INDIAN STATISTICAL INSTITUTE

POST-GRADUATE DIPLOMA IN BUSINESS ANALYTICS (PGDBA): 2024–26

Course: Stochastic Processes and Applications

Time Series Project Report

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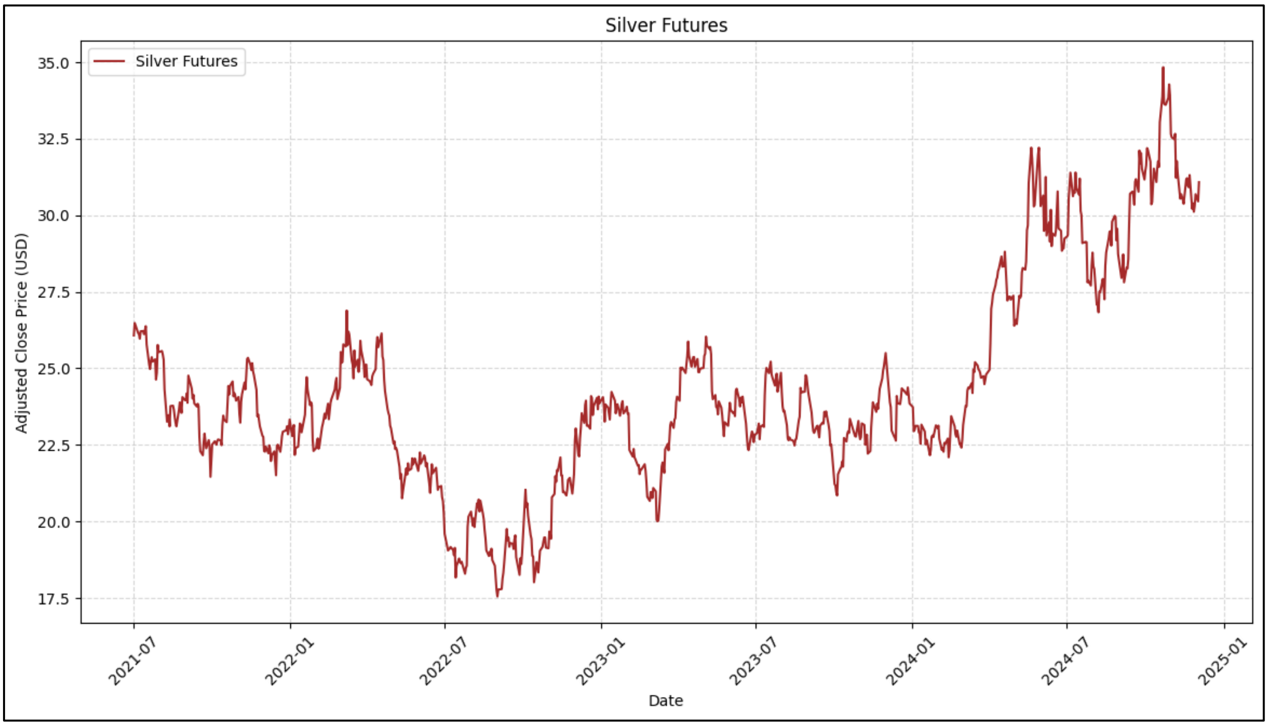
Dataset Description

For this project, we have chosen **silver futures** as our dataset to conduct a time series analysis. Silver futures represent an essential financial instrument widely traded in commodity markets, and understanding their price trends is crucial for market participants, including investors, traders, and policymakers.

The dataset comprises **daily closed prices of silver** recorded over a period of **861 days**, spanning from **July 1, 2021, to December 4, 2024**. The closed price indicates the market price at the end of each trading day, providing a consistent measure for analyzing trends and patterns over time.



Upon visual inspection of the time series plot, distinct patterns were observed:

Trend: A persistent upward movement in the data indicates long-term progression in silver prices.

Seasonality: Recurrent fluctuations at regular intervals suggest cyclical patterns, possibly influenced by external factors like market demand, macroeconomic conditions, or seasonal events.

