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Abstract

This article presents the simplest creeping gait (creeping gait with one center-of-gravity movement in a cycle) for a quadruped robot. The creeping gait with one center-of-gravity movement is efficient in reducing the complexity of gait planning and the control of quadrupeds. To find the simplest creeping gait, the geometrical model of a quadruped is constructed, and the omni-directional stability margin is derived to determine the stability. Based on the features of creeping gaits, the simplest possible gait is analyzed. The mathematical description is used to describe the simplest gait with the maximum omni-directional stability margin. Details of the creeping gait, including its initial pattern and its sequences, are provided. In a cycle of the creeping gait with one center-of-gravity movement, the center of gravity needs to move only once. Only 16 commands are required to move a quadruped with two degrees of freedom in each leg. An experiment conducted on the THU-WL robot proves that the gait is reliable and stable. The creeping gait with one center-of-gravity movement is a remarkable simplification for the creeping gait.

Keywords

Quadruped, creeping gait, center of gravity

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Introduction

Various quadruped robots have been developed and possible gaits have been studied in the past. There is an extensive literature on analyzing and computing stable walking patterns for legged robots, including bipeds, quadrupeds, and hexapods. Yi[1] proposed a gait and a control algorithm for the quadruped robot pet commercialized by Dasarob. Chang et al.[2] analyzed the kinematics-based gait planning of a gecko-like quadruped. Igarashi et al.[3] provided a free gait with posture control of a quadruped. The creeping gait, which is simple and stable, is well known in gait planning for quadrupeds.

A creeping gait requires at least three legs of a quadruped to be in contact with the ground at all times, and its support patterns are either triangles or quadrilaterals.[4] This gait is widely used when the quadruped is cumbersome or when the quadruped cannot walk very fast. The creeping gait is the most studied because it is a highly stable gait for a quadruped. Several researchers have analyzed its stability. McGhee[4] and McGhee and Frank[5] started their work in this field in the 1960s. They found that only 6 in 5040 kinds of gaits were non-singular creeping gaits for the quadrupeds. McGhee used the longitudinal stability margin to analyze the quadrupeds and obtained the optimum stability based on the duty factor. Song and Waldron[6] provided an analytical approach to study the locomotion. Their approach was more efficient and could be applied to a $2n$-legged wave gait. Chen et al.[7] considered the adjustment of the center of gravity (COG) and proposed adaptive gait planning for multi-legged robots. They used the compensation and reachable areas to determine the area for the COG and the leg which improved on McGhee's work. A graph search with a diagonal line was used by Pack and Kak[8] and Pack and Kang[9] to present a simplified forward gait control for a quadruped. In their work, the COG of the quadruped in the creeping gait moves only twice, and this was simpler than McGhee's proposed gait. Pack and Kak[8] observed

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that the COG of a quadruped should move eight times in a cycle of McGhee's gait planning and 68 commands are required. They believed that McGhee's gait is too complex and proposed a simplified gait with the sequence 'leg move – leg move – COG move – leg move – leg move – COG move' which requires only two movements of the COG.[8] The robot in Pack's experiment had three degrees of freedom (DOFs) for each leg. Therefore, the total number of commands needed in a cycle is 28, which is less than in McGhee's gait.

The creeping gait's complexity, from McGhee's eight COG movements with 68 commands to Pack's two COG movements with 28 commands in a cycle, has been significantly reduced. However, it is believed that there must be a gait with one COG movement which is supposed to be the simplest creeping gait for quadrupeds. A creeping gait, which is called the Creeping gait with One Center-of-gravity Movement in a cycle (COCM), is presented in this article. In the COCM, the COG needs to move only once in a cycle. Moreover, the details of the COCM with the maximum omni-directional stability margin are analyzed as well.

In the following section, a typical quadruped model with the necessary assumptions is provided and the omni-directional stability margin is developed as the criterion of a quadruped's stability. Then, the method used to solve COCM is described. Next are the results with the initial pattern and sequence of COCM, as well as the experiments on THU-WL quadruped. Subsequently, analyses of the robot dimensions and an irregular workspace are discussed. Finally, 'Conclusions' are given.

Model

Assumptions for a quadruped model

Many different types of quadrupeds have been developed, and it is impossible to explain all the characteristics of them here. To study the general rules of quadrupeds, however, a typical quadruped model with some widely accepted characteristics is adopted to simplify the analysis. These characteristics include the following.

