NCERT DISCRETE 11.9.5 Q9

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Question:
The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of G.P.

Solution:
G.P. in terms of its z-transform:

Symbol	Description	Value
$x[n]$	General term	ar^n
a	First term	1
r	Common ratio	-
ar^2	third term	-
ar^5	Fifth term	-
$x[2]+x[5]$	Sum of 3rd and 5th terms	90

TABLE I
GIVEN PARAMETERS LIST

$$X(z) = 1 + arz^{-1} + (ar)^2 z^{-2} + (ar)^3 z^{-3} + \dots \quad (1)$$
$$= 1 + arz^{-1} + (ar)^2 z^{-2} + (ar)^3 z^{-3} + \dots \quad (2)$$

$$X(z) = \frac{1}{1 - arz^{-1}} \quad (3)$$

The z-transform of the third term is:

$$X_3(z) = (ar)^2 z^{-2} \quad (4)$$

The z-transform of the fifth term is:

$$X_5(z) = (r)^4 z^{-4} \quad (5)$$
$$X_3(z) + X_5(z) = (ar)^2 z^{-2} + (ar)^4 z^{-4} \quad (6)$$

Inverse z-transform:

$$x[n] = \mathcal{Z}^{-1}\{X_3(z) + X_5(z)\} \quad (7)$$
$$x[n] = \mathcal{Z}^{-1}\{(ar)^2 z^{-2} + (ar)^4 z^{-4}\} \quad (8)$$
$$x[n] = \mathcal{Z}^{-1}\{(ar)^2 z^{-2}\} + \mathcal{Z}^{-1}\{(ar)^4 z^{-4}\} \quad (9)$$

the inverse z-transform of z^{-k} is $u[k]$, where $u[k]$ is the unit step function. So, Therefore,

$$x[n] = a^2 r^2 u[n-2] + a^4 r^4 u[n-4] \quad (10)$$
$$x[3] + x[5] = a^2 r^2 u[3-2] + a^4 r^4 u[5-4] \quad (11)$$
$$90 = a^2 r^2 + a^4 r^4 \quad (12)$$
$$r^4 + r^2 - 90 = 0 \quad (13)$$
$$r^2 = x \quad (14)$$
$$x^2 + x - 90 = 0 \quad (15)$$
$$x = \frac{-1 \pm 19}{2} \quad (16)$$
$$x = 9 \quad (17)$$
$$r^2 = 9 \quad (18)$$
$$r = \pm 3 \quad (19)$$