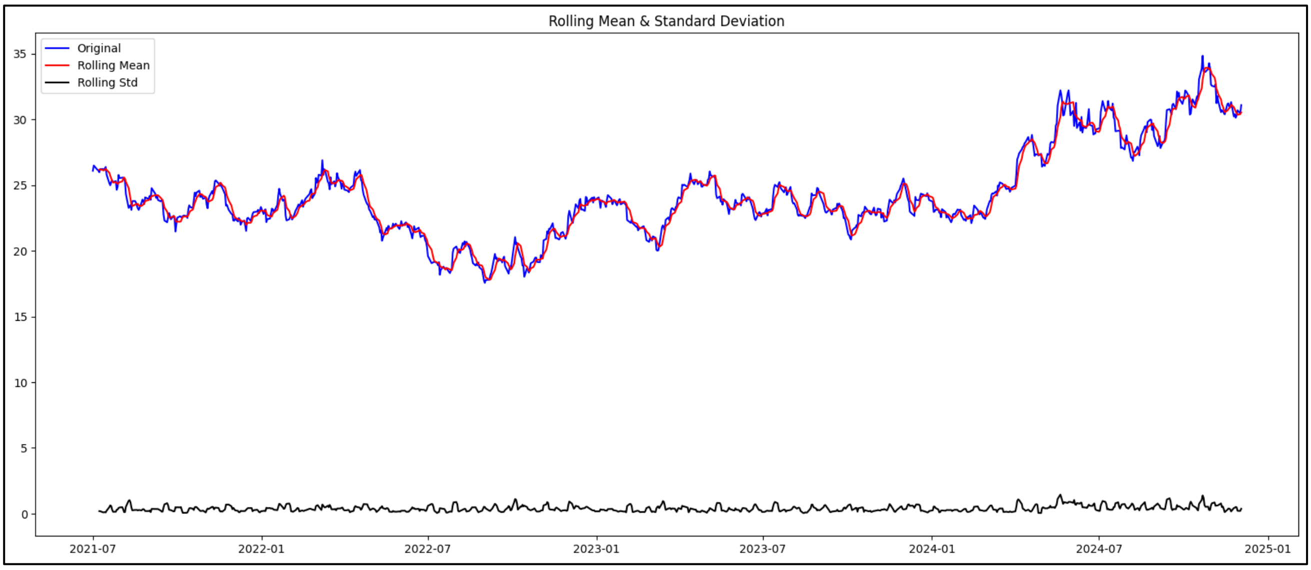
The identified trend and seasonality suggest the need for advanced time series models that can effectively capture these characteristics to forecast future prices accurately. In subsequent sections, we will delve deeper into preprocessing, model selection, and analysis.

Stationarity check of Data:

Before proceeding with modelling, it is critical to assess whether the data is stationary, as many time series models require stationarity for reliable predictions. A stationary time series has constant statistical properties—mean, variance, and autocovariance—over time.

To evaluate the stationarity of the dataset, the following methods were applied:

Rolling Mean and Rolling Variance Plots



A rolling mean plot with a window size of **5 days** was generated alongside a rolling variance plot.

The rolling mean exhibited noticeable fluctuations over time, indicating that the dataset does not maintain a constant mean. These visual cues strongly suggested that the data is **not stationary**.

Augmented Dickey-Fuller (ADF) test

Results of Dickey-Fuller Test:	**Values**
Test Statistic	-1.510263
p-value	0.528428
#Lags Used	0.000000
Number of Observations Used	862.000000
Critical Value (1%)	-3.437959
Critical Value (5%)	-2.864899
Critical Value (10%)	-2.568558

- The Augmented Dickey-Fuller (ADF) test, a statistical test to confirm stationarity, was conducted.
- The null hypothesis of the ADF test is that the data is not stationary (i.e., it contains a unit root).
- The **p-value** from the test was observed to be **0.52**, which is significantly higher than the typical threshold (e.g., 0.05).

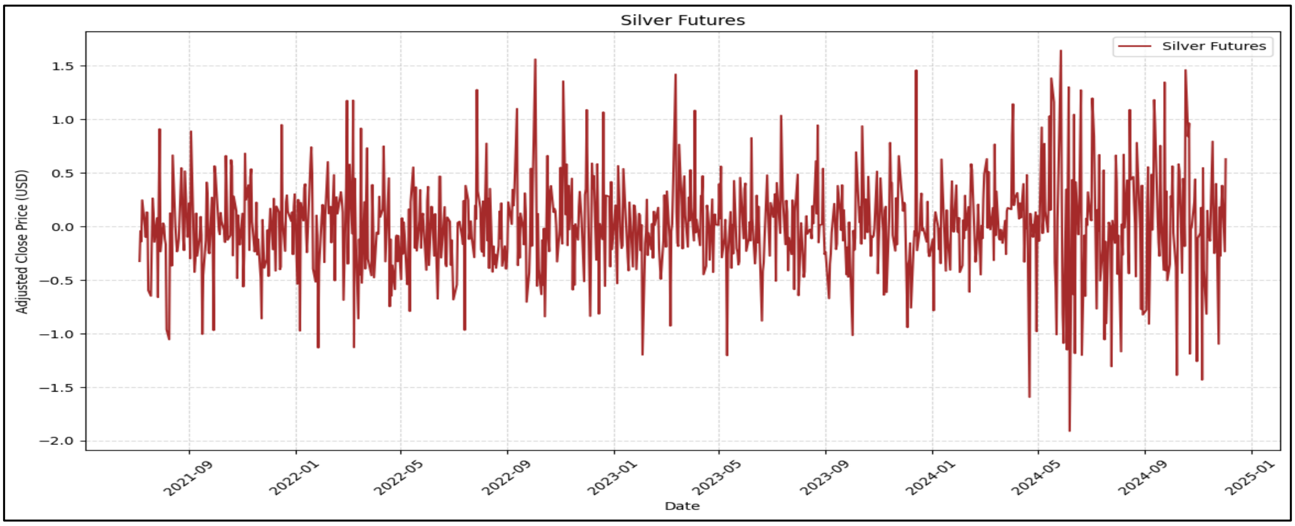
As a result, we failed to reject the null hypothesis, reaffirming that the data is **not stationary**. The findings from both the rolling plots and the ADF test demonstrate that the dataset lacks stationarity in its original form. This outcome emphasizes the necessity for transformation techniques, such as differencing, to achieve stationarity, a prerequisite for effective time series modelling. Further analysis and transformations will be discussed in subsequent sections.

