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Centroid-based Analysis of Quadruped-Robot Walking Balance

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***Abstract*— Multi-legged robots are very useful to do tasks in rough environment. A challenging task in their applications is to manage its walking balance. To analyze such a walking balance, we consider a simple model of quadruped robotic walking and try to identify the centroid of foot polygons formed in every step. Also, we propose a performance index to estimate the walking balance. Simulation studies show that an indispensable waddling motion of the quadruped walking can be estimated by the robot's centroid trajectory and the proposed balance index. The walking balance of the robot depends on the walking pattern employed. Finally, we discuss on a strategy to maintain the best walking balance and to facilitate an excessive dynamic walking motion. A useful walking style for the quadruped robot is also addressed in the bio-mimetic aspect.**

I. INTRODUCTION

Mobile robots are useful for performing various services at industrial work spaces, hospitals, exhibition halls, homes, and so on. So, many research groups are studying on the mechanism, motion planning, and control methods for their applications. In general, locomotion is necessary additionally for the complement of manipulation. Thus, a locomotion mechanism that enable a mobile robot to move throughout its environment is very important in the design aspect of mobile robots. There are various types of mobile robots developed in world wide. Those mobile robots can be classified as three types of locomotion mechanisms [1]: wheel-based, legged, and hybrid of the two mechanisms. They have individual features by employing different locomotion mechanisms. Specifically, a wheel-based mobile robot is very useful to do delivery services on flat ground [2]. In fact, wheeled locomotion is more efficient than legged locomotion on flat surfaces. However, a legged walking robot is more adaptable in rough terrain, relatively [3]. The hybrid type of mobile robot can utilize the advantages of wheel and leg mechanisms [4][5].

Especially, a walking robot is very recommended to do a task passing through a stairway or some obstacles. It is because legged locomotion is capable of overcoming such inconveniences. Of course, mechanism complexity increases when a robot has several legs. If any leg of the robot is not compatible with the robot's motion, its maneuverability and equilibrium may not be guaranteed. So, leg coordination is very important for achieving the desired walking performance. In order to walk, a robot must lift its leg in what order. Thus, it is required to consider a gait for effective walking. A proper walking strategy for all legs is also needed. Some researchers are interested in six-legged configurations because of their static stability during walking [6]-[9]. When a six-legged robot is being walked by using four legs in a case, it can be considered as a four-legged mechanism. For additional performance, it is desirable that a robot with large degrees of freedom can cover the problems expected in a lower system. In this sense, we interested in a quadruped robotic walking. In fact, a quadruped robot is usually able to stand easily on four legs, but its balance may be uncertain in walking [10][3]. For a walk without falling, the robot's center of gravity is needed to be actively shifted to an equilibrium position during walking. Therefore, management of walking balance is an essential issue in the four-legged walking. Thus, we need to consider how to manage the balance of legged robot during standing and walking. To this end, there exists an approach that the robot tries to adjust its center of mass to be projected inside of the polygon formed by supporting foots.

The objective of this paper is to analyze the walking balance in a quadruped robotic motion according to walking patterns. For the analysis, we identify the centroid trajectory for every foot polygon made in a quadruped robotic walking process by several simulations. We also estimate the quadruped walking balance based on the identification and a proposed performance index.

II. MODELING A QUADRUPED WALKING

In order to analyze a quadruped robotic walking, this section considered some representative four-legged walking robots shown in Fig. 1. For example, the Titan VIII robot shown in Fig. 1(a) is a representative quadruped robot developed at Tokyo Institute of Technology [10]. The Arikawa and Hirose's research provided the potential for additional researches in multi-legged locomotion as effective artifacts. An industrial company, Sony in Japan, interested in the area of entertainment robot developed the dog robot in Fig. 1(b) [11][12]. Recently, some researchers have an interest in the robots for military applications. For the purpose of military delivery services, the robot in Fig. 1(c) was developed by Boston Dynamics Company [13]. Actually, this robot can be used to deliver some military loads needed in dangerous environment including irregular terrain.

