Assignment 12

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AI1110

Outline

Problem Statement

Papoulis Pillai Probability Random Variables and Stochastic Processes
Exercise : 6-65

The random variables X_i are i.i.d. and uniform in the interval (0,1). Show that if $Y = max(X_i)$ then $F_Y(y) = y^n$ for $0 \leq y \leq 1$.

Theory

If X_1 and X_2 are independent, then

$$P(X_1 \leq x, X_2 \leq x) = P(X_1 \leq x) \cdot P(X_2 \leq x) \tag{1}$$

A random variable X is said to be uniformly distributed over an interval if the probability density function is given by

$$f_X(x) = 1, \text{ for x in given interval} \tag{2}$$

X_i are said to be i.i.d.(independent and identically distributed) if they are independent and have same pmf.

Solution

Given,

$$Y = max(X_i) \implies \{Y \leq y\} = \{X_1 \leq y, X_2 \leq y, \ldots, X_n \leq y\} \tag{3}$$

Hence,

$$\begin{aligned} F_Y(y) &= P(Y \leq y) && (4) \\ &= P(X_1 \leq y, X_2 \leq y, \ldots, X_n \leq y) && (5) \end{aligned}$$

From (1) and (5),

$$\begin{aligned} F_Y(y) &= P(X_1 \leq y)P(X_2 \leq y)\ldots P(X_n \leq y) && (6) \\ &= (P(X_1 \leq y))^n \text{..........(Since } X_i \text{ are i.i.d)} && (7) \\ \implies F_Y(y) &= (F_{X_i}(y))^n && (8) \end{aligned}$$

Given that X_i are uniformly distributed over $\{0 \leq x \leq 1\}$
From (2)

$$f_{X_i}(x) = 1 \tag{9}$$

$$\implies F_{X_i}(x) = x \qquad 0 \leq x \leq 1 \tag{10}$$

From (8) and (10),
For $0 \leq y \leq 1$

$$F_Y(y) = (F_{X_i}(y))^n = y^n \tag{11}$$

$$\implies F_Y(y) = y^n \tag{12}$$