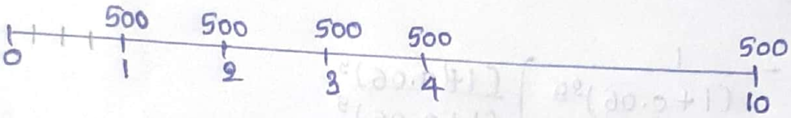
$$PV = R \cdot a_{\overline{n|}}$$

$$3000 = R \cdot a_{\overline{20|}}$$

$$3000 = R\left[\frac{1 - \frac{1}{(1+0.024695)^{20}}}{0.024695}\right]$$

$$R = \text{EUR } \underline{191.89}$$

(3)



(a) $$EAR = \left(1 + \frac{0.08}{4}\right)^4 - 1 \qquad i^{(4)} = 8\%$$

$$= 0.08243$$

$$PV = R \cdot a_{\overline{10|}}$$

$$PV = 500\left[\frac{1 - \frac{1}{(1+0.08243)^{10}}}{0.08243}\right]$$

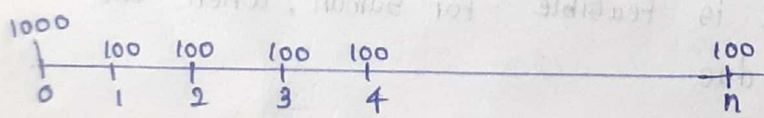
$$= \$\ \underline{3,318.58}$$

(b) $$FV = PV(1+i)^n$$

$$FV = 3318.58\,(1 + 0.08243)^{10}$$

$$= \$\ \underline{7,327.41}$$

(4)



$$EAR = \left(1 + \frac{0.07}{2}\right)^2 - 1 \qquad i^{(2)} = 7\%$$

$$= 0.071225$$

$$PV = R \cdot a_{\overline{n}|}$$

$$1000 = 100 \cdot a_{\overline{n}|}$$

$$10 = a_{\overline{n}|} = \left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

$$10 \times 0.071225 = 1 - \frac{1}{(1+0.071225)^n}$$

$$\frac{1}{1.071225^n} = 0.28775$$

$$1.071225^n = 3.475239$$

$$n \cdot \ln(1.071225) = \ln(3.475239)$$

$$n = \frac{\ln[3.475239]}{\ln(1.071225)} = 18.105$$

There are 18 regular Payments.

$$1000 = PV \text{ of regular Payments} + PV \text{ of smaller Payments}$$

$$1000 = 100 \cdot a_{\overline{18}|} + R \cdot v^{18}$$

$$1000 = 100\left[\frac{1 - \frac{1}{(1+0.071225)^{18}}}{0.071225}\right] + R \cdot \frac{1}{(1+0.071225)^{18}}$$

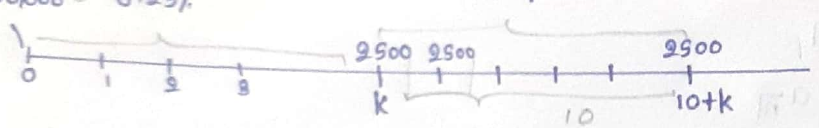
$$1000 \times 1.071225^{18} = \frac{100}{0.071225}\left[1.071225^{18} - 1\right] + R$$

$$R = 10.089$$

$$\text{Total Final Payment} = 100 + R$$
$$= 100 + 10.089$$
$$= \$\ 110.089$$

⑥

$PV = 50{,}000 \quad i^{(4)} = 8.25\%$ $\quad i^{(2)} = 4.8\%$ compounded semiannually



$PV = 2500 \cdot \ddot{a}_{\overline{120}|i_1} (1+i)^{-k}$

Effective interest rate per month (i_1) $= \left(1+\frac{0.048}{2}\right)^{1/6} - 1 = 0.003552$

$$50{,}000 = 2500 \cdot \left[\frac{1 - \frac{1}{(1+0.003552)^{120}}}{\left(\frac{0.003552}{1.003552}\right)}\right]\left(1+\frac{0.0825}{4}\right)^{-k}$$

$$20 = \left[\frac{1 - \frac{1}{(1+0.003552)^{120}}}{\left(\frac{0.003552}{1.003552}\right)}\right]\left(1+\frac{0.0825}{4}\right)^{-k}$$

$$\left(1+\frac{0.0825}{4}\right)^{-k} = 0.204268$$

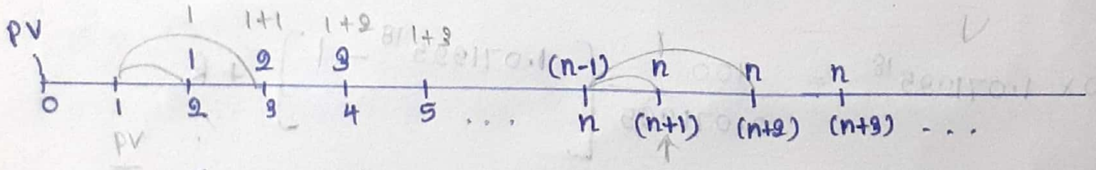
$$(-k)\ln\left(1+\frac{0.0825}{4}\right) = \ln(0.204268)$$

