

turn-in answers as a Word document (A5.docx) and commit/push it to your TAMU GitHub

1. Translate the following sentences into First-Order Logic. Remember to break things down to simple concepts (with short predicate and function names) and make use of quantifiers. Foreexample, don't say "tasteDelicious(someRedTomatos)", but rather: " $\exists x \text{tomato}(x) \wedge \text{red}(x) \wedge \text{taste}(x, \text{delicious})$ ". See the lecture slides for more examples and guidance.

Tomatoes are either a fruit or vegetable.

$$\forall x \text{tomato}(X) \rightarrow (\text{fruit}(X) \wedge \neg \text{vegetable}(X)) \vee (\neg \text{fruit}(X) \wedge \text{vegetable}(X))$$

Because Tomato can either be fruit or vegetable but not both!

Some mushrooms are poisonous.

$$\exists x \text{Mushroom}(x) \wedge \text{Poisonous}(x)$$

No student at TAMU likes every class.

$$\text{For every } X, \text{student}(X), \text{tamu}(X) \rightarrow \text{Some } Y, \text{classes}(Y) \wedge \text{dislikes}(X, Y)$$

Every king has a crown and some subjects. (How many is not specified)

$$\text{For every } X, \text{king}(X), \text{has_crown}(X) \rightarrow \text{For some } Y, \text{subject}(Y)$$

An isosceles triangle is a triangle with 2 sides of equal length (but not 3).

$$\text{For all } X, Y, Z, \text{side}(X), \text{side}(Y), \text{side}(Z), \text{triangle}(\text{side}(X), \text{side}(Y), \text{side}(Z)) \rightarrow (\text{equals}(X, Y) \wedge \neg \text{equals}(X, Z)) \vee (\text{equals}(Y, Z) \wedge \neg \text{equals}(X, Y)) \vee (\text{equals}(X, Z) \wedge \neg \text{equals}(X, Y))$$

The left front tire of Henry's car is flat.

$$\text{For some } t, c, \text{Car}(c) \wedge \text{owner}(\text{Henry}, c) \wedge \text{Tire}(t) \wedge \text{count}(\text{Tire}(t)) = 4 \wedge \text{flat}(t) \wedge \text{partOf}(t, \text{leftfront}(c))$$

All disk drives are electronic; they are either mechanical (HDDs) or solid-state (SSDs). Solid-state drives are faster than mechanical drives, in terms of read-access time (e.g., msper byte), but SSDs are also more expensive (in dollars per Gb).

$$\text{For all } X, \text{DiskDrive}(x) \wedge \text{Electronic}(x) \rightarrow \text{Mechanical}(x) \vee \text{SolidStateDrive}(x)$$

$$\text{For all } X, Y \text{ SSD}(X), \text{MechanicalDrive}(Y) \rightarrow \text{ReadSpeed}(X) > \text{ReadSpeed}(Y)$$

$$\text{For all } X, Y \text{ SSD}(X), \text{MechanicalDrive}(Y) \rightarrow \text{cost}(X) > \text{cost}(Y)$$

All laptops sold by Dell in 2022 have at least 4GB of RAM and cost at least \$1000.

$$\text{For all } X, \text{laptop}(X), \text{sellingYEAR}(2022), \text{sold}(\text{DELL}, X) \rightarrow \text{ramInGB}(X) \geq 4 \wedge \text{costInDollar}(X) > 1000$$

John's favorite Sci-Fi movie is Star Wars (hint: instead of saying *favoriteSciFiMovie(John, StarWars)* which is too complex, break it down into simpler concepts such as predicate 'likes(P,M)', and function 'score(P,M)', which denotes P's internal rating system for things (e.g., 1-100). Ensure you are comparing SciFi movies, because a person might use a different rating system for other items, such as toasters.

$$\text{For all } X, \text{movie}(X) \wedge \text{sciFi}(X) \rightarrow \text{score}(\text{John}, X) \leq \text{score}(\text{John}, \text{starwars})$$

2. Determine whether or not the following pairs of predicates are unifiable. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Terms that are variables are in capital letters; constants and function names are lowercase. For example, 'loves(A, hay)' and 'loves(horse, hay)' are unifiable, the unifier is $u=\{A/horse\}$, and the unified expression is 'loves(horse, hay)' for both.

