

# Assignment 13

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# Problem Statement

## Papoulis Pillai Probability Random Variables and Stochastic Processes Exercise : 8-13

We plan a poll for the purpose of estimating the probability  $p$  of Republicans in a community. We wish our estimate to be within  $\pm 0.02$  of  $p$ . How large should our sample be if the confidence coefficient of the estimate is 0.95?

# Definitions

## Sample Proportion

If  $X$  is a binomial random variable, then  $X \sim B(n, p)$  where  $n$  is the number of trials and  $p$  is the probability of a success. To form a sample proportion, take  $X$ , the random variable for the number of successes and divide it by  $n$ , the number of trials (or the sample size). The random variable  $P'$  is the sample proportion

$$P' = \frac{X}{n} \quad (1)$$

And  
 $p'$  = the estimated proportion of successes or point estimate for  $p$

# Youth percentile or Z score

## Z-score

Z-score indicates how much a given value differs from the standard deviation. The Z-score, or standard score, is the number of standard deviations a given data point lies above or below mean.

$$\implies Z_u = \frac{x - \mu}{\sigma} = \frac{p - p'}{\sigma} \quad (2)$$

Where,

$Z_u$  = Normal (Youth) percentile or Z score

$x$  = Observed value

$\sigma$  = Standard deviation

## Confidence interval for a population proportion

The confidence interval for a population proportion ( $p$ )

$$|p - p'| \leq \sigma Z_u \quad (3)$$

$$p' - \sigma Z_u \leq p \leq p' + \sigma Z_u \quad (4)$$

Where

$$\sigma_{p'} = \sqrt{\frac{(1 - p')(p')}{n}} \quad (5)$$

Therefore,

$$p' - Z_u \times \sqrt{\frac{(1 - p')(p')}{n}} \leq p \leq p' + Z_u \times \sqrt{\frac{(1 - p')(p')}{n}} \quad (6)$$

# Solution

Given,

$$p \leq \pm 0.02 \text{ of } p' \quad (7)$$

$$\implies |p - p'| \leq 0.02 \text{ and,} \quad (8)$$

$$\text{Confidence Coefficient (CF)} = 0.95 \implies Z_u = 2 \quad (9)$$

From (6),(8) and (9),

$$\sqrt{\frac{(1 - p')(p')}{n}} \times 2 \leq 0.02 \quad (10)$$

As  $n > 0$ , From (10)

$$\frac{(1 - p')(p')}{n} \leq \left(\frac{1}{100}\right)^2 \quad (11)$$

$$\implies n \geq (1 - p')(p') \times 100^2 \quad (12)$$

$$A.M \geq G.M \quad (13)$$

$$\frac{(p') + (1 - p')}{2} \geq \sqrt{(p')(1 - p')} \implies p'(1 - p') \leq \frac{1}{4} \quad (14)$$

From (12),(14)

$$n \geq \frac{100^2}{4} \quad (15)$$

$$\implies n \geq 2500 \quad (16)$$

Therefore, the size of sample( $n$ ) must be greater than equal to 2500.