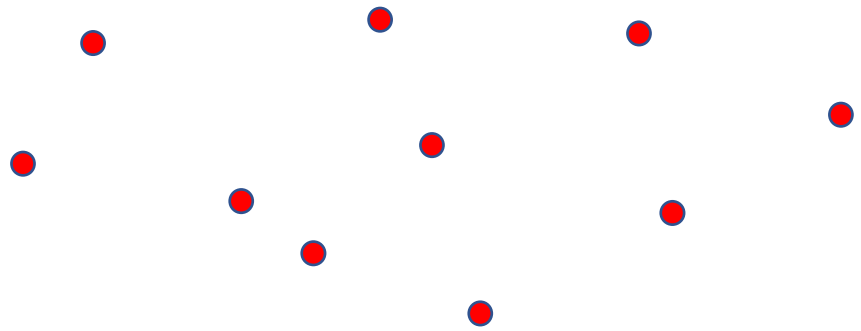
Closest Pair

P12. Closest pair of n points

I/P: Given $P=\{p_1, p_2,\ldots, p_n\}$ a set of n points. Each $p_i=(x_i,y_i)$.

O/P: Find the closest pair of points.



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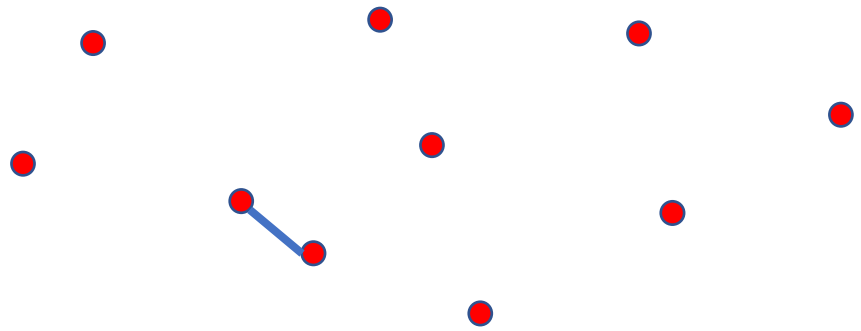

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I/P: Given $P=\{p_1, p_2,\ldots, p_n\}$ a set of n points. Each $p_i=(x_i,y_i)$.

O/P: Find the closest pair of points.

Naïve algorithm:

1. For each point p in P, check all points in P \ {p}.
2. Compute the minimum distances with respect to p, i.e., min_p
3. Repeat step 1 and 2 for all points in P,
4. Finally return min of all min_p's.

Let’s do the problem in 1-Dimension

y-coordinate of all points are same.

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1.Sort each point with respect to x-coordinate.

2. For each point p_i , check two neighbours p_{i-1} and p_{i+1}.

3. Do it for all points and return the minimum.

Let’s do the problem in 1-Dimension

y-coordinate of all points are same.

1.Sort each point with respect to x-coordinate. (doing this to know two adjacent points)

2. For each point p_i , check two neighbours p_{i-1} and p_{i+1}.

3. Do it for all points and return the minimum.

Let’s do the problem in 1-Dimension

y-coordinate of all points are same.

1.Sort each point with respect to x-coordinate. O(nlogn)

2. For each point p_i , check two neighbours p_{i-1} and p_{i+1}.
3. Do it for all points and return the minimum.

O(n)

Can we do via divide and conquer?

Here input is not sorted….. Means the point are not sorted with respect to x-coordinate…. ??

How to apply D&C??

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Intuition is to divide points set in equal halves ??– But how??

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Here input is not sorted….. Means the point are not sorted with respect to x-coordinate…. ??

How to apply D&C??

Intuition is to divide points set in equal halves ??– But how??

We need median to partition in two halves….??

Lets apply the Select(P,n/2) algorithm to partition point set in two halves.

P split in two halves P_L and P_R

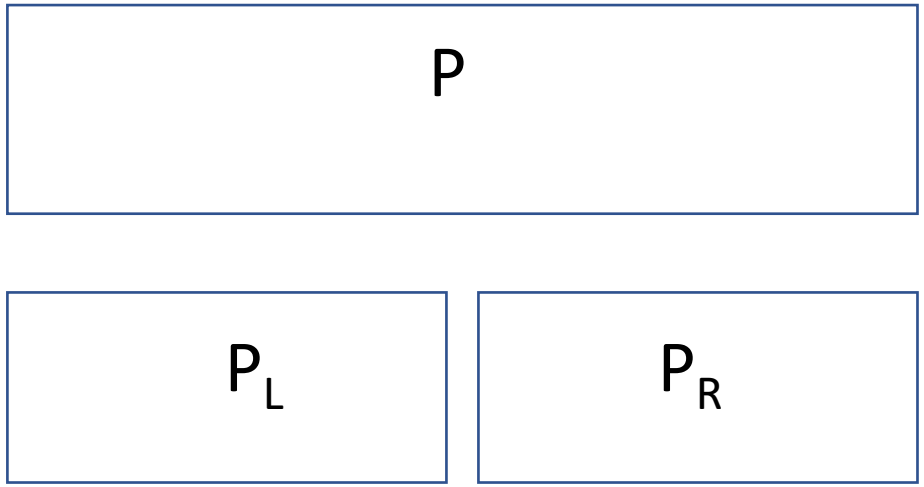
Recurse on both halves.

Lets apply the unknown algorithm to partition point set in two halves.

P split in two halves P_L and P_R.

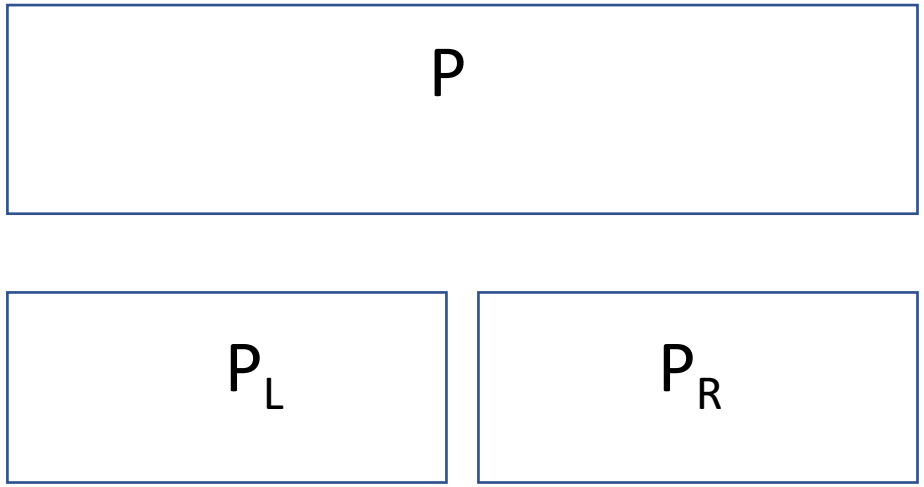
Recurse on both halves.

P_L contains the median point (W.L.O.G)…



Suppose the closest pair in P_L is d_L

the closest pair in P_R is d_R



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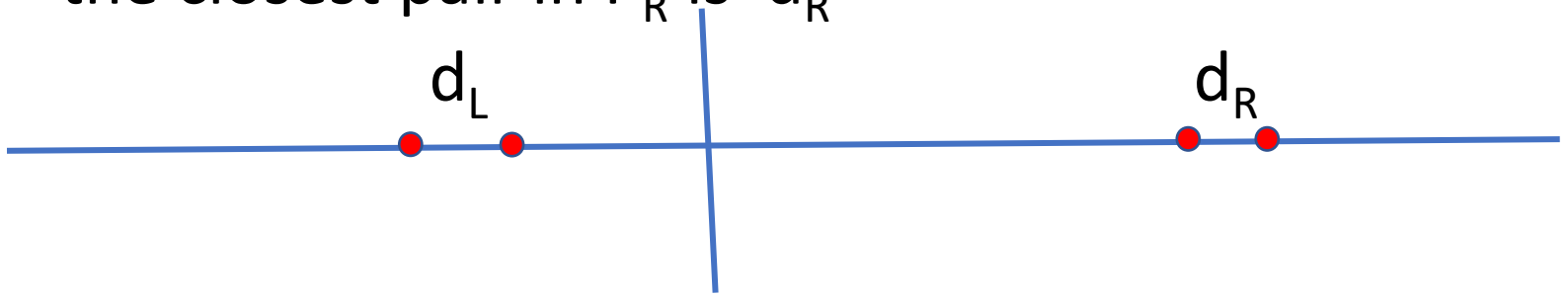
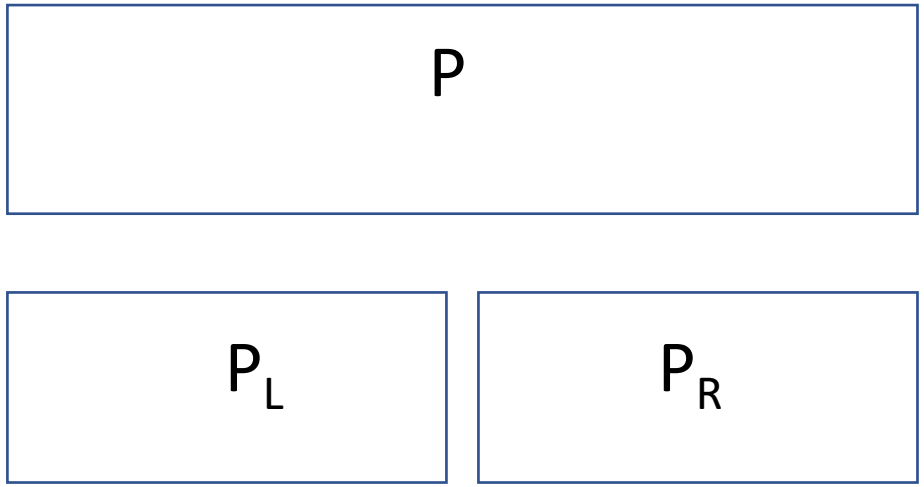
$s=\min(d_L, d_R)$





