Assignment 11

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AI1110

Outline

Problem Statement

Papoulis Pillai Probability Random Variables and Stochastic Processes
Exercise : 6-65

For any two random variables x and y with $E[x^2] < \infty$ Show that, $Var\{x\} \geq E[Var\{x|y\}]$ and $Var\{x\} = E[Var\{x|y\}] + Var\{E[x|y]\}$

Solution

As

$$Var(x|y) = E(x^2|y) - [E(x|y)]^2 and \tag{1}$$

$$Var[E(x|y)] = E([E(x|y)]^2) - (E[E(x|y)])^2 \tag{2}$$

And From (1) and (2)

$$E[Var(x|y)] = E[E(x^2|y) - [E(x|y)]^2] \tag{3}$$

$$= E[E(x^2|y)] - E[(E[x|y])^2] \text{ and} \tag{4}$$

$$Var(E[x|y]) = E([E(x|y)]^2) - (E[E(x|y)])^2 \tag{5}$$

Using (3) and (5)

$$E[Var(x|y)] + Var(E[x|y]) = E[E(x^2|y)] - E[(E[x|y])^2] + E([E(x|y)]^2) - (E[E(x|y)])^2 \tag{6}$$

$$\implies E[Var(x|y)] + Var(E[x|y]) = E[E(x^2|y)] - (E[E(x|y)])^2 \tag{7}$$

$$= E[x^2] - (E[x])^2 = Var(x) \tag{8}$$

Therefore

$$Var(x) = E[Var(x|y)] + Var(E[x|y]) \tag{9}$$

From (9), using $Var(x) \geq 0$

$$Var(x) = E[Var(x|y)] + Var(E[x|y]) \tag{10}$$

$$\implies Var(x) \geq E[Var(x|y)] \tag{11}$$