For their successful applications, stable walking is essential and thus we focused on the walking problem for four-legged robots. To deal with such a quadruped walking problem, we considered the simple walking model shown in Fig. 2, which implies a typical four-legged locomotion mechanism.

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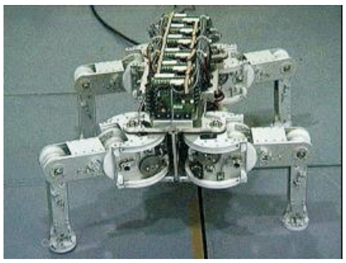
In general, if a mobile robot has k legs, the number of possible gaits N is determined by

$$N = (2k-1)! \tag{1}$$

where the symbol ! means a factorial expression [1]. In particular, a quadruped robot can make 5040 gaits. So, there exists so many gait patterns and they can construct various styles of foot configuration.



(a) Titan VIII [10]

(b) An entertainment robot, AIBO [12]

(c) A military service robot, BigDog [14]

Fig. 1. Quadruped walking robots.

In this paper, we considered one of sequential walking patterns as shown in Fig. 3. Specifically, the walking process depicted in Fig. 3 is as follows. The robot is supported by four foots stayed in the ground at the initial state. As time goes by, each foot steps sequentially to an adjacent position arbitrarily planned within its motion boundary as shown in Fig. 4. If the robot lifts a leg in the order to walk, other three legs should support the body for the walking. In such a walking situation, it is necessary to consider the balance of the walking system. In fact, a quadruped robot is usually able to stand easily on four legs, but its balance may be uncertain in walking. Therefore, management of walking balance is an essential point in the four-legged walking. Thus, we concerned how to manage the balance of legged robot during standing and walking.

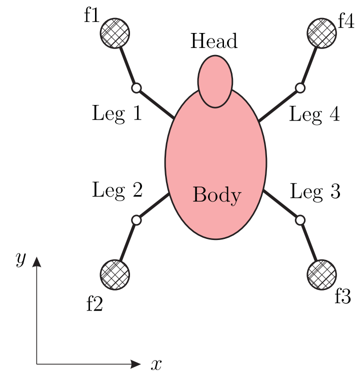


Fig. 2. Model of a quadruped robot.

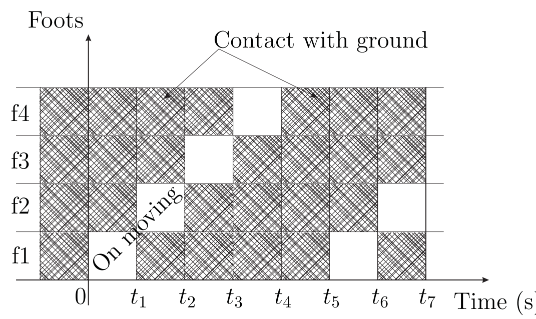


Fig. 3. Walking pattern of a quadruped robot.

III. FOOT POLYGON AND CENTROID

For the centroid-based analysis of the quadruped robot in Fig. 2, a foot polygon is basically considered. Actually, the alternatives of foot polygon in the four-legged walking are a rectangular and a triangular styles. They were illustrated in Fig. 5. For the analysis, this section describes a computation method to find the centroid of those foot polygons.

If a leg is lifted in the four-legged walking, a triangular foot configuration is made. For example, Fig. 5(a) illustrates such a triangular foot polygon and its centroid coordinate position $C_g(x_g(t), y_g(t))$ can be represented by

$$x_g(t) = \{x_1(t) + x_2(t) + x_3(t)\}/3 \tag{2}$$

$$y_g(t) = \{y_1(t) + y_2(t) + y_3(t)\}/3 \tag{3}$$

where $x_i(t)(i = 1, 2, 3)$ and $y_i(t)(i = 1, 2, 3)$ are the x- and y-directional positions of each foot, respectively.