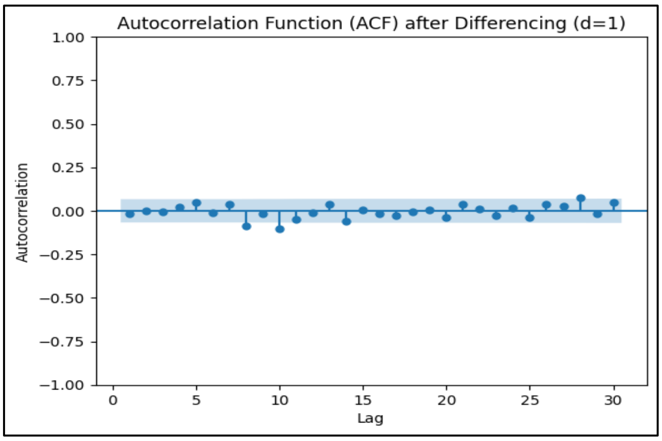
Data Transformation for Stationarity

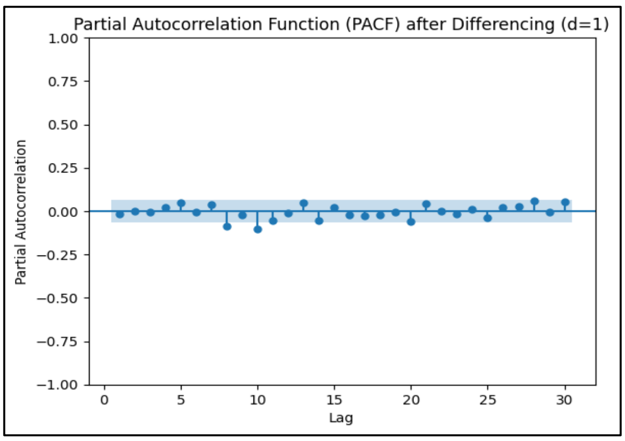
Since all tests conducted indicated that the data was not stationary, a transformation was applied to ensure stationarity—a prerequisite for accurate time series modelling. The method of **first differencing** was employed to achieve this transformation. First differencing involves subtracting the previous observation from the current observation, effectively removing trends and stabilizing the mean of the time series. The differenced data was then analysed further to confirm the success of the transformation. By eliminating the trend component and reducing variability, first differencing provided a stationary dataset suitable for subsequent analysis and modelling. This step was crucial for ensuring the reliability and validity of the time series forecasting models used in later stages of the project. Subsequent sections will detail the results of this transformation and the modelling process.

Results after 1St Differencing

Analysing Trend After 1st Differencing







ACF and PACF Plots

Observations from ACF Plot:

- Almost all autocorrelation coefficients fall within the **shaded confidence interval**, which is consistent with white noise.
- There are no significant spikes, which indicates no persistent correlation between the data points.

Observations from PACF Plot:

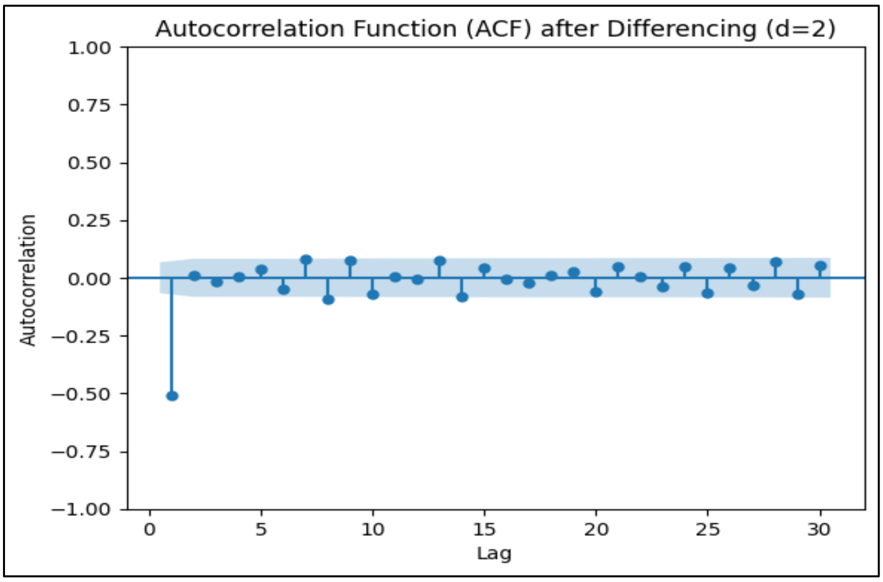
- Similarly, all partial autocorrelation coefficients are within the confidence interval.
- There is no sharp cutoff or decay, further supporting the white noise hypothesis.