1. The typical quadruped moves in a steady-state, constant-speed locomotion, in a straight line over a horizontal supporting surface with the legs of the system cycling periodically in both space and time. This rule is given in the works of many authors.[5,6,8]
2. The typical quadruped is symmetrical, both in the longitudinal and lateral directions. This rule is adopted because the same structure for each leg can reduce the complexity of the mechanical design and the cost.[2,8,10]
3. The mass of the quadruped is centered in the geometric center of the quadruped, which is known as the COG. This assumption is reasonable because the symmetrical structure and the mass of the body is usually several times more than that of the legs; it is used in the works of many authors.[3,9,11]

Figure 1 shows a model of a typical quadruped. It possesses the above-described characteristics. Its workspace is symmetrical both in the longitudinal and lateral directions; the center of mass of the typical quadruped is located at its geometrical center. The quadruped remains stable if and only if the omni-directional stability margin is greater than zero. In Figure 1, the vertical projection of the workspace for each leg is a line. These lines are oriented in the longitudinal direction to satisfy the requirement of forward motion. Similar to Pack's research, the line workspace is used as an example to study the COCM in this article.

In Figure 1, a is the distance from the COG of the quadruped to the nearest point of the workspace for each leg, b the distance from the COG to the farthest point of the leg's workspace, and c the width of the two front or rear legs' workspace, and the workspace for each leg is $(b-a)$. Therefore, the largest stride for this quadruped is $(b-a)$. For most quadrupeds, $2b$ and $2c$ are approximately the length and the width of the robot, respectively. Hence, the length-to-width ratio is c/b.

Omni-directional stability margin

To find a possible creeping gait, stability is the first consideration. The stability margin is a criterion used to describe the stability of a robot, and it is crucial for a

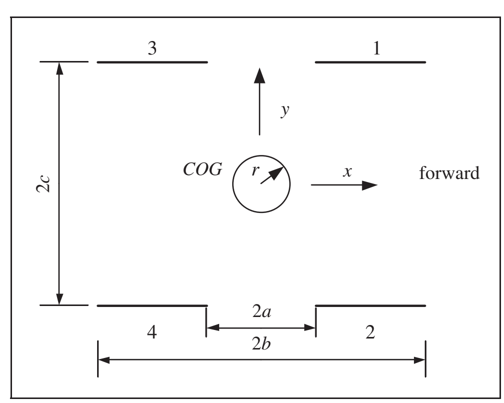

Figure 1. A typical quadruped model.
The model is symmetrical and each leg has a similar workspace.

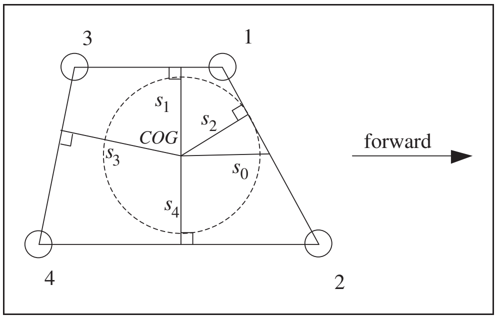

Figure 2. The omni-directional and longitudinal stability margins.

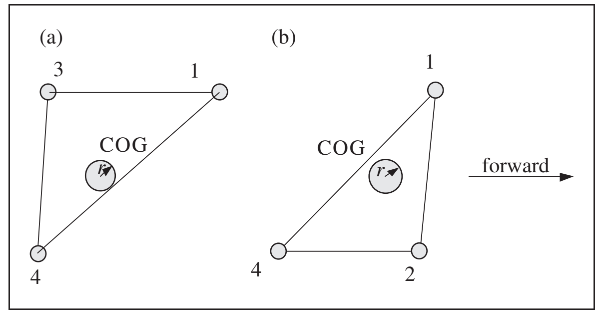

Figure 3. (a) When leg 2 is lifted, the COG is on the left side of the diagonal 1–4 and (b) when leg 3 is lifted, the COG is on the right side of the diagonal 1–4.
COG: center of gravity.

practical gait. Researchers have adopted various standards to assess robot stability. Igarashi et al.[3] and McGhee and Frank[5] used the longitudinal stability margin which was defined as the shortest distance from the body's COG to the boundary of the support pattern in the forward direction. Pack and Kak[8] believed that the diagonals of the support pattern are critical to the stability of a quadruped, and they used the diagonals as the stability admitting lines to determine the stability of a quadruped. However, when walking on the ground, a quadruped may also fall in other directions. The omni-directional stability margin used in this article is the shortest distance between the COG of a robot and the boundary of its support pattern. The mathematical model of a quadruped is shown in Figure 2, where s_0 is the longitudinal stability margin used by McGhee and s_1–s_4 the stability margins for each edge of the support pattern.

