

Artificial Intelligence

(Handling Uncertainty and structured knowledge representation)

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Syllabus-

- **Non Monotonic Reasoning-** Logics for Non Monotonic Reasoning, Justification based Truth Maintenance Systems,
- **Statistical Reasoning.**
 - **Fuzzy logic:** fuzzy set definition and types, membership function, designing a fuzzy set for a given application.
 - **Probability** and Bayes' theorem, Bayesian Networks.
- **Structured Knowledge representation-** Semantic net and Conceptual dependency

Non-monotonic reasoning

- **Definition:-**
- Reasoning effectively even when complete, consistent and constant model of the world is not available. (Read ABC murder story from Rich and knight page 195)
- **Conventional systems** (first-order predicate logic) are designed to work systems that has three properties.
 - **Completeness:-** It is complete wrt to domain of interest
i.e all the facts necessary to solve a problem are either present in the system or can be derived
 - It is **consistent**.
 - New facts added never **contradicts** previous one
- If any **one of the property** is not satisfied then **conventional logic system** becomes **inadequate**.
- Reasoning under **uncertainty** is called **non-monotonic reasoning/Defeasible Reasoning**.



Non-monotonic and statistical reasoning definitions

- **Non-monotonic Reasoning:-** in which the **axioms and/or rules of inference** are extended to make it possible to reason with **incomplete information**. At any moment a statement is believed to be **true, false** or not believed to be either.
- **Statistical reasoning-** in which representation is extended to allow some kind of **numeric measure** of certainty (rather than only **true** or **false**) to be associated with each statement.



What is Monotonicity???

Let Σ be the set of formulae and Σ' be the superset of Σ , (i.e $\Sigma \subset \Sigma'$) (Σ' is Σ plus some other facts) then

If $\Sigma \models \theta$ (i.e if θ is logically followed from Σ) then it is also true that $\Sigma' \models \theta$.

Interpretation : - Σ and Σ' are consistent (monotonicity)

<https://plato.stanford.edu/entries/logic-nonmonotonic/>

Logics for Non-monotonic reasoning

- Non-monotonic logic
- Default logic
- Abduction
- Inheritance
- Closed world assumption

<https://plato.stanford.edu/entries/logic-nonmonotonic/>

The term "**non-monotonic logic**" covers a family of formal frameworks devised to capture and represent **defeasible inference**, i.e., that kind of inference of everyday life in which reasoners draw conclusions **tentatively, reserving the right to retract them in the light of further information**. Such inferences are called "**non-monotonic**" because the set of conclusions warranted on the basis of a given knowledge base.

Logics for Non-monotonic reasoning

- **Non-monotonic logic**:- is extension to the language of **predicate logic** with model **operator M**, which can be read as “**is consistent**”

$$\forall x,y: \text{Related}(x,y) \wedge M \text{ GetAlong}(x,y) \rightarrow \text{WillDefend}(x,y)$$

Problems:-

1. What is meaning of “**is consistent**”? is un-decidable in **first order logic**
2. Multiple **non-monotonic** statements taken together may be **inconsistent**

Logics for Non-monotonic reasoning

2. Multiple non-monotonic statements taken together may be inconsistent

$$\forall x: \text{Republican}(x) \wedge M \neg \text{Pacifist}(x) \rightarrow \neg \text{Pacifist}(x)$$

$$\forall x: \text{Quaker}(x) \wedge M \text{Pacifist}(x) \rightarrow \text{Pacifist}(x)$$

$$\text{Republican}(\text{Joe}) \longrightarrow \neg \text{Pacifist}(\text{Joe})$$

$$\text{Quaker}(\text{Joe}) \longrightarrow \text{Pacifist}(\text{Joe})$$

In-
consistent

Pacifist- A person believing that war and violence are unjustifiable

Quaker- A peaceful religious person

Logics for Non-monotonic reasoning

- **Default Logic**:- An alternative logic used for performing the **reasoning based on default assumptions**. Proposed by Canadian logician **Raymond Reiter**. It provides a new inference rule of the form.

$$\frac{A : B}{C}$$

If **A is provable** and it is consistent to **assume B** then **conclude C**

$$\frac{Bird(x) : Flies(x)}{Flies(x)}$$

Default rule about the birds, which fails for Penguin

Bird(Penguin) \rightarrow Flies(Penguin)



Logics for Non-monotonic reasoning

- **Abduction**:- is using the implication in reverse order

$$\forall x : Engineer(x) \rightarrow Literate(x)$$

Engineer(Ram) we can deduce *Literate(Ram)*

Given *Literate(Shyam)* we can infer *Engineer(Shyam)*

by Abduction (Abductive reasoning)

Abduction- deriving conclusions even though they are not more appropriate using implication in reverse

Malaria (parasitic infection) \Rightarrow Hydroxychloroquine

Covid19(viral infection) \Leftarrow Hydroxychloroquine

Logics for Non-monotonic reasoning

- **Inheritance:-** “ An object inherits **attribute values** from their classes”. In a broader way

$$\frac{BaseBall - Player(x) : height(x, 6)}{height(x, 6)}$$

- **Closed world assumption**

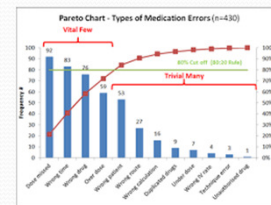
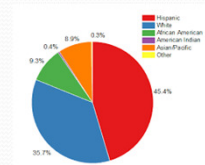
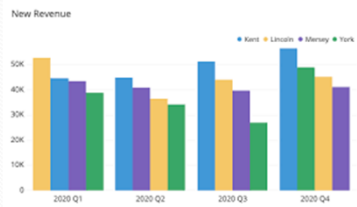
(i.e Reasoning based on **everything you don't know is false**)

e.g. 1 Is there water on planet Mars? **No**

2. If there is no direct flight between Pune to Kanpur in the database; travel agent will say no flight assuming database as complete .