Analyzing Differenced Data and Additional Transformation

After applying first differencing to the dataset, the **Autocorrelation Function (ACF)** and **Partial Autocorrelation Function (PACF)** plots were analyzed. The patterns in these plots revealed characteristics of **white noise**, suggesting that the first differencing alone was insufficient to fully address the non-stationarity in the data. To further stabilize the dataset and capture the underlying structure, **second differencing** was performed. This additional differencing step helped to account for the residual trend and variability in the time series. Upon reviewing the original plot of silver prices, it was observed that the dataset appeared to follow a **quadratic trend**. This observation indicated that second differencing was particularly appropriate, as it is effective in handling quadratic or higher-order trends in time series data. The application of second differencing resulted in a dataset that was better suited for time series analysis and forecasting. The transformed data was then ready for model identification and selection, as will be detailed in subsequent sections.

Results after 2^{ND} Differencing

ACF PLOT



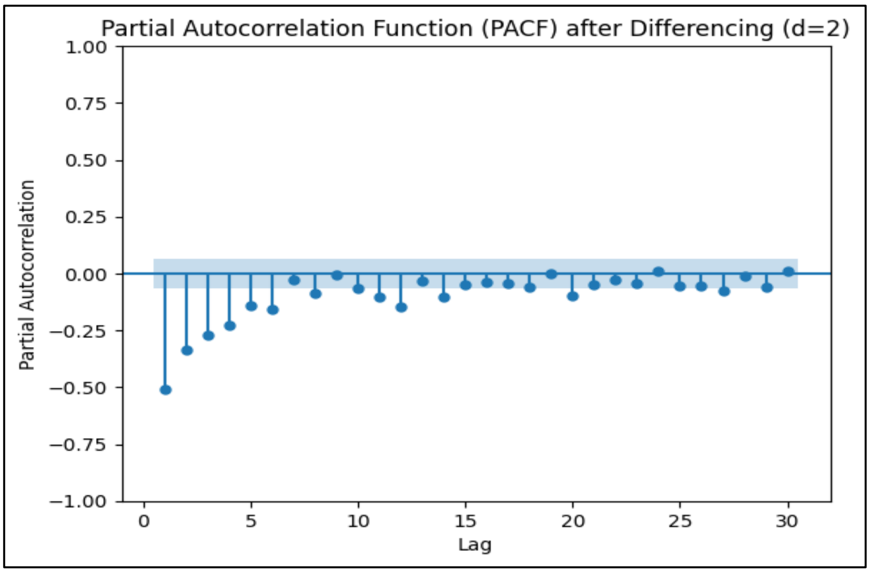
Observations from ACF Plot:

Significant spike at lag 1: The ACF has a **large significant spike at lag 1**. This means that the value at time t is highly correlated with the value at t−1, even after applying differencing twice. This suggests some short-term memory in the series that hasn't been fully removed by differencing.

Gradual decay for higher lags: The ACF gradually decays toward zero for larger lags (lags 2 and beyond).This decay indicates the **presence of moving average (MA) components**, suggesting the data exhibits residual dependencies that could be modelled with an MA process.

Oscillations: The ACF plot shows slight oscillations (alternating positive and negative correlations) after the first few lags. This behaviour can sometimes indicate a combination of trend removal and seasonal effects or noise in the data.

PACF PLOT



Observations from PACF Plot:

Sharp cutoff at lag 1: The PACF shows a **significant spike at lag 1**, followed by a sharp drop-off, with no significant spikes at higher lags.

This is a hallmark of an **AR (1)** process, where the series is primarily influenced by its immediate past value.

No significant spikes beyond lag 1: The lack of significant spikes after lag 1 suggests that higher-order autoregressive terms (e.g., AR (2), AR (3)) do not contribute much to explaining the variation in the series.

Interpretation of ACF and PACF:

- **Autoregressive (AR) Component:** The sharp cutoff in the PACF after lag 1 points to **AR (1)**.
- **Moving Average (MA) Component:** The ACF's gradual decay suggests the potential for a **low-order MA process (possibly MA (0) or MA (1))**.

Model Identification and Selection

Following the second differencing of the data, the **Autocorrelation Function (ACF)** and **Partial Autocorrelation Function (PACF)** plots were examined to identify potential models for forecasting. Based on the observed patterns, the data appeared to align with the characteristics of the following ARIMA models:

1. **ARIMA (0,2,1)**: Incorporating no autoregressive (AR) terms, second-order differencing, and one moving average (MA) term.
2. **ARIMA (1,2,1)**: Including one autoregressive (AR) term, second-order differencing, and one moving average (MA) term.

