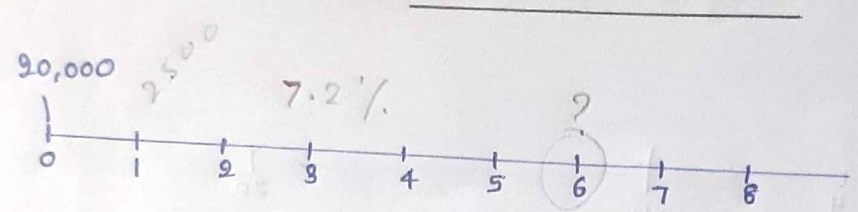
MAT 327B - Introduction to Financial Mathematics

Tutorial NO. 04 - Answers

(1)



Principal amount for each year = \$ 2,500

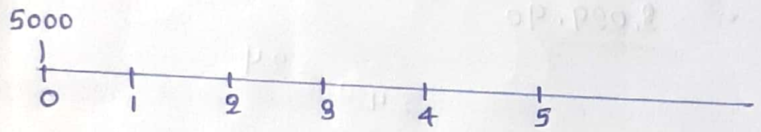
outstanding balance at the end of 5th year = 20,000 − 2500×5

= \$ 7,500

Interest due in the 6th payment = 7,500 × 0.072

= \$ 540

(2)



$$5000 = R.a_{\overline{5}|}$$

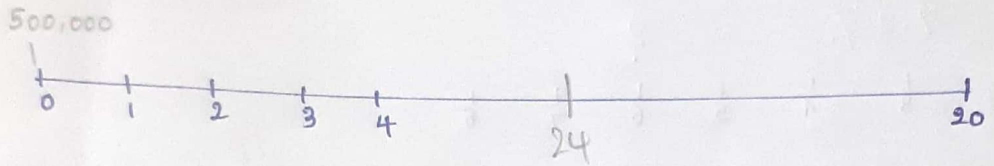
$$5000 = R.\left[\frac{1 - \frac{1}{(1+0.09)^5}}{0.09}\right]$$

R = EUR 1,285.46

Year	Begining Balance	Payment	Interest Paid	Principal Paid	Ending Balance
0					5000.00
1	5000.00	1,285.46	450	835.46	4,164.54
2	4,164.54	1,285.46	374.81	910.65	3,253.89
3	3,253.89	1,285.46	292.85	992.61	2,261.28
4	2,261.28	1,285.46	203.52	1,081.94	1,179.34
5	1,179.34	1,285.46	106.14	1,179.32	0
Total		6,427.30	1,427.32	5000	

Since final balance is zero, Jane is pay off the loan.

(3)



Monthly installment for bank A $= R$ $\quad i = 4\%$

$$500{,}000 = R . a_{\overline{240}|\frac{4\%}{12}}$$

$$500{,}000 = R\left[\frac{1-\frac{1}{(1+\frac{0.04}{12})^{240}}}{(\frac{0.04}{12})}\right]$$

$$R = 3{,}029.90$$

$240 - 24$

The outstanding balance after Paying the 24th installment $= R . a_{\overline{216}|}$

$$= 3{,}029.90\left[\frac{1-\frac{1}{(1+\frac{0.04}{12})^{216}}}{(\frac{0.04}{12})}\right]$$

$$= \$\ 465{,}996.73$$

If the man re-finances with bank B, he needs to borrow

$465{,}996.73\ (1+0.015) = 472{,}986.681$ → Penalty amount

Then, the monthly installment is

$$472{,}986.681 = R_1\ a_{\overline{216}|}$$

$$472{,}986.681 = R_1\left[\frac{1-\frac{1}{(1+\frac{0.035}{12})^{216}}}{(\frac{0.035}{12})}\right]$$

$R_1 = 2,954.57$

This amount is less the installment of \$ 3,029.90 for the bank A. Therefore, he should re-finance.

(4)

$PRin_t = R.v^{n-t+1}$

$PRin_3 = R.v^{n-3+1} = R.v^{n-2}$

$PRin_7 = R.v^{n-7+1} = R.v^{n-6}$

$$PRin_7 = \frac{PRin_3}{v^{n-2}}.v^{n-6} = \frac{845.25}{v^6}.v^{2}$$

$= 845.25(1+0.063)^4$

$= 1,079.24$

$L = 30,000$

$i = 8\%$

$t = 30$ years

$R = 1000$

(5) (a) outstanding balance at the end of the 10th year $= 30,000 - 10 \times 1000$
$=$ \$ 20,000

installment loan

The interest due in the 11th Payment $= 20,000 \times 0.08$
$=$ \$ 1,600

(b) The actual level Payment $= 1600 + 1000$
$=$ \$ 2,600

(6)