Statistical reasoning

- **Statistical reasoning tools**
 - Fuzzy logic
 - Probability and Bay's theorem



Probability - Basic Rules

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$P(A \text{ OR } B) \text{ vs } P(A \text{ AND } B)$$

Need of Fuzzy Logic

- Human brain computes with imprecise information. ($9.1435 + 10.1111234 \approx 20$)
- Uncertainty is related with most of the information used for computation.
- Precise information is rarely available and not needed for most day to day problem solving
- Some facts involve vague terms like “More”, “Better”, “Taller” etc.
- Some task can not be performed in a precise manner. e.g classifying Good/bad persons

Use of Fuzzy sets and fuzzy Logic

- To provide a way for representing imprecise things
- To handle uncertain situations other than randomness.
- To capture vagueness in natural languages.

In a domain with n events randomness is

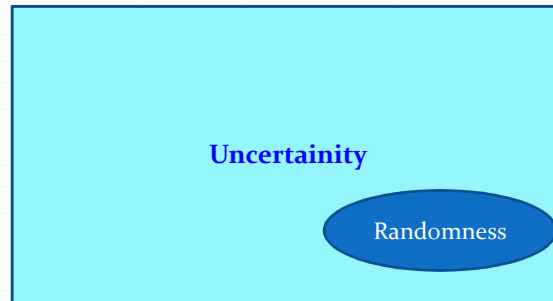
$$1. P(E_1) = P(E_2) = \dots = P(E_n)$$

$$2. P(E_1) + P(E_2) + \dots + P(E_n) = 1$$

Use probability

Uses of Fuzzy sets and fuzzy Logic

- Handling uncertainty other than randomness



What happens ???if

$$1. P(E_1) \neq P(E_2) \neq \dots \neq P(E_n)$$

$$2. P(E_1) + P(E_2) + \dots + P(E_n) \neq 1$$

**Can not use Probability.
Use Fuzzy Logic**

Introduction to Fuzzy sets

- Set theory; which we have been using was proposed by a German mathematician **George Cantor** in 1870.

□ Sets with which we are already familiar are termed as **crisp set**.

□ The **membership function** of **crisp set** returns only two values, either 1 or 0.

1- member

$$A = \{1, 2, 3\}$$

0- Non member.

$$\mu_A(2) = 1 \text{ and } \mu_A(5) = 0$$



Introduction to Fuzzy sets

- **Fuzzy** means **imprecise/vague/difficult to perceive**
- **Prof. Lotfi Zadeh**, 1965, university of California, Berkley, proposed “**Fuzzy sets**”, which are **generalization** of crisp sets, such that membership function returns values in **interval [0,1]**, including ZERO and ONE.
 - 1- means a **full member**
 - 0-**non member**
 - $0 < \text{membership} < 1$ -**Partial members**

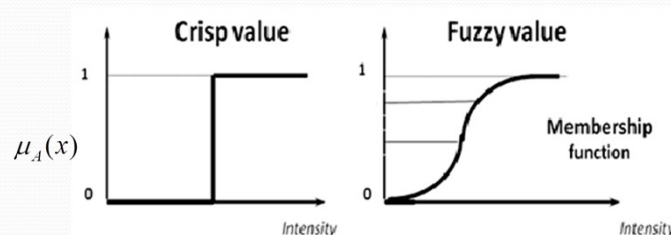


Fuzzy membership function

- A function that takes input **x** and returns out put as **membership grade in [0,1]**, for **x**, is called as **fuzzy membership function**.

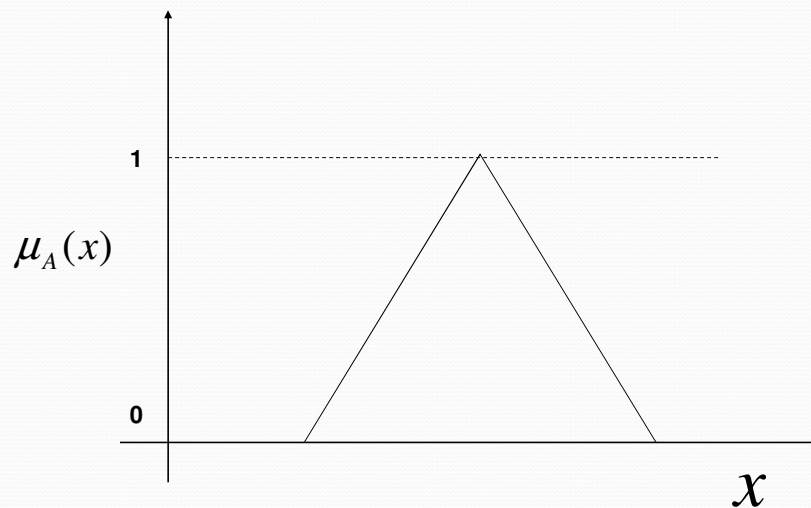
$$\mu_A(x)$$

-Read it as “ **membership of input x in the fuzzy set A**”



https://www.researchgate.net/figure/Difference-between-Crisp-and-Fuzzy-Sets_fig6_311419824

