Assignment 13

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AI1110

Outline

Problem Statement

Papoulis Pillai Probability Random Variables and Stochastic Processes Exercise : 8-10

Among 4000 newborns, 2080 are male. Find the 0.99 confidence interval of the probability $p = P\{male\}$.

Definitions

Sample Proportion

If X is a binomial random variable, then $X \sim B(n, p)$ where n is the number of trials and p is the probability of a success. To form a sample proportion, take X, the random variable for the number of successes and divide it by n, the number of trials (or the sample size). The random variable P' is the sample proportion

$$P' = \frac{X}{n} \tag{1}$$

And
p' = the estimated proportion of successes or point estimate for p

Youth percentile or Z score

Z-score

Z-score indicates how much a given value differs from the standard deviation. The Z-score, or standard score, is the number of standard deviations a given data point lies above or below mean.

$$\implies Z_u = \frac{x - \mu}{\sigma} = \frac{p - p'}{\sigma_{p'}} \tag{2}$$

Where,
Z_u = Normal (Youth) percentile or Z score
x = Observed value
σ = Standard deviation

Confidence interval for a population proportion

The confidence interval for a population proportion (p)

$$|p - p'| \leq \sigma Z_u \tag{3}$$

$$p' - \sigma Z_u \leq p \leq p' + \sigma Z_u \tag{4}$$

Where

$$\sigma_{p'} = \sqrt{\frac{(1 - p')(p')}{n}} \tag{5}$$

Therefore,

$$p' - Z_u \times \sqrt{\frac{(1 - p')(p')}{n}} \leq p \leq p' + Z_u \times \sqrt{\frac{(1 - p')(p')}{n}} \tag{6}$$

Solution

Given,

$$\text{No.of newborns } (n) = 4000 \quad (7)$$
$$\text{No.of males } = 2080 \quad (8)$$
$$\implies p' = P\{male\} = \frac{2080}{4000} = 0.52 \quad (9)$$
$$\text{Confidence coefficient } (CF) = 0.99 \quad (10)$$
$$\implies Z_u = 2.326 \quad (11)$$

$\therefore$ From (6),(7),(9) and (11),

$$0.52 - 2.326\sqrt{\frac{(0.48)(0.52)}{4000}} \leq p \leq 0.52 + 2.326\sqrt{\frac{(0.48)(0.52)}{4000}} \quad (12)$$
$$\implies 0.502 \leq p \leq 0.538 \quad (13)$$