The omni-directional stability margin r is defined as $r = \min(s_1, s_2, s_3, s_4) = s_2$, and the stability margin cycle is drawn with a dashed line. In the ideal case, the quadruped is stable when it walks if and only if $r > 0$. However, this never happens in reality for several reasons, such as the differences between the actual robot and the mathematical model, e.g. the COG of the quadruped cannot be exactly the same as that in mathematical model. Therefore, an allowance is necessary when planning a gait. If the accepted allowance between the COG of the actual robot and the COG in the mathematical model is $r_0 (r > 0)$, the mathematical model may be used to replace the real robot. In that case, the criterion changes to the following: the gait is stable if and only if $r > r_0$ during the entire time of locomotion; otherwise, the gait is unstable.

Method

In a COCM cycle, the COG moves only once, followed by the four legs moving one by one. As in Pack's approach, the legs and the COG should not move at the same time.[8] For a better understanding of the model, the time at which the COG stops moving is chosen as the start of a cycle, and the origin of the coordinates is the location of the COG. The forward direction is along the x-axis, and the y-axis is perpendicular to the x-axis. In the original support pattern, the locations of the four legs are (x_1, c), $(x_2, -c)$, (x_3, c), and $(x_4, -c)$, and the stride of the gait is l. Therefore, the locations of each leg after the movements are $(x_1 + l, c)$, $(x_2 + l, -c)$, $(x_3 + l, c)$, and $(x_4 + l, -c)$. Obviously, all the locations should be in the workspace

$$\begin{cases} a \leqslant x_1 \leqslant b \\ a \leqslant x_2 \leqslant b \\ -b \leqslant x_3 \leqslant -a \\ -b \leqslant x_4 \leqslant -a \\ a \leqslant x_1 + l \leqslant b \\ a \leqslant x_2 + l \leqslant b \\ -b \leqslant x_3 + l \leqslant -a \\ -b \leqslant x_4 + l \leqslant -a \end{cases} \tag{1}$$

In the creeping gait, there are at least three legs on the ground and the COG will move one stride length in one cycle. Considering the movement of the COG in a cycle (Figure 3), when leg 2 is lifting off the supporting surface, the COG must be located within the triangle pattern 1–3–4; in other words, the COG is on the left side of the diagonal 1–4. When leg 3 is lifting off the supporting surface, the COG is within the triangle pattern 1–2–4 or on the right side of the diagonal 1–4. This suggests that there must be a time when the COG moves from the left side of the diagonal 1–4 to the right side.

In Figure 3(a), the support pattern is 1–3–4, and the locations of legs 1 and 4 are (x'_1, c) and $(x'_4, -c)$, respectively. Once the support pattern changes to

1–2–4, the locations are (x_1'', c) and $(x_4'', -c)$. The stride is l, so the coordinates meet the following requirements

$$x_1' x_1'' \in \{x_1, x_1 + l\} \tag{2}$$

$$x_4' x_4'' \in \{x_4, x_4 + l\} \tag{3}$$

When the COG is on the left of diagonal 1–4, the distance from the COG to the diagonal, or the stability margin, is

$$d' = \frac{c\left|x_1' + x_4'\right|}{\sqrt{(x_1' - x_4')^2 + 4c^2}} \tag{4}$$

When the COG is on the right side of diagonal 1–4, the stability margin is

$$d'' = \frac{c\left|x_1'' + x_4''\right|}{\sqrt{(x_1'' - x_4'')^2 + 4c^2}} \tag{5}$$

The omni-directional stability margin in a cycle is

$$r = \min(d', d'') \tag{6}$$

Various x_1, x_2, x_3, x_4 and l would define various omni-directional stability margins. For the maximum stability, the maximum value of the omni-directional stability margin is analyzed which can be described as

$$r_{\max} = \max(\min(d', d'')) \tag{7}$$

Results

The initial state pattern

Equations (1) to (7) give the coordinates of x_1, x_2, x_3, x_4 and the stride l that lead to the maximum stability margin $r_{\max}$. Equation (8) shows optimal values of x_1, x_2, x_3, x_4 and l, which are calculated for the maximum stability margin by the MATLAB program. In equation (8), the stride l is maximized on the premise of requirements of equation (7), thus the gait has high motion efficiency

$$\begin{cases} x_1 = x_p \\ x_2 = a \\ x_3 = -b \\ x_4 = -a - b + x_p \\ l = b - x_p \\ r_{\max} = \dfrac{c(x_p - a)}{\sqrt{(a + x_p)^2 + 4c^2}} \end{cases} \tag{8}$$

