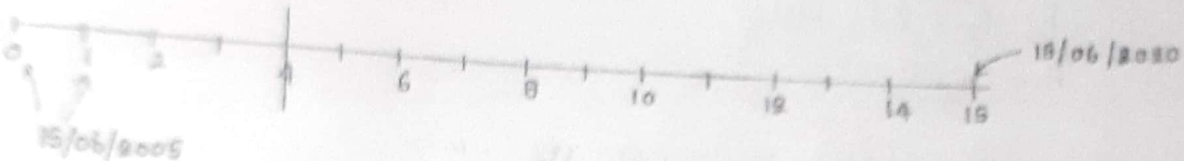
~~1213 - Financial Mathematics I~~

Tutorial No. 05 - Answers

①



$F = 100$, $C = 100$, $r = \frac{0.042}{2} = 0.021$, $i = \frac{0.04}{2} = 0.02$, $n = 22$

$$P = (Fr)a_{\overline{n|}} + Cv^n$$

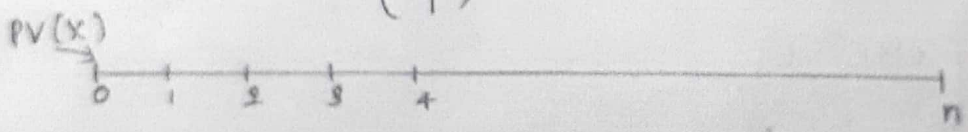
$$P = (100 \times 0.021)\, a_{\overline{22|}} + 100v^{22}$$

$$= (2.1)\left[\frac{1 - \frac{1}{(1+0.02)^{22}}}{0.02}\right] + 100 \times \frac{1}{(1+0.02)^{22}}$$

$$= \$\ \underline{101.77}$$

②

$F = \$1{,}000$ $\quad \left(\frac{r}{i}\right) = 1.03125$ $\quad C = 381.50$, $v^n = 0.5889$



The Present Value of the bond (X) = Present Value of the Coupons + Present Value of the redemption Value

$$= 1000r\, a_{\overline{2n|}} + 381.50$$

$$= 1000r\left[\frac{1 - v^{2n}}{i}\right] + 381.50$$

$$= 1000\left(\frac{r}{i}\right)\left[1 - (v^n)^2\right] + 381.50$$

$$= 1000 \times 1.03125\left[1 - (0.5889)^2\right] + 381.50$$

$$= 673.61 + 381.50$$

$$X = \$\ \underline{1{,}055.11}$$

(3) Since the bond is a Par bond priced at the same nominal rate, the Price of the bond is 1000.

$P = F + F(r-i)a_{\overline{n}|}$

$P = \$1000$

Therefore, the initial investment is 1,000.

Future amount of the investment $= PV(1+i)^n$

$= 1000(1+0.07)^{10}$

$= \$1{,}967.15$

1,967.15 = 1000 repayment of the face value of the bond + future amount of the reinvestment account

$1{,}967.15 = 1000 + FV_1$

$FV_1 = 967.15$

Coupon Payment $= 1000 \times 0.03 = \$30$

$FV_1 = R \cdot s_{\overline{n}|}$

$FV_1 = R \cdot \left[\frac{(1+i)^{20} - 1}{i} \right]$

$967.15 = 30 \left[\frac{(1+i)^{20} - 1}{i} \right]$

$\frac{967.15}{30} i = (1+i)^{20} - 1$

Sem-annual yield is 4.7596%

The annual effective yield is $(1.047596)^2 - 1 = 0.097458$

Pro

(4) $P = (Fr)a_{\overline{n}|} + Cv^n$

$t = 10$

$F = \$100$

$r^{(2)} = 8\%$

$i^{(2)} = 6\%$

$118.2 = (100 \times 0.04)a_{\overline{20}|} + Cv^{20}$

$$118.2 = 4\left[\frac{1 - \frac{1}{(1+0.03)^{20}}}{0.03}\right] + C \times \frac{1}{(1+0.03)^{20}}$$

$$118.2 = 59.51 + \frac{C}{(1.03)^{20}}$$

$$58.69 = \frac{C}{(1.03)^{20}}$$

$$C = \$106.0$$

(5) The purchase price of the bond

$i = 5\%$

$P = (Fr)a_{\overline{n}|} + Fv^n$

$P(PV) = (1000 \times 0.04)a_{\overline{20}|} + 1000.v^{20}$

$$= 40\left[\frac{1 - \frac{1}{(1+0.03)^{20}}}{0.03}\right] + \frac{1000}{(1+0.03)^{20}}$$

$PV = \$1,148.77$

Loan amount $= 1,148.77$

The investor will borrow 1148.77 at 5% and repay in 10 years the amount

Future value of the loan $= PV(1+i)^{10}$

$= 1,148.77(1+0.05)^{10}$

$= \$1,871.23$

Future value of all reinvested coupon payments $= R.S_{\overline{20}|}$

$$= 40\left[\frac{(1+0.02)^{20} - 1}{0.02}\right]$$

$= \$971.89$

At the end of 10 years,

redemption value of the bond + future value of all coupon payments $= 1000 + 971.89$