At the standing state by four legs, a rectangular foot configuration as shown in Fig. 5(b) is constructed. The centroid of rectangular polygon can be obtained by

$$x_g(t) = \frac{1}{s_1(t) - s_2(t)}\{s_1(t)x_{g1}(t) - s_2(t)x_{g3}(t) + y_{g3}(t) - y_{g1}(t)\} \tag{4}$$

$$y_g(t) = \begin{cases} s_1(t)\{x_g(t) - x_{g1}(t)\} + y_{g1}(t), \text{ or} \\ s_2(t)\{x_g(t) - x_{g3}(t)\} + y_{g3}(t) \end{cases} \tag{5}$$

where $x_i(t)(i = 1, 2, 3, 4)$ and $y_i(t)(i = 1, 2, 3, 4)$ are the x- and y-directional positions of each foot, respectively. The parameter $s_1(t)$ implies the slope of the line passing through the centroid of triangle $\triangle f1f2f3$, $C_{g1}(t)$, and the centroid

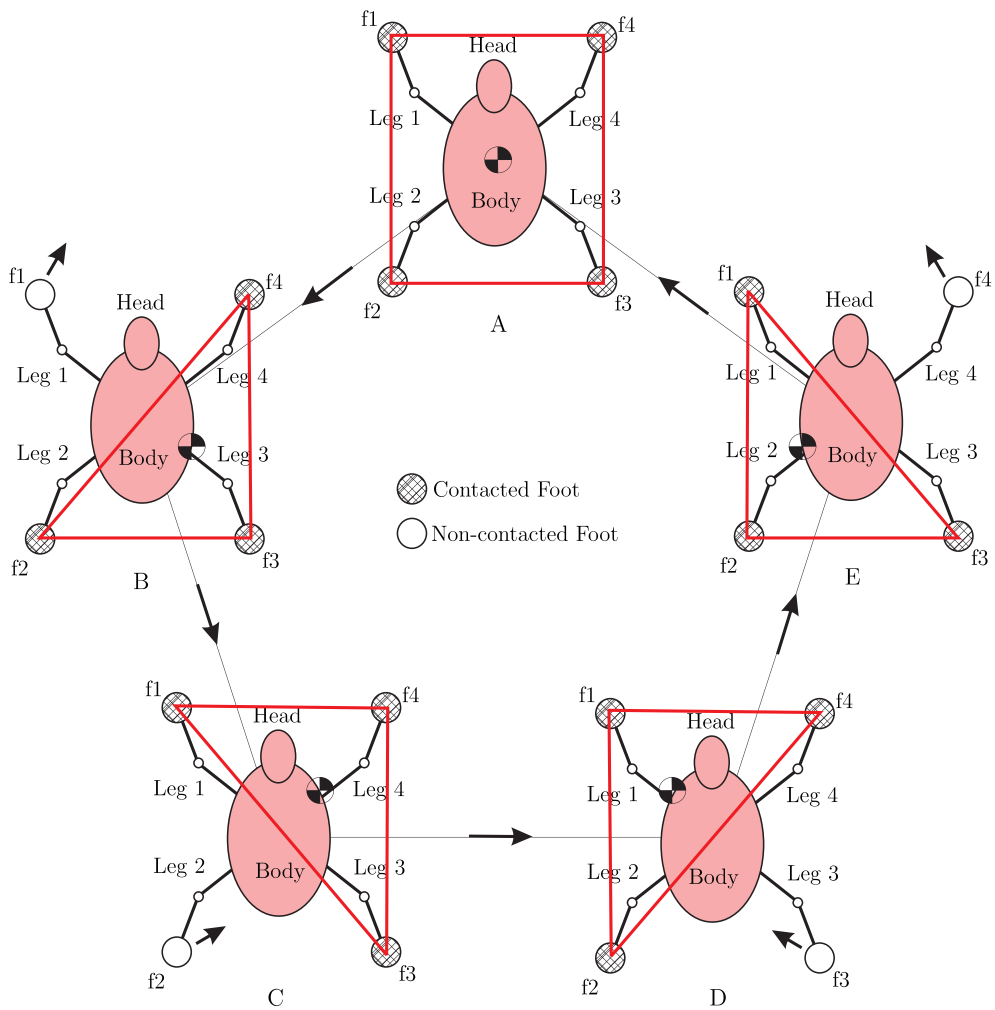
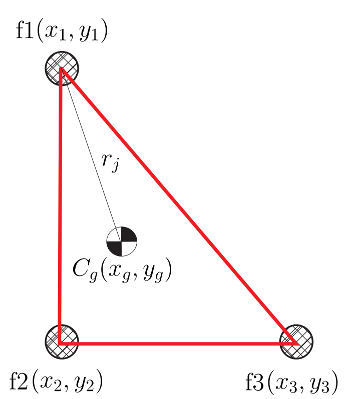
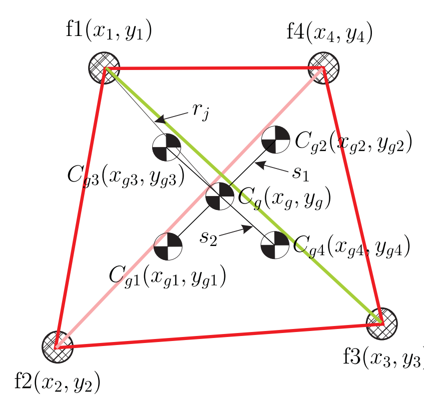