$k = 77.80103104$ quartely compounded

years $= \frac{77}{4} = 19.25 =$ 19 years and 3 months

$0.80103104 \times 91 = 72.8938 = $ 73 days

⑦



$$\cancel{PV = [v + 2v^2 + 3v^3 + 4v^4 + \dots + n v^n + n v^{(n+1)} + n v^{n+2} + \dots](1+i)}$$

$$\cancel{77.1 = v[1 + 2v + 3v^2 + 4v^3 + \dots + n v^{n-1} + n v^n + n v^{n+1} + \dots](1+i)}$$

$$PV = 1 \cdot v^2 + 2 \cdot v^3 + 3 \cdot v^4 + \dots + (n-1)v^{n-1} + n v^n + n/2$$

$$PV = v \cdot [v + 2v^2 + 3v^3 + 4v^4 + \dots + n \cdot v^n] + [nv + nv^2 + nv^3 + \dots] v^n$$

$v((IA)_{\overline{n}|})$

$$= v\left[\frac{\ddot{a}_{\overline{n}|} - nv^n}{i}\right] + nv\left[1 + v + v^2 + \dots\right]v^n$$

Increasing annuity differed one period

perpetuity of n dollars differed (n+1) years

$$= v\left[\frac{\ddot{a}_{\overline{n}|} - nv^n}{i}\right] + \frac{nv^{n+1}}{i}$$

77.1 =

$$PV = \frac{V.\ddot{a}_{\overline{n}|}}{i}$$

$$= \frac{a_{\overline{n}|}}{i}$$

$$77.1 = \frac{1}{i}\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

$$77.1 = \frac{(1+i)^n - 1}{(1+i)^n\, i^2}$$

$$(1.105)^n\, 77.1 \times (0.105)^2 = (1+0.105)^n - 1$$

$$(1.105)^n = 6.667889113$$

$$n.\ln(1.105) = \ln(6.667889113)$$

$$n = \underline{19}$$

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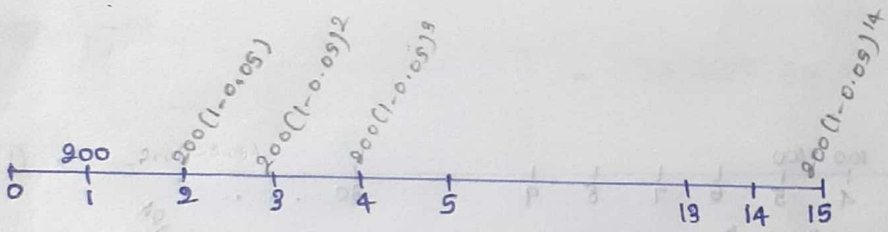
(a)



$$\cancel{FV = 200(1-0.05)^{14} + 200(1-0.05)^{13}(1+i) + 200(1-0.05)^{12}(1+i)^2 + \dots +}$$
$$\cancel{200(1-0.05)(1+i)^{13} + 200(1+i)^{14}}$$

$$FV = PV(1+i)^{15}$$

$$= 200\left[\frac{1 - \left(\frac{1+k}{1+i}\right)^{15}}{i-k}\right](1+i)^{15}$$

$k = -0.05$

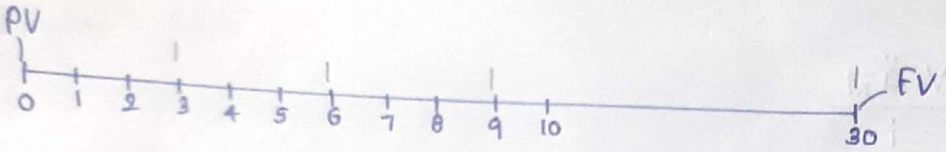
$i = 0.09$

$$= 200\left[\frac{1 - \left(\frac{1-0.05}{1+0.09}\right)^{15}}{0.09+0.05}\right](1+0.09)^{15}$$

$$= \frac{200}{0.14}\left[1.09^{15} - 0.95^{15}\right]$$

$$= \underline{4,541.702}$$

(b)



$$PV = v^3 + v^6 + v^9 + v^{12} + \dots + v^{30}$$

$$= v^3\left[1 + v^3 + v^6 + v^9 + \dots + v^{27}\right]$$

$$= v^3\left[\frac{1-(v^3)^{10}}{1-v^3}\right]$$

$$= \frac{v^3 \cdot a_{\overline{30}|}}{a_{\overline{3}|}} = \underline{\underline{\frac{a_{\overline{30}|}}{s_{\overline{3}|}}}}$$

$$FV = PV(1+i)^{30}$$

$$= \frac{a_{\overline{30}|}}{s_{\overline{3}|}} \cdot (1+i)^{30} = \underline{\underline{\frac{s_{\overline{30}|}}{s_{\overline{3}|}}}}$$