	200	200	200	200	200	200	200	200
0	1	2	3	4	5	6	7	8

In the amortization schedule, outstanding balance at the end of the 3rd year $= 200 \cdot a_{\overline{5}|}$

$$= 200\left[\frac{1 - \frac{1}{(1+0.05)^5}}{0.05}\right]$$

$=$ \$ 865.90

Interest component of the 4th Payment $= 865.90 \times 0.05$

$= \$\ 43.295$

For the sinking fund method,

Loan amount $= 200 \cdot a_{\overline{8|}}$

$$= 200 \cdot \left[\frac{1 - \frac{1}{(1+0.05)^8}}{0.05} \right]$$

$= \$\ 1,292.64$

Sinking fund Payment $= \dfrac{1,292.64}{s_{\overline{8|}}}$ Explain bit

$$= \frac{1,292.64}{\left[\frac{(1+0.05)^8 - 1}{0.05}\right]} = \$\ 135.37$$

Sinking fund balance at the end of 4th year $= 135.37\ s_{\overline{4|}}$

$$= 135.37 \left[\frac{(1+0.05)^4 - 1}{0.05} \right]$$

$= \$\ 583.46$

The interest credited to the sinking fund at the end of year 5 $= 583.46 \times 0.05$

$= \$\ 29.17$

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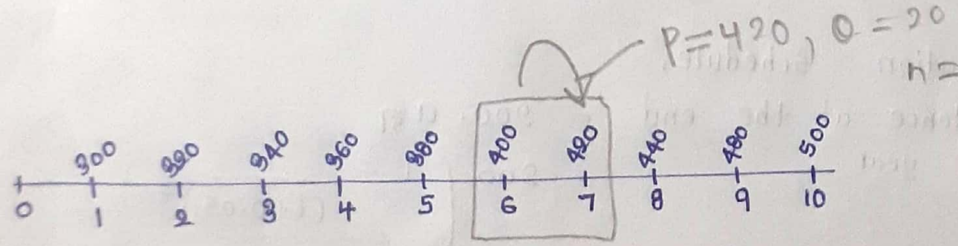


When using the retrospective method,

The loan balance after the 6th Payment $= Pa_{\overline{n}|} + Q\left[\dfrac{a_{\overline{n}|} - nv^n}{i}\right]$

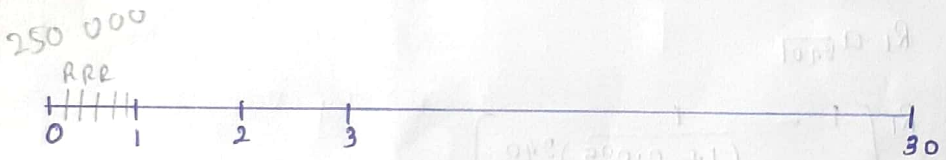
$$= 420 . a_{\overline{4}|} + 20\left[\frac{a_{\overline{4}|} - 4v^4}{0.065}\right]$$

$$= 420 . \left[\frac{1 - \frac{1}{(1+0.065)^4}}{0.065}\right] + 20\left[\frac{\frac{1-v^4}{i} - 4v^4}{0.065}\right]$$

$$= \frac{420}{0.065}\left[1 - \frac{1}{(1.065)^4}\right] + \frac{20}{0.065}$$

$$\left[\left(\frac{1 - \frac{1}{(1.065)^4}}{0.065}\right) - 4 \times \frac{1}{(1.065)^4}\right]$$

$$= \underline{1,536.22}$$



$$250,000 = R . a_{\overline{30\times 12}|}$$

$z^{(12)} =. 6.3\ \%$

$$250,000 = R\left[\frac{1 - \frac{1}{\left(1+\frac{0.063}{12}\right)^{360}}}{\left(\frac{0.063}{12}\right)}\right]$$

0.529

$$R = \$\ 1,547.43$$

242,292.57

The actual loan amount $= 250,000 - 250,000 \times \dfrac{2.5}{100}$

$$= 250,000 - 6,250$$

$$= \$\ 243,750$$

$$PV = R . a_{\overline{360}|}$$

$$243,750 = 1,547.43\left[\frac{1 - \frac{1}{(1+i)^{360}}}{i}\right]$$

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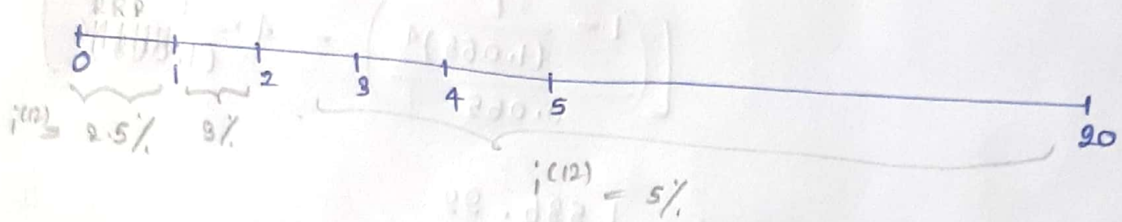
(a)



