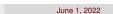
Assignment 11

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Outline

Problem Statement

Solution



Problem Statement

Papoulis Pillai Probability Random Variables and Stochastic Processes Exercise: 6-65

For any two random variables x and y with $E[x^2] < \infty$ Show that, $Var\{x\} \ge E[Var\{x|y\}]$ and $Var\{x\} = E[Var\{x|y\}] + Var\{E[x|y]\}$



Solution

As

$$Var(x|y) = E(x^2|y) - [E(x|y)]^2$$
 and (1)

$$Var[E(x|y)] = E([E(x|y)]^{2}) - (E[E(x|y)])^{2}$$
 (2)

And From (1) and (2)

$$E[Var(x|y)] = E[E(x^2|y) - [E(x|y)]^2]$$
(3)

$$= E[E(x^{2}|y)] - E[(E[x|y])^{2}] \text{ and } (4)$$

$$Var(E[x|y]) = E([E(x|y)]^{2}) - (E[E(x|y)])^{2}$$
(5)

Using (3) and (5)

$$E[Var(x|y)] + Var(E[x|y]) = E[E(x^2|y)] - E[(E[x|y])^2] + E([E(x|y)]^2) - (E[E(x|y)])^2$$
(6)

$$\implies E[Var(x|y)] + Var(E[x|y]) = E[E(x^{2}|y)] - (E[E(x|y)])^{2}$$

$$= E[x^{2}] - (E[x])^{2}$$

$$= Var(x)$$
(8)

Therefore

$$Var(x) = E[Var(x|y)] + Var(E[x|y])$$
(9)

From (9), using $Var(x) \ge 0$

$$Var(x) = E[Var(x|y)] + Var(E[x|y])$$
(10)

$$\implies Var(x) \ge E[Var(x|y)]$$
 (11)