Plot of fuzzy Membership function

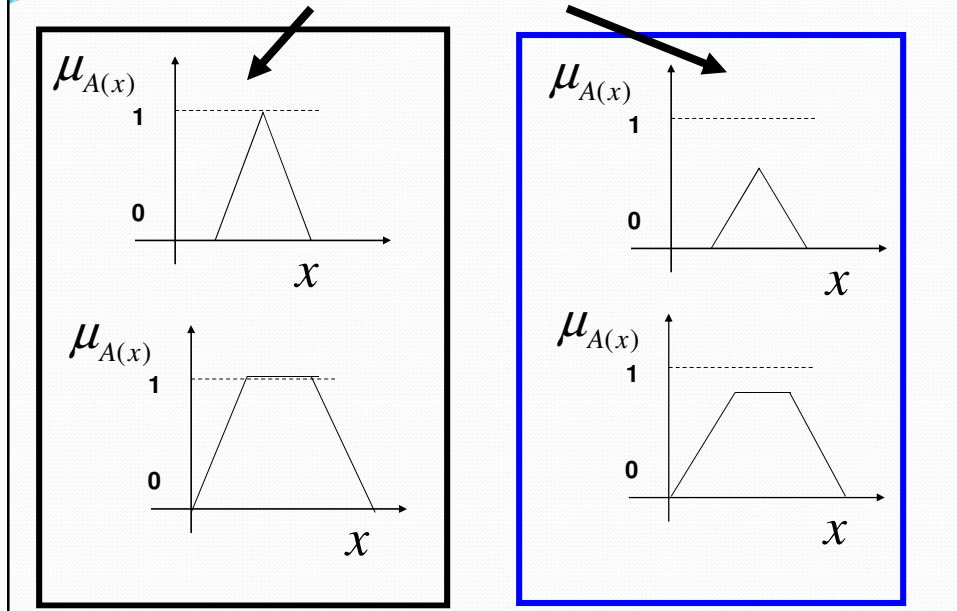


Types of fuzzy sets

- There are 4 types of fuzzy sets based on the nature of fuzzy membership values.
 - Normal fuzzy sets
 - Subnormal fuzzy sets
 - Convex fuzzy sets
 - Non-convex fuzzy sets

Normal fuzzy set:- A fuzzy set is called as normal fuzzy set, if at least one of its member has attained highest membership value 1, other wise it is called as subnormal fuzzy set.

Normal and Subnormal fuzzy sets



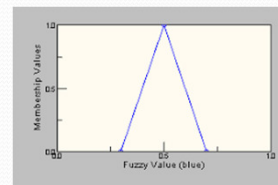
Convex and non-convex fuzzy sets

- **Convex fuzzy set:-** A fuzzy set is called as convex fuzzy set if the membership values are either monotonically increasing or monotonically decreasing, otherwise it is called as non-convex fuzzy set.
- The property of monotonically increasing or decreasing is called convexity.

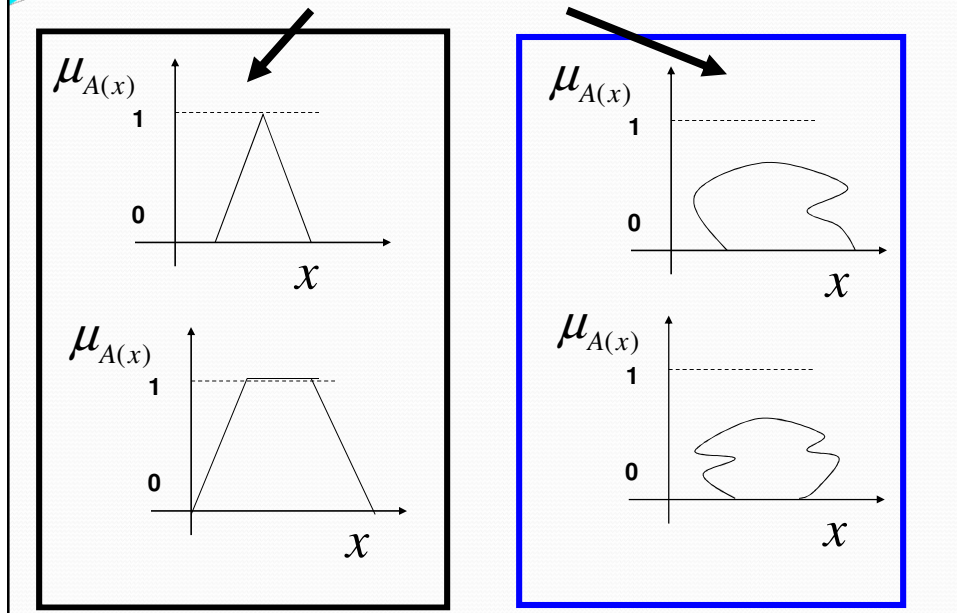
$$f(ax_1 + bx_2) \leq af(x_1) + bf(x_2)$$

$$\forall a, b \in [0, 1] \text{ and}$$

$$\forall x_1, x_2 \in X (\text{vector space})$$



Convex and non-convex fuzzy sets



How to define Fuzzy sets?

- A fuzzy set with **finite** members having non-zero membership is represented as

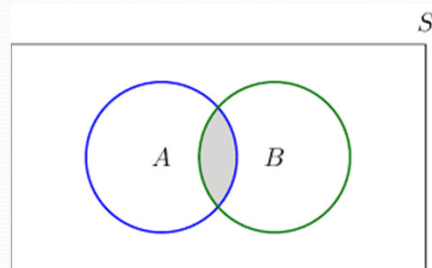
$$A = \left\{ \frac{0.4}{1}, \frac{0.6}{2}, \frac{1}{3} \right\}$$

If there are **infinite** members having non-zero membership then use a **fuzzy membership function** to define the fuzzy set as shown below

$$\mu_A(x) = \begin{cases} 1 - & \text{if condition} \\ 0 < \mu_A(x) < 1 - & \text{if conditions} \\ 0 - & \text{if conditions} \end{cases}$$

Operations on fuzzy sets

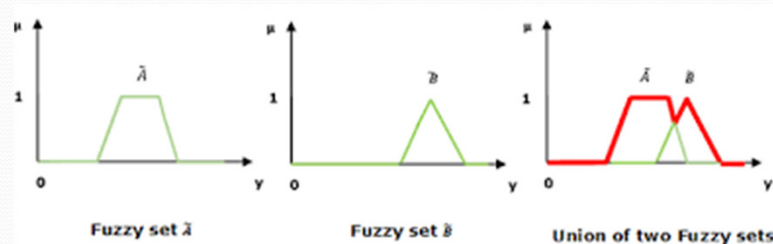
- Following common **operations** that can be performed on **fuzzy sets**
 1. **Union**
 2. **Intersection**
 3. **Compliment**



