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Thresholding based on Fisher linear discriminant

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Abstract

In this paper, a new thresholding objective function is formulated for segmenting small object based on histogram projection by Fisher discrimination. The proposed method determines the optimal threshold is selected by the Fisher discriminant criterion; namely, by maximizing the measure of separability of the resultant classes in gray levels. The method was compared with several classic thresholding methods on a variety of small medicals images, and the experimental results show the effectiveness of the method.

Keywords: Segmentation; Thresholding ; Histogram; Fisher discrimination.

1. Introduction

Image segmentation is one of the most important and fundamental tasks in image processing and techniques based on image thresholding are typically simple and computationally efficient [1]. The popular technique in image segmentation is threshold segmentation (thresholding) [2], which is computationally simpler than other existing algorithms, such as boundary detection [3] or region dependent techniques [4, 5]. Its aim is to find an appropriate threshold for separating the object of interest from the background. The output of a thresholding process is a binary image where all pixels with gray levels higher than the determined threshold are classified as object and the rest of pixels are assigned to background, or vice versa. This technique can serve a variety of applications, such as biomedical image analysis, handwritten character identification, automatic target recognition and change detection see for example: [6, 7, 8].

Threshold methods can be classified into parametric and nonparametric approaches. The parametric approach assumes gray level distribution of each class obey a given distribution, usually a normal distribution, and calculate a threshold by estimating the parameters of the distribution using the given histogram. For example, Lee and Yang [9] estimated the parameters of normal distribution corresponding to the object and background from a truncated normal distribution. The optimal threshold is then determined by the Bayes decision rule. Kittler and Illingworth [10] suggested a minimum error thresholding method. A modification to Kittler and Illingworth's minimum error thresholding method presented by [11]. Tsai's parametric method [12] calculates the optimal threshold on the condition that the threshold image has the same moment as the original one. A parametric and global method [13], assumes that the object and background classes follow a generalized Gaussian distribution, and finds the threshold from the estimation of the parameters of the two classes by the expectation maximization algorithm. This typically leads to a nonlinear estimation problem of expensive computation. Recently in [14], Elguebaly et al. proposed a highly efficient unsupervised segmentation and spot detection of cDNA microarray images, based on generalized Gaussian mixture models.

The nonparametric approaches usually determines the optimal threshold by optimizing certain (objective function). For example, Pun's method [15], propose a threshold criterion based on maxim entropy. Kapur et al. [16] found some flaws in Pun's derivations and presented a corrected entropy-based method. Cheng et al. [17] selected the threshold criteria based on maximizing fuzzy entropy. Otsu's method [18] chooses a threshold criteria by maximizing the between-class variance of gray levels in the object and background portions. Otsu's method is one of the better threshold selection approaches for general real world images with regard to uniformity and shape measures [19]. Hou et al. [20] presented an approach based on minimizing class variance, where the term of class variance follows the precise definition of variance for a class. Interested readers may refer to [21] for a survey over threshold methods. Among these methods, statistical threshold approaches are popular for the simplicity and efficiency. The methods presented in [18] and [20] are two typical examples. The former tends to dichotomize an image in to object and background of similar sizes. Unfortunately, both methods subject to a limitation of being unable to obtain satisfactory results when segmenting some images of small object and when the distribution of gray intensities of the background is complex. In attempt to overcome the weakness of the statistical criteria, a novel statistical criterion for threshold selection is proposed in [22]. It takes class variance sum and variance discrepancy into account at the same time. Qiao et al. [23] suggest a thresholding criterion based on the convex combination of within-class variance and intensity contrast between the object and background. It has a critical parameter that is difficult to set.

Alternative we propose a simple and easy implementation method with new thresholding objective function based on histogram projection by Fisher discriminantant. This method is taken intensity contrast and within class variance into account at the same time. The threshold is obtained by maximization the proposed criteria.

