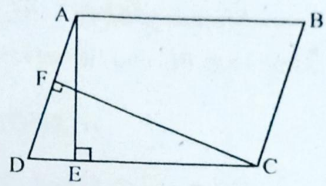
AREAS OF PARALLELOGRAMS AND TRIANGLES

Short Answer Type Questions

Q. 1. In figure, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, then find AD.



Sol. We know that,

Area of parallelogram = Base × Altitude

∴ Area of parallelogram $ABCD = AB \times AE$

$$= (16 \times 8) \text{ cm}^2$$

$$= 128 \text{ cm}^2 \quad ...(1)$$

Also, Area of parallelogram $ABCD = AD \times CF$

$$= (AD \times 10) \text{ cm}^2 \quad ...(2)$$

From eqns. (1) and (2),

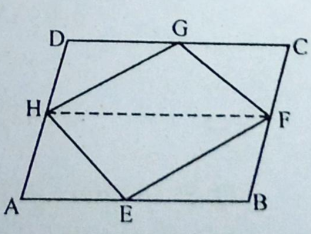
$$AD \times 10 = 128$$

$$\Rightarrow \quad AD = \frac{128}{10} = 12.8 \text{ cm.}$$

Ans.

Q. 2. If E, F, G and H are the mid-points of sides of parallelogram $ABCD$, then show that :

$$ar(EFGH) = \frac{1}{2} ar(ABCD).$$



Sol. ΔHGF and parallelogram $HDCF$ stand on the same base HF and lie between the same parallels HF and DC.

$$\therefore \quad ar(HGF) = \frac{1}{2} ar(HDCF) \quad ...(1)$$

Similarly, ΔHEF and parallelogram $ABFH$ stand on the same base HF and lie between the same parallels HF and AB.

$$\therefore \quad ar(HEF) = \frac{1}{2} ar(ABFH) \quad ...(2)$$

∴ Adding eqns. (1) and (2), we get

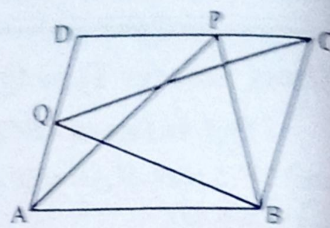
$$ar(HGF) + ar(\Delta HEF) = \frac{1}{2}[ar(HDCF) + ar(ABFH)]$$

$$\Rightarrow \quad ar(EFGH) = \frac{1}{2} ar(ABCD).$$

Proved.

Q. 3. ***P*** **and** ***Q*** **are any two points lying on the sides** ***DC*** **and** ***AD*** **respectively of a parallelogram** ***ABCD*****. Show that** $ar\,(APB) = ar\,(BQC)$**.**

Sol. $\Delta\, APB$ and parallelogram $ABCD$ lie between the same base AB and the same parallels AB and DC.



$$\therefore \qquad ar(APB) = \frac{1}{2} ar(ABCD) \qquad \text{...(1)}$$

Similarly, ΔBQC and parallelogram $ABCD$ stand on the same base BC and lie between the same parallels BC and AD.

$$\therefore \qquad ar(BQC) = \frac{1}{2} ar(ABCD) \qquad \text{...(2)}$$

From eqns. (1) and (2), we have

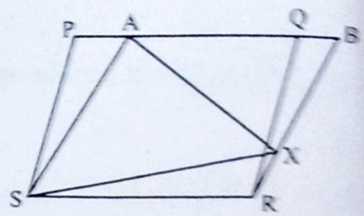
$$ar(APB) = ar(BQC).$$

Proved.

Q. 4. In figure, ***PQRS*** **and** ***ABRS*** **are parallelograms and** ***X*** **is any point on side** ***BR*****. Show that :**

(i) $ar(PQRS) = ar(ABRS)$

(ii) $ar(AXS) = \frac{1}{2} ar(PQRS)$**.**



Sol. (i) Parallelogram $PQRS$ and parallelogram $ABRS$ stand on the same base RS and lie between the same parallels SR and $PAQB$.

$$\therefore \qquad ar(PQRS) = ar(ABRS) \qquad \text{...(1)}$$

Proved.

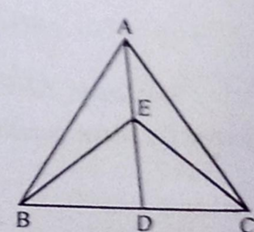
(ii) ΔAXS and parallelogram $ABRS$ stand on the same base AS lie between the same parallels AS and RB.

$$\therefore \qquad ar(AXS) = \frac{1}{2} ar(ABRS)$$

$$\Rightarrow \qquad ar(AXS) = \frac{1}{2} ar(PQRS),$$

[using eqn. (1)] **Proved.**

Q. 5. In figure, ***E*** **is any point on median** ***AD*** **of a** $\Delta\, ABC$**. Show that** $ar(ABE) = ar(ACE)$ **.**



Sol. Given : AD is median of ΔABC and E is any point on AD.

