Assignment

G Harsha Vardhan Reddy (CS21BTECH11017)

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1 Uniform Random Numbers

et U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    exrand.c
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.1}$$

Solution: The following code plots Fig. 1.2

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    cdf_plot.py
```

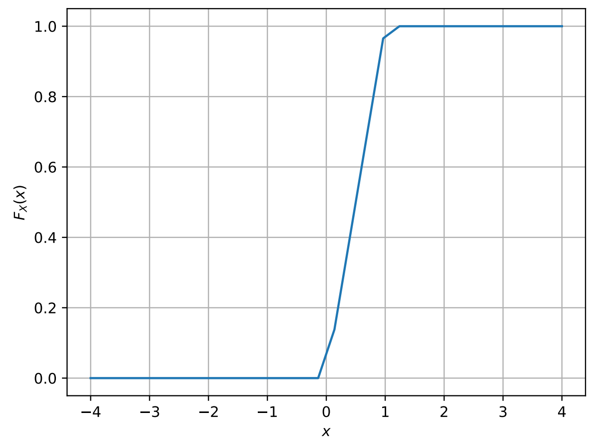

Fig. 1.2: The CDF of U

1.3 Find the theoretical expression for $F_U(x)$
Solution:
Given,
U is an uniform random variable

$$\implies P(U = x) = k \qquad \forall x \in [0, 1] \tag{1.2}$$

$$\implies F_U(x) = \int_0^x k dx \tag{1.3}$$

$$= kx \tag{1.4}$$

As $F_U(1) = 1$

$$k(1) = 1 \implies k = 1 \tag{1.5}$$

$$\implies F_U(x) = x \qquad \forall x \in [0, 1] \tag{1.6}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.7}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.8}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program.

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    mean_variance.c
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.9}$$

Solution: As $F_U(x) = x \; \forall x \in [0,1]$

a) k=1

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \tag{1.10}$$
$$\implies E[U] = \int_0^1 x dx \tag{1.11}$$
$$\implies E[U] = \frac{1}{2} \tag{1.12}$$

b) k=2

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{1.13}$$
$$= \int_0^1 x^2 dx = \frac{1}{3} \tag{1.14}$$
$$\text{As Variance } Var[U] = E\left[U^2\right] - (E[U])^2 \tag{1.15}$$
$$Var[U] = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \tag{1.16}$$

Therefore,

$$\text{Theoritical mean} = \frac{1}{2} \tag{1.17}$$
$$\text{Experimental mean} = 0.500007 \tag{1.18}$$
$$\text{Theoritical Variance} = \frac{1}{12} \tag{1.19}$$
$$\text{Experimental Variance} = 0.083301 \tag{1.20}$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat
Solution: Download the following files and execute the C program.

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    exrand_gaussian.c
```

2.2 Load "gau.dat" in python and plot the empirical CDF of X using the samples in "gau.dat". What properties does a CDF have?
Solution: The CDF of X is plotted in Fig. 2.2 using code below

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    cdf_gau_plot.py
```

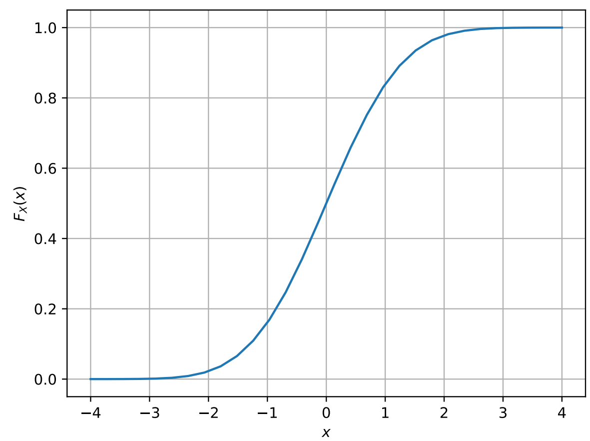

Fig. 2.2: The CDF of X

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

What properties does the PDF have?
Solution: The PDF of X is plotted in Fig. 2.3 using the code below

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    pdf_gau_plot.py
```

2.4 Find the mean and variance of X by writing a C program.

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{2.3}$$

repeat the above exercise theoretically.
Solution: As $dF_X(x) = p_X(x)dx$

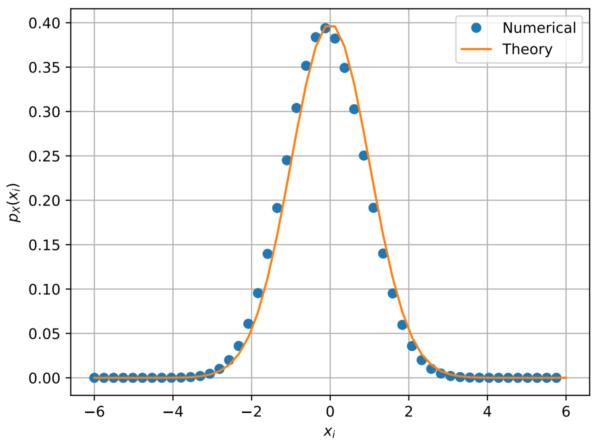

Fig. 2.3: The CDF of X

a) Mean :

$$E[X] = \int_{-\infty}^{\infty} x p_X(x) dx \tag{2.4}$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x exp\left(-\frac{x^2}{2}\right) dx \tag{2.5}$$
$$= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\left(exp\left(-\frac{x^2}{2}\right)\right) \tag{2.6}$$
$$= 0 \tag{2.7}$$

b) Variance

$$E\left[X^2\right] = \int_{-\infty}^{\infty} x^2 p_X(x) dx \tag{2.8}$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 exp\left(-\frac{x^2}{2}\right) dx \tag{2.9}$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 exp\left(-\frac{x^2}{2}\right) dx \tag{2.10}$$
$$= \frac{1}{\sqrt{2\pi}} \sqrt{2\pi} = 1 \tag{2.11}$$

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1-U) \tag{3.1}$$

and plot its CDF.
Solution: The CDF of X is plotted in Fig. 3.1 using the code below

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    exrand_other.c
```

```
wget https://github.com/
    HarshaVardhanReddyG/
    A1110_Assignment/blob/main/codes/
    cdf_other_plot.py
```

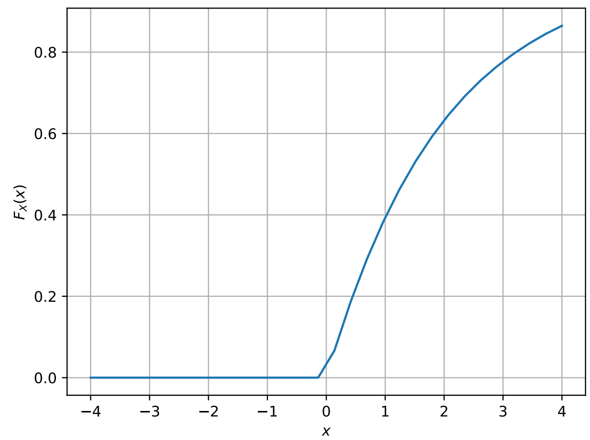

Fig. 3.1: The CDF of X

3.2 Find a theoretical expression for $F_V(x)$.
Solution:
Given,
$V = -2\ln(1-U)$ and from (1.6)

$$F_V(x) = P(V \le x) \tag{3.2}$$
$$= P(-2\ln(1-U) \le x) \tag{3.3}$$
$$= P\left(U \le 1 - e^{-\frac{x}{2}}\right) \tag{3.4}$$
$$\implies F_V(x) = 1 - e^{-\frac{x}{2}} \tag{3.5}$$