Tests against real and synthetic images show that small objects can be extracted successfully irrespective of the complexity of background and difference in class sizes. The rest of this paper is organized as follows: Section 2 the proposed threshold algorithm. Experimental results presented in Section 3. Conclusion appear in Section 4.

2. The proposed Threshold Algorithm

In this section, Fisher linear discriminant method discussed, some statistical thresholding methods are reviewed. A new thresholding objective function and the corresponding algorithm are then proposed.

2.1 Fisher Linear discriminant

Fisher linear discriminant seeks directions efficient for discrimination by yielding the maximum ratio of between-class scatter to within-class scatter. For each image Fisher linear discriminant finds a projection orientation of intensity by which two classes (object and background) are well separated. For any image, there is a set H including N intensity.

$$H = \{C_1, C_2\} = \{h_1, h_2, ..., h_N\}, \qquad n_1 + n_2 = N$$

where n_1 and n_2 are cardinality of subset $C_1$and subset C_2, respectively. If we form a linear combination of the components of h_i, we obtain the scalar dot product

$$x_i = v^t h_i$$

and a corresponding set of $X = \{x_1, x_2, ..., x_N\}$ of N projected points divided into the subsets X_1 and X_2, where v denotes an n-dimensional column vector. Geometrically, if $||v||= 1$, each X_i is the projection of the corresponding h_i onto a line in the direction of v. How do we judge a projection vector v is good? In fact, the total variance of the projected classes can be characterized by

$$J(v) = \frac{v^t S_B v}{v^t S_W v} \tag{1}$$

whereS_W is called the within-class scatter defined by
$S_W = s_1^2 + s_2^2$, where s_i^2 is the scatter of class i, i=1,2 and S_B is called the between-class scatter defined by

$$S_B = (m_1 - m_2)(m_1 - m_2)^t.$$

where m_i is the mean of all elements of set X_i.
This criterion is called Fisher linear projection criterion. The criterion function $J(v)$ in (1) can be written [24, 25]

$$J(v) = \frac{|m_1 - m_2|^2}{s_1^2 + s_2^2}. \tag{2}$$

The unitary vector v that maximizes $J(v)$ is called the Fisher optimal projection. The projection maximizes the distance between the means of the two classes while minimizing the variance within each class. The optimal projection v_{opt} is chosen when the criterion is maximized, i.e.,

$$v_{opt} = \arg \max_v J(v),$$

2.2 New objective function

Let I denote a gray scale image with L gray levels [0, 1, ..., $L-1$]. The number of pixels with gray level i is denoted by n_i and the total number of pixels by $N = n_0 + n_1 + ... + n_{L-1}$. The probability of gray level i appeared in the image is defined as:

$$p_i = \frac{n_i}{N}, \quad p_i \geq 0, \quad \sum_{i=0}^{L-1} p_i = 1.$$

Suppose that the pixels in the image are divided into two classes C_1 and C_2 by a gray level t; C_1 is the set of pixels with levels[0, 1, ..., k], and the rest of pixels belong to C_2. C_1 and C_2 normally correspond to the object class and the back ground one, or vice versa. Then the probabilities of the two classes are given by within

$$w_1(k) = \sum_{i=0}^{k} p_i, \qquad w_2(k) = 1 - w_1(k)$$

The mean gray levels of the two classes can be defined as:

$$m_1(k) = \sum_{i=0}^{k} \frac{ip_i}{w_1}, \qquad m_2(k) = \sum_{i=k+1}^{L-1} \frac{ip_i}{w_2},$$

corresponding class variances are given by

$$\sigma_1^2 = \sum_{i=0}^{k} \frac{(i-m_1)^2 p_i}{w_1}, \qquad \sigma_2^2 = \sum_{i=k+1}^{L-1} \frac{(i-m_2)^2 p_i}{w_2}.$$

The within-class variance, can be defined [18]

$$\sigma_w^2 = w_1\sigma_1^2 + w_2\sigma_2^2,$$

In [23] Qiao et al. suggest a threshold criterion to overcome the weakness of the statistical criteria. Their criteria based on the convex combination of within-class variance and intensity contrast between the object and background as

$$J(\alpha, k) = (1-\alpha)\sigma_W^2 - \alpha\,|m_2(k) - m_1(k)| \tag{3}$$

Theoretical bounds of the weight α are generally difficult to obtain as reported in [23]. So they assuming the uniformly distributed background and object, followed by the procedure to estimate the weight parameter from prior knowledge.