To determine the most appropriate model, both combinations of these *pp*p, *dd*d, and *qq*q values were tested. The models were evaluated based on their performance metrics, including AIC, BIC, and RMSE, to identify the model with the best fit. Additionally, the possibility of introducing a **seasonality component** was considered. If seasonal patterns were detected in the residuals or observed data, a Seasonal ARIMA (SARIMA) model would be employed to enhance the predictive capability. The seasonal parameters (*PP*P, *DD*D, *QQ*Q, and *ss*s) would be identified and integrated to address these patterns effectively. In the subsequent section, the results of model testing and the potential inclusion of seasonal components will be detailed, along with insights into the final selected model and its implications for forecasting.

ARIMA (1,2,1)

Model Results and Evaluation

To begin the modelling process, the **ARIMA (1,2,1)** model was tested. The results from the SARIMAX implementation provided the following key metrics and observations:

Model Summary:

AIC: 1092.269 **BIC**: 1106.519 **HQIC**: 1097.726

Parameter Estimates:

Component	Coefficient	Std. Error	z-value	P-value	Confidence Interval (95%)
AR (1)	-0.0166	0.033	-0.507	0.612	[-0.081, 0.048]
MA (1)	-1.0000	0.458	-2.182	0.029	[-1.898, -0.102]
Sigma^2	0.2072	0.095	2.172	0.030	[0.020, 0.394]

Diagnostics:

1. **Ljung-Box (Q)**: The p-value of 0.94 suggests that the residuals exhibit no significant autocorrelation at lag 1.
2. **Jarque-Bera (JB)**: A p-value of 0.00 indicates that the residuals deviate from normality.
3. **Heteroskedasticity (H)**: The p-value of 0.00 indicates the presence of heteroskedasticity, with a skew close to zero and a kurtosis of 4.57.

Analysis of Results:

1. The p-value for the AR (1) coefficient was **0.612**, suggesting that the AR component was statistically insignificant.
2. Conversely, the MA (1) component was significant with a p-value of **0.029**, indicating its contribution to the model.
3. The sigma² term was also statistically significant, reflecting variability in the residuals.

Given the insignificance of the AR (1) component, it was deemed appropriate to simplify the model by dropping the autoregressive component and testing the **ARIMA(0,2,1)** model. The subsequent section will present the results and evaluation of this revised model.

ARIMA (0,2,1)

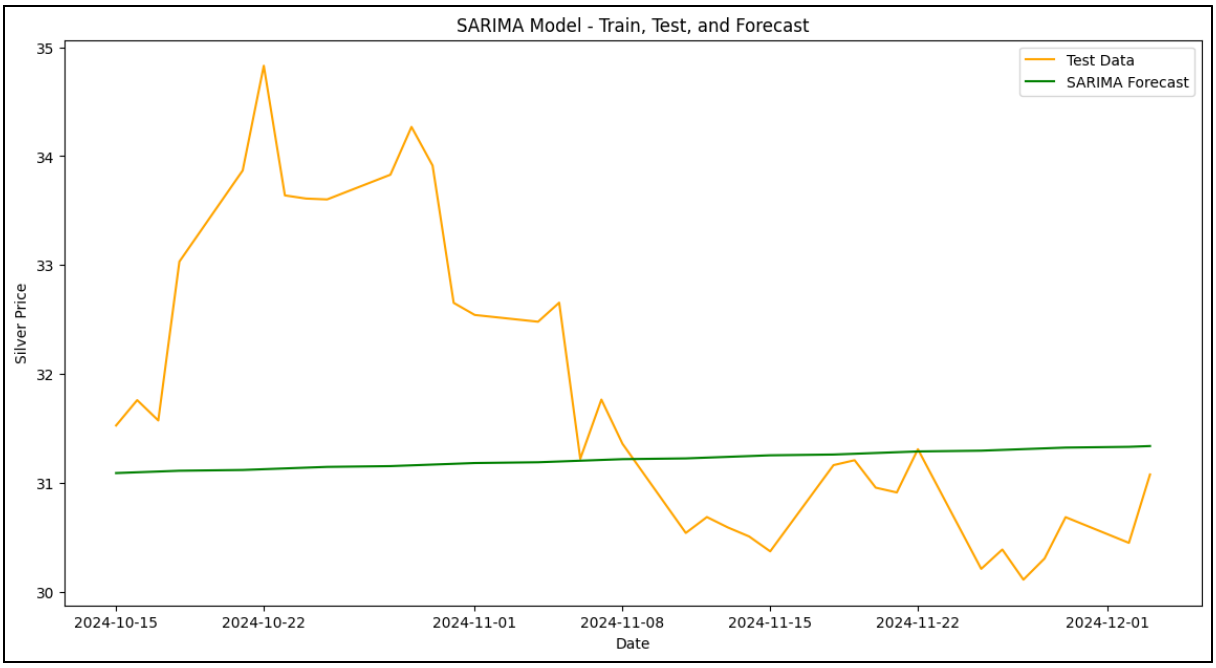
Model Results and Evaluation

Following the ARIMA (1,2,1) evaluation, the ARIMA (0,2,1) model was tested. The **Root Mean Square Error (RMSE)** for the test dataset was calculated as **1.5390**, and the model's results were visualized using a plot comparing the test data and forecasted values.