To prove : $ar(ABE) = ar(ACE)$

Proof : AD is median of ΔABC

$$ar(ABD) = ar(ACD) \qquad \text{...(1)}$$

Also, ED is median of triangle EBC

$$\therefore \qquad ar(BED) = ar(CED) \qquad \text{...(2)}$$

Subtracting eqn. (2) from eqn.(1),

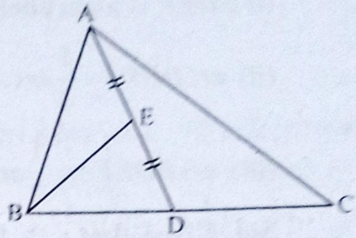
$$ar(ABD) - ar(BED) = ar(ACD) - ar(CED)$$

$$\Rightarrow \qquad ar(ABE) = ar(ACE).$$

Proved.

Q. 6. In ΔABC, E is mid-point of median AD. Show that $ar(BED) = \frac{1}{4} ar(ABC)$.

Sol. Given : In ΔABC, E is mid-point of median AD.



To prove : $ar(BED) = \frac{1}{4} ar(ABC)$.

Proof : Since AD is median of ΔABC and median divides a triangle into two triangles having equal area.

$\therefore$ $$ar(ABD) = ar(ADC)$$

$\Rightarrow$ $$ar(ABD) = \frac{1}{2} ar(ABC) \quad ...(1)$$

In ΔABD, BE is median

$\therefore$ $$ar(BED) = ar(BAE) \quad ...(2)$$

$\Rightarrow$ $$ar(BED) = \frac{1}{2} ar(ABD)$$

$\Rightarrow$ $$ar(BED) = \frac{1}{2} \times \frac{1}{2} ar(ABC), \quad \text{[from eqn.(1)]}$$

$\Rightarrow$ $$ar(BED) = \frac{1}{4} ar(ABC).$$ **Proved.**

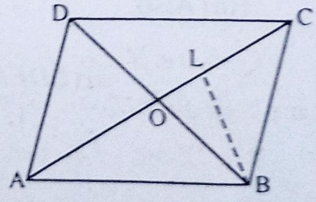
Q. 7. Show that both the diagonals of a parallelogram divide it into four triangles of equal area.

Sol. Given : A parallelogram $ABCD$.

To prove : Diagonal AC and BD divide parallelogram $ABCD$ into four triangles of equal area.

Construction : Draw $BL \perp AC$.

Proof : Since $ABCD$ is a parallelogram, hence its diagonals AC and BD bisect at point O.



$\therefore$ $$AO = OC \text{ and } BO = OD$$

Now, $$ar(AOB) = \frac{1}{2} \times AO \times BL$$

$$ar(OBC) = \frac{1}{2} \times OC \times BL$$

But $$AO = OC$$

$\therefore$ $$ar(AOB) = ar(OBC)$$

Similarly, we can show that,

$$ar(OBC) = ar(OCD); ar(OCD) = ar(ODA);$$

$$ar(ODA) = ar(OAB); ar(OAB) = ar(OBC)$$

$$ar(OCD) = ar(ODA)$$

Thus, $$ar(OAB) = ar(OBC) = ar(OCD) = ar(OAD).$$ **Proved.**

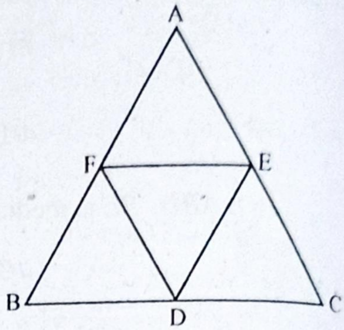
Q. 8. ***D, E*** **and** ***F*** **are mid-points of sides** ***BC, CA*** **and** ***AB*** **of triangle** ***ABC*** **respectively. Show that :**

(i) ***BDEF*** **is a parallelogram.**

(ii) $ar(DEF) = \frac{1}{4} ar(ABC)$

(iii) $ar(BDEF) = \frac{1}{2} ar(ABC)$.

Sol. Given that : D, E and F are mid-points of sides BC, CA and AB.