Their algorithm composed of two steps:

1. Estimate an appropriate weight α either through supervised learning or through theoretical bounds by assuming the uniformly distributed object and background.
2. Find the threshold T by minimizing the criteria.

The computation of the weight α is complicated either through supervised learning or through theoretical bounds.

As we have seen in Section 2.1, the FLD seeks directions efficient for discrimination by yielding the maximum ratio of between-class scatter to within-class scatter. Thus, based on the function defined by (2) we propose the following criterion as objective function to evaluate the separability of the threshold at level k.

$$\lambda(k) = \sigma_W^{-2} * (m_2(k) - m_1(k))^2, \tag{4}$$

where

$$\sigma_w^2 = w_1(k)\sigma_1^2 + w_2(k)\sigma_2^2,$$

such that

$$0 < w_1(k)(1 - w_1(k)) < 1, \quad \text{and} \quad (m_2(k) - m_1(k))^2$$

represents the intensity contrast. It characterizes the intensity difference between the object and background. That is, k maximize $\lambda(k)$ leads to the best separation between object and background.

2.3 Algorithm

Following steps describe the proposed algorithm for image threshloding:

1. Let $max_\lambda = 0$, be the maximum value of the *objective* function.
2. For k *=1* to Maximum of gray intensities
3. Compute the *objective* function value corresponding to the gray level k?
   If $max_\lambda < \lambda(k)$, then $max_\lambda = \lambda(k)$, $Topt = k$, end

Take $Topt$ as the optimal threshold for segmenting the image.

3. Experimental results

In [23], the limitation of most commonly used methods namely, MinError and Otsu methods discussed in detail. For example, these methods are unable to obtain satisfactory results for the following cases:

(a) Segmenting some images of small object

(b) The distribution of background intensity is complex.

To demonstrate the effectiveness of the proposed method, we have applied the method to synthetic and real images which introduced by [23] for the same goals.

The optimum threshold values T are determined by, the Otsu thresholding method, MinError, Qiao et al. method [23] and the proposed method. See the Figs 1-6.

Example 1: The original image (Figure 1(a)) is a slice of three-dimensional (3D) MR angiography (MRA) along the axial direction. In the predefined ROI marked by a white rectangle (Fig. 1(b)), there are three cerebral vessels (the object) whose intensities are significantly brighter than those of the background.

