

NCERT DISCRETE 10.5.4 Q1

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Question: Which term of the AP : 121, 117, 113, ..., is its first negative term?

consider the region of convergence (ROC) for which $|z| > 1$.

Solution: Let's denote this sequence as $x[n]$. Then $x[n]$ can be represented as:

$$x[n] = 121 - 4(n - 1) \quad (1)$$

$$X(z) = \frac{z - 1}{121z - 121 - 4z^2} \quad (11)$$

$$x[n] = 121 - 4(n - 1) < 0 \quad (12)$$

$$125 - 4n < 0 \quad (13)$$

$$4n > 125 \quad (14)$$

$$n > \frac{125}{4} \quad (15)$$

To find the z-transform of this sequence, we'll apply the definition of the z-transform:

Since n must be an integer, the first negative term in the sequence occurs at $n = 32$.

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n} \quad (2)$$

$$= \sum_{n=0}^{\infty} (121 - 4(n - 1))z^{-n} \quad (3)$$

$$= \sum_{n=0}^{\infty} (121z^{-n} - 4z^{-n+1}) \quad (4)$$

$$= \sum_{n=0}^{\infty} 121z^{-n} - \sum_{n=0}^{\infty} 4z^{-n+1} \quad (5)$$

$$= 121 \sum_{n=0}^{\infty} z^{-n} - 4z \sum_{n=0}^{\infty} z^{-n} \quad (6)$$

Applying the formula for the sum of an infinite geometric series, we get:

$$X(z) = 121 \left(\frac{1}{1 - z^{-1}} \right) - 4z \left(\frac{1}{1 - z^{-1}} \right) \quad (7)$$

$$= 121 \left(\frac{z}{z - 1} \right) - 4z \left(\frac{z}{z - 1} \right) \quad (8)$$

$$= \frac{121z - 121}{z - 1} - \frac{4z^2}{z - 1} \quad (9)$$

$$= \frac{121z - 121 - 4z^2}{z - 1} \quad (10)$$

The pole is at $z = 1$.