$= \$1,971.89$

After repayment of the loan, the investor will have a net gain of

$$1971.89 - 1871.23 = \$100.66$$

(6)

Future value of all coupon payments $= (1000 \times 0.045) s_{\overline{20|}}$

$$= 45\left[\frac{(1+0.035)^{20}-1}{0.035}\right]$$

$$= \$1,272.59$$

At the end of 10 years,

redemption value of the bond + future value of all coupon payments $= 1,000 + 1,272.59$

$= \$2,272.59$

original investment was \$925

His semiannual yield on the investment $= \left(\frac{2272.59}{925}\right)^{1/20} - 1$?

$$= 0.046$$

The nominal yield convertible semiannually is $2 \times 0.046 = 0.092$

$= 9.2\%$

(7) Semiannual coupon Payment = 1.100×0.02
= \$ 22

FV = \$ 1,100

PV = \$ 1,021.50

Since the Par value of 1100 is greater than the Price of 1,021.5, this is a discount bond. So, n = 20 Payments.

Price = PV of all coupon Payments + PV of redemption Value

$$1021.5 = 22\, a_{\overline{20}|} + 1100\, v^{20}$$

$$1021.5 = 22\left[\frac{1 - v^{20}}{i}\right] + 1100\, v^{20}$$

$$1021.5 = \frac{22}{i}\left[1 - \frac{1}{(1+i)^{20}}\right] + 1100 \times \frac{1}{(1+i)^{20}}$$

$$i^{(2)} = 2.456\%$$

The minimal nominal annual yield to May is 4.912%

(8) $r^{(2)} = 4\%$

F = \$ 1,000 , r = 0.02 i = 0.025

n	Redemption value (C)	Price
15	1000	938.09
16	1000	934.72
17	1000	931.44
18	1000	928.23
19	1000	925.11
20	1000	922.05

n	Redemption Value (C)	Price
21	1,010	925.03
22	1,020	927.79
23	1,030	930.34
24	1,040	932.69
25	1,050	934.85
26	1,060	936.82
27	1,070	938.62
28	1,080	940.25
29	1,090	941.71
30	1,100	943.02

5%

Thus, the minimum price is \$ 922.05. So, the price of 922.05 is an investor is willing to pay for a yield of at least 5% compounded semiannually.

(9)

$$FV = PV(1+i)^n$$

$$FV = 4.1(1+0.025)^5$$

$$= \$\ 4.64$$

?

(10) Since the semiannual yield rate of 3% is less than the semiannual coupon rate of 4%, this is a premium bond. Since the bond is callable in 15 years, it is priced as if it will be redeemed in 15 years.

$$P = (Fr)a_{\overline{n}|} + Cv^n$$

$$1,722.25 = (0.04X)a_{\overline{30}|} + Xv^{30}$$

$$1,722.25 = (0.04X)\left[\frac{1 - \frac{1}{(1+0.03)^{30}}}{0.03}\right] + \frac{X}{(1+0.03)^{30}}$$

$$1,722.25 = (0.04X) \times 19.6 + \frac{X}{(1.03)^{30}}$$

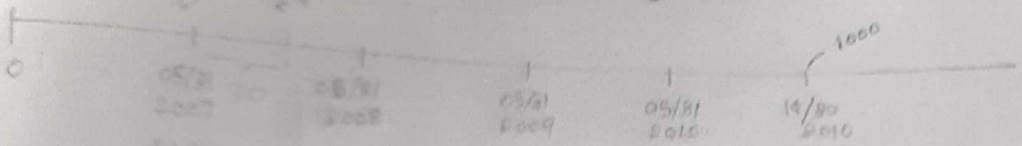
$$4,180.35 = (1.90)X + X$$

$$2.9X = 4,180.35$$

$$X = \$\ 1,440.39$$

(11)

$F = \$1{,}000$ $\quad r = 4\%$ $\quad i = \frac{7.4}{2}\%$, $n = 5$



$P = (Fr)a_{\overline{n}|} + Cv^n$

$P = (1000 \times 0.04)a_{\overline{5}|} + 1000v^5$

$= (40)\left[\frac{1 - \frac{1}{(1+0.037)^5}}{0.037}\right] + 1000 \times \frac{1}{(1+0.037)^5}$

$P_0 = \$1{,}013.47$

$t = \frac{70}{183}$ There are 183 days between the coupon payment dates of May 31 and November 30.

Price plus accrued interest $= P_0(1+i)^t$

$= 1{,}013.47(1+0.037)^{70/183}$

$= \$1{,}027.65$

Accrued interest $= t(Fr)$

$= \frac{70}{183} \times 40 = 15.3$

Price = Price plus accrued − Accrued interest

$= 1{,}027.65 - 15.3$

$= \$1{,}012.35$