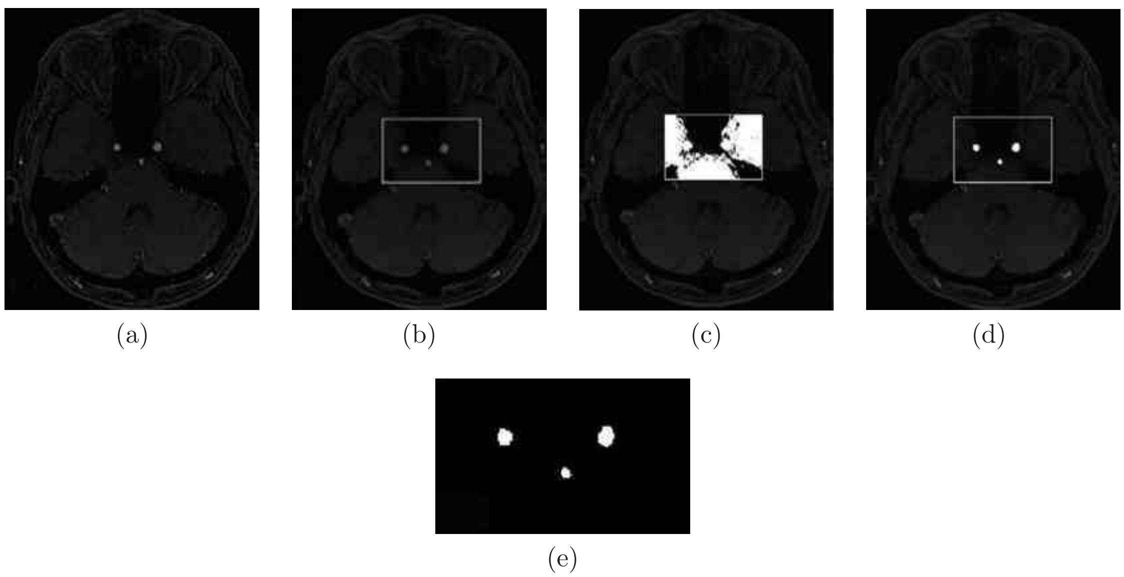


Fig. 1: (a) original MRA slice; (b) ROI selected using prior knowledge; (c) segmentation obtained by Otsu's method with a threshold of $T = 19$; and (d) segmentation using Qiao et al. method [23] ($\alpha = 0.054$) with a threshold $T = 69$: and (e) the proposed method with a threshold T= 60.

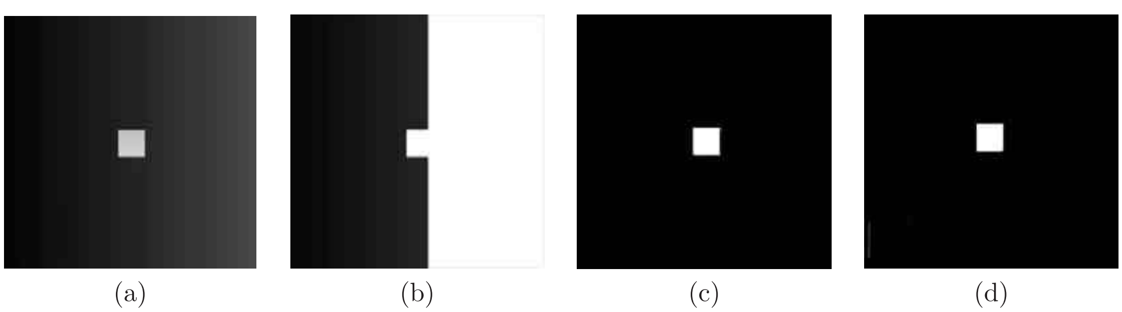
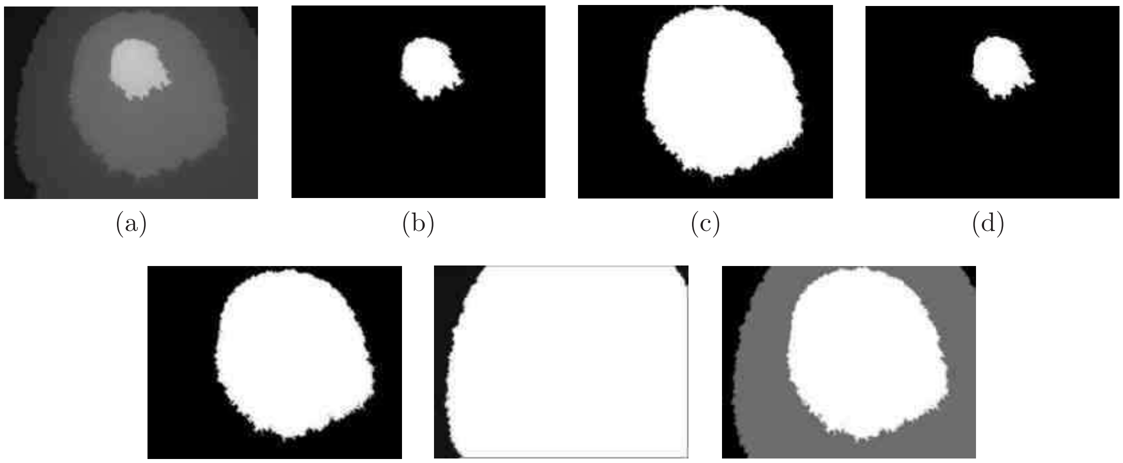


Fig. 2: (a) original image; (b) segmentation obtained by Otsu's method with a threshold of T =52; (c) segmentation using Qiao et al. method [23] ($\alpha = 0.205$) with a threshold $T = 90$. and (e) the proposed method with a threshold T= 120.