To prove : (i) $BDEF$ is a parallelogram

(ii) $ar(DEF) = \frac{1}{4} ar(ABC)$

(iii) $ar(BDEF) = \frac{1}{2} ar(ABC)$

Proof : (i) In ΔABC,

$EF \parallel BC$, [from mid-point theorem]

$\therefore$ $EF \parallel BD$ ...(1)

Also $ED \parallel AB$, [from mid-point theorem]

$\therefore$ $ED \parallel FB$ ...(2)

From eqns.(1) and (2), $BDEF$ is a parallelogram. **Proved.**

(ii) Similarly, $FDCE$ and $AFDE$ are parallelograms.

$\therefore$ $ar(FBD) = ar(DEF)$, [$\because$ FD is a diagonal of parallelogram $BDEF$]

$ar(DEC) = ar(DEF)$, [$\because$ ED is a diagonal of parallelogram $FDCE$]

and $ar(AFE) = ar(DEF)$, [$\because$ FE is a diagonal of parallelogram $AFDE$]

$\therefore$ $ar(FBD) = ar(DEC) = ar(AFE) = ar(DEF)$ ...(3)

$\Rightarrow$ $ar(DEF) = \frac{1}{4} ar(ABC)$. **Proved.**

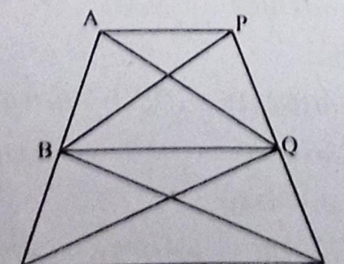
(iii) Also,

$$ar(BDEF) = 2 \times ar(\ldots) = 2 \times \frac{1}{4} ar(ABC)$$

$$= \frac{1}{2} ar(ABC).$$ **Proved.**

Q. 9. In figure, ***AP*** $\parallel$ ***BQ*** $\parallel$ ***CR*****. Prove that :**

$$ar(AQC) = ar(PBR).$$



Sol. From figure,

$$ar(AQC) = ar(AQB) + ar(BQC) \qquad ...(1)$$

and $$ar(PBR) = ar(PBQ) + ar(QBR) \qquad ...(2)$$

But, $$ar(AQB) = ar(PBQ), \qquad ...(3)$$

[$\because$ These triangles are on the same base BQ and between same parallel lines AP and BQ]

Also, $$ar(BQC) = ar(QBR), \qquad ...(4)$$

[$\because$ These triangle are on the same base BQ and between same parallels BQ and CR]

Using eqns. (3) and (4) in eqns. (1) and (2), we get

$$ar(AQC) = ar(PBR).$$

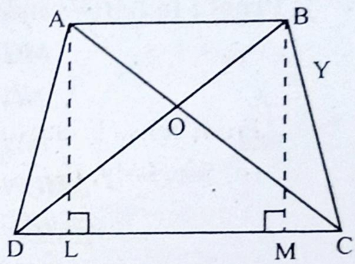
Proved.

Q. 10. Diagonals AC and BD of a quadrilateral $ABCD$ intersected at in O such a way that $ar(AOD) = ar(BOC)$. Prove that $ABCD$ is trapezium.

Sol. Diagonals AC and BD of a quadrilateral $ABCD$ intersect at O, such that

$$ar(AOD) = ar(BOC) \qquad ...(1)$$

Adding $ar(ODC)$ both sides,



$$ar(AOD) + ar(ODC) = ar(BOC) + ar(ODC)$$

$$\Rightarrow \quad ar(ADC) = ar(BDC)$$

$$\Rightarrow \quad \frac{1}{2} \times DC \times AL = \frac{1}{2} \times DC \times BM$$

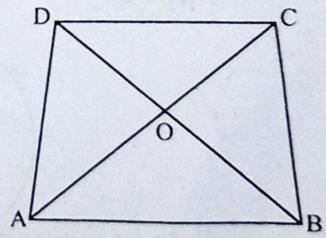
$$\Rightarrow \quad AL = BM$$

$$\Rightarrow \quad AB \parallel DC$$

Hence, $ABCD$ is a trapezium. **Proved.**

Q. 11. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at O. Prove that $ar(AOD) = ar\ (BOC)$

Sol. Diagonals AC and BD of trapezium $ABCD$ intersect at O.



Since $\Delta\, ABC$ and $\Delta\, ABD$ lie on same base and between same parallels.

$$\therefore \quad ar(ABD) = ar(ABC)$$

$$\Rightarrow \quad ar(ABD) - ar(AOB) = ar(ABC) - ar(AOB),$$

[by subtracting both sides of $ar(AOB)$]

$$\Rightarrow \quad ar(AOD) = ar(BOC).$$

Proved.

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