Fig. 4. Step sequence of four foots.



(a) Triangular style



(b) Rectangular style

Fig. 5. Foot polygons.

of triangle $\triangle f1f3f4$, $C_{g2}(t)$. The $s_2(t)$ represents the slope of the line passing through the centroid of triangle $\triangle f1f2f4$, $C_{g3}(t)$, and the centroid of triangle $\triangle f2f3f4$, $C_{g4}(t)$.

IV. WALKING BALANCE INDEX

Even though the center of mass of the robot has been adjusted to the centroid of foot polygon, the balance level of the walking configuration may be different as the shape of the polygon. It is acceptable usually that the more the shape is spacious, the more increases the walking balance. In this sense, we propose a simple performance index for the walking balance based on a distance as follows.

$$I_B = \min(r_j(t)), \ j = 1, \cdots, n \tag{6}$$

where $\min(r_j(t))$ gives the minimum value of $r_j(t)$. The n is assigned by 3 for a triangular polygon and by 4 for a rectangular case. For instance, the distance between the centroid and the jth vertex of a polygon in Fig. 5, $r_j(t)$, is calculated by

$$r_j(t) = \sqrt{\{x_g(t) - x_j(t)\}^2 + \{y_g(t) - y_j(t)\}^2}. \qquad (7)$$

Practically, the value I_B in (6) indicates the minimum distance between the support polygon's centroid and each foot position in a walking configuration. It is finally estimated that the case with large I_B in a comparative study considering many walking styles has better walking balance potentially.

V. WALKING SIMULATIONS

Various fundamental simulations were performed to analyze the quadruped walking performance. Particularly, we focused on the centroid trajectory of every foot polygon for the sequential quadruped-robotic walking specified in Section II. Based on the centroid trajectory and the proposed performance index, we tried to estimate the robot's walking balance.

A. Task Assigned

The assigned task to the robot in Fig. 2 was to walk on a planar space by employing some particular walking patterns as shown in Fig. 3. Though many walking patterns can be made in the four-legged robotic walking, this paper particularly considers the walking examples starting from the first leg shown in Fig. 3. Those are classified as six cases summarized in Table I.

TABLE I
WALKING EXAMPLES STARTING FROM THE FIRST LEG.