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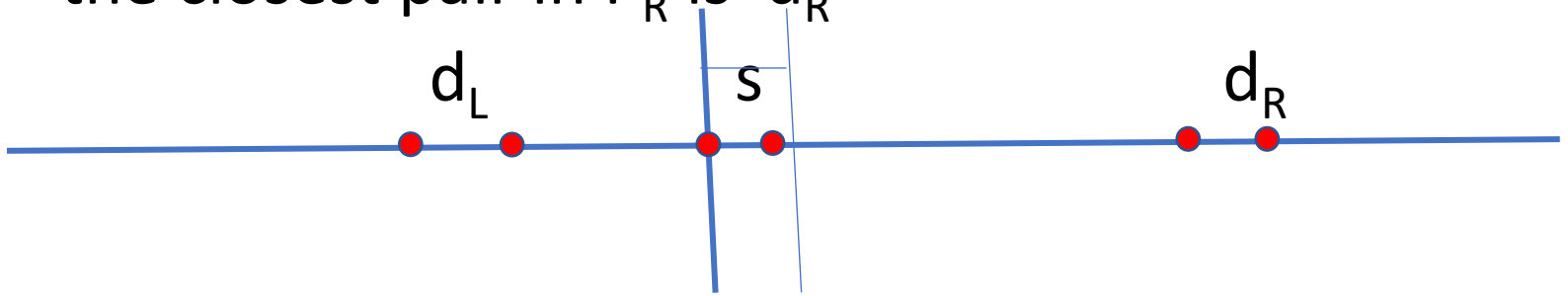

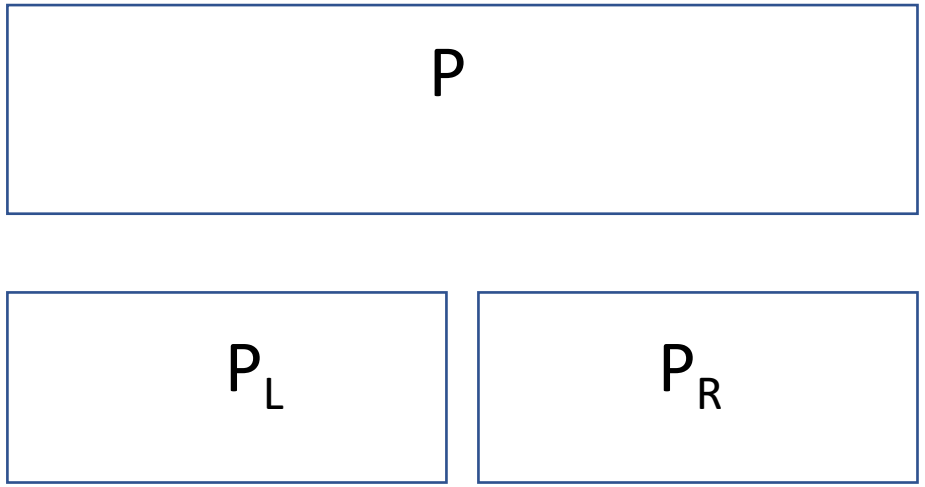

complexity??

Suppose the closest pair in P_L is d_L

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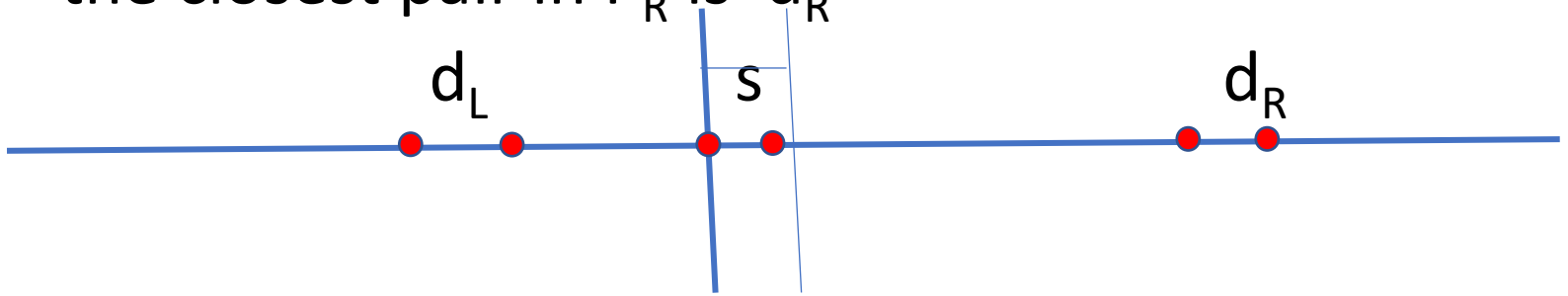

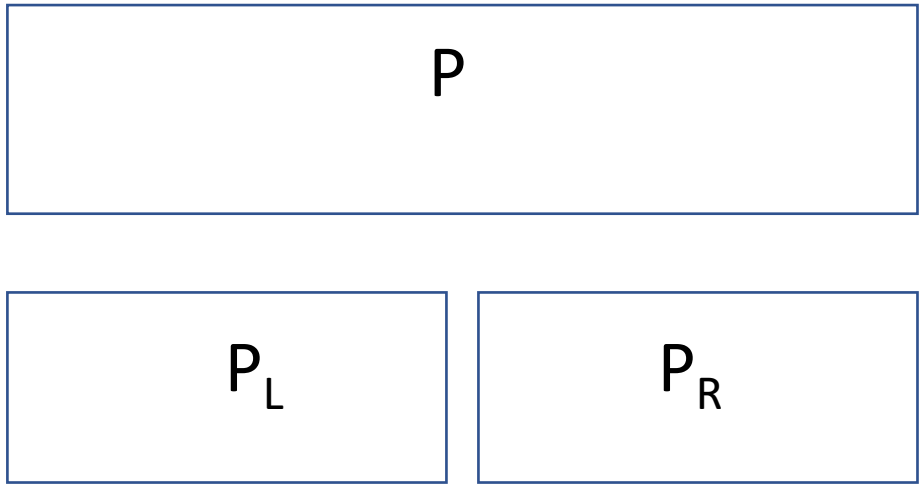

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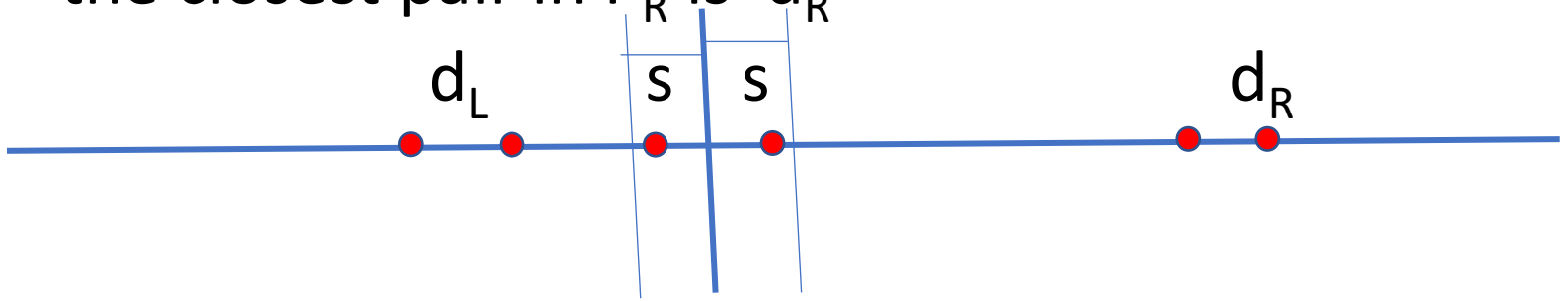
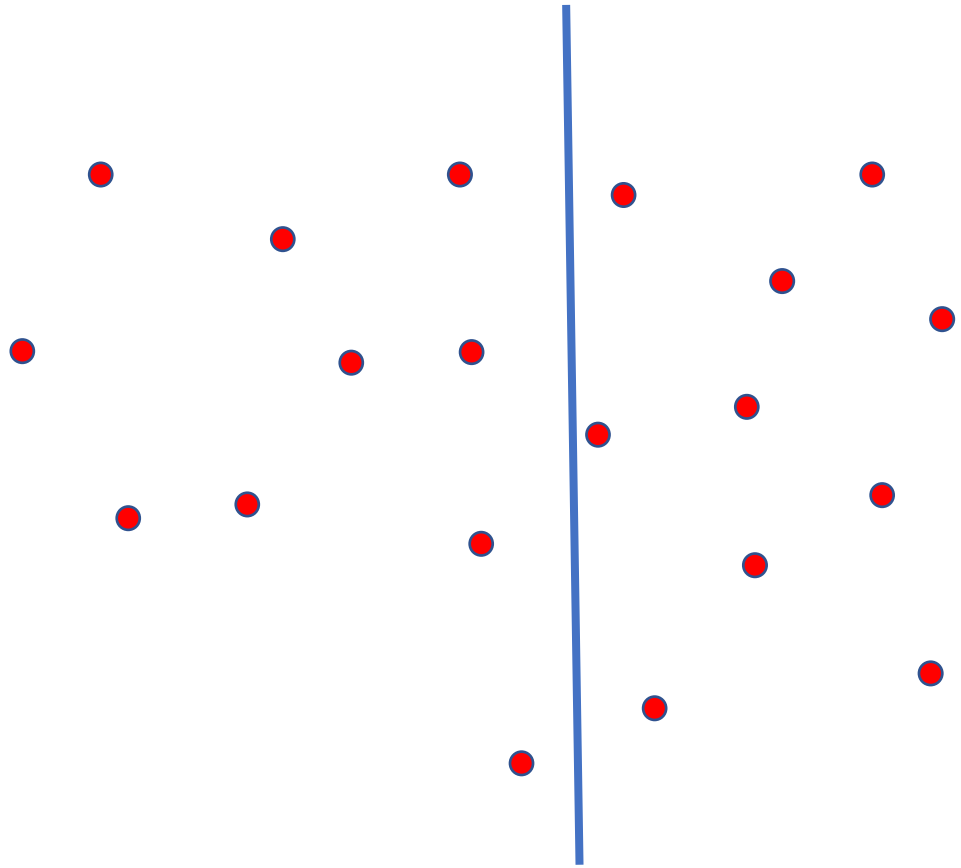