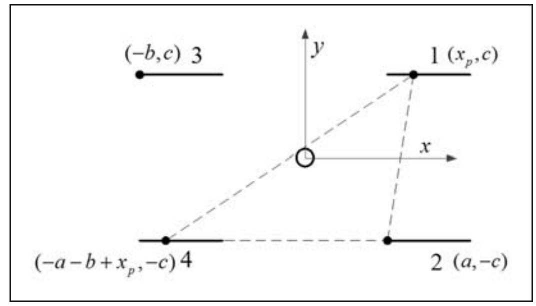

Figure 4. The initial state pattern of the COCM.
COCM: Creeping gait with One Center-of-gravity Movement.

where

$$x_p = \left(\sqrt{q^2 - p^3} - q\right)^{\frac{1}{3}} - \frac{a}{3} + \frac{p}{\left(\sqrt{q^2 - p^3} - q\right)^{\frac{1}{3}}} \tag{9}$$

$$p = \frac{4}{9}a^2 + \frac{ab}{3} - c^2 \tag{10}$$

$$q = \frac{a(a^2 + ab - 3c^2)}{6} - ac^2 + \frac{bc^2}{2} - \frac{a^3}{27} \tag{11}$$

If $r_{\max} \geqslant r_0$, then the COCM will be stable. The starting support pattern is shown in Figure 4 with the following coordinates: COG (0, 0), leg 1 (x_p, c), leg 2 $(a, -c)$, leg 3 $(-b, c)$, and leg 4 $(-a - b + x_p, -c)$.

The sequence of the gait

Figure 5 shows the details of COCM, with the initial state pattern shown in Figure 4. From the initial state pattern, the quadruped moves its legs in sequence 'leg 3–leg 4–leg 2–leg 1–COG (along with the legs reposition).' The stride follows the distance shown in equation (8). There are only five phases in a cycle of COCM. Compared with McGhee's and Pack's gaits, it is easy to observe that the COG only moves once in a cycle of the COCM. The reduction in the complexity of the gait will make the quadruped easier to control. Considering the symmetrical geometry of a quadruped and periodicity of the gait, there are other initial state patterns that will have the maximum omni-directional stability margin using the COCM. For example, one may exchange the x-coordinates of leg 3 and leg 1 with leg 4 and leg 2 (Figure 4), then the sequence turns to 'leg 4–leg 3–leg 1–leg 2–COG'; or one may choose the pattern in Figure 5(b) as an initial state pattern, then the sequence turns to 'leg 4–leg 2–leg 1–COG–leg 3.' However, all these sequences share the same stability margin and stride length determined in equation (8).

