Assignment 8

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Outline

Abstract

This document contains 16^{th} problem from the chapter 4 in the book **Papoulis Pillai Probability RandomVariables and Stochastic Processes.**

Problem Statement

Problem

Show that if $X(\zeta) \leq X(\zeta)$ for every $\zeta \in S$ then $F_X(\omega) \geq F_Y(\omega)$ for every ω.

Random Variables

We are given an experiment specified by the space S, the field of subsets of S called events, and the probability assigned to these events. To every outcome ζ of this experiment, we assign a number $X(\zeta)$. We have thus created a function X with domain the set S and range a set of numbers. This function is called random variable

Cumulative Distribution Function

The cumulative distribution function for a random variable X $F_X(\omega)$ can be defined as below:

$$F_X(\omega) = \Pr(X \leq \omega) \text{ for every } \omega \in S \tag{1}$$

Solution

Given

$$X(\zeta) \leq Y(\zeta) \ \forall \zeta \in S \tag{2}$$

Let's say

$$Y(\zeta_i) \leq \omega \tag{3}$$

then, From (2)

$$X(\zeta_i) \leq \omega \tag{4}$$

Hence,

$$\{Y \leq \omega\} \subseteq \{X \leq \omega\} \tag{5}$$

And we know that

$$A \subseteq B \implies P(A) \leq P(B) \tag{6}$$

Therefore from (5),(6)

$$P(Y \leq \omega) \leq P(X \leq \omega) \tag{7}$$

And From (1)

$$P(Y \leq \omega) \leq P(X \leq \omega) \implies F_X(\omega) \leq F_Y(\omega) \tag{8}$$

If $Y(\zeta_i) > \omega$ then

$$\{Y \leq \omega\} = \Phi \implies P(Y \leq \omega) = F_Y(\omega) = 0 \tag{9}$$

And as $P(x) \geq 0$

$$F_X(\omega) \geq 0 \tag{10}$$

$$\implies F_X(\omega) \geq F_Y(\omega) \tag{11}$$

From (8) and (11) it is clear that

$$X(\zeta) \leq Y(\zeta) \implies F_X(\omega) \geq F_Y(\omega) \tag{12}$$