0)	loves(A, hay)	loves(horse, hay)	yes	$u=\{A/horse\}$	loves(horse, hay)
1)	$p(a, X)$	$p(b, X)$	no	reason: regardless of X, a and b conflict	
2)	$p(a, X, f(g(Y)))$	$p(Z, f(Z), f(U))$	yes	$u=\{Z/a, X/f(a), U/g(Y)\}$	$p(a, f(a), f(g(Y)))$
3)	$q(f(a), g(X))$	$q(Y, Y)$	no	Because if we unify the Y to f(a), we cannot unify it to g(X).	
4)	$r(f(Y), Y, X)$	$r(Z, f(a), f(V))$	yes	$u=\{Z/f(Y), Y/f(a), X/f(V)\}$	$r(f(f(a)), f(a), f(V))$
5)	$p(a, Y, f(X))$	$p(Z, f(b), f(b))$	yes	$u=\{Z/a, Y/f(b), X/b\}$	$p(a, f(b), f(b))$
6)	$q(g(f(a)), g(X), Z)$	$q(Y, Y, f(W))$	yes	$u=\{Y/g(f(a)), X/f(a), Z/f(W)\}$	$q(g(f(a)), g(f(a)), f(W))$
7)	$p(x, f(X), X)$	$p(Y, f(a), b)$	no	Because X/b then X/a is not unifiable	
8)	$q(f(a, a), V, Z)$	$q(X, f(X, X), Y)$	yes	$u=\{X/f(a, a), V/f(f(a, a), f(a, a)), Y/Z\}$	$q(f(a, a), f(f(a, a), f(a, a)), Z)$

3. Caesar

Consider the following situation:

Marcus is a Pompeian.

All Pompeians are Romans.

Caesar is a ruler.

All Romans are either loyal to Caesar or hate Caesar (but not both).

Everyone is loyal to someone.

People only try to assassinate rulers they are not loyal to.

Marcus tries to assassinate Caesar.

Translate these sentences to First-Order Logic.

1. pompeiiian(marcus)
2. For all x, pompeiiian(X) \rightarrow roman(X)
3. ruler(caesar)
4. For all x, roman(X) \rightarrow [loyal(X,caesar) \wedge \sim hate(X,caesar)] \vee [\sim loyal(X,caesar) \wedge hate(X,caesar)]
5. For all x, Some y, people(X), people(Y) \rightarrow loyal(X, Y)
6. For all x,y, people(X), ruler(Y), tryToAssassinate(X, Y) \rightarrow \sim loyal(X, Y)
7. tryToAssassinate(marcus, caesar)

Prove that **Marcus hates Caesar** using Natural Deduction. Label all derived sentences with the ROI and prior sentences and unifier used.

Implicitly We Know

- 1) Some x, marcus \wedge people(X)
- 2) Some x, caesar \wedge people(X)

Marcus is a roman because {Marcus is Pompeian and Pompeians are Romans}

- 3) roman(marcus) MP on 1 and 2
u={X/marcus} pompeiiian(marcus) \rightarrow roman(marcus)

Marcus is not loyal to Caesar because {People only try to assassinate rulers they are not loyal to and Marcus tries to assassinate Caesar}

- 4) people(marcus) AE on 8
u={X/marcus}
- 5) \sim loyal(marcus, caesar) MP on 11, 7, 3 on 6
u={X/marcus, Y/caesar}
people(marcus), ruler(marcus), tryToAssassinate(marcus, caesar)

Marcus hates Caesar because {Marcus is a roman and Marcus is not loyal to Caesar}

- 6) \sim loyal(marcus, caesar) \wedge hates(marcus, caesar) MP 10, 4
u={X/marcus}
- 7) hates(marcus, caesar) AE on 13

Convert all the sentences into CNF

1. pompeiiian(marcus)
2. \sim pompeiiian(x) \vee roman(x)

3. ruler(caesar)

$\sim \text{roman}(x) \vee \{ [\text{loyal}(x, \text{caesar}) \wedge \sim \text{hate}(x, \text{caesar})] \vee [\sim \text{loyal}(x, \text{caesar}) \wedge \text{hate}(x, \text{caesar})] \}$

//Now distribute the $\sim \text{roman}(X)$ inside the big { } bracket!

