Problem Sheet 2, Algorithms, 2024[1,2]

1. Prove: $5n^2 + 7n + 6 \in \theta(n^2)$.
2. Prove: $4n^2 + 6n - 11 \in \theta(n^2)$.
3. Prove: $6n^3 + 3n^2 - 14n + 2 \in \theta(n^3)$.
4. Prove: $(2n^6 - 4n + 3)^2 \in \theta(n^{12})$.
5. Prove: $\log(3n^2 + n - 5) \in \theta(logn)$.
6. Prove: $n^{sinn} \in O(n)$. Can we prove $\Omega(n)$ for the same function?
7. Prove: $\sum_{i=1}^{n} i^{1/2} = \theta(n^{3/2})$.
8. Prove: $\log(n!) = \theta(n \log n)$
9. Prove: $\sum_{i=1}^{n} i \log i = \theta(n^2 \log n)$.
10. Prove using induction that $T(n) = \theta(n)$ for the below recurrence.

$$T(n) = c, n = 1$$

$$T(n) = T(n-1) + d, n > 1$$

11. Prove using induction that $T(n) = \theta(\log n)$ for the below recurrence.

$$T(n) = c, n = 1$$

$$T(n) = T(n/2) + d, n > 1$$

12. Prove using induction that $T(n) = \theta(n)$ for the below recurrence.

$$T(n) = c, n = 1$$

$$T(n) = 2T(n/2) + d, n > 1$$

13. Prove using induction that $T(n) = \theta(n^2)$ for the below recurrence.

$$T(n) = c, n = 1$$

$$T(n) = T(n-1) + dn, n > 1$$

14. Prove that $T(n) = O(\log n)$ for $T(n) = T(n/3) + T(2n/3) + n$.

[1] log means the base is 2
[2] Prepared by Pawan K. Mishra