In 2D



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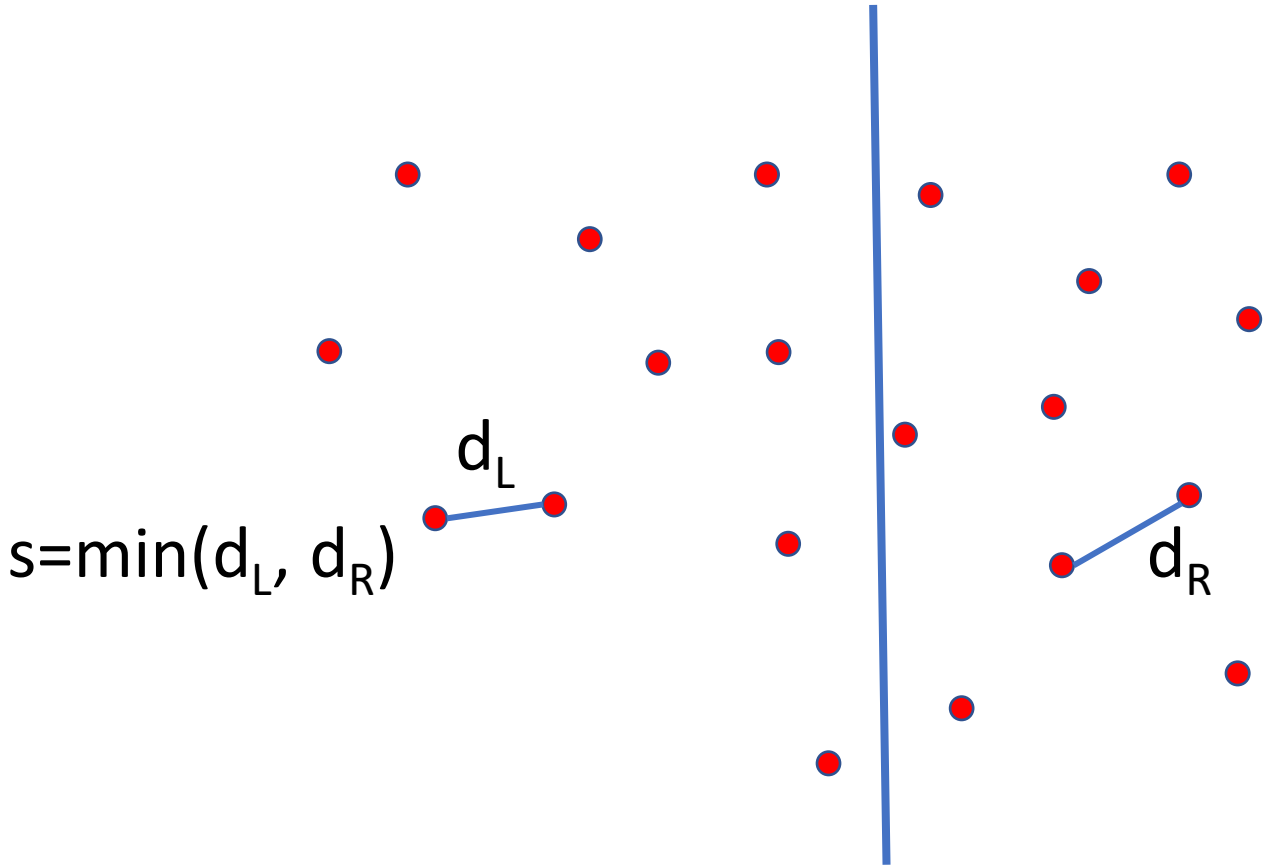

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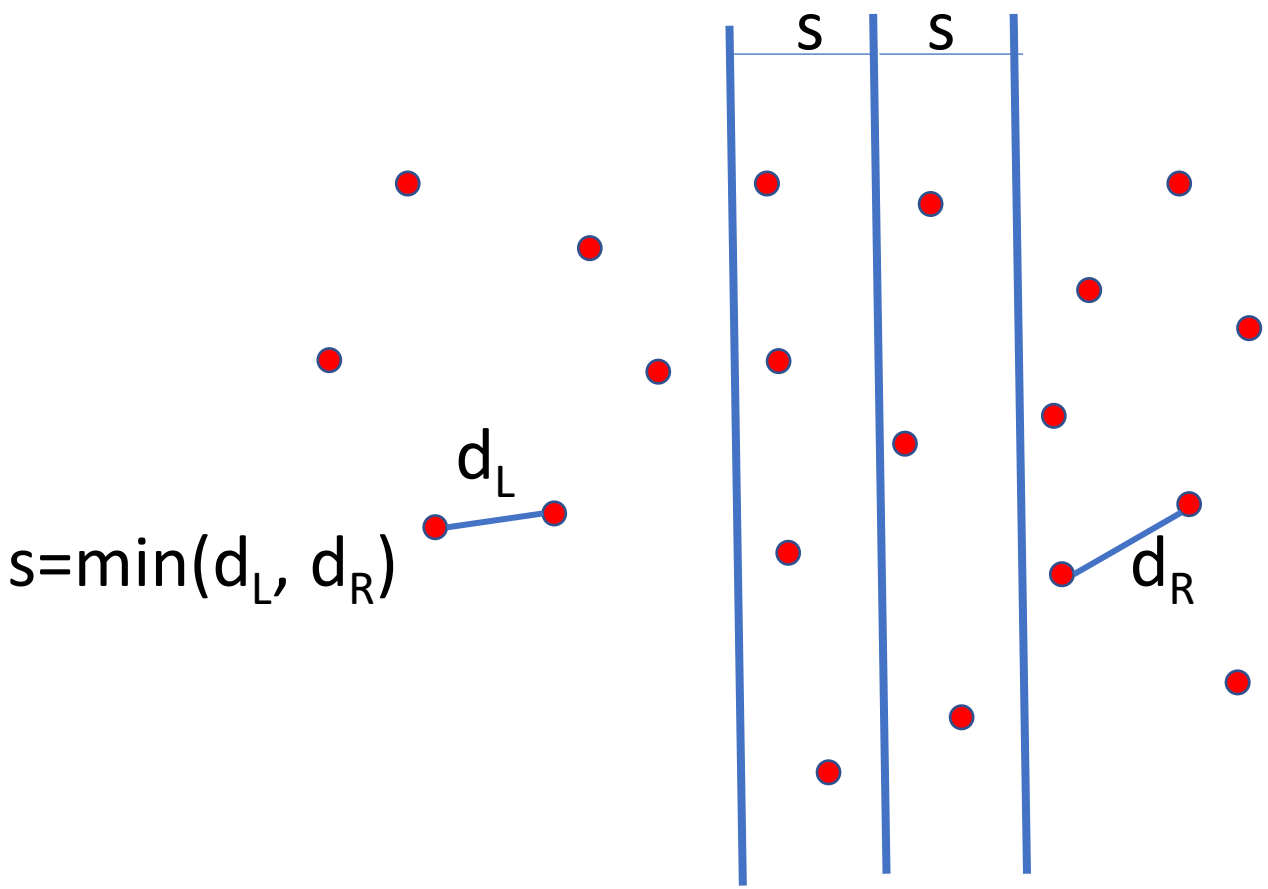

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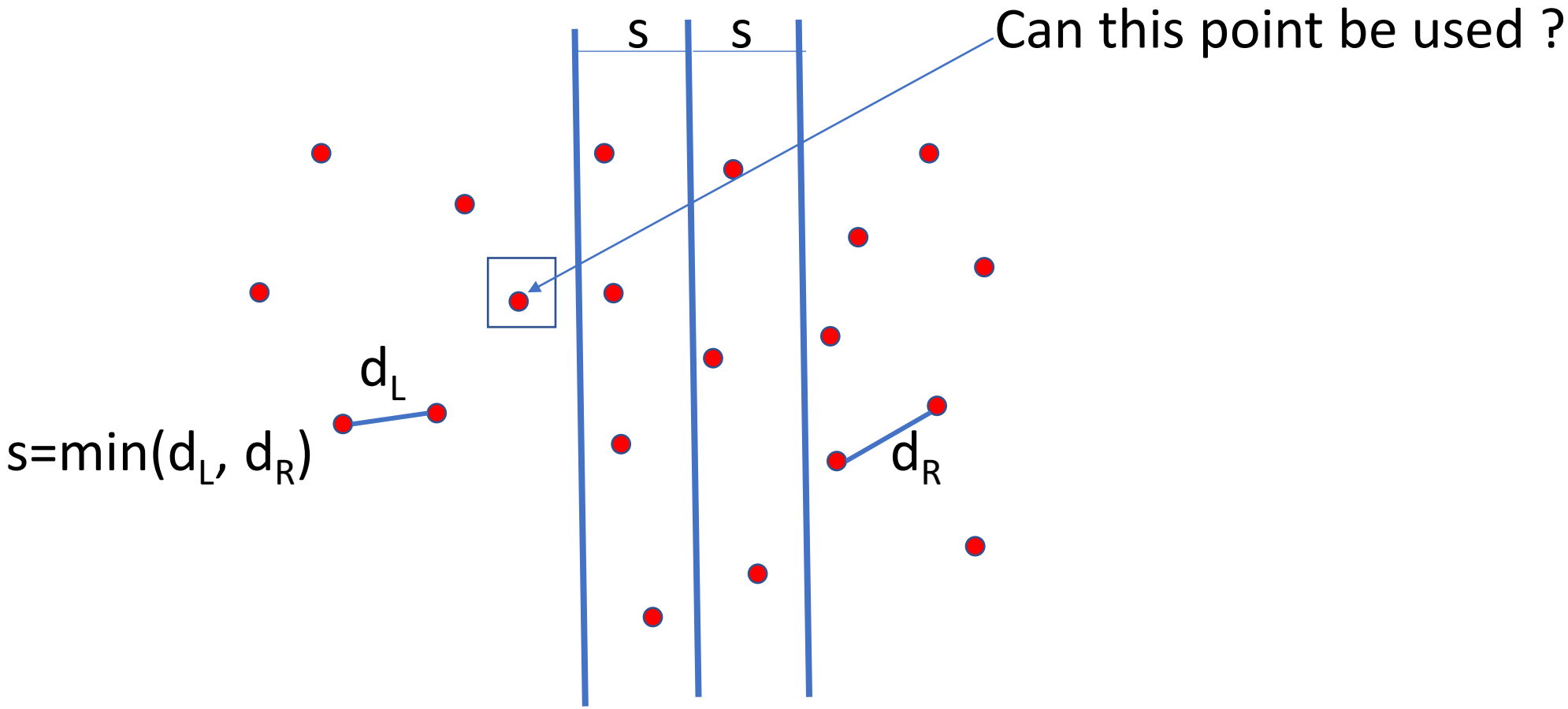