$[\sim \text{roman}(x) \vee \text{loyal}(x, \text{caesar}) \wedge \sim \text{roman}(x) \vee \sim \text{hate}(x, \text{caesar})] \vee [\sim \text{loyal}(x, \text{caesar}) \wedge \text{hate}(x, \text{caesar})]$

//Applying distributivity and removing the unnecessary ones

$\sim \text{roman}(x) \vee \{ [\sim \text{loyal}(X, \text{caesar}) \vee \text{loyal}(X, \text{caesar})] \wedge [\sim \text{loyal}(X, \text{caesar}) \vee \sim \text{hate}(X, \text{caesar})] \wedge [\text{hate}(x, \text{caesar}) \vee \text{loyal}(x, \text{caesar})] \wedge [\text{hate}(x, \text{caesar}) \vee \sim \text{hate}(x, \text{caesar})] \}$

4. $\sim \text{roman}(x) \vee \{ \text{hate}(x, \text{caesar}) \vee \text{loyal}(x, \text{caesar}) \}$

5. $\sim \text{people}(X) \vee \sim \text{people}(F(x)) \vee \text{loyal}(X, F(x))$

6. $\sim \text{people}(X) \wedge \sim \text{ruler}(Y) \wedge \sim \text{tryToAssassinate}(X, Y) \vee \sim \text{loyal}(X, Y)$

7. $\text{tryToAssassinate}(\text{marcus}, \text{caesar})$

Prove that **Marcus hates Caesar** using Resolution Refutation.

8. $\sim \text{hates}(\text{marcus}, \text{caesar})$

Distribution on 4

9. $[\sim \text{roman}(\text{marcus}) \vee \text{hate}(\text{marcus}, \text{caesar})] \vee [\sim \text{roman}(\text{marcus}) \vee \text{loyal}(\text{marcus}, \text{caesar})]$

Unification on 9 using 8 AND Resolution Refutation on 9

10. $\sim \text{roman}(\text{marcus}) \vee \text{loyal}(\text{marcus}, \text{caesar})$

Resolution Refutation from 10 onto 2

11. $\sim \text{pompeiiian}(\text{marcus})$

//11 contradicts with statement 1 and therefore they cancel out by Resolution Refutation

12. {null-set}

4. Writing KBs in FOL

Write a KB (rules/axioms) in FOL for...

Map-colouring – every state must be exactly 1 colour, and adjacent states must be different colours. Assume possible colours are states are defined using unary predicate like colour(red) or state(WA). To say a state has a colour, use a binary predicate, e.g. 'colour(WA, red)'.

state(WA)	neighbor(WA,SA)
state(NT)	neighbor(NT,SA)
state(SA)	neighbor(NT,QU)
state(QU)	neighbor(QU,SA)
state(NSW)	neighbor(QU,NSW)
state(VI)	neighbor(SA,NSW)
state(TA)	neighbor(SA,VI)
neighbor(WA,NT)	neighbor(NSW,VI)

For every X, state(X) \rightarrow colour(X, red) \vee colour(X, green) \vee colour(X, blue)

For every X, Y, P, Q state(X) \wedge state(Y) \wedge neighbor(X,Y) \wedge color(X, P) \wedge color(Y, Q) \rightarrow P \neq Q

Sammy's Sport Shop – include implications of facts like obs(1,W) or label(2,B), as well as constraints about the boxes and colours. Use predicate 'cont(x, q)' to represent that box x contains tennis balls of colour q (where q could be W, Y, or B).

box(1)	observe(1, yellow)
box(2)	observe(2, white)
box(3)	observe(3, yellow)
color(white)	label(1, white)
color(yellow)	label(3, both)
color(both)	label(1, white)

//If you observe a ball of that color drawn, then it contains either that color alone or both

For every X,Y, box(X), color(Y), observe(Y) \rightarrow contains(X, Y) \vee contains(X, B)

//If the box is labeled that color, then it does not contain that color

For every Y, X, box(Y), color(X), label(Y, X) \rightarrow \sim contains(Y, X)