Case	Order of foot step	Remarks
I	f1 → f2 → f3 → f4 → f1	See Fig. 3
II	f1 → f2 → f4 → f3 → f1	
III	f1 → f3 → f4 → f2 → f1	
IV	f1 → f3 → f2 → f4 → f1	
V	f1 → f4 → f2 → f3 → f1	
VI	f1 → f4 → f3 → f2 → f1	

The assigned task requires each foot to follow a planar trajectory controlled by each leg. The planar x- and y-directional trajectories are planned by

$$x_i(t+dt) = x_i(t) + p_i\text{rand}(1), \; i = 1, 2, \cdots, 4 \qquad (8)$$
$$y_i(t+dt) = y_i(t) + p_i\text{rand}(1), \; i = 1, 2, \cdots, 4 \qquad (9)$$

where rand(1) is a random function to generate an arbitrary value in between 0 and 1. The p_i plays a role to adjust the pace of walking, and it was set to 0.05. A sampling time dt for each step motion of a leg was set to 5ms. The initial positions of each foot were assigned in Table II.

B. Case Studies for Quadruped Walking

Since the walking motion of a multi-legged robot is basically made by multiple legs, effective coordination of legs is required for stable walking. Usually, four legs contributes to a quadruped robotic walking and their internal coordination is

TABLE II
INITIAL POSITIONS OF FOUR FOOTS.

Foot	x position(m)	y position(m)	Remarks
f1	0.5	0.9	A rectangular posture :See Fig. 4-A
f2	0.5	0.5	
f3	0.8	0.5	
f4	0.8	0.9	

also very important for constructing a stable posture. If a leg in the quadruped robotic walking is up in the air for a step, the stability of the current walking posture is mainly related to the other three legs. In this situation, if the coordination of three legs is not balanced properly, the walking motion may be fallen.

To deal with such a walking problem, we basically considered a centroid-based walking strategy that the robot tries to adjust its center of mass to be projected inside of the polygon formed by supporting foots for the robot's structural balance. We also focused on the centroid of foot polygon to estimate the quadruped walking behavior. In particular, when the walking speed of a multi-legged robot is slow, it is reasonable to consider its walking in a static mode. In this case, if the entire center of mass of a multi-legged robot should be projected to the centroid of its foot polygon, it is possible for the robot to be with a balanced gait posture. In this point of view, we performed various walking simulations to analyze the behavior of the quadruped robot shown in Fig. 2. For comparative analysis of the walking behavior for all cases in Table I, a reference walking trajectory with 100 steps for respective foot was assigned as shown in Fig. 6. The reference trajectory was actually made by (8) and (9), and it was used for all case studies.