Example 2: The original image (Figure 2(a)) is a synthesized image containing a small bright object at the image center. The intensities of the object and background vary smoothly.
Example 3: The original image (Figure 3(a))is a synthesized image, the background is composed of three regions. The object is the small area whose intensities are normally distributed around 200, [23].



(a) (b) (c) (d) (e) (f) (g)

Fig. 3: (a) the original synthesized image; (b) segmentation using Qiao et al. method [23]) with $\alpha = 0.287$; (c) with $\alpha = 0.05$ with a threshold $T = 137$; (d) segmentation using the proposed method with a threshold $T = 151$; (e) segmentation obtained by Otsu's method; (f) segmentation obtained by 2-mode MinError and (g) segmentation obtained by 3-mode MinError.

Example 4: The original image is an axial slice of MR image (Figure 4(a)). It contains the lateral ventricles (the left and right deep dark areas), the interhemispheric fissure (the deep dark region in the middle), the gray matter (light dark part surrounding the interhemispheric fissure) and the white matter (bright background). We need to extract the lateral ventricles and the interhemispheric fissure.

We summarize the results obtained by Otsu's method, MinError, Qiao et al. method [23] and the proposed method in Table 1.

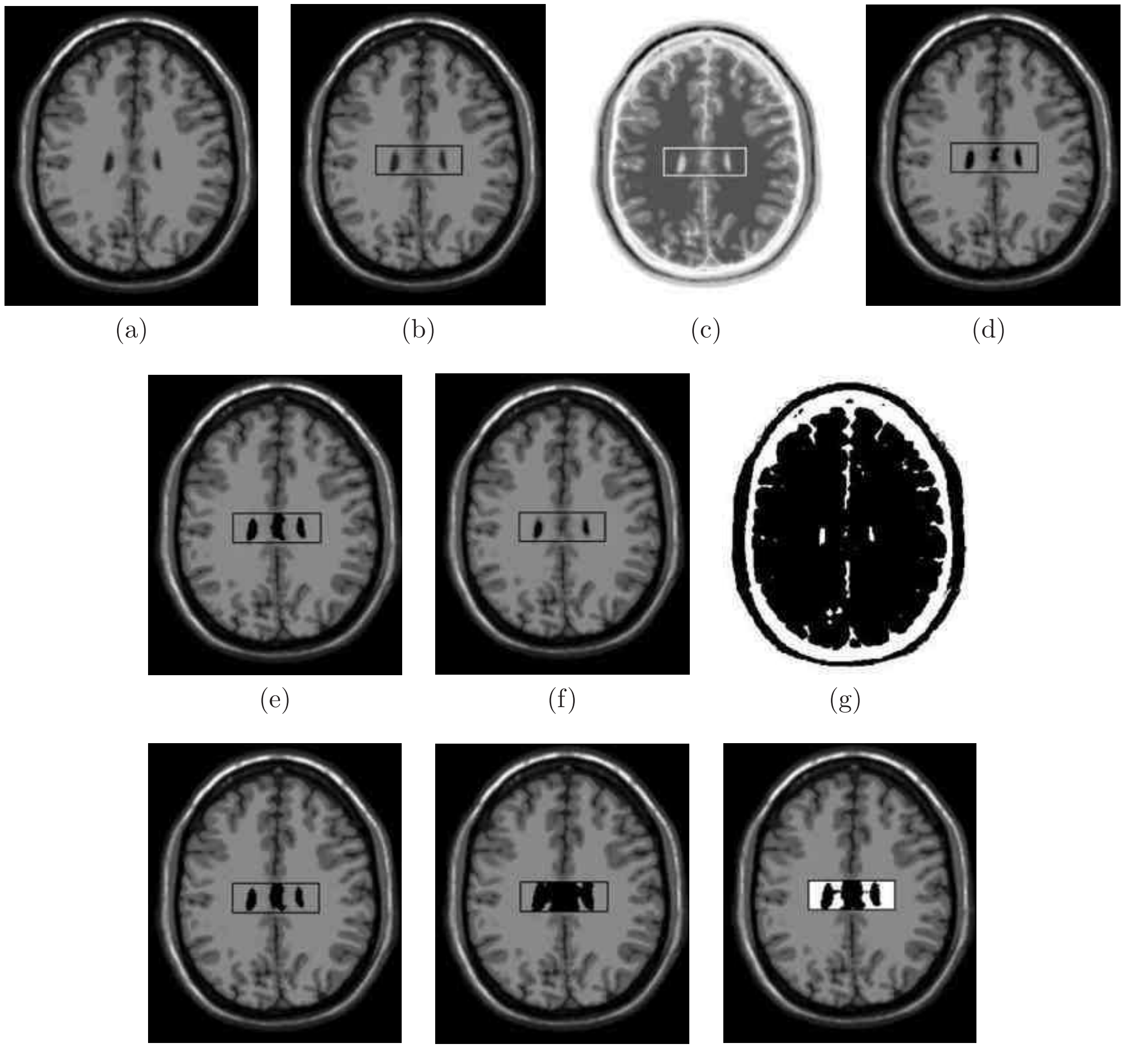
Table 1: A threshold numbers according Otsu's method, MinError, Qiao and et al. method and the proposed method