Union of two fuzzy sets

Let **A** and **B** be two fuzzy sets, then their union is defined as

$$\mu_{A \cup B}(x) = \max (\mu_{A(x)}, \mu_{B(x)})$$



Intersection of two fuzzy sets

Let **A** and **B** be two fuzzy sets, then their intersection is defined as

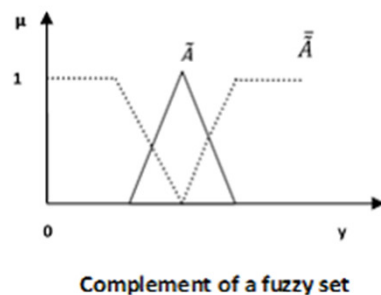
$$\mu_{A \cap B}(x) = \min(\mu_{A(x)}, \mu_{B(x)})$$



Compliment of a fuzzy set

Let **A** be a fuzzy set then its compliment can be defined as

$$\mu_{\bar{A}}(x) = 1 - \mu_{A(x)}$$



Exercise 1 on Fuzzy sets

Consider following fuzzy sets A and B

$$A = \left\{ \frac{0.4}{1}, \frac{0.1}{2}, \frac{0.55}{3}, \frac{0.6}{4}, \frac{.7}{5} \right\} \quad B = \left\{ \frac{0.5}{4}, \frac{0.6}{5}, \frac{0.7}{6}, \frac{0.7}{7} \right\}$$

1. Compute following things

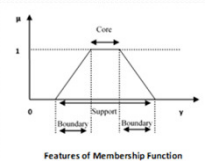
1. Intersection of A and B
2. Union of A and B
3. Compliment of A and B

2. Which of the fuzzy set A and B is Normal? Why?

3. How can we prove that fuzzy set B is convex?

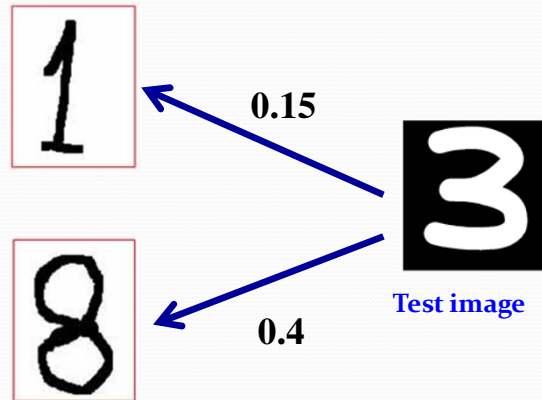
Uses of Fuzzy Membership value

- Are used to **reason** about **similarity** of a test condition with **previously known situations**.
- Can be used to **reason** about **class label** of a test pattern using **previously learned patterns**.
- Washing machine to **decide initial torque** based on weight of cloths.
- Fuzzy **controllers** to take better decisions than crisp controllers.



1. Fuzzy sets for pattern recognition

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9



Fuzzy sets for 1 and 8

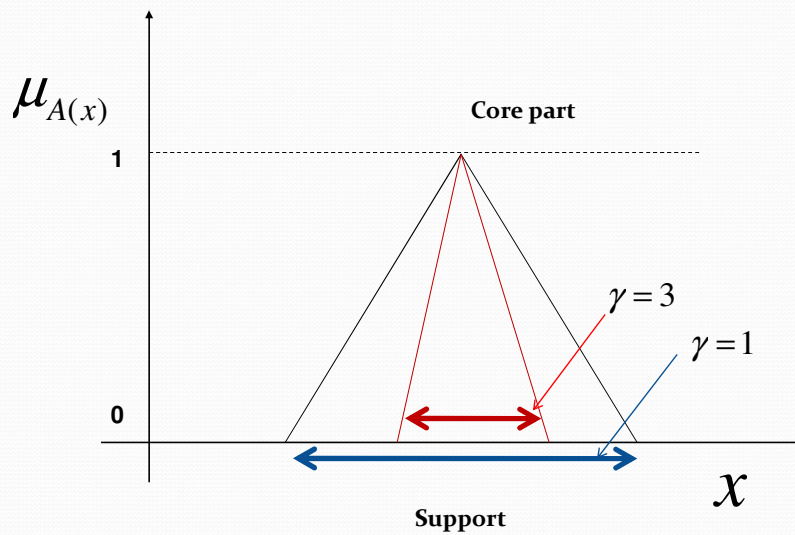
Design a fuzzy set for numbers close to 5

Let A be the fuzzy set & x be a number whose membership is to be calculated

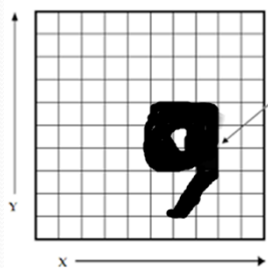
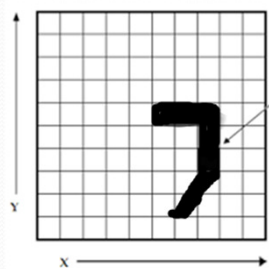
$$\mu_A(x) = \begin{cases} 1 & \text{if } \gamma \left| \frac{5}{\max(x,5)} - \frac{x}{\max(x,5)} \right| = 0 \\ 1 - \gamma \left| \frac{5}{\max(x,5)} - \frac{x}{\max(x,5)} \right| & \text{if } 0 < \gamma \left| \frac{5}{\max(x,5)} - \frac{x}{\max(x,5)} \right| \leq 1 \\ 0 & \text{if } \gamma \left| \frac{5}{\max(x,5)} - \frac{x}{\max(x,5)} \right| > 1 \end{cases}$$

where, $\gamma > 0$ – is sensitivity parameter

Role of sensitivity parameter gamma



2. Fuzzy set for HCR matching



$$V_{ref} = (a_1, a_2, \dots, a_k)$$

a_i - is the i th feature

$$V_{test} = (b_1, b_2, \dots, b_k)$$

Distance d between two images
can be computed as

$$d = \sqrt{\sum_{i=1}^k (a_i - b_i)^2}$$

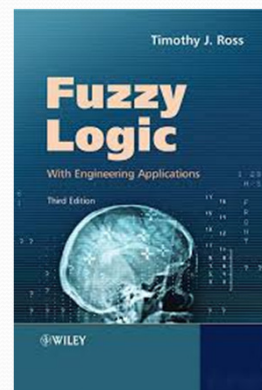
2. Fuzzy set for character matching

A and B are character patterns to be matched

$$\mu_A(x) = \begin{cases} 1 & \text{if } \gamma * d(A, B) = 0 \\ 1 - \gamma * d(A, B) & \text{if } 0 < \gamma * d(A, B) \leq 1 \\ 0 & \text{if } \gamma * d(A, B) > 1 \end{cases}$$

d- is the Euclidean distance between features of character A and B

1. Problems on operations on fuzzy sets
2. Problems on fuzzy set design.



Reference Book

- Timothy J. Ross, “*fuzzy logic with engineering applications*”, Wiley publication

Structured Knowledge Representation

- **Weak Slot and Filler structures**
 - Semantic Nets
 - Frames.
- **Strong Slot and Filler structures**
 - Conceptual dependency
 - Scripts.