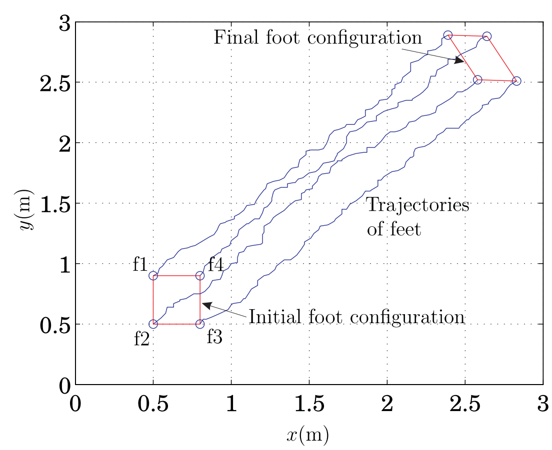
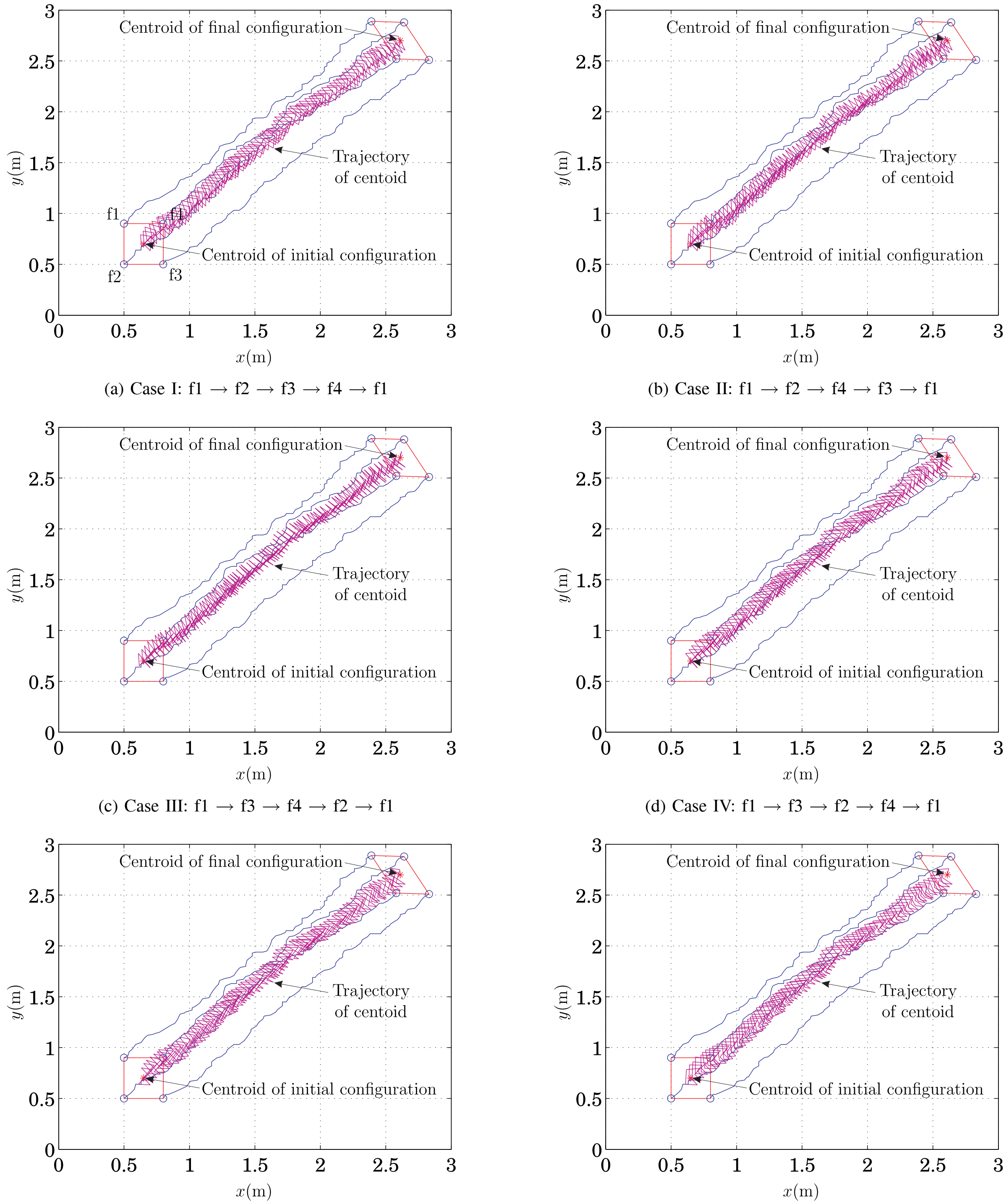


Fig. 6. Reference foot trajectory for the walking task.

Fig. 7 shows the centroid trajectory for each walk assigned in Table I. The centroid was actually obtained by using (2) ∼ (5) for each step. Specifically, Fig. 7(c) shows the result for the Case I where the robot goes on the following step order: f1 → f2 → f3 → f4 → f1, repeatedly. Since four legs are stationary at the initial state, it is definitely found that the initial centroid of the foot polygon is located in the



(a) Case I: f1 → f2 → f3 → f4 → f1

(b) Case II: f1 → f2 → f4 → f3 → f1

(c) Case III: f1 → f3 → f4 → f2 → f1

(d) Case IV: f1 → f3 → f2 → f4 → f1

(e) Case V: f1 → f4 → f2 → f3 → f1

(f) Case VI: f1 → f4 → f3 → f2 → f1

Fig. 7. Centroid trajectory of foot polygons during the walking task.

center of the rectangle. As the robot walks, all of centroid trajectories are being wriggled in another form. This means that the centroid trajectory depends on the walking foot order. Theoretically, the wriggling trajectories imply that the four-legged walking robot walks with a waddle as a duck during those walks assigned. In fact, they are not strange behavior, and such a wriggling behavior is naturally occurred for the structural balance of the robot. It is because the walking robot is basically necessary to adjust its whole motion for a walk so as not to be fallen. Nevertheless, those motions are not desirable in practice. That is why abrupt change of the centroid trajectory is not allowable for dextrous walking and such behavior requires an excessive walking. Consequently, it is noted from the trend of Fig. 7 that the walking behavior of the robot can be more balanced by proper scheduling of the walking order.