	Otsu	MinError	Qiao et al.	Proposed Method
Example 1	19	-	69	60
Example 2	52	-	90	120
Example 3	97	32	137	151
Example 4	137	103	171	175

The segmentation of all examples by the proposed method clearly illustrates its advantage in extracting small objects from complex background.

4. Conclusion

Statistical thresholding on within - class variance tends to classify images with regard to uniformity and shape measures. In order to eliminate the above limitation for segmenting



(a) (b) (c) (d) (e) (f) (g) (h) (i) (j)

Fig. 4: Example 4: (a) the original MR image; (b) the ROI in the original image; (c) the inverted ROI image; (d) segmentation using Qiao et al. method [23] with ($\alpha = 0.364$); (e) with (($\alpha = 0.051$); (f) with ($\alpha = 0.46$); and (g) segmentation using the proposed method with a threshold T= 151; (h) segmentation obtained by Otsu's method; (i) segmentation obtained by 2-mode MinError and (j) segmentation obtained by 3-mode MinError.

small objects, a new statistical criterion is proposed in this paper, which considers class variance sum and intensity contrast at the same time. The proposed method selects a threshold from a gray level histogram which derived from the viewpoint of Fisher linear discriminant analysis. It maximize the measure of separability of the resultant classes in gray levels. The proposed method has the following advantages:

1. Characterized by its nonparametric and unsupervised nature of threshold selection.

2. The proposed algorithm balance the within-class variance and the intensity contrast automatically i.e. without a weight parameter. Thus it separates the small object from background.

3. The implementation of the method is very simple.

4. Extension of the proposed method into the multi-level thresholding and color images is an open problem and being studied.

