NCERT ANALOG - 11.15 - Q20

EE23BTECH11214
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Question: A travelling harmonic wave on a string is described by

$$\text{y(x, t)} = 7.5 \sin (0.0050\text{x} + 12\text{t} + \pi/4)$$

(a)what are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s ? Is this velocity equal to the velocity of wave propagation?
(b)Locate the points of the string which have the same transverse displacements and velocity as the x = 1 cm point at t = 2 s, 5 s and 11 s.
solution:

(a) Displacement and Velocity at x=1cm, t=1s

Displacement Substituting x = 0.01 m (1 cm) and t = 1 s in the given equation,
we get:

$$y(0.01, 1) = 7.5 \sin(0.0050 \times 0.01 + 12 \times 1 + \pi/4) \approx 7.46\,\text{cm} \tag{1}$$

Velocity of Oscillation

we have:

$$v = \frac{\partial y}{\partial t} = 90 \cos(0.0050x + 12t + \pi/4) \tag{2}$$

Substituting x = 0.01 m and t = 1 s,

$$v(0.01, 1) = 90 \cos(0.0050 \cdot 0.01 + 12 \cdot 1 + \pi/4) \approx -54.03\,\text{cm/s} \tag{3}$$

Velocity of Wave Propagation

The wave's velocity is given by:

$$v_{\text{wave}} = \frac{\omega}{k} = \frac{12}{0.0050} = 2400\,\text{cm/s} \tag{4}$$

Therefore velocity of oscillation is not equal to the velocity of wave propagation

(b) Points with Same Displacement and Velocity

To have the same displacement and velocity, the argument of the sine and cosine functions must remain constant.
Therefore, we need:

$$0.0050x + 12t + \pi/4 = 0.0050 \cdot 0.01 + 12 \cdot 1 + \pi/4 \tag{5}$$

Solving for x at t = 2 s, 5 s, and 11 s, we get:

t = 2 s: x = 0.015 m (1.5 cm)
t = 5 s: x = 0.065 m (6.5 cm)
t = 11 s: x = 0.115 m (11.5 cm)

These points on the string will have the same displacement and velocity as the point x = 1 cm at t = 1 s.