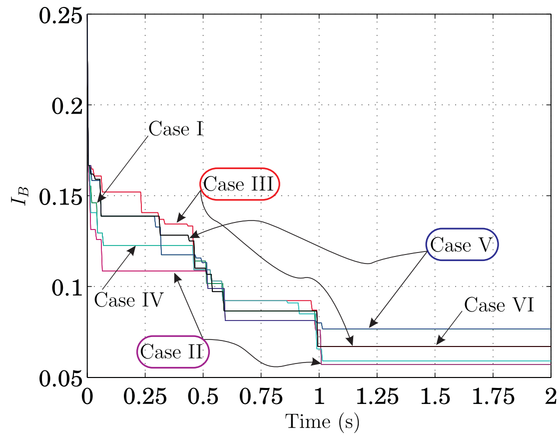


Fig. 8. Balance index for the walking task.

Next, to estimate what style gives better walking balance, we tried to check the performance index defined in (6) for the walking task. Fig. 8 shows the walking balance indices for the six walks assigned in Table I. From Fig. 8, we can confirm that the Case III (f1 → f3 → f4 → f2 → f1) walking style defined in Table I maintains the best balance in the front part of time, but the Case V (f1 → f4 → f2 → f3 → f1) is better than the others in the rear part. Also, the overall walking balance of the Case II (f1 → f2 → f4 → f3 → f1) is the lowest for the walking task.

C. Discussions

In a multi-legged task, we basically desire that the walking motion of the robot should be controlled by minimizing any unnecessary waddling. One of major reasons is that such a waddling may disturb the control performance of the walking robot. Theoretically, if the center of mass of the robot is to be projected to the centroid of foot polygon in every step, it is possible to guarantee the walking balance in each step. So, in order to identify such a waddling phenomenon in a quadruped robotic walking. it is valuable to check the centroid trajectory of foot polygon formed in every walking step. On the other hand, it is noted from Fig. 7 that a waddling body motion is required intentionally to adjust its walking balance. Also, the walking balance of the quadruped robot may be different according to the walking pattern employed. Therefore, for the best walking balance, it is required to find a realtime walking pattern during the walking process. From Fig. 8, it is noted that the Case III walking style can be considered as a reasonable walking pattern for the quadruped robot.

VI. CONCLUSIONS AND FUTURE WORKS

We presented a model of quadruped walking and provided a centroid-based analysis of the model's walking balance. The centroid of foot polygons formed in a quadruped robotic walking. was identified to analyze the walking balance. Also, the walking balance of the quadruped robot was estimated by employing a simple performance index. Through various simulations, we confirmed that a quadruped locomotion requires a proper waddling motion to walk in a balanced way. Also, a quadruped walking balance depends on the walking pattern of the robot employed. As a result, it is concluded that a real-time reasonable selection of walking pattern is necessary to maintain the best walking balance and facilitate an excessive dynamic walking motion. This is actually related to the scheduling of multiple legs for the stable performance of simple entertainment robots [12] as well as multi-legged walking robots for complex tasks [9]. Especially, it is expected that the changeover walking style such as the Case III can be utilized as a useful walking pattern for the quadruped robot. In the bio-mimetic aspect, it is somewhat interesting observation that a baby usually crawls by the walking style. We also expect that the current quadruped walking analysis is available for dextrous walking of hexapedal robots [7].

In addition, overall analysis of four- or six-legged walking mechanism including optimal motion planning, energy expenditure and additional performance index is available for legged mobile manipulations.

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