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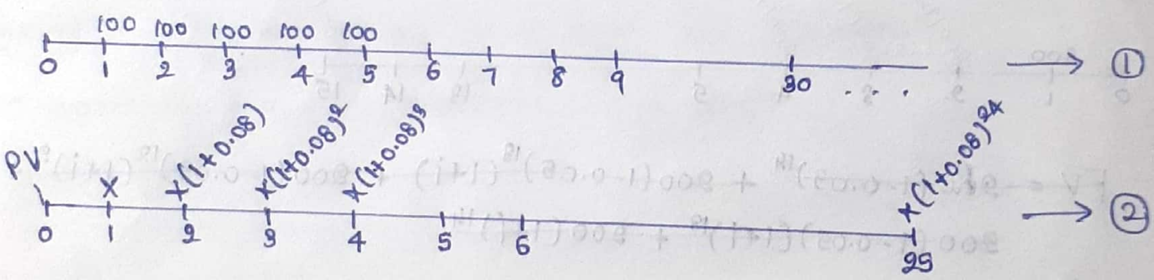


$$PV \text{ of the Perpetuity} = \frac{R}{i} = \frac{100}{0.08} = 1250$$

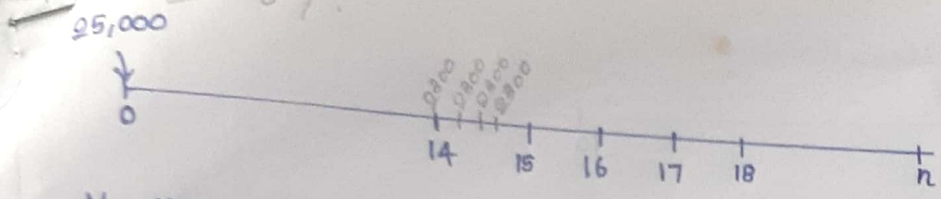
$$PV \text{ of the annuity} = Xv + X(1.08)v^2 + X(1.08)^2 \cdot v^3 + X(1.08)^3 v^4 + \dots + X(1.08)^{24} \cdot v^{25}$$

$$= X\left[\frac{1}{1.08} + \frac{(1.08)}{(1.08)^2} + \frac{(1.08)^2}{(1.08)^3} + \dots + \frac{(1.08)^{24}}{(1.08)^{25}}\right]$$

$$= X \cdot \frac{25}{(1.08)}$$

$$1250 = \frac{25X}{(1.08)}$$

$$X = \underline{54}$$



Monthly interest rate of the income annuity $= \left(1+\frac{0.0325}{2}\right)^{1/6}-1$

$= 0.00269$

PV of the annuity $= R \cdot \ddot{a}_{\overline{n|}}$

$= 2300\, \ddot{a}_{\overline{n|}}$

FV of the investment $= PV(1+i)^n$

$= 25{,}000(1+0.08)^{14}$

$= 73{,}429.84$

$$2300 \cdot \ddot{a}_{\overline{n|}} = 73{,}429.84$$

$$2300 \times \left[\frac{1-\frac{1}{(1+0.00269)^n}}{\left(\frac{0.00269}{1.00269}\right)}\right] = 73{,}429.84$$

$$1-\frac{1}{(1.00269)^n} = 0.08565$$

$$\frac{1}{1.00269^n} = 0.91435$$

$$1.00269^n = 1.093673101$$

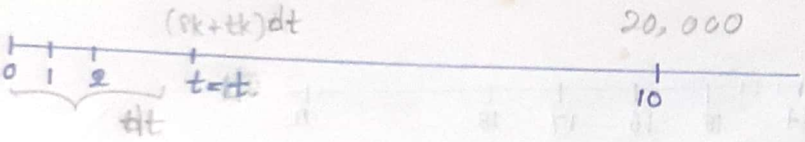
$$n \cdot \ln(1.00269) = \ln(1.093673101)$$

$$n = \frac{\ln(1.093673101)}{\ln(1.00269)}$$

$= 33.33$

$= 33$ Monthy Payments

5



Deposited amount at time t = $(8k + tk)\,dt$

Accumulation factor for growth from time t_1 to time 10.

$$e^{\int_t^{10} \delta_u\, du} = e^{\int_t^{10} \frac{1}{(8+u)}\, du} = e^{\ln(18) - \ln(8+t)} = \frac{18}{8+t}$$

Accumulation value for deposited amount = $k(8+t)\,dt \cdot \frac{18}{(8+t)}$

$$= 18k\,dt$$

Total accumulation value $= \int_0^{10} 18k\,dt$

$$= 18k\,[t]_0^{10} = 180k$$

$180k = 20{,}000$

$k = \underline{111.11}$