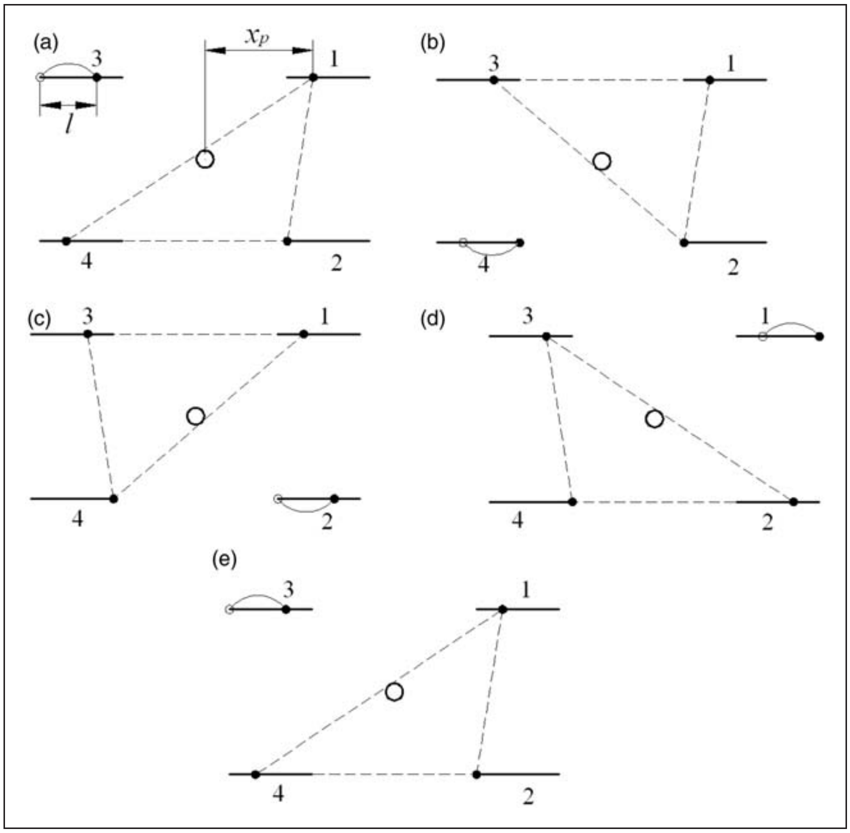

Figure 5. The COCM with the maximum omni-directional stability margin.
The sequence of the movements is 'leg3–leg 4–leg 2–leg 1–COG' based on the initial support pattern being chosen. (a) The initial state pattern. The coordinates of the parts are COG (0, 0), leg 1 (x_p, c), leg 2 $(a, -c)$, leg 3 $(-b, c)$, leg 4 $(-a - b + x_p, -c)$. The first moving leg is leg 3. Leg 3 moves forward one stride length while the other legs and the COG remain stationary, and the coordinates change to: COG (0, 0), leg 1 (x_p, c), leg 2 $(a, -c)$, leg 3 $(-b + l, c)$, and leg 4 $(-a - b + x_p, -c)$. (b) In the second step, leg 4 moves forward one stride length while the other legs and the COG remain stationary, and the coordinates change to: COG (0, 0), leg 1 (x_p, c), leg 2 $(a, -c)$, leg 3 $(-x_p, c)$, and leg 4 $(-a, -c)$. (c) In the third step, leg 2 moves forward one stride length while the other legs and the COG remain stationary, and the coordinates change to: COG (0, 0), leg 1 (x_p, c), leg 2 $(a + b - x_p, -c)$, leg 3 $(-x_p, c)$, and leg 4 $(-a, -c)$. (d) In the fourth step, leg 1 moves forward one stride length while the other legs and the COG remain stationary, and the coordinates change to: COG (0, 0), leg 1 (b, c), leg 2 $(a + b - x_p, -c)$, leg 3 $(-x_p, c)$, leg 4 $(-a, -c)$. (e) After all four legs have moved, the COG moves forward one stride length while all the legs remain stationary (only the feet remain stationary; the joints of each leg will rotate through some angle to adjust the stability of the COG) and the coordinates, relative to COG, change to (x_p, c), leg 2 $(a, -c)$, leg 3 $(-b, c)$, leg 4 $(-a - b + x_p, -c)$, so that they have returned to the starting support pattern and the quadruped can repeat the cycle.
COCM: Creeping gait with One Center-of-Gravity Movement and COG: center of gravity.

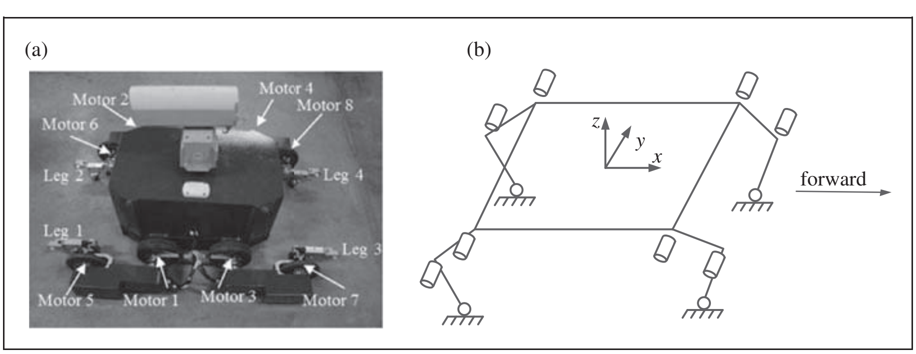

Figure 6. THU-WL robot (a) and its sketch (b).
It has eight motors, two for each leg.

Experiments

Figure 6 shows the THU-WL robot which has four legs. It additionally shows a sketch of the geometry of the THU-WL. There are three rotary joints (shoulder, knee, and foot) in each of its legs. According to the Grübler–Kutzbach criterion, if the THU-WL has four legs on the ground, the DOFs of the quadruped's body total $3(10-12-1)+12=3$, and if it has three legs on the ground, the DOFs total $3(8-9-1)+9=3$. Therefore, when it walks, the THU-WL can only walk forward or backward because it has no freedom in the lateral direction.

The dimensions of the THU-WL are as follows: $2b=750$ mm, $2c=400$ mm, and $2a=20$ mm. The legs are labeled in Figure 6. The total mass of the THU-WL is 60.7 kg, while the mass of the each leg is 3.4 kg, and the mass distribution is approximately symmetrical which means that the weight of the THU-WL can be regarded as in its geometric center.

From equation (8), the omni-directional stability margin is $r_{\max}=52.3$ mm, where $x_p=120.0$ mm and the stride is $l=255.1$ mm. The starting support pattern is $x_1(120.0, 200.0)$, $x_2(10.0, -200.0)$, $x_3(-375.0, 200.0)$, and $x_4(-265.1, -200.0)$. Figure 7 shows the THU-WL walking with the COCM. COCM has been applied on THU-WL at least 20 times and it works very well. These experiments shows the stability of COCM.

