MAT 327β - Introduction to Financial Mathematics

Tutorial No. 01 - Answers

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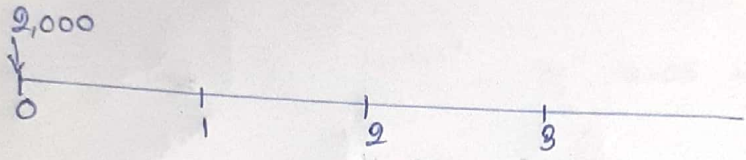

(a) Compound interest method :-

Interest charge for year 1 : $2000 \times \frac{8}{100} = \underline{\$160}$

Accumulated amount at the end of year 1 $= 2000 + 160$

$= \$\,2,160$

Interest charge for year 2 : $2160 \times \frac{8}{100} = \underline{\$\,172.8}$

Accumulated amount at the end of year 2 $= 2160 + 172.8$

$= \$\,2,332.8$

Accumulated amount at the end of year 3 $= 2,332.8 \times \frac{8}{100} + 2,332.8$

$= 186.624 + 2332.8$

$= \underline{\$\,2,519.424}$ or $2000(1+0.08)^3$

(b) Simple interest method :-

Interest charge for year 1 : $2000 \times \frac{8}{100} = \underline{\$\,160}$

Interest charge for year 2 : $2000 \times \frac{8}{100} = \underline{\$\,160}$

Accumulated amount at the end of year 3 $= 2000\,(1 + 3 \times 0.08)$

$= \underline{\$\,2,480}$

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②

$FV = \$\ 50{,}000 \qquad i = 8\%$

$$FV = PV(1+i)^t$$

$$PV = \frac{FV}{(1+i)^t}$$

$$= \frac{50{,}000}{(1+0.08)^5} = \underline{\$\ 34{,}029.16}$$

③

$FV = \$\ 40{,}000 \qquad i = 8\%$

$$PV = \frac{FV}{(1+i)^t}$$

$$= \frac{40{,}000}{(1+0.08)^2} = \underline{\$\ 34{,}293.55}$$

④

$PV = 10{,}000 \qquad FV = \$\ 17{,}910$

$$FV = PV(1+i)^t$$

$$(1+i)^n = \frac{FV}{PV}$$

$$i = \left(\frac{FV}{PV}\right)^{1/t} - 1$$

$$i = \left[\frac{17{,}910}{10{,}000}\right]^{1/10} - 1$$

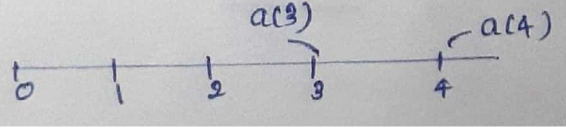
$$= 0.06 = \underline{\underline{6\%}}$$

⑤ (a)

Accumulated amount at time t $[a(t)]$ $= (1+i)^t$

$$a(4) = (1+0.05)^4 \qquad a(3) = (1+0.05)^3$$

$$= 1.05^4 \qquad = 1.05^3$$



$$i_4 = \frac{a(4) - a(3)}{a(3)}$$

$$= \frac{1.05^4 - 1.05^3}{1.05^3} = 0.05 = 5\%$$

Accumulated amount at time t = $(1+it)$
$[a(t)]$

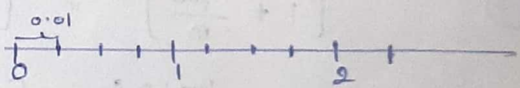
$a(4) = (1 + 0.05 \times 4)$
$= 1.2$

$a(3) = 1 + 0.05 \times 3$
$= 1.15$

$$i_4 = \frac{a(4) - a(3)}{a(3)}$$

$$= \frac{1.20 - 1.15}{1.15} = 0.0435$$
$$= 4.35\%$$

Num of quarters within 25 months = $\frac{25}{3}$



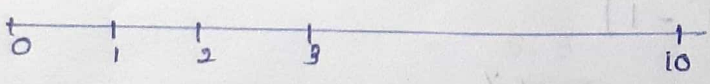
$$FV = PV\left(1+\frac{i^{(m)}}{m}\right)^{tm}$$