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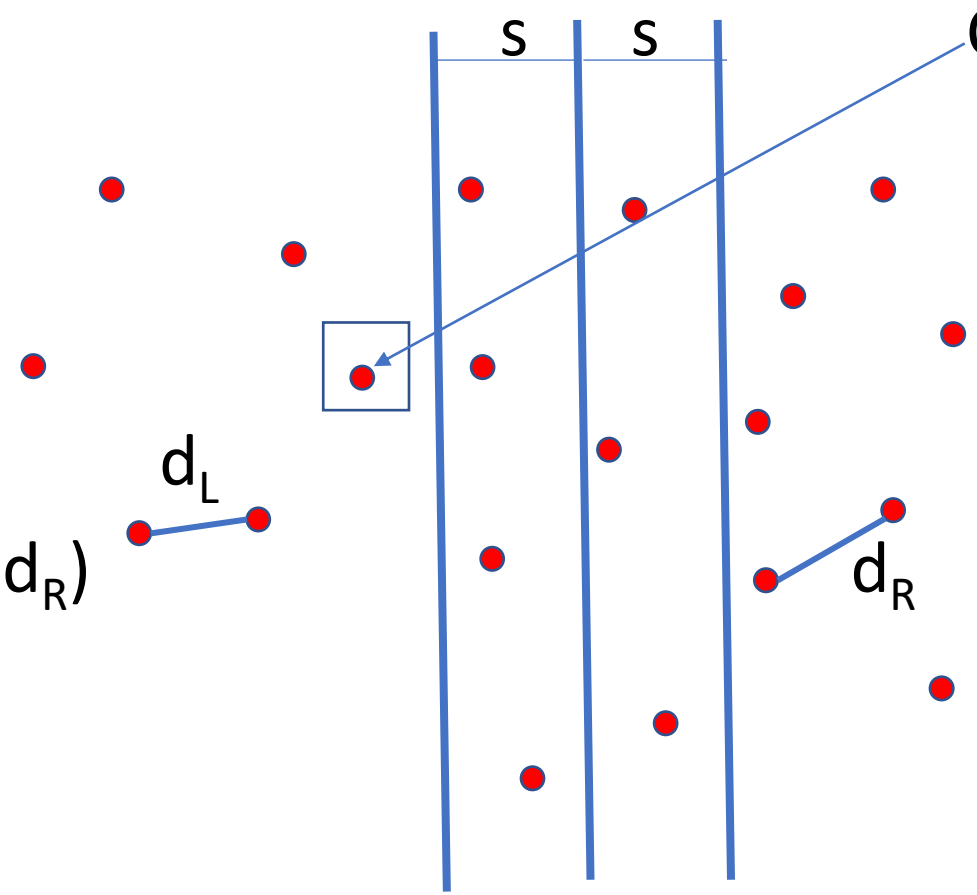
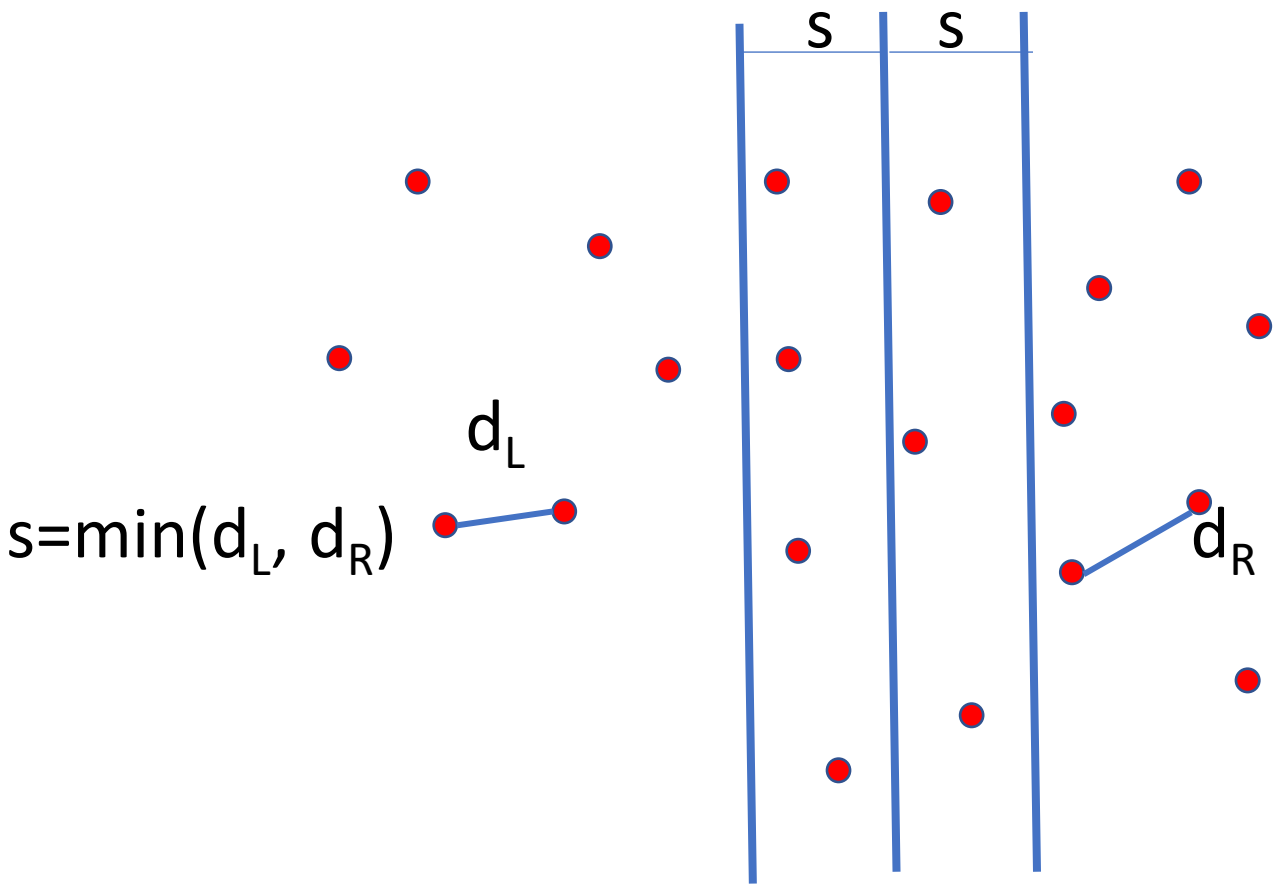


Can this point be used ?

No, so check points in strip.

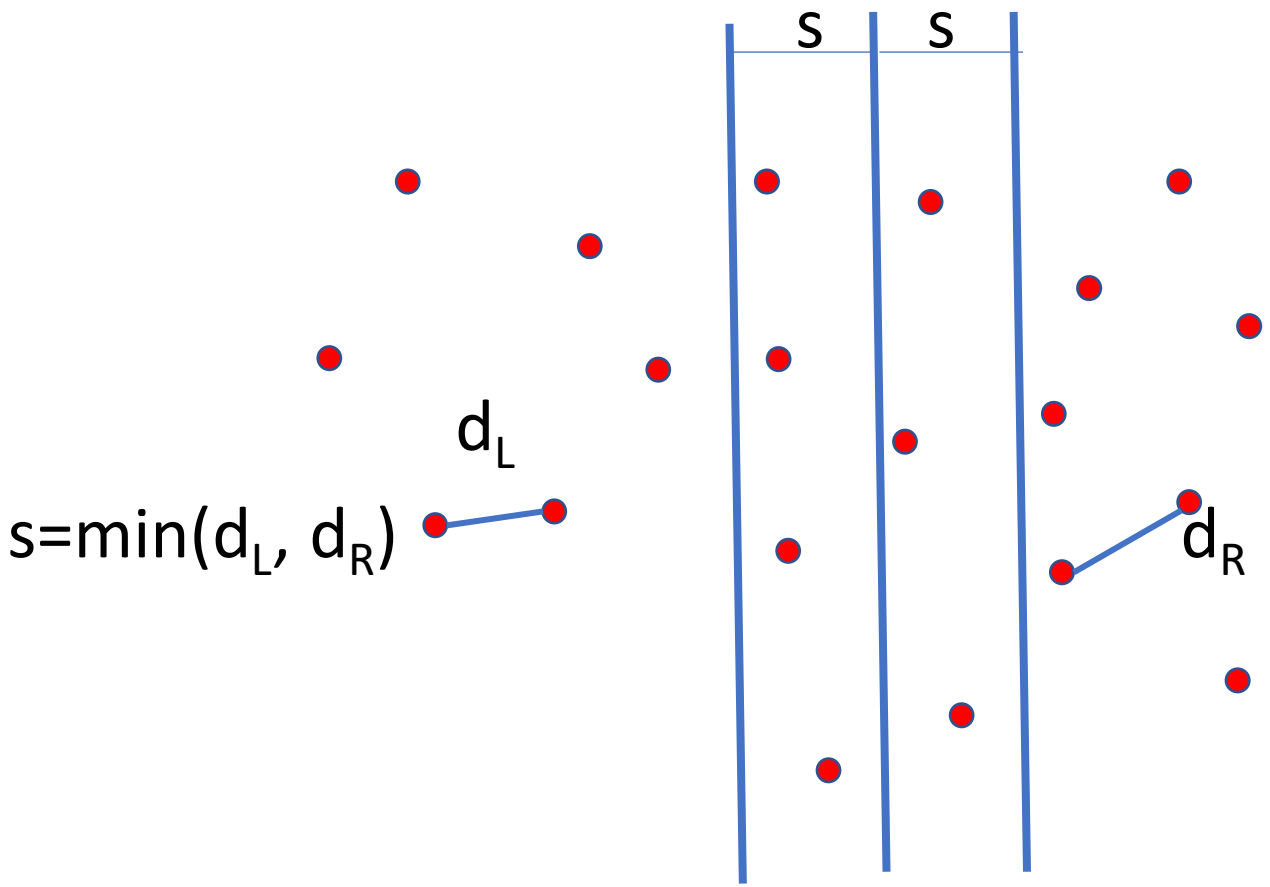
$s=\min(d_L, d_R)$

In 2D



So, we need to find the closest pair in strip only.