The relative angle of motor 4 when performing the COCM is shown in Figure 8. The angle is obtained from the motor encoder on each leg. When the robot is at the starting support pattern, the angle is zero. In Figure 8, motor 4 moves twice in a cycle. The first time is in step two when leg 4 is forward (interval a) and the second in step five when the COG is moving (interval b). Because there are four legs, the total number of commands for the robot in a cycle is 16.

When THU-WL is performing the COCM, it requires 16 commands per cycle. However, if the quadruped is similar to Pack's robot, with three DOFs for

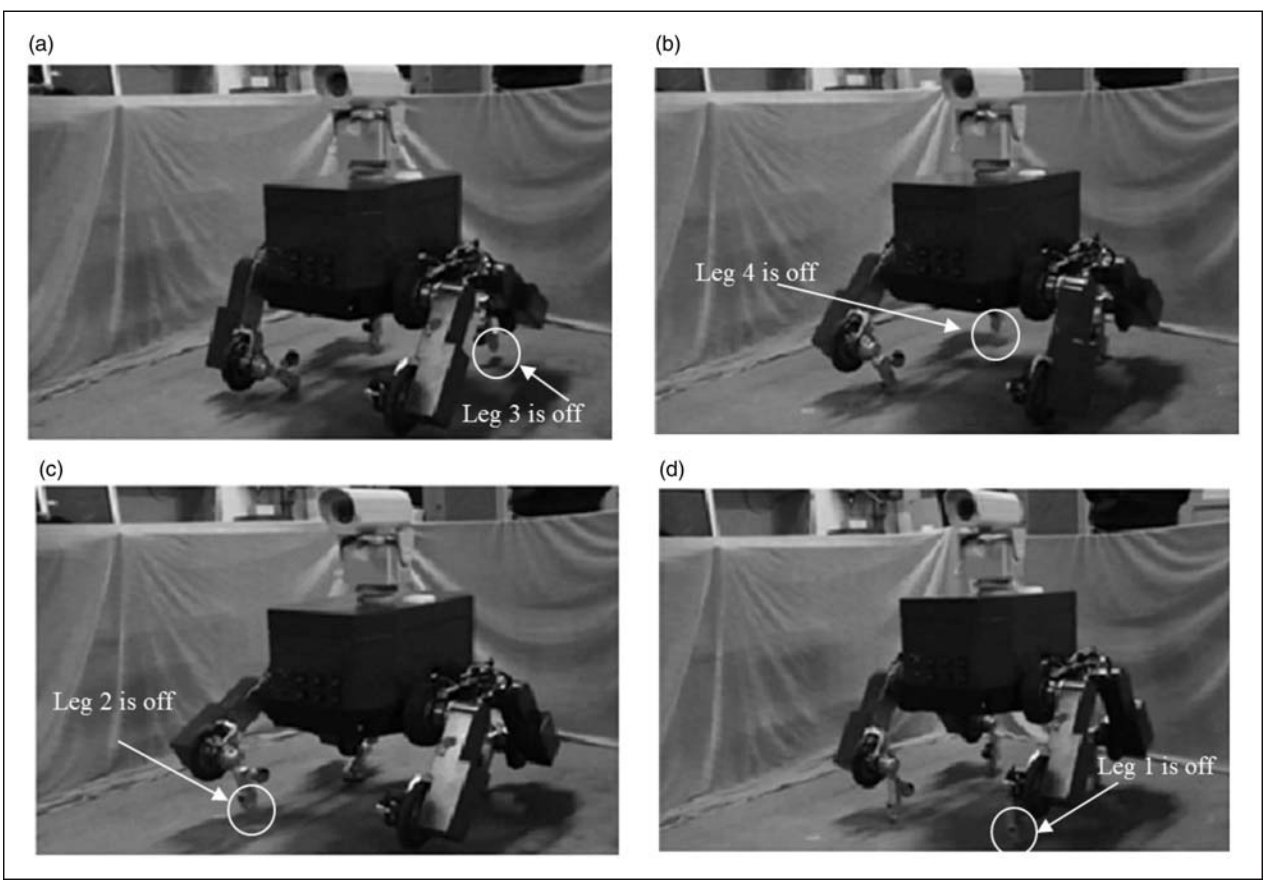

Figure 7. The experiment on the THU-WL with the COCM: (a) in the first step, leg 3 is lifted and about to move a stride; (b) in the second step, leg 4 is lifted and about to move a stride; (c) in the third step, leg 2 is lifted and about to move a stride; and (d) in the fourth step, leg 1 is lifted and about to move a stride.
COCM: Creeping gait with One Center-of-gravity Movement.