Key Observations:

1. While the ARIMA(0,2,1) model showed some improvement in terms of simplicity and lower AIC compared to ARIMA(1,2,1), the performance metrics and forecast accuracy were still not satisfactory.
2. The plot indicated that the forecasted values failed to capture significant variations in the test data, highlighting the model's inability to adapt to potential patterns or seasonality in the data.

To address these shortcomings, it was decided to introduce **seasonal components** into the model. Seasonal patterns often play a crucial role in financial time series data, such as silver futures, as they can capture repetitive cycles or trends occurring over fixed intervals.



In the next step, a **Seasonal ARIMA (SARIMA)** model will be implemented. This will include seasonal parameters to account for potential cyclic behaviour, thereby improving the model's forecasting capabilities. Results from the SARIMA implementation and its evaluation will be presented in the subsequent section.

Power Spectrum

To investigate seasonality in the silver price data, the Power Spectrum was analyzed against the time period using a periodogram. This analysis revealed various periodic components within the dataset, which provided insights into the dominant cycles present in the data. The periodogram results were as follows:

Period (1/f)	Power Spectrum
inf	1.60E-27
862.00	5.13E+03
431.00	1.01E+03
287.33	1.44E+03
215.50	4.08E+02
172.40	1.38E+02
143.67	6.67E+00
123.14	3.26E+02
107.75	2.47E+02
95.78	1.88E+02
86.20	2.75E+01

Secondary peaks at 431, 287, 215,107, days and so on suggest significant seasonality, but the 107-day period provided the best results for forecasting. Therefore, we will incorporate the 107-day seasonal component into the SARIMA model to enhance prediction accuracy and capture the observed cyclical behaviour. The power at **107 days** is notably high indicating possible seasonality over this time intervals.

Test Seasonal Models:

Based on the identified seasonality of approximately **107 days**, we will introduce this seasonality into our modelling approach by adjusting the **seasonal order** in the **SARIMA** or **SARIMAX** models. This will allow us to determine whether incorporating this seasonal component improves the model's performance (e.g., reducing RMSE or MAPE).

Justification of 107 as the Seasonality Period:

Visual Evidence: The sharp peak at 107 days in the periodogram is consistent with the observed time series data, making it a valid candidate for seasonality.

Physical Meaning: In the context of silver prices, a **107-day cycle** could align with quarterly or biannual patterns in market behaviour, potentially driven by fiscal cycles, supply chain events, or broader commodity market cycles. By integrating this seasonality parameter (with **m=107**) into our SARIMA model, we will evaluate if the inclusion of this seasonal pattern significantly enhances the model's ability to forecast silver price trends accurately.

Seasonal Parameter Optimization

To enhance the model's performance, we experimented with various combinations of seasonal parameters **(P, D, Q)** along with the seasonal period **m=107**, and evaluated the results using **Root Mean Square Error (RMSE)** and **Mean Absolute Percentage Error (MAPE)**. After testing several combinations, we identified the optimal parameters that provided the best forecasting accuracy.

The following table summarizes the seasonal parameter combinations tested:

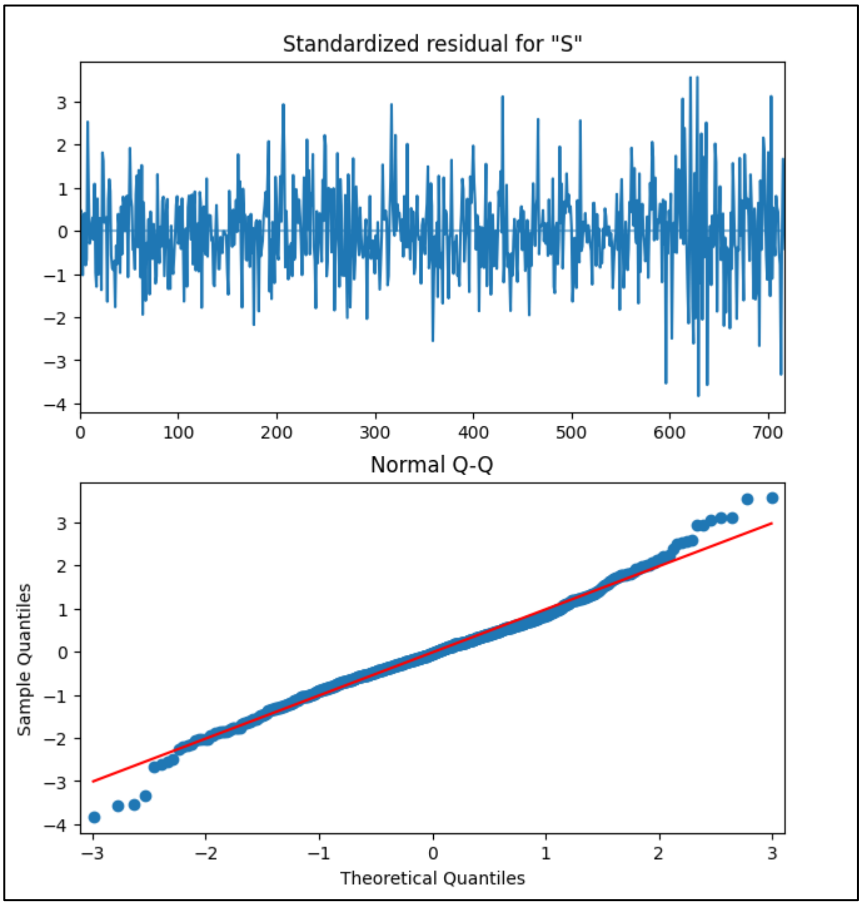
P	D	Q	m (Seasonal Period)	RMSE	MAPE
0	0	0	107	1.932	5.02%
0	1	0	107	2.712	4.67%
0	1	1	107	1.589	4.31%
1	0	0	107	3.845	4.92%
1	1	0	107	2.563	4.16%
1	1	1	107	4.529	6.98%
2	0	0	107	1.587	4.08%
2	1	0	107	**1.046**	**2.93%**
2	1	1	107	1.21	3.55%

Based on the results, the best combination of seasonal parameters was found to be **(P=2, D=1, Q=0, m=107)**, which yielded the lowest **RMSE (1.046)** and **MAPE (2.93%)**. This combination provided the best fit for the seasonal patterns in the silver price data, making it the optimal choice for the SARIMA model. Thus, the SARIMA model with **(2,1,0,107)** seasonal parameters will be used for the final forecasting. This will allow us to capture both the seasonal behaviour and underlying trends in the silver price data more effectively.

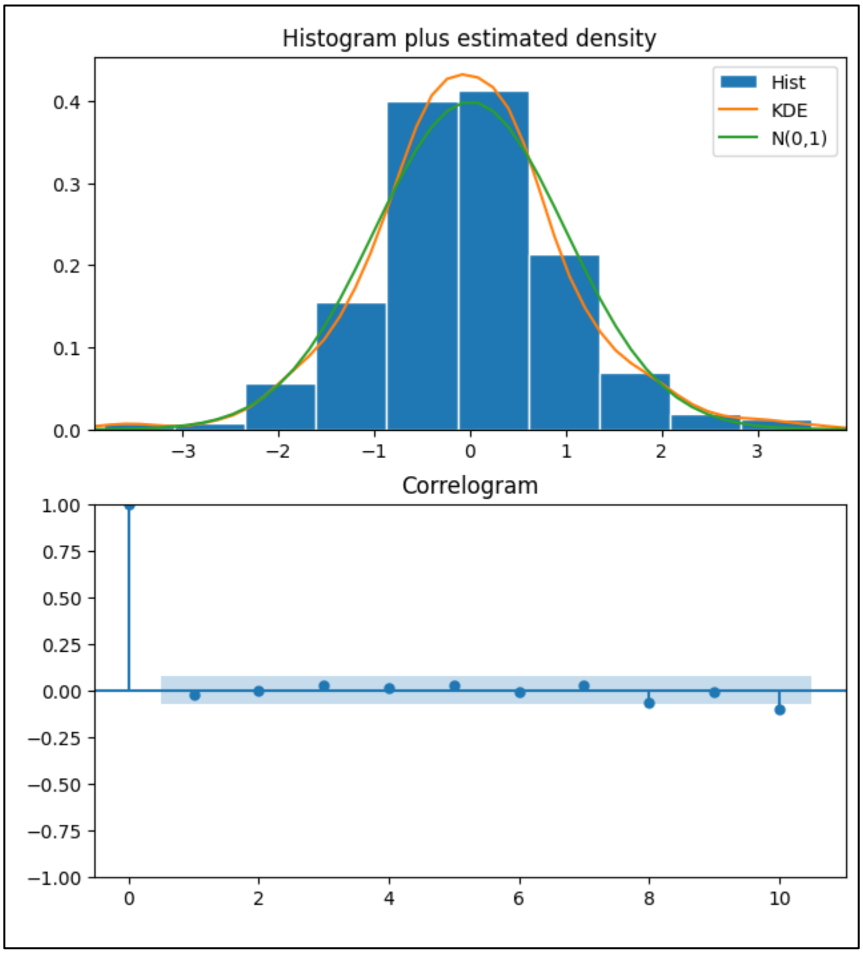
SARIMA model with **(0,2,1) (2,1,0,107)**