$$FV = 100\left[1 + \frac{0.04}{4}\right)^{25/3}$$

$$= 100\,(1+0.01)^{25/3} = \$\ 108.65$$

$$EAR = \left(1 + \frac{4\%}{4}\right)^{1/3} - 1$$

$$FV = 100\,(1+i_{EAR})^{25}$$
$$= 108.645$$



$$FV = PV\left(1+\frac{i^{(m)}}{m}\right)^{tm}$$

$$FV = 1000\left(1 + \frac{0.1}{4}\right)^{4 \times 10}$$

$$= 1000\,(1 + 0.025)^{40}$$

$$= \$\ 2{,}685.06$$

$$FV = PV\left(1+\frac{i^{(m)}}{m}\right)^{mt}$$

$$FV = 1000\left[1+\frac{0.1}{12}\right]^{12\times 10}$$

$$= \$\ 2{,}707.04$$

$$FV = PV\left(1+\frac{i^{m}}{m}\right)^{mt}$$

$$FV = 1000\left[1+\frac{0.1}{365}\right]^{365\times 10}$$

$$= \$\ 2{,}717.91$$

$$FV = PV\cdot \exp\left(i^{(m)}t\right)$$

$$FV = 1000\cdot \exp(0.1\times 10)$$

$$= \$\ 2{,}718.28$$

$$\left[1+\frac{i^{(m)}}{m}\right]^{m} = \left[1-\frac{d^{(p)}}{p}\right]^{-p}$$

$$\left[1+\frac{i^{(4)}}{4}\right]^{4} = \left[1-\frac{0.06}{12}\right]^{-12}$$

$$1+\frac{i^{(4)}}{4} = \left[1-\frac{0.06}{12}\right]^{-3}$$

$$1+\frac{i^{4}}{4} = (0.995)^{-3}$$

$$i^{(4)} = 4\left[(0.995)^{-3}-1\right]$$

$$= 0.0606 = 6.06\%$$

For the simple interest method,

The force of interest of an accumulation function is defined by $\delta(t) = \dfrac{d \cdot \ln(1+i)^{t}}{dt}$

Since $a(t) = (1+i)^t$

$$\delta(t) = \frac{d \ln a(t)}{dt}$$

$$= \frac{a'(t)}{a(t)}$$

$a(t) = a(0)(1+it)$

$a'(t) = a(0)i$

$$= \frac{a(0)i}{a(0)(1+it)}$$

$$\delta(t) = \frac{i}{1+it}$$

$$\delta(t) = \frac{0.05}{1+0.05t}$$

For the compound interest method,

$$\delta(t) = \ln(1+i)$$

$$\delta(t) = \ln(1+0.04)$$

Since, the two funds have the same force of interest,

$$\frac{0.05}{1+0.05t} = \ln(1+0.04)$$

$$\frac{0.05}{\ln(1.04)} = 1+0.05t$$

$$0.05t = \frac{0.05}{\ln(1.04)} - 1$$

$$t = \frac{1}{0.05}\left[\frac{0.05}{\ln(1.04)} - 1\right]$$

$$= 5.5$$

After this time, compound interest fund has a higher force of interest.

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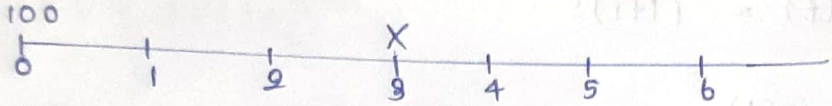


Accumulated fund amount at time 3 [A(3)]