A baby dog is called as

What are “Slot and filler structures”?

- They provide a way of representing real world facts in a more structured manner than predicate logic (unstructured).
- Used for representing entities and their attributes.
- They supports inheritance along *isa* and *instance* relationship
- Inheritance can be performed more easily than the pure logic.
- They support object orientation.

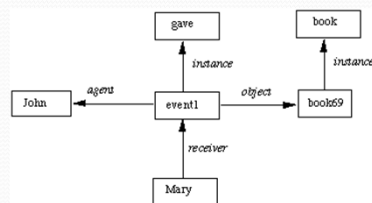
Weak and strong structures.

- **Weak slot and filler structures:-**

They do not have straight forward method of reasoning (semantic nets, frames)

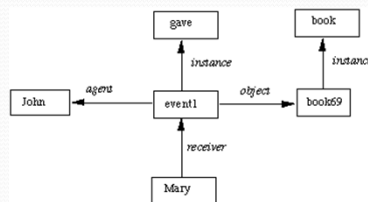
- **Strong slot and filler structures:-**

They have straight forward method of reasoning (CD, Scripts)



Semantic net

- **Basic philosophy-** “Meaning of a concept comes from the way in which it is connected to other concepts”.
- In semantic net knowledge is stored as set of nodes connected to each other by a set of arcs.
- Arcs represents relationship and nodes represents class, subclass or object.



Represent above facts using semantic net.

isa(person, mammal)

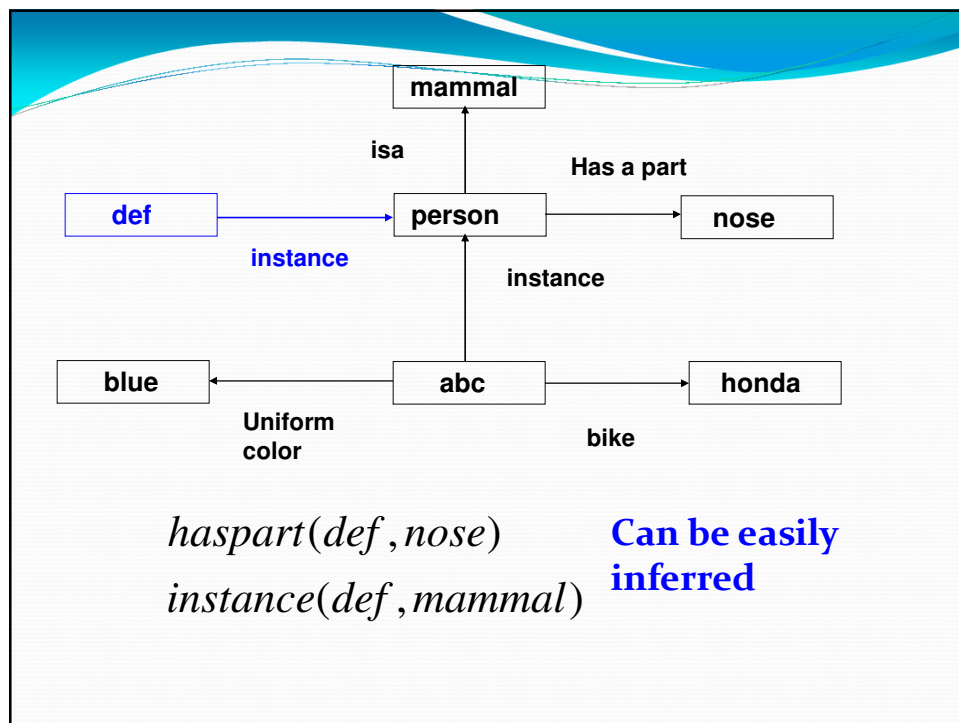
insatnce(abc, person)

haspart(person, nose)

bike(abc, honda)

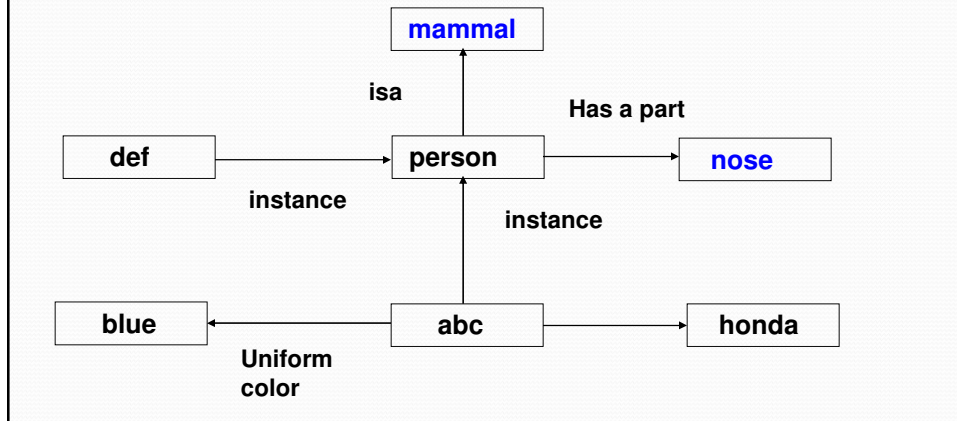
uniformcolor(abc, blue)