In 2D

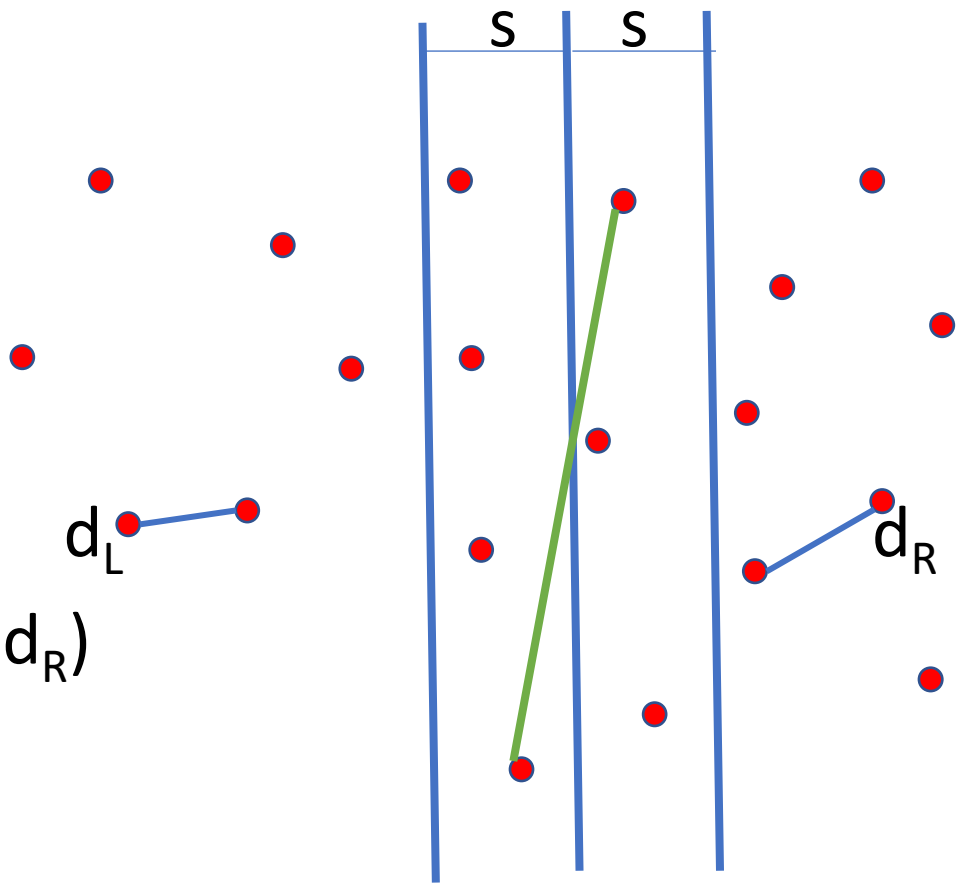


So, we need to find the closest pair in strip only.

naïve way: $O(n^2)$

$T(n)=T(n/2)+ O(n^2)$

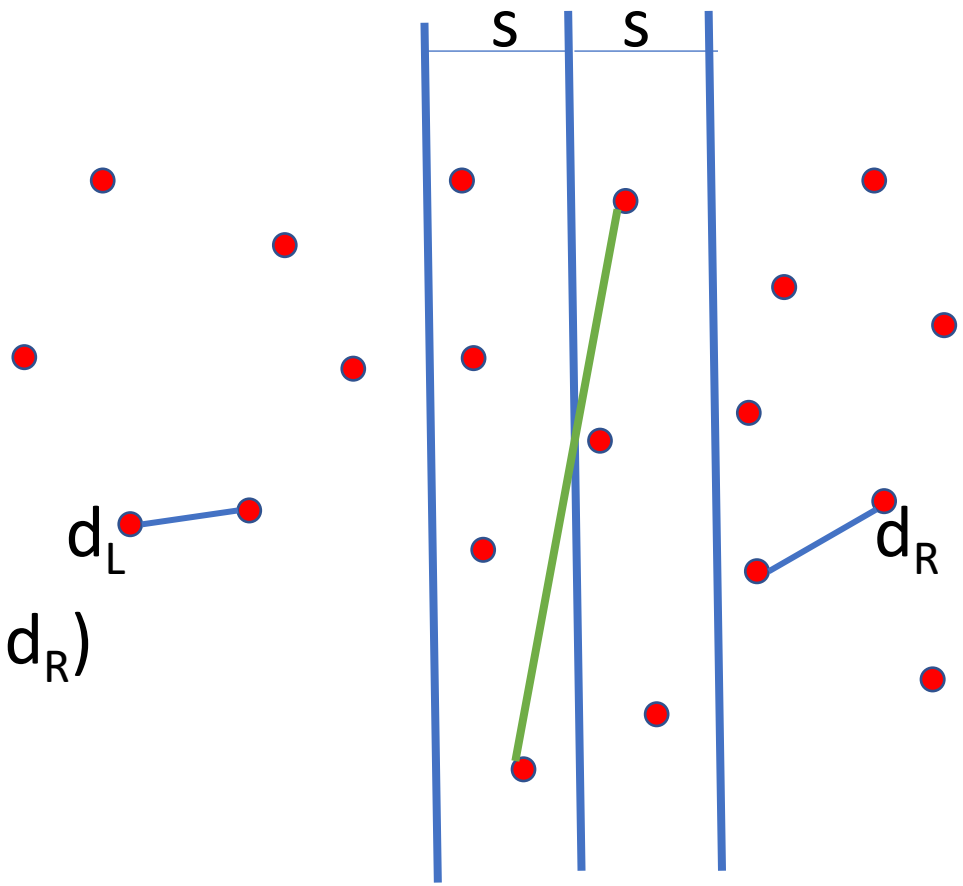
In 2D



$s=\min(d_L, d_R)$

Do we need to compare this pair?

In 2D



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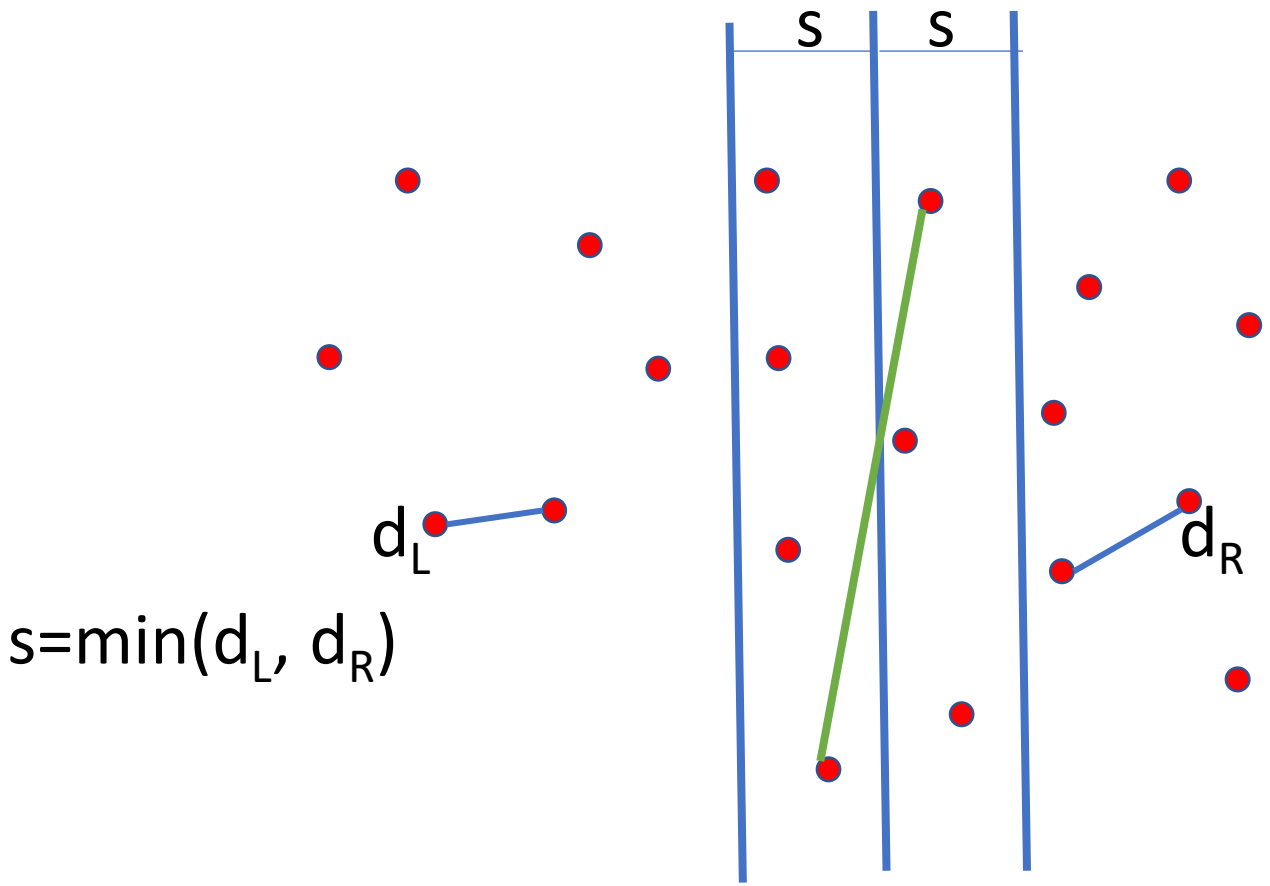

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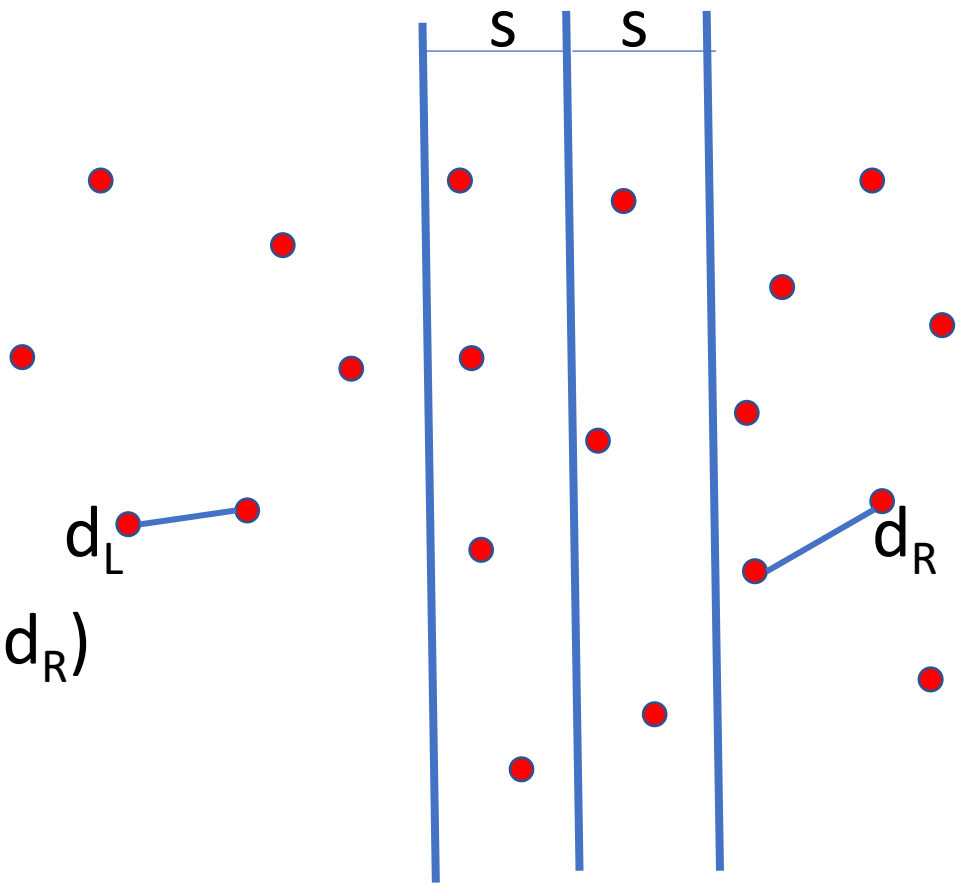
Let's dissects the strip.

