UNIVERSITY OF RUHUNA
DEPARTMENT OF MATHEMATICS

Bachelor of Science (General) Degree (Level III)
MATHEMATICS
MAT 324β: Mathematical Models in Ecology

Tutorial 07 **Semester II, 2023**

Submit the answers only for Question no (1) and (4) on or before: 30/10/2023

---

1. (a) Define,

(i) Harvesting

(ii) Sustainable Yield

(b) Consider the species with growth function

$$g = 0.8u - 0.0001u^2.$$

Assume that the harvest is given by the equation $h = pu$ in usual notation and each year's harvest is 4%.

(i) Find the intersection points of the graphs g and h.

(ii) Find the positive equilibrium population size and sustainable harvest.

---

2. Consider the species with growth function,

$$g = 0.3u - 0.00004u^2$$

The line $h = pu$ is going through the vertex of the parabola $(1000, 100)$. Assume that the harvest is given by the equation $h = pu$ in the usual notation.

(i) Find p.

(ii) Find the intersection points.

(iii) Find the positive equilibrium population size and sustainable harvest.

---

3. Suppose intrinsic growth rate and carrying capacity are $b = 0.8$, $l = 8000$.

(i) Find growth rate r.

(ii) Find growth rate function g.

(iii) Find minimum viable population and stable population when $h = 200$.

---

4. Suppose $b = 0.1$ and $l = 6000$. Assume the growth rate r is linearly decreasing.

(i) Find growth rate r.

(ii) Find growth rate function g.

(iii) Find the equilibrium population size h interms of p.

(iv) Find the maximum sustainable harvest.

---

5. Consider a population of fish where the growth rate function is given by;

$$g = 0.3u - 0.00005u^2.$$

Take the harvest per year as 8%. Also, assume that the harvest is given by the equation $h = pu$ in the usual notation.

(a) Find the positive equilibrium population size and sketch a graph using the functions given above.

(b) Does the species safe, if the harvest changes to 40%? Explain the reason for your answer.

---

6. Consider a population of fish in which the growth rate increases initially from the point $(0, 0)$ until the population size reaches 300, then decreases until the population size reaches the carrying capacity of 600. Take the intrinsic growth rate as $b = 0.3$. Assume that the harvest is given by the equation $h = pu$ in the usual notation. Assume one kilogram of the fish can be sold for $Rs.1000$ and the cost function is $C = 200p$.

(a) Find the growth function and sketch the graph of it.

(b) What positive fraction p should be harvested to achieve no profit?

(c) Find the sustainable harvest and equilibrium population size for the value p, obtained in part b) above.

****