Binary predicates



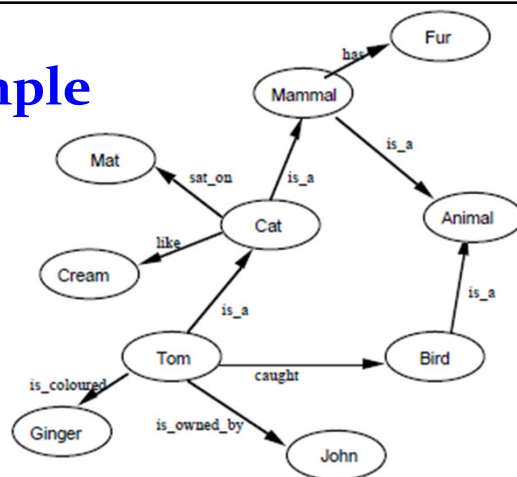
Intersection search in semantic net

- In semantic net **intersection search** is used to find out relation between any two nodes.
e.g. what is relationship between “mammal” and “nose”.



Another example

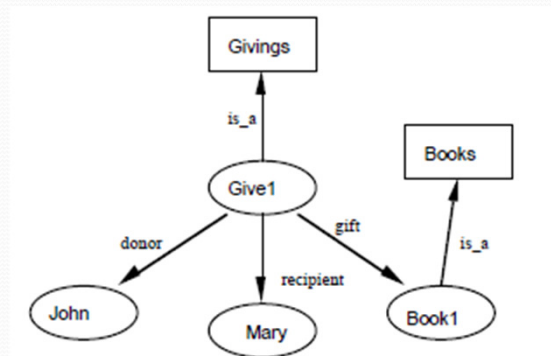
Tom is a cat.
Tom caught a bird.
Tom is owned by John.
Tom is ginger in colour.
Cats like cream.
The cat sat on the mat.
A cat is a mammal.
A bird is an animal.
All mammals are animals.
Mammals have fur.



<http://www.eecs.qmul.ac.uk/~mmh/AINotes/AINotes4.pdf>

Representing multi-arity predicates in semantic net

gives(John, Mary, Book)



<http://www.eecs.qmul.ac.uk/~mmh/AINotes/AINotes4.pdf>

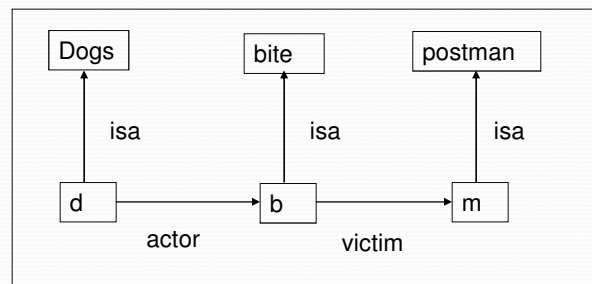
Problem on semantic net

- Represent following fact using semantic net.
- *“Hilbert is a hippo. Like other hippos he eats grass and likes swimming”.*

Partitioned Semantic net.

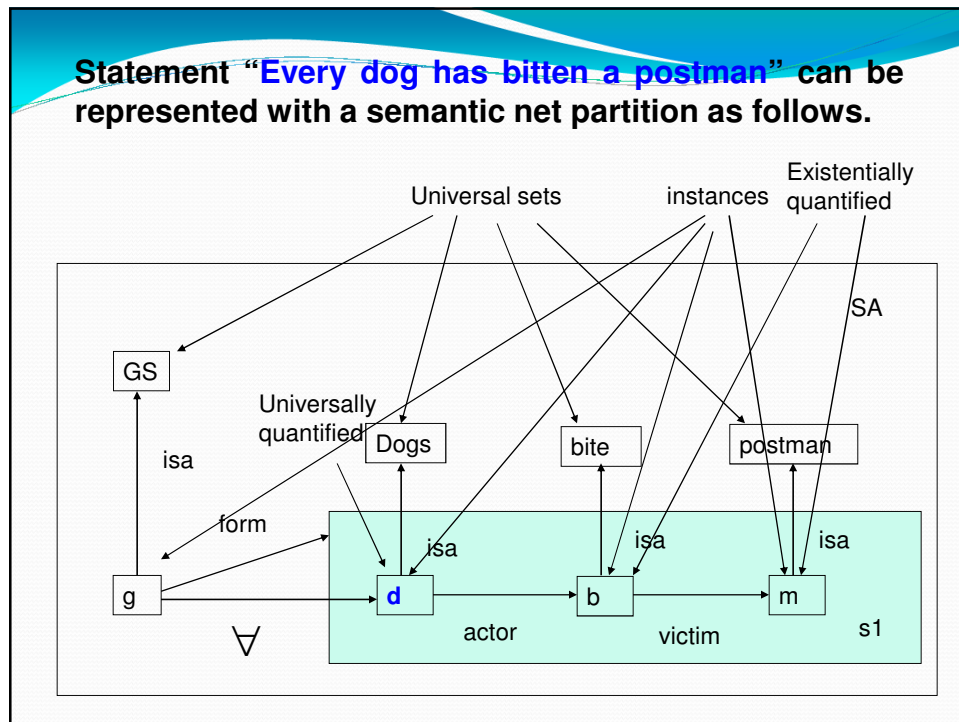
- They are used to represent universally quantified facts.
- In Partitioned Semantic net, each partition corresponds to scope of variable and spaces are hierarchically classified.

Partitioned Semantic net.



Statement “**The dog has bitten a postman**” can be represented with a semantic net without partition as follows.

Statement “**Every dog has bitten a postman**” can be represented with a semantic net partition as follows.



Partitioned Semantic net.

GS is a set of *general statements* in this universe.

-Every element of GS has two attributes

1. *A form*:- which states a relation.
2. *one or more “for all”* \forall *connectives*,
for each universal quantifier.

Advantages and drawbacks of Semantic net.

Advantages

1. They are **explicit** and **easy** to understand
2. They support **default reasoning** in finite time

Drawbacks

1. They are **incomplete** to handle procedural knowledge
2. They are not **temporal** – and thus, can not define sequence and time

Conceptual Dependency (CD)

- **CD** is proposed as a model of **Natural Language Understanding** (NLP) By **Roger Schank**, in **1969** at Stanford university.