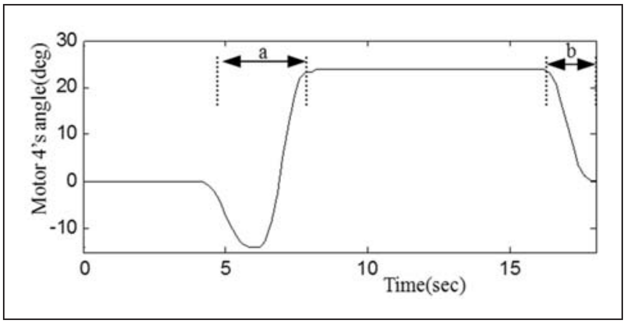

Figure 8. The relative angle of motor 4 in the COCM. In interval a, leg 4 is moving forward, while in interval b, the COG is moving. There are two commands for motor 4 in a cycle. COCM: Creeping gait with One Center-of-gravity Movement and COG: center of gravity.

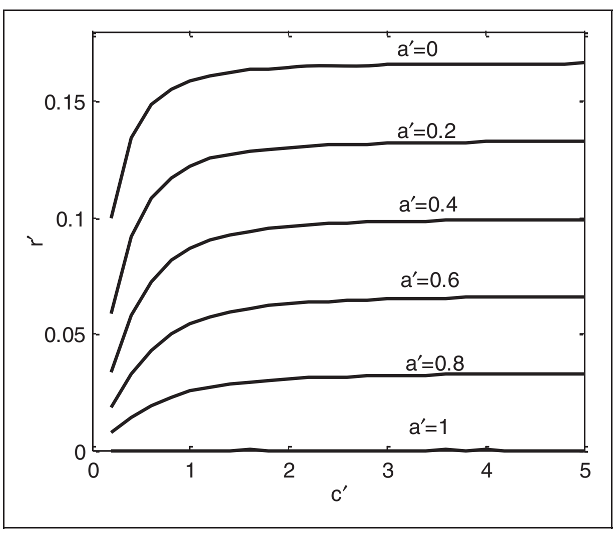

Figure 9. The relationship between the omni-directional stability margin r' and the quadruped's dimensions a' and c', where $b = 1$.

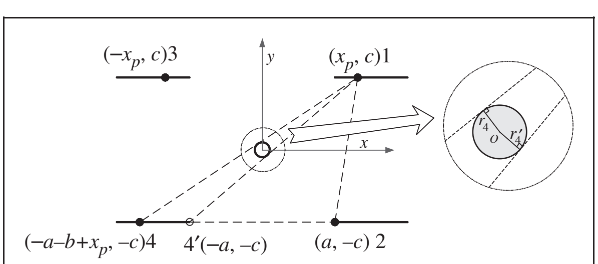

Figure 10. The geometrical analysis of COCM. COCM: Creeping gait with One Center-of-gravity Movement.

each leg and six DOFs for the body, it requires 20 commands to complete a cycle. Compared to the 68 commands in McGhee's gait and 28 commands in Pack's, the COCM requires fewer commands and reduces the control complexity.

Discussion

The influence of the dimensions

In equation (7), $r_{\max}$ depends on the robot dimensions a, b, and c. To find the relationships among these parameters, set $b = 1$ and then a', c', and r' are related to b as follows

$$a' = \frac{a}{b};\ c' = \frac{c}{b};\ r' = \frac{r_{\max}}{b} \tag{12}$$

From equation (1), it is clear that $0 \leqslant a' \leqslant 1$. Here, c' is the length-to-width ratio of a quadruped. Considering a quadruped's geometry in real life, the value of c' is discussed in interval [0.2, 5]. Figure 9 shows the relationship between r', a', and c'.

In Figure 9, r' grows as c' grows, but there exists a turning point at approximately $c' = 1$. After this point, c' has little effect on r'. This characteristic indicates that the width of the quadruped should be approximately equal to its length. This choice of the design ensures that a quadruped will have a sufficiently large r' without increasing the value of c'. In other words, the optimum length-to-width ratio is $c/b = 1$. Figure 9 shows that the smaller the value of a' is, the larger the value of $r_{\max}$ is. Therefore, another criterion for an optimum design is that $a = 0$.