In 2D



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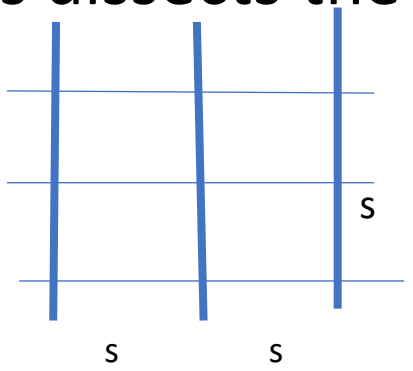

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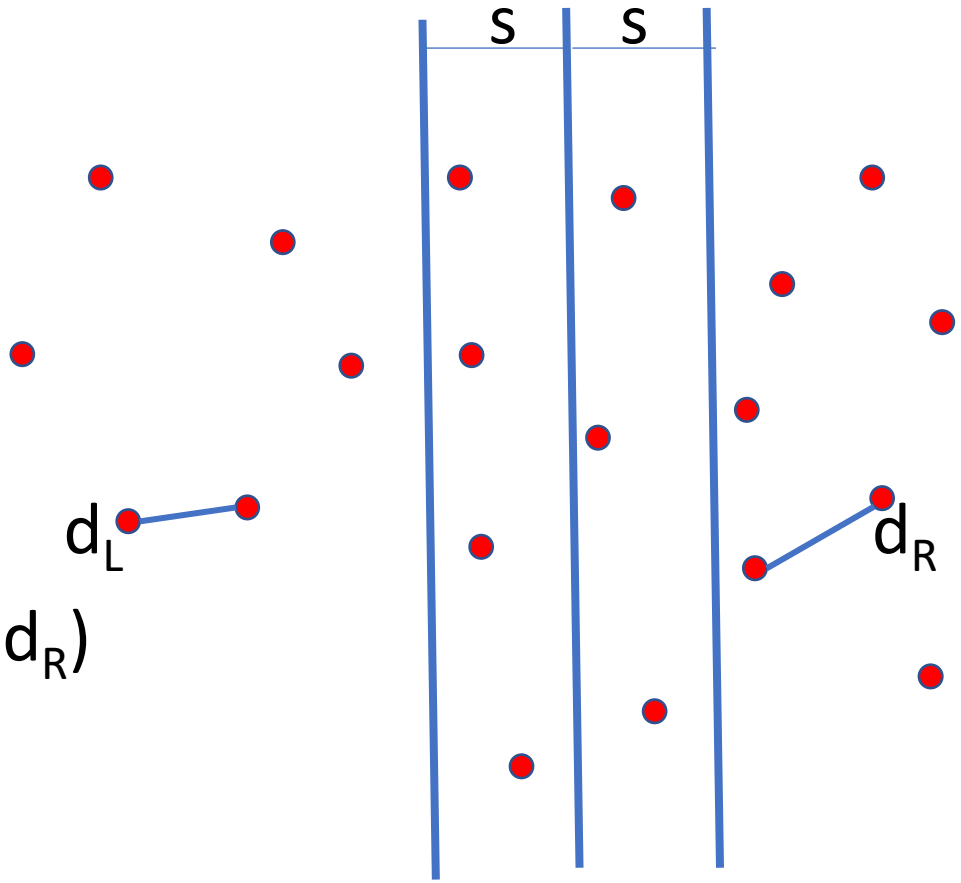

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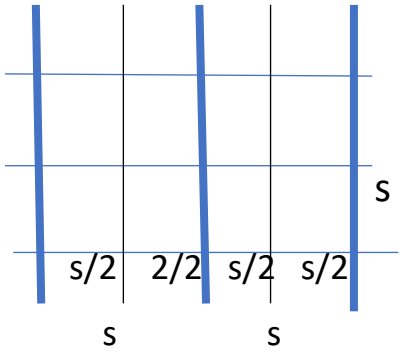
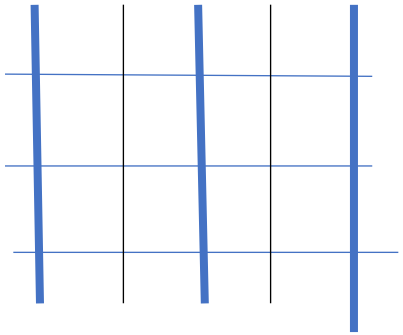
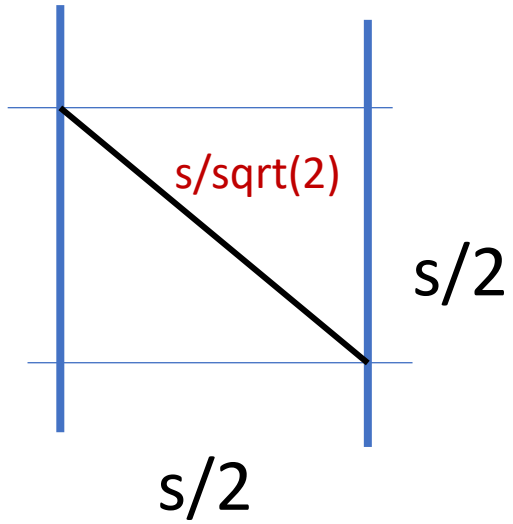
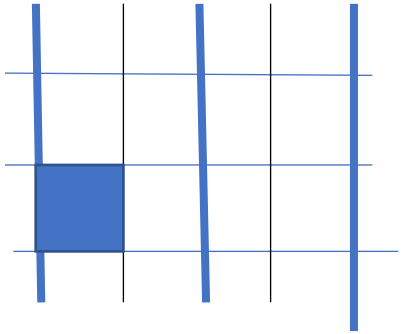
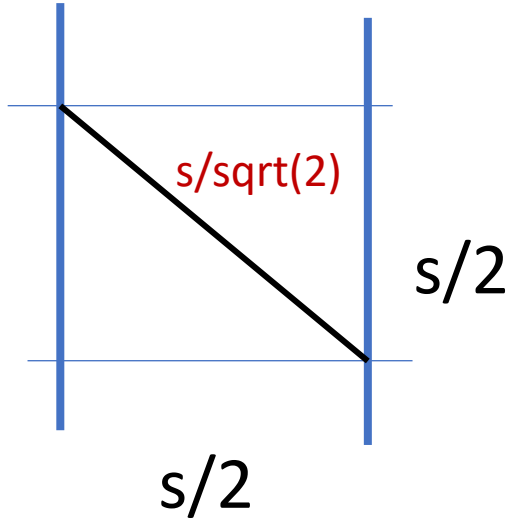


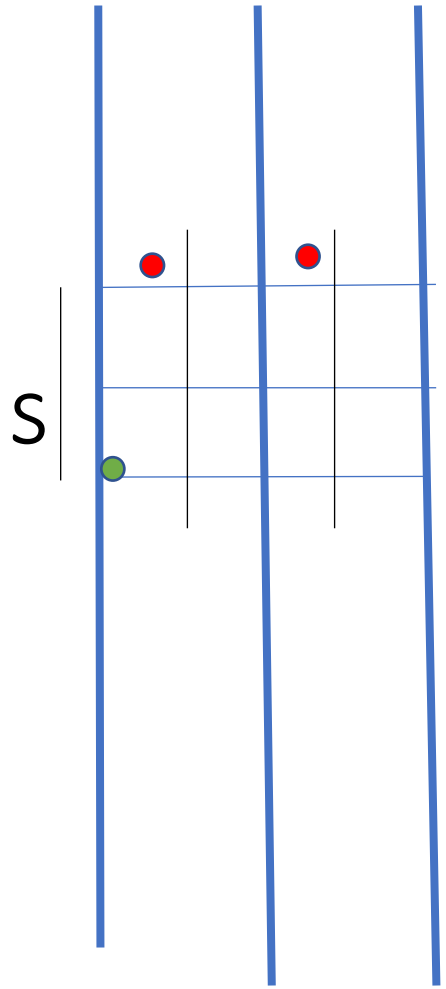



s/sqrt(2)
s/2
s/2



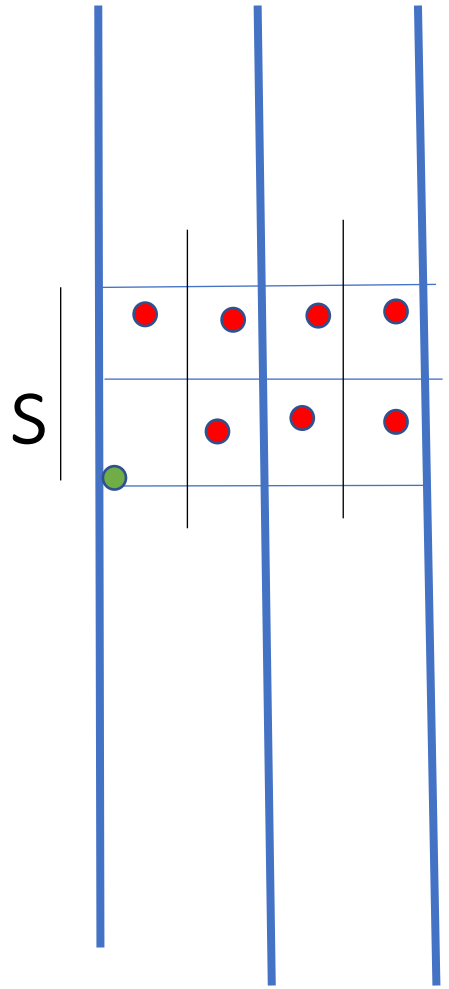
so, in a box, can have one point.





for green point, do we need to check these red points ?

NO? Why?