$$= 100\, e^{\int_0^3 \delta_t \cdot dt} + X$$

$$= 100 \cdot \exp\left[\int_0^3 \frac{t^2}{100} \cdot dt\right] + X$$

$$= 100 \cdot \exp\left[\frac{1}{300}\left(t^3\right)_0^3\right] + X$$

$$= 100\left[\cdot \exp\left[\left(\frac{27}{300}\right) - 0\right]\right] + X$$

$$= 100 \cdot \exp\left(\frac{27}{300}\right) + X$$

$$= 109.42 + X$$

At time 6 the account grows to

$$A(6) = (109.42 + X) \exp \int_3^6 \delta_t \cdot dt$$

$$= (109.42 + X) \exp\left[\int_3^6 \frac{t^2}{100} dt\right]$$

$$= (109.42 + X) \cdot \exp\left[\frac{1}{300}\left(t^3\right)_3^6\right]$$

$$= (109.42 + X) \cdot \exp\left[\frac{1}{300}(216 - 27)\right]$$

$$= (109.42 + X) \exp\left(\frac{189}{300}\right)$$

$$= (109.42 + X)\, 1.8776$$

The amount of interest earned from time three to time six

$$= A(6) - A(3) = X$$

$$(109.42 + X) 1.8776 - (109.42 + X) = X$$

$$96.03 + 0.8776X = X$$

$$0.1224\, X = 96.03$$

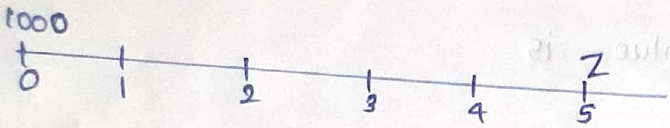
$$X = \frac{96.03}{0.1224}$$

$$= \$\ 784.56$$

Achini's Account :-

$$\delta(t) = \ln(1+i)$$
$$= \ln\left(1+\frac{0.1}{2}\right)$$

Kamal's Account :-



$$\delta(t) = \frac{i}{1+it}$$

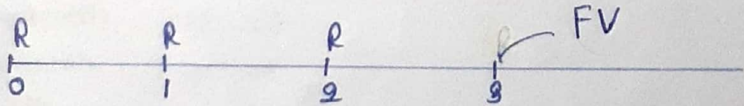
At the end of five years, the forces of interest on the two accounts are equal,

$$\frac{i}{1+5i} = \ln\left(1+\frac{0.1}{2}\right)$$

$$\frac{i}{1+5i} = 0.0488$$
$$i = 0.0488(1+5i)$$
$$i(1-0.244) = 0.0488$$
$$0.756i = 0.0488$$
$$i = 0.065$$

$$A(5) = 1000 \cdot e^{\int_0^5 \delta_t \cdot dt}$$

$$Z = 1000 \cdot e^{\int_0^5 \left[\frac{i}{1+it}\right] dt}$$
$$= 1000 \cdot \exp\left[0.065 \int_0^5 \left(\frac{1}{1+it}\right) dt\right]$$
$$= 1000 \cdot \exp\left[0.065 \left[\frac{\ln(1+0.065t)}{0.065}\right]_0^5\right]$$
$$= 1000 \cdot \exp\left[\ln(1.325) - \ln(1)\right]$$
$$= 1000 \times 1.325 = \underline{\$\ 1{,}325}$$



The equation of value is

$$FV = \sum_{j=0}^{n} C_j (1+i)^{n-j}$$

$$10{,}000 = R(1+0.05)^3 + R(1+0.05)^2 + R(1+0.05)$$
$$10{,}000 = R\left[1.05^3 + 1.05^2 + 1.05\right]$$

$$10{,}000 = R \times 3.31$$

$$R = \$\ \underline{3{,}021.15}$$

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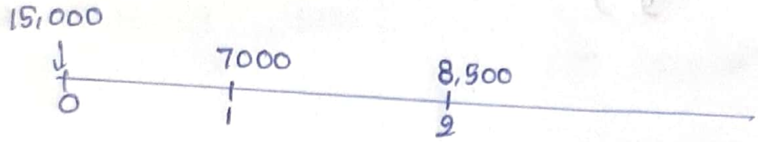

The equation of value is

$$PV = \sum_{j=0}^{n} c_j v^j$$

$$15{,}000 = \frac{7000}{(1+i)} + \frac{8500}{(1+i)^2}$$

$$150(1+i)^2 = 70(1+i) + 85$$

$$30(1+i)^2 - 14(1+i) - 17 = 0$$

$$(1+i) = \frac{14 \pm \sqrt{14^2 - 4 \times 30\,(-17)}}{60}$$

$$= \frac{14 \pm \sqrt{196 + 2040}}{60}$$

$$= \frac{14 \pm \sqrt{2236}}{60}$$

$$1+i = 1.02144$$

$$i = 0.02144 = \underline{2.144\%}$$