The geometrical analysis

Figure 5 shows that x_p is geometrically related to the initial state pattern, as well as the COCM. It affects both the stability margin and the stride length. The value of x_p determined in COCM is based on the idea that the stability has a higher priority. Figure 10 is the geometrical analysis of x_p. When the support pattern is $1-2-3$, leg 4 will move from the position $(-a-b+x_p, -c)$ to $(-a, -c)$. Computing the distances from the COG to the diagonals $1-4$ and $1-4'$, it shows that the two diagonals are two tangents of the stability margin cycle, and the related stability margins r_4 and r'_4 are equal to $r_{\max}$ which is described in equation (8). It can be observed from Figure 10, on one hand, if x_p increases, r_4 will decrease, and when $x_p = (a+b)/2$, r_4 is zero. Once $x_p > (a+b)/2$, the gait would not be a stable one. On the other hand, if x_p decreases, r'_4 will decrease, and when $x_p = a$, r'_4 is zero. Therefore, from the geometrical point, x_p in equation

(8) also ensures a maximum value for the stability margin.

Conclusions

In this article, the simplest creeping gait for a quadruped robot is analyzed. A possible gait is proposed based on the research of McGhee and Pack. The omni-directional stability margin is used to describe its stability. This gait requires the COG to move only once in a cycle, and it is called as COCM. This gait is easier to apply than the previous gaits and is considered to be the simplest creeping gait for quadrupeds. Given the geometrical model of a quadruped, the starting support pattern, and the stride length, the maximum omni-directional stability margin can be computed. The results show that the length-to-width ratio should be appropriately equal to 1 and the workspace of the leg should close to the COG for the optimum design. The experiment conducted with the THU-WL robot confirmed these findings. These results are applicable to the design of a quadruped or the planning of quadruped walking.

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References

1. Yi S. Reliable gait planning and control for miniaturized quadruped robot pet. *Mechatronics* 2010; 20(4): 485–495.
2. Chang D, Son D, Seo T, et al. Kinematics-based gait planning of a quadruped gecko-like model. In: *Proceedings of the IEEE international conference on robotics and biomimetics (ROBIO)*, Guilin, China, 19–23 December 2009, pp.233–238. New York: IEEE.
3. Igarashi H, Machida T, Harashima F, et al. Free gait for quadruped robots with posture control. In: *Proceedings of the 9th IEEE international workshop on advanced motion control*, Istanbul, 30 May 2006, pp.433–438. New York: IEEE.
4. McGhee RB. Some finite state aspects of legged locomotion. *Math Biosci* 1968; 2(1–2): 67–84.
5. McGhee RB and Frank A. On the stability properties of quadruped creeping gaits. *Math Biosci* 1968; 3(1–2): 331–351.
6. Song SM and Waldron KJ. An analytical approach for gait study and its applications on wave gaits. *Int J Rob Res* 1987; 6(2): 60–71.
7. Chen W, Low K and Yeo S. Adaptive gait planning for multi-legged robots with an adjustment of center-of-gravity. *Robotica* 1999; 17(4): 391–403.
8. Pack DJ and Kak A. A simplified forward gait control for a quadruped walking robot. In: *Proceedings of the IEEE/RSJ/GI international conference on intelligent robots and systems*, Munich, Germany, 12–16 September 1994, vol. 1012, pp.1011–1018. New York: IEEE.
9. Pack DJ and Kang HS. An omnidirectional gait control using a graph search method for a quadruped walking robot. In: *Proceedings of the IEEE international conference on robotics and automation*, Nagoya, Japan, 21–27 May 1995, vol. 981, pp.988–993. New York: IEEE.
10. Albers A and Ottnad J. Integrated structural and controller optimization in dynamic mechatronic systems. *J Mech Des* 2010; 132: 041008.
11. Ma S, Tomiyama T and Wada H. Omni-directional walking of a quadruped robot. In: *Proceedings of the IEEE/RSJ international conference on intelligent robots and systems*, EPFL, Switzerland, 30 September–5 October 2002, vol. 2603, pp.2605–2612. New York: IEEE.

Appendix

Notation

a	the distance from the COG of the quadruped to the nearest point of the workspace
a'	the proportion of a/b
b	the distance from the COG to the farthest point of the leg's workspace
c	the width of the two front or rear legs' workspace
c'	the proportion of c/b
l	stride length
r	the omni-directional stability margin for a creeping gait
$r_{\max}$	the omni-directional stability margin for the creeping gait with the maximum omni-directional stability margin. $r_{\max} = \max(r)$
r'	the proportion of $r_{\max}/b$
x_p, p, q	substitutions for the clarity of the equations
x_1, x_2, x_3, x_4	the x-coordinates for each leg
x'_1, x'_4, d'	the x-coordinates for leg 1 and leg 4, and the distance from the COG to the diagonal when the support pattern is 1–3–4
x''_1, x''_4, d''	the x-coordinates for leg 1 and leg 4, and the distance from the COG to the diagonal when the support pattern is 1–2–4