-Motto /Aim

- Used to *acquire knowledge* from natural language *input*
- “*Two sentences identical in meaning, would have a single representation*”
- Knowledge representation is *independent of language*.
- Allow *inferences*
- Knowledge is represented using conceptual primitives and dependencies

https://en.wikipedia.org/wiki/Conceptual_dependency_theory

<https://users.cs.cf.ac.uk/Dave.Marshall/AI2/node69.html>

Conceptual Dependency (CD)

Following are the **primitive acts** used in CD

ATRANS -- Transfer of an abstract relationship. *e.g. give.*

PTRANS-- Transfer of the physical location of an object. *e.g. go.*

PROPEL-- Application of a physical force to an object. *e.g. push.*

MTRANS-- Transfer of mental information. *e.g. tell.*

MBUILD-- Construct new information from old. *e.g. decide.*

SPEAK -- Utter a sound. *e.g. say.*

<https://www.slideshare.net/JismyKJose/conceptual-dependency-70129647>

Conceptual Dependency (CD)

Following are the **primitive acts** used in CD

ATTEND-- Focus a sense on a stimulus. *e.g. listen, watch.*

MOVE-- Movement of a body part by owner. *e.g. punch, kick.*

GRASP-- Actor grasping an object. *e.g. clutch.*

INGEST-- Actor ingesting an object. *e.g. eat.*

EXPEL-- Actor getting rid of an object from body. *e.g. Cry.*

Conceptual Dependency (CD)

Following are the **Conceptual categories** used in CD

PP-- Real world objects. (picture producers)

ACT-- Real world actions. (any one **primitive act**)

PA -- Attributes of objects. (picture aiders)

AA -- Attributes of actions. (action aiders)

T-- Times.

LOC-- Locations.

Conceptual Dependency (CD)

Other components used in CD

- **Arrows** indicates direction of dependency (\rightarrow)
- **Double arrow** indicate two way link between **actor** and **action** (\Leftrightarrow)
 - **O**- for object case relation
 - **R**- Recipient case relation
 - **P**-for past tense
 - **D**-for destination

Conceptual Dependency (CD)

Following **tense** and **moods** are used in CD

p	Past tense
f	Future tense
t	transition
ts	Start transition
tf	Finish transition
k	continuing
?	interrogative
/	negative
	Absence of any modifier indicates present tense

Rules in Conceptual Dependency (CD)

Rule 1: PP \Leftrightarrow ACT

Relationship between **actor** and **action**. A p/f written on link indicates **tense**.

ex1. John ran John $\overset{p}{\Leftrightarrow}$ PTRANS

ex2. John will run John $\overset{f}{\Leftrightarrow}$ PTRANS

Rule 2: ACT \leftarrow PP

Relationship between **act** and PP (object).

ex1. John pushing bike John \Leftrightarrow PROPEL $\overset{o}{\leftarrow}$ bike

Rules in Conceptual Dependency (CD)

Rule 3: $PP \leftrightarrow PP$

Relationship between **two PPs** (objects)

ex1. John is a doctor $John \leftrightarrow doctor$

Rule 4: $PP \leftarrow PP$

Relationship between **two PPs** (objects), one provides info to other of 3 types

1. POSS-BY- possession

2. LOC- location

3. CONT- physical containment

-Arrow is towards concept

ex1. John's dog $\overset{\text{POSS-BY}}{\text{dog}} \leftarrow John$

2. Train at Pune $\overset{\text{LOC}}{\text{Pune}} \leftarrow \text{train}$

3. Pen is in the pocket $\overset{\text{CONT}}{\text{pocket}} \leftarrow pen$

Rules in Conceptual Dependency (CD)

Rule 5: $PP \Leftrightarrow PA$

Relationship between **PP** (object) and **PA** (**attribute** or **state** of PP)

ex1. John is fat $John \Leftrightarrow \text{weight}(> 80)$

Rule 6: $PP \leftarrow PA$

Relationship between **PP** (object) and **PA** (**attribute** of PP)

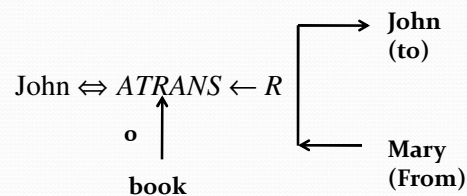
ex1. Smart John $John \leftarrow \text{smart}$

Rules in Conceptual Dependency (CD)

$$\text{Rule 7: ACT} \leftarrow R \left| \begin{array}{l} \rightarrow PP(to) \\ \leftarrow PP(from) \end{array} \right.$$

Relationship between **ACT** and **source** and **recipient** of the act **PP** (object) . R indicate **recipient case** relation

ex1. John take the book from mary



<https://www.slideshare.net/JismyKJose/conceptual-dependency-70129647>

Rules in Conceptual Dependency (CD)

$$\text{Rule 8: PP} \leftarrow \left| \begin{array}{l} \rightarrow PA(to) \\ \leftarrow PA(from) \end{array} \right.$$

Relationship between **attributes** (PA) of an **object** (PP)

ex1. Tree grows

$$\text{tree} \leftarrow \left| \begin{array}{l} \rightarrow size > c \\ \leftarrow size = c \end{array} \right.$$

Rules in Conceptual Dependency (CD)

Rule 9: $\begin{matrix} \Leftrightarrow\{x\} \\ \Uparrow \\ \Leftrightarrow\{y\} \end{matrix}$

Relationship between one concept
to other that causes it
i.e $\{x\}$ causes $\{y\}$

$\{x\}$ Bill shot Bob
 \Uparrow
 $\{y\}$ Bobs health is poor

<https://www.slideshare.net/JismyKJose/conceptual-dependency-70129647>

Rules in Conceptual Dependency (CD)

Rule 10: $\begin{matrix} \Leftrightarrow\{x\} \\ \Downarrow \\ \Leftrightarrow\{y\} \end{matrix}$