//Each box is of different color

For every Y, contains (box(Y), color(white)) \rightarrow contains (box(Y), \sim color(yellow)) \wedge tains (box(Y), \sim color(both))

For every Y, contains (box(Y), color(yellow)) \rightarrow contains (box(Y), \sim color(white)) \wedge tains (box(Y), \sim color(both))

For every Y, contains (box(Y), color(both)) \rightarrow contains (box(Y), \sim color(yellow)) \wedge tains (box(Y), \sim color(white))

```
//Each box contains one of the three colors
For every X, Y,    box(Y) ^ color(X)
```

Wumpus World - (hint start by defining a helper concept 'adjacent(x,y,p,q)' which defines when a room at coordinates (x,y) is adjacent to another room at (p,q). Don't forget rules for 'stench', 'breezy', and 'safe'.

```
//Define the starting square
For all X,Y    starting_square(X,Y) -> X = 1 ^ Y = 1
```

```
//Declare the adjacent squares for any given coordinates X, Y in the Wumpus world!
For all X, Y,    adjacent(X, Y, P, Q) <-> [P = X+1 ^ Q = Y] v [P = X-1 ^ Q = Y] v
[P = X ^ Q = Y+1] v [P = X ^ Q = Y-1]
```

```
//Rooms next to rooms with Wumpus have stench
For all X, Y, P, Q    adjacent(X, Y, P, Q) ^ wumpus(P,Q) -> stench(X, Y)
```

```
//Rooms next to rooms with Breeze have Pit
For all X, Y, P, Q    adjacent(X, Y, P, Q) ^ breeze(P,Q) -> pit(X, Y)
```

```
//Rooms next to rooms with Breeze have Pit
For all P, Q    ~pit(P, Q) ^ ~Wumpus(P, Q) -> safe(P, Q)
```

4-Queens – assume row(1)...row(4) and col(1)...col(4) are facts; write rules that describe configurations of 4 queens such that none can attack each other, using 'queen(r,c)' to represent that there is a queen in row r and col c.

row(0)	col(0)
row(1)	col(1)
row(2)	col(2)
row(3)	col(3)

```
//No Queen in both the left diagonal
For all X, Y,    queen(row(X), col(Y)), X >= 0, X <= 3, Y >= 0, Y <= 3 -> ~queen(row(X+1), col(Y+1)) ^
~queen(row(X-1), col(Y-1)) ^ ~queen(row(X+2), col(Y+2)) ^ ~queen(row(X-2), col(Y-2)) ^ ~queen(row(X+3),
col(Y+3)) ^ ~queen(row(X-3), col(Y-3))
```

```
//No Queen in both the right diagonal
For all X, Y,    queen(row(X), col(Y)), X >= 0, X <= 3, Y >= 0, Y <= 3 -> ~queen(row(X+1), col(Y-1)) ^
~queen(row(X-1), col(Y+1)) ^ ~queen(row(X+2), col(Y-2)) ^ ~queen(row(X-2), col(Y+2)) ^ ~queen(row(X+3),
col(Y-3)) ^ ~queen(row(X-3), col(Y+3))
```

```
//No Queen in the top or bottom col
For all X, Y,    queen(row(X), col(Y)), X >= 0, X <= 3, Y >= 0, Y <= 3 -> ~queen(row(X+1), col(Y)) ^
~queen(row(X-1), col(Y)) ^ ~queen(row(X+2), col(Y)) ^ ~queen(row(X-2), col(Y)) ^ ~queen(row(X+3), col(Y)) ^
~queen(row(X-3), col(Y))
```

//No Queen in the left or right row

For all X, Y, $\text{queen}(\text{row}(X), \text{col}(Y))$, $X \geq 0, X \leq 3, Y \geq 0, Y \leq 3 \rightarrow \sim \text{queen}(\text{row}(X), \text{col}(Y+1)) \wedge$
 $\sim \text{queen}(\text{row}(X), \text{col}(Y-1)) \wedge \sim \text{queen}(\text{row}(X), \text{col}(Y+2)) \wedge \sim \text{queen}(\text{row}(X), \text{col}(Y-2)) \wedge \sim \text{queen}(\text{row}(X), \text{col}(Y+3)) \wedge$
 $\sim \text{queen}(\text{row}(X), \text{col}(Y-3))$