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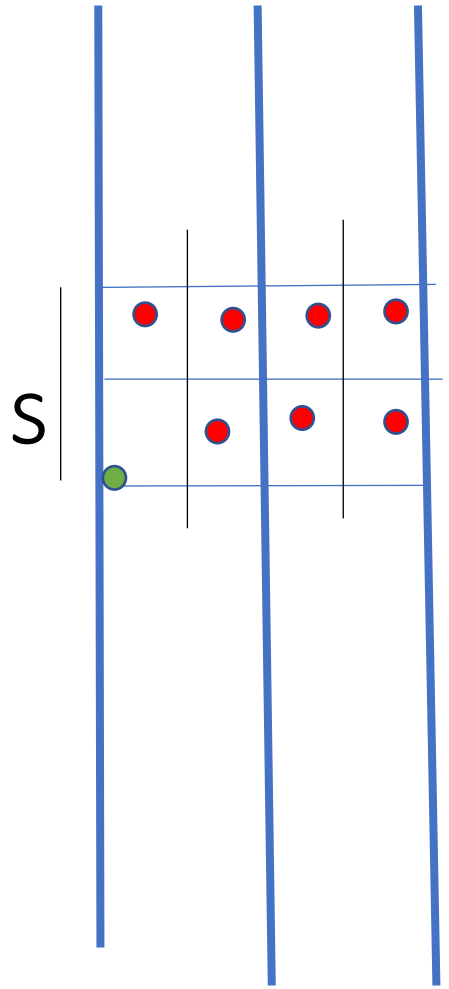

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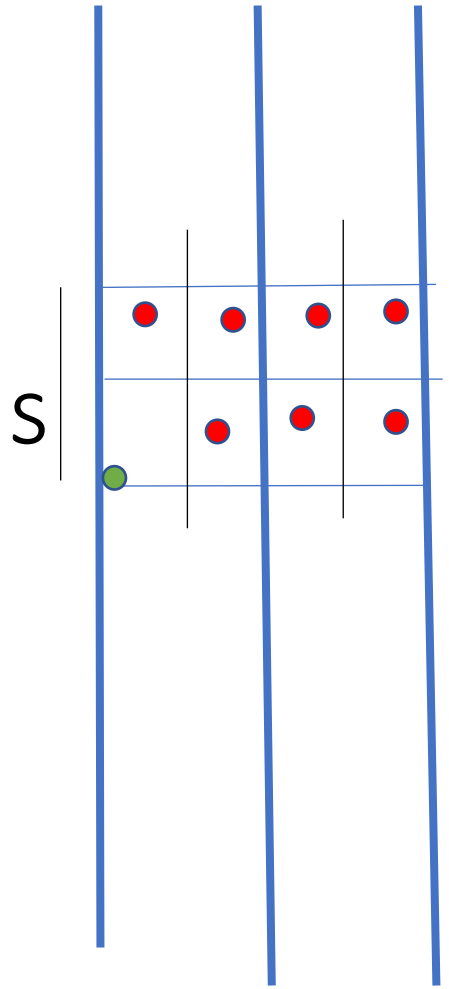
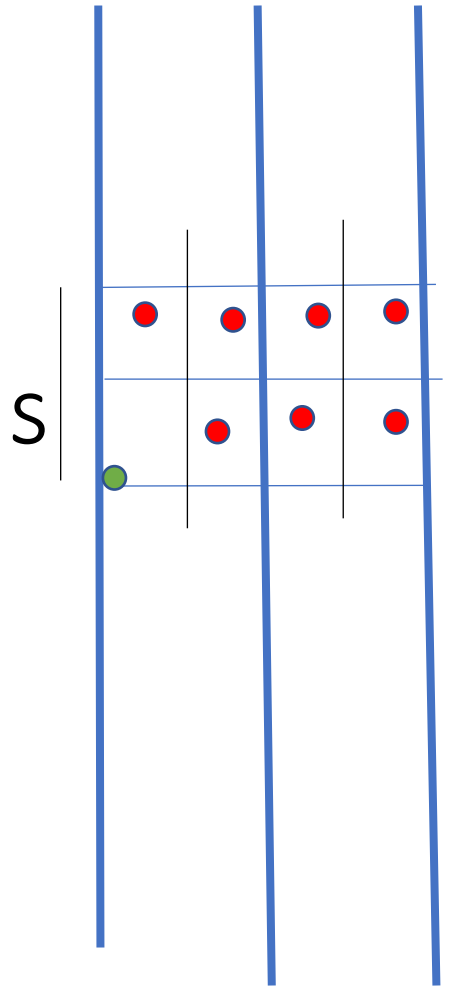
Yes? Why?

7 points



how to find seven points for each point?



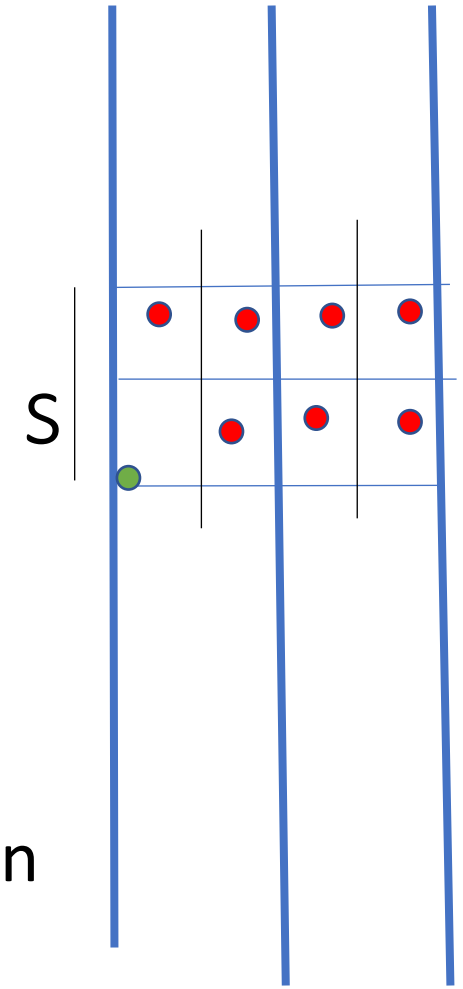
for green point, we need to check these red points ?

Yes? Why?

7 points

so, for each point, we need to check at most next 7 points.

To get next 7, need to sort



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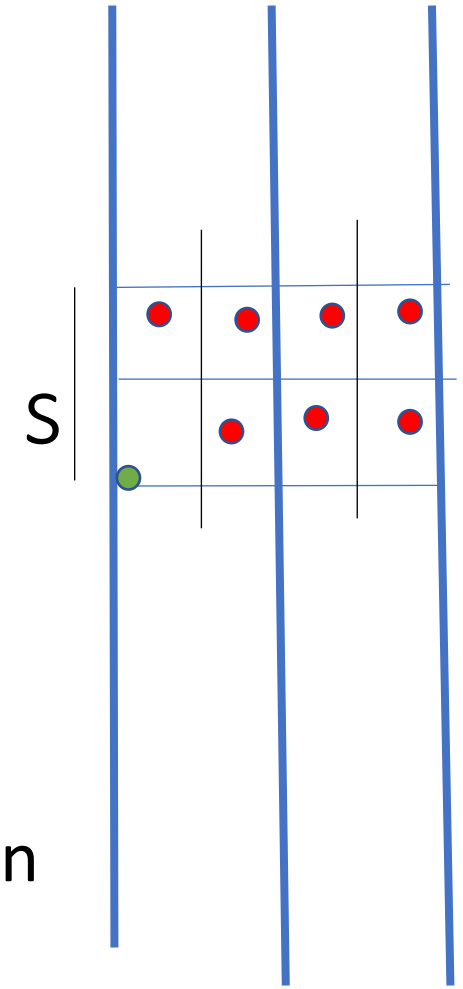
To get next 7, need to sort

T(n)=2T(n/2)+nlogn

$O(n\log^2 n)$

$T(n)=2T(n/2)+n\log n$



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2 dimensions, divide and conquer

- Split set of points into two halves by vertical line
- Recursively compute closest pair in left and right half
- Need to then compute closest pairs across separating line
- How can we do this efficiently?

Sorting points by x and y

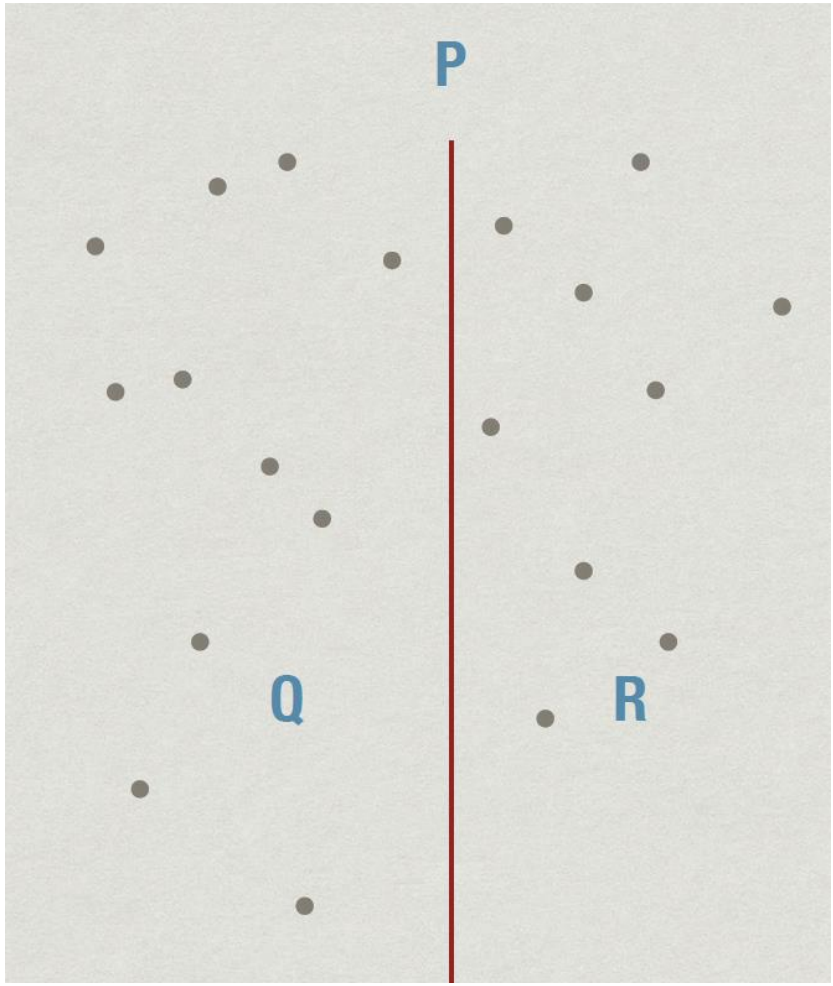
Given n points P = $\{p_1, p_2, \ldots, p_n\}$, compute

- P_x : P sorted by x coordinate
- P_y : P sorted by y coordinate

Divide P by vertical line into equal size sets Q and R.