Relationship between one concept
to other that is happening when one is in progress
i.e $\{y\}$ is happening while $\{x\}$ in progress

$\{x\}$ while going home
 \Downarrow
 $\{y\}$ I saw snake

Reference-

<https://www.slideshare.net/JismyKJose/conceptual-dependency-70129647>

Inferences in Conceptual Dependency (CD)

Inferences can be taken directly from primitive ACT

-Inferences are stored with primitive acts itself e.g

-Action INGEST has following inferences

- Object is no longer in **previous form**
- Actor has less **hunger now**
- **Physical position** of object can be **changed** and thus **PTRANS** can be inferred

Conceptual Dependency (CD)

Advantages

- Using primitives need **fewer inference rules** (IR) and IR are **represented** in CD **structure itself**
- Stores knowledge in a **language independent** manner
- Multiple statements using different words having same meaning use **same representation**

Disadvantages

- Difficult to find **correct set of primitives** for a situation
- Knowledge must be decomposed in to low **level primitives**
- Representation of even a **simple sentence** can be **very complex** in CD

<http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.73.9307&rep=rep1&type=pdf>

Conceptual Dependency (CD)

CD theory is used in **MARGIE** (**M**emory, **A**nalysis, **R**esponse **G**eneration and **I**nference in **E**nglish) **s**ystem developed for NLP- By **Roger Schank, Stanford University, USA, 1973**

<https://www.ijcai.org/Proceedings/73/Papers/028.pdf>

Conditional probability and Bay's theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \text{ where } P(B) > 0$$

$P(A|B)$ – is conditional probability of event A occurring,
if event B is true/ already occurred

$P(B|A)$ – is conditional probability of event B occurring,
if event A already occurred

$P(A), P(B)$ – are probabilities of event A and B, respectively

A and B must be **different events** and are **dependent** events

Probabilistic reasoning using Bay's theorem

Drug testing for Cannabis user. Cannabis is psycho active drug.

Given data

1. 5% population uses cannabis $P(\text{User})=0.05$
2. $P(\text{Positive} | \text{User})=0.9$, the actual user of cannabis tested positive i.e *true positive rate is* 90%
3. $P(\text{Positive})=0.235$

Find the probability that someone is a cannabis user given that they test positive $P(\text{User} | \text{Positive})$

Probabilistic Reasoning using Bay's theorem

$$P(\text{User} | \text{Positive}) = \frac{P(\text{Positive} | \text{User})P(\text{User})}{P(\text{Positive})}$$

$$\begin{aligned} P(\text{User} | \text{Positive}) &= \frac{P(\text{Positive} | \text{User})P(\text{User})}{P(\text{Positive})} = \frac{0.9 \times 0.05}{0.235} \\ &= \frac{0.045}{0.235} = 0.1915 = 19\% \end{aligned}$$

https://en.wikipedia.org/wiki/Bayes%27_theorem

Bayesian Networks

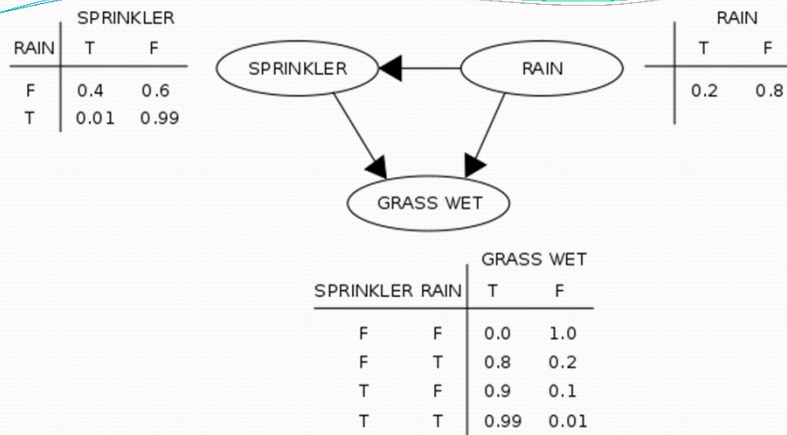
- It is one of the approach to reason in Uncertainty using the Probability
- Also termed as Bays Network, Belief network or Decision network.
- It is a graphical model that represents set of variables and conditional dependencies using Directed Acyclic Graph (DAG)
- They are most suited to know the likelihood of probable causes of an event that occurred
- There can be probabilistic relationship between symptoms and disease, given symptoms a disease can be predicted (*probabilistic inference*)

Bayesian Networks

Definition:- They contain *set of nodes* representing *variables* describing observable *parameters* or *hypothesis*. *Edges* represents *conditional dependencies*.

- Nodes that are not connected assumed *conditionally independent* of each other
- Each node is associated with the *probability function* that takes as *input* from the *nodes parents* and provide *output* that is *probability* of that variable represented by that node
- For example, if *m* parent nodes represent *m Boolean variables*, then the probability function could be represented by a table of 2^m entries, one entry for each of the 2^m possible *parent combinations*

Bayesian Networks



A simple Bayesian network with [conditional probability tables](#)

It reasons by using [posterior probability](#) of unobserved variable when an evidence of a cause is available

[https://en.wikipedia.org/wiki/Bayesian_network#:~:text=A%20Bayesian%20network%20\(also%20known,directed%20acyclic%20graph%20\(DAG\).](https://en.wikipedia.org/wiki/Bayesian_network#:~:text=A%20Bayesian%20network%20(also%20known,directed%20acyclic%20graph%20(DAG).)

Bayesian Networks-Pros and Cons

- Produce [accurate](#) and [valid](#) results
- Sensitive to [prior probabilities](#)
- We need an [expert in probability/ great expertise in statistics](#) to define Bayesian Networks, than other approaches.
- Designing them for [more variables](#) is a [challenging](#) task in [machine learning](#)
- [BayesiaLab](#)- a s/w for design of Bayesian Networks (bayesia.com)

Reading list

- Justification based truth maintenance system
- Many **equivalent ways** to write and apply Bays theorem

<https://www.statisticshowto.com/bayes-theorem-problems/>

Probability and Bays' theorem