First year installment = R_1

$$PV = R_1 \cdot a_{\overline{12 \times 20}|}$$

$$300{,}000 = R_1 \, a_{\overline{240}|}$$

$$300{,}000 = R_1 \left[\frac{1 - \dfrac{1}{\left(1 + \frac{0.025}{12}\right)^{240}}}{\left(\frac{0.025}{12}\right)} \right]$$

$$R_1 = \$\ \underline{\underline{1{,}589.71}}$$

Retrospective method

The loan balance after 12 Payments $= R_1 \cdot a_{\overline{228}|\frac{0.025}{12}}$

240 − 12

$$= 1{,}589.71 \left[\frac{1 - \dfrac{1}{\left(1 + \frac{0.025}{12}\right)^{228}}}{\left(\frac{0.025}{12}\right)} \right]$$

$$= \$\ 288{,}290.16$$

Second year installment $= R_2$

$$288,290.16 = R_2 \cdot a_{\overline{228|}\frac{0.03}{12}}$$

$$288,290.16 = R_2 \left[\frac{1 - \frac{1}{\left(1+\frac{0.03}{12}\right)^{228}}}{\left(\frac{0.03}{12}\right)} \right]$$

$$R_2 = \$\,1,660.38$$

The loan balance after 24 Payments $= 1,660.38 \cdot a_{\overline{216|}\frac{0.03}{12}}$

$240 - 24$

$$= 1660.38 \left[\frac{1 - \frac{1}{\left(1+\frac{0.03}{12}\right)^{216}}}{\left(\frac{0.03}{12}\right)} \right]$$

$$= \$\,276,857.68$$

Third year installment $= R_3$

$$276,857.68 = R_3\, a_{\overline{216|}\frac{0.05}{12}}$$

$$276,857.68 = R_3 \left[\frac{1 - \frac{1}{\left(1+\frac{0.05}{12}\right)^{216}}}{\left(\frac{0.03}{12}\right)} \right]$$

$$R_3 = \$\,1,946.40$$

(b) The total amount of Payments in the first year $= 1589.71 \times 12$

$= \$\,19,076.52$

Principal amount in the first year $= 300,000 - 288,290.16$

$= \$\,11,709.84$

The interest Payment in the first year $= 19,076.52 - 11,709.84$

$= \$\ 7,366.68$

The total amount of Payments in the 2nd year $= 1660.38 \times 12$

$= \$\ 19,924.56$

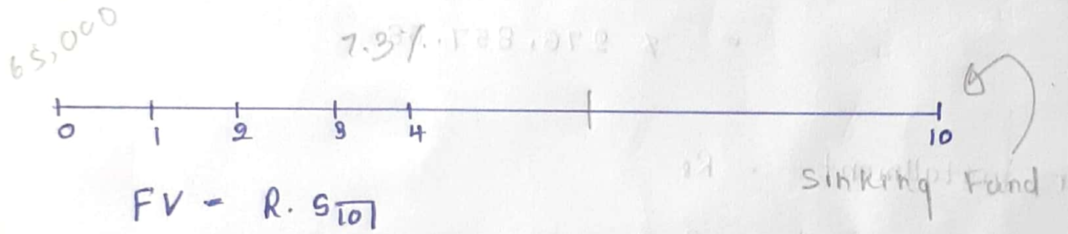
Principal amount in the 2nd year $= 288,290.16 - 276,857.68$

$= \$\ 11,432.48$

The interest Payment in the second year $= 19,924.56 - 11,432.48$

$= \$\ 8,492.08$

(10)



$$FV = R \cdot s_{\overline{10|}}$$

$$65{,}000 = R\left[\frac{(1+0.048)^{10}-1}{0.048}\right]$$

$$R = \$\ 5,216.23$$

Balance at time 5 $= 5216.23\ s_{\overline{5|}}$

$$= 5216.23\left[\frac{(1+0.048)^{5}-1}{0.048}\right]$$

$$= \$\ 28,708.03$$