Need to efficiently compute

Q_x, Q_y, R_x, R_y



Sorting points by x and y

- Need to efficiently compute

Q_x, Q_y, R_x, R_y

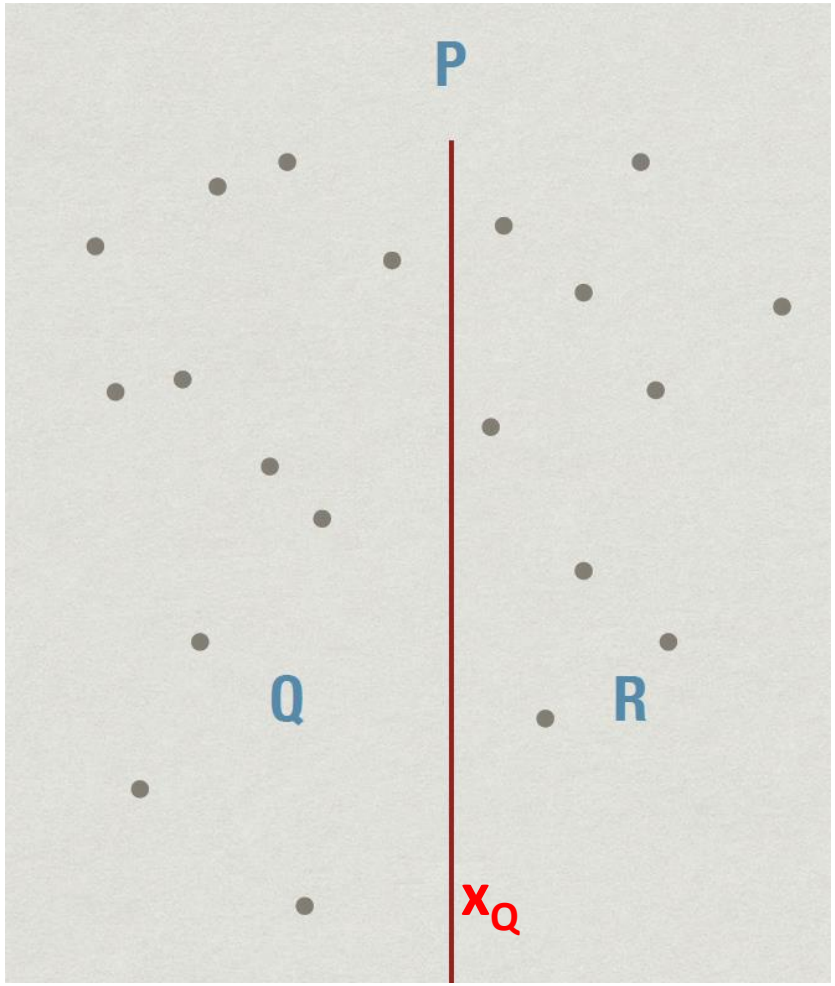
➢ Q_x is first half of P_x

➢ R_x is second half of P_x

- When splitting P_x,

note the largest x coordinate in Q, x_Q

- Separate P_y as Q_y, R_y by checking x coordinate with x_Q.
- All O(n)



2 dimensions, divide and conquer

- Basic recursive call is ClosestPair(P_x, P_y)

- Set up recursive calls ClosestPair(Q_x, Q_y)

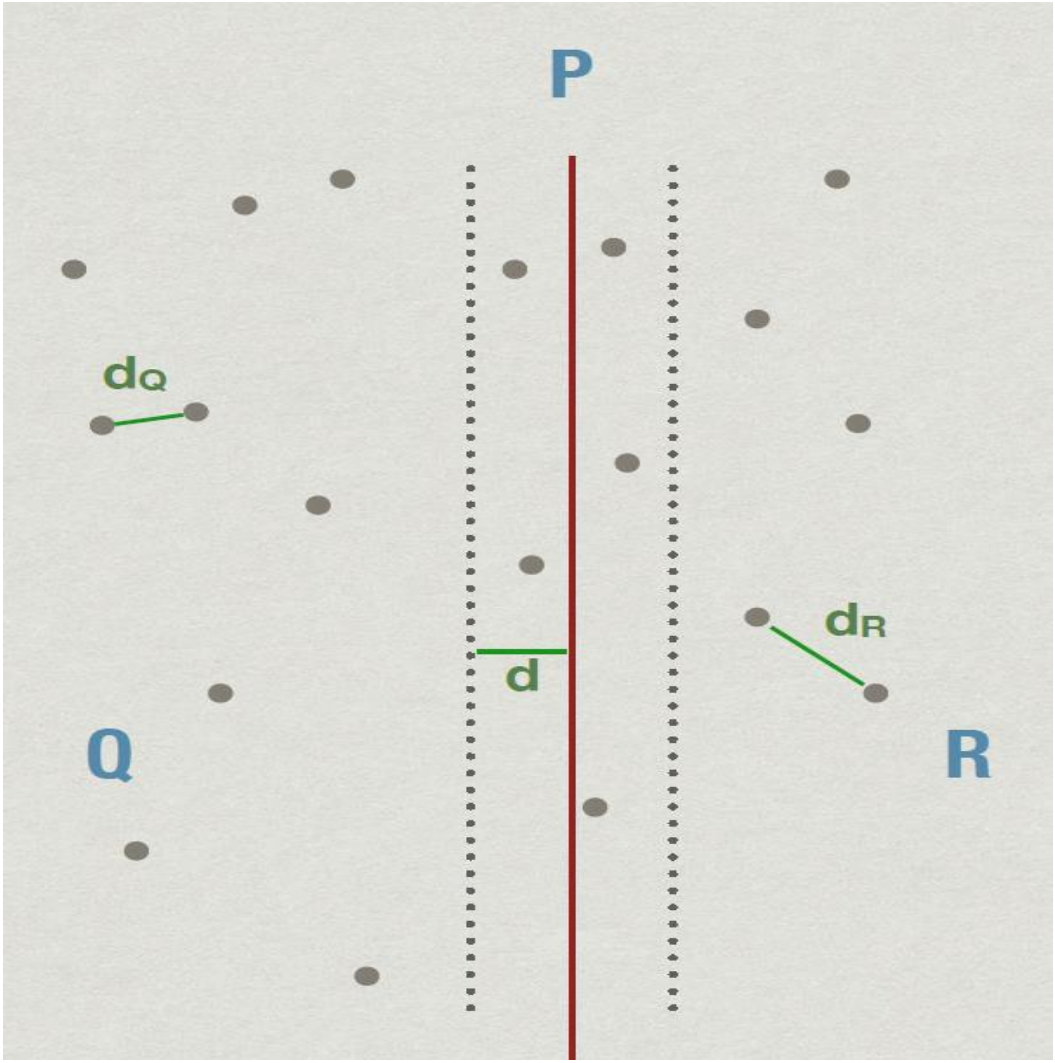
  and ClosestPair(R_x, R_y) for left and right half of P in time O(n)

- How to combine these recursive solutions?

Combining solutions

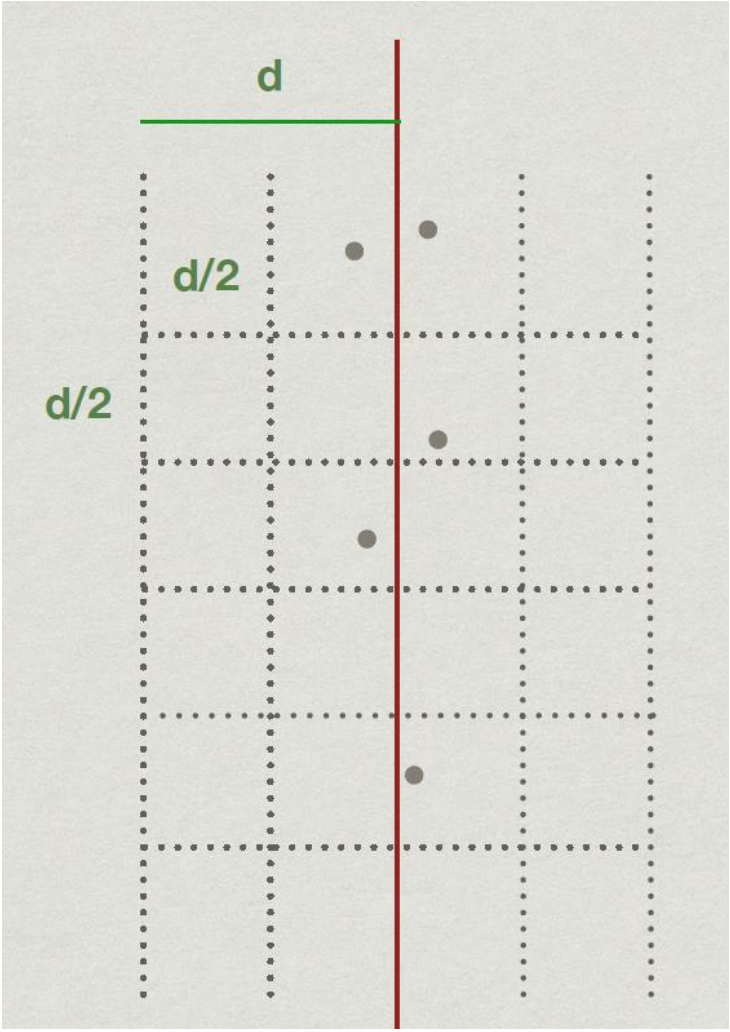
- Let d_Q be closest distance in Q and d_R be closest distance in R
- Let d be min(d_Q, d_R)
- Only need to consider points across the separator at most distance d from separator

➢ Any pair outside this strip cannot be closest pair overall



Combining solutions

- From Q_y, R_y, extract S_y,
  points in d-band sorted by y coordinate
- Scan S_y from bottom to top,
  comparing each point against next
  7 points in S_y
- Linear scan



Algorithm

ClosestPair(P_x,P_y)

if (|P_x| <= 3)

compute pairwise distances and

return the closest pair and distance

Construct (Q_x, Q_y, R_x, R_y)

(d_Q,q_1, q_2) = ClosestPair(Q_x, Q_y)

(d_R ,r_1, r_2) = ClosestPair(R_x, R_y)

Construct S_y and scan to find (d_S,s_1, s_2)

Return (d_Q,q_1, q_2) , (d_R ,r_1, r_2) , (d_S,s_1, s_2) depending on which among (d_Q,d_R, d_S) is minimum

Analysis

- Computing (P_x, P_y) from P takes O(n log n) (before recursion call)
- Recursive algorithm
  - Setting up (Q_x, Q_y, R_x, R_y) from (P_x, P_y) is O(n)
  - Setting up S_y from Q_y, R_y is O(n)
  - Scanning S_y is O(n)

$$T(n) = 2T(n/2) + O(n) = O(n \log n)$$