Residual Analysis (Diagnostic Plots)

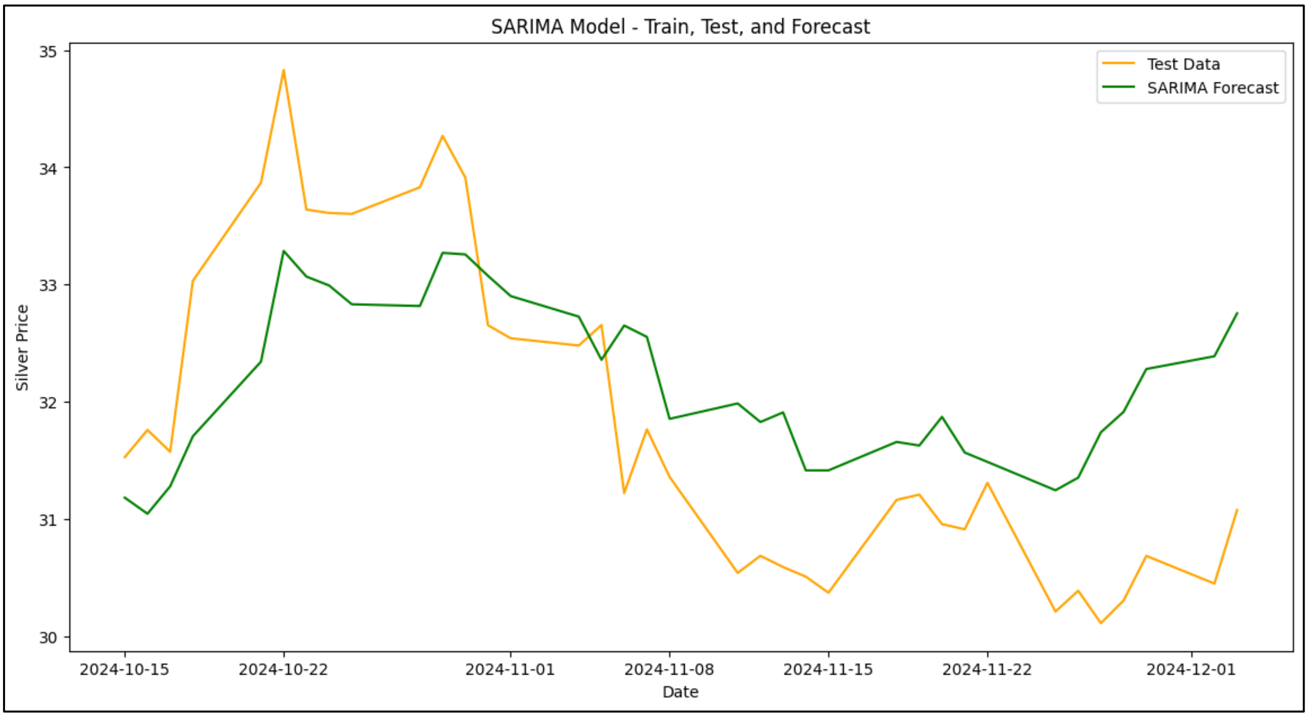
1. The standardized residuals appear random and are centered around zero, which is indicative of a well-fitted model. No apparent trends or patterns suggest that the SARIMA model has effectively captured the seasonality and trends in the data.
2. The histogram with a density curve closely aligns with a normal distribution, confirming that the residuals are approximately normally distributed. This validates the assumption of homoscedasticity.
3. The Normal Q-Q plot shows that most residuals lie along the 45-degree reference line, with minor deviations at the tails, further supporting the normality of the residuals.
4. The correlogram indicates that the residuals are not autocorrelated, as the lag coefficients mostly lie within the confidence bands. This suggests that the model has accounted for temporal dependencies effectively.



Standardized residual for "S"
Normal Q-Q
Sample Quantiles
Theoretical Quantiles



Histogram plus estimated density
Hist
KDE
N(0,1)
Correlogram



Test vs. Forecast Plot

- The plot comparing the test data (actual prices) and forecasted values shows that the SARIMA model captures the general trend and seasonal variation in silver prices well.
- There is slight divergence in certain periods, particularly during high volatility in the test data. However, the model remains close to the observed data, indicating reasonable predictive performance.

Performance Metrics

- **Root Mean Squared Error (RMSE):** 1.046
- **Mean Absolute Percentage Error (MAPE):** 2.93%
  These metrics highlight the model's accuracy in capturing both the magnitude and variability of silver price movements.

Conclusion

The SARIMA model with parameters (0, 2, 1); (2, 1, 0, 107) provides an excellent balance between simplicity and predictive power. By incorporating a seasonal component (m=107), the model captures the underlying periodic behavior of silver prices effectively.