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References

[1] S. Wang, F. Chung, F. Xiong, A novel image thresholding method based on parzen window estimate. Pattern Recognition 2008, 41:117–29.
[2] D. L. Pham, C. Xu, and J. L. Prince, “Current methods in medical image segmentation,” Annu. Rev. Biomed. Eng., 2000, 2:315-337.
[3] S. Mallat and S. Zhong, “Characterization of Signals from Multiscale Edges,” IEEE Trans. on Pattern Analysis and Machine Intelligence, 1992, 14:710-732.
[4] Y. Zhang, “A survey of evaluation methods for image segmentation.” Pattern Recogn., 29:1335-1346, 1996.
[5] P. K. Saha, and J. K. Udupa, Optimum image thresholding via class uncertainty and region homogeneity. IEEE Trans Pattern Anal Mach Intell., 2001, 23:689–706.
[6] T. Sund and K. Eilertsen, An algorithm for fast adaptive binarization with applications in radiotherapy imaging. IEEE Trans Med Imaging , 2003, 22:22–28.
[7] Y. Solihin and C. G. Leedham, Integral ratio: a new class of global thresholding techniques for handwriting images. IEEE Trans Pattern Anal Mach Intell. 1999, 21:761–768.
[8] B. Bhanu, Automatic target recognition: state of the art survey. IEEE Trans Aerosp Electron Syst. 1986, 22:364–379.
[9] J. S. Lee and M. C. K. Yang, Threshold selection using estimates from truncated normal distribution, IEEE Trans. Syst. Man Cybern., 1989, 19: 422–429.
[10] J. Kittler and J. Illingworth, “Minimum error thresholding,” *Pattern Recogn.*, 19, 41–47 , 1986.
[11] S. Cho, R. Haralick, and S. Yi, “Improvement of Kittler and Illingworths's minimum error thresholding,, *Pattern Recogn.,* 1989, 22:609-617.
[12] W. H. Tsai, Moment-preserving thresholding: a new approach. Comput Vis Graph Image Process (1985) 29:377–93.
[13] Y. Bazi, L. Bruzzone and F. Melgani, Image thresholding based on the EM algorithm and the generalized Gaussian distribution. Pattern Recognition, 2007, 40:619–634.
[14] T. Elguebaly, N. Bouguila , F. Schwenker and El Gayar, N. (Eds.) “Bayesian Learning of Generalized Gaussian Mixture Models on Biomedical Images”, Artificial Neural Networks in Pattern Recognition, Springer, Berlin/ Heidelberg, 5998**,** 207-218, 2010.
[15] T. Pun, A new method for grey-level picture thresholding using the entropy of the histogram. Signal Process (1980) 2:223–237.
[16] J. N. Kapur, P. K. Sahoo, Wong AKC. A new method for grey-level picture thresholding using the entropy of the histogram. Comput Vis Graph Image Process, 1985, 29:273–285.
[17] H. d. Cheng, J. R. Chen and J. G. Li, Threshold selection based on fuzzy c-partition entropy approach, Pattern Recognition, 1998, 31:857–870.
[18] N. Otsu, A threshold selection method from gray-level histograms. IEEE Trans Syst Man Cybern, 1979, 9:62–66.
[19] P. K. Sahoo, S. Soltani and A. K. C. Wong, A survey of thresholding techniques. Comput Vis Graph Image Process (1988) 41:233–260.
[20] Z. Hou, Q. Hu, W. l. Nowinski, On minimum variance thresholding. Pattern Recognition Lett., 2006, 27:1732–1743.
[21] M. Sezgin and B. Sankur, Survey over image thresholding techniques and quantitative performance evaluation. J Electron Imaging, 2004, 13:146–65.
[22] Z. Li, C. Liu, G. Liu, Y. Cheng, X. Yang and C. Zhao, A novel statistical image thresholding. Int J Electron Commun. 2011 64:1137–1147.
[23] Y. Qiao, Q. Hu, G. Qian, S. Luo and W. L. Nowiniski, Thresholding based on variance and intensity contrast. Pattern Recognition, 2007, 40:596–608.
[24] H. Wang, P. Li and T. Zhang, Histogram feature-based Fisher linear discriminant for face detection. Neural Comput & Appl. 2008, 17:49-58.
[25] A. Hadid, M. Pietikainen and T. Ahonen, A discriminative feature space for detecting and recognizing faces. Proc IEEE Conf Comput Vision Pattern Recogn 2004, 2:797–804.