Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Frequency Response Analysis

1.1 Polar Plot

1.1.1. Sketch the Polar Plot for

$$G(s) = \frac{1}{(1+s)(1+2s)} \tag{1.1.1}$$

Solution: The following code generates Fig. 1.1.1

```
codes/ee18btech11012.py
```

The polar plot is to the right of $(-1,0)$. Hence the closed loop system is stable.

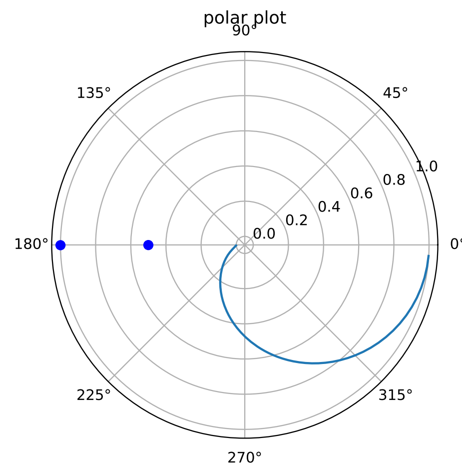

Fig. 1.1.1

1.1.2. Sketch the polar plot of

$$G(s) = \frac{1}{(s^2)(s+1)(s+2)}. \tag{1.1.2}$$

Solution: Substituting $s = \jmath\omega$ in (1.1.2), Now the magnitude will be

$$r = |G(\jmath\omega)| = \frac{1}{(\omega^2)(\sqrt{1+\omega^2})(\sqrt{1+4\omega^2})} \tag{1.1.3}$$

$$\theta = \angle G(\jmath\omega) = -\tan^{-1}(0) - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{1.1.4}$$

$$= 180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{1.1.5}$$

The polar plot is the (r,θ) plot for $\omega \in (0,\infty)$. The following python code generates the polar plot in Fig. 1.1.2

```
codes/ee18btech11028.py
```

The location of $(-1,0)$ with respect to the polar plot provides information regarding the stability of the system.

- If $(-1,0)$ is not enclosed, then it is stable.
- If $(-1,0)$ is enclosed by polar plot then it is unstable.
- If $(-1,0)$ is on the polar plot then it is

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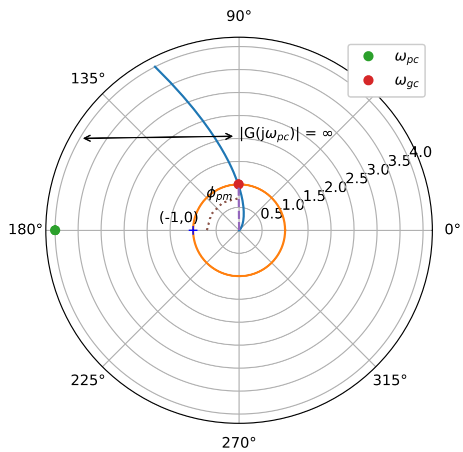

Fig. 1.1.2

marginally stable

In Fig. 1.1.2, the point $(-1, 0)$ is enclosed by the polar plot, which implies system is not stable. The polar plot also provides info on the GM and PM, which can then be used for determining the stability of the system.

- If the $GM > 1 \cap PM > 0$, then the control system is **stable**.
- If the $GM = 1 \cap PM = 0$, then the control system is **marginally stable**.
- If the $GM < 1 \cup PM < 0$, then the control system is **unstable**.

Therefore, our system is unstable $\because GM < 1 \cap PM < 0$.

1.1.3. Sketch the Polar Plot of

$$G(s) = \frac{1}{s(1+s^2)} \tag{1.1.6}$$

Solution: From (1.1.6),

$$G(\jmath\omega) = \frac{1}{\jmath\omega(1-\omega^2)} \tag{1.1.7}$$

$$\left|G(\jmath\omega)\right| = \frac{1}{\left|\omega(1-\omega^2)\right|} \tag{1.1.8}$$

$$\angle G(\jmath\omega) = \begin{cases} \frac{\pi}{2} & \omega > 1 \\ -\frac{\pi}{2} & 0 < \omega < 1 \end{cases} \tag{1.1.9}$$

The corresponding polar plot is generated in Fig. 1.1.3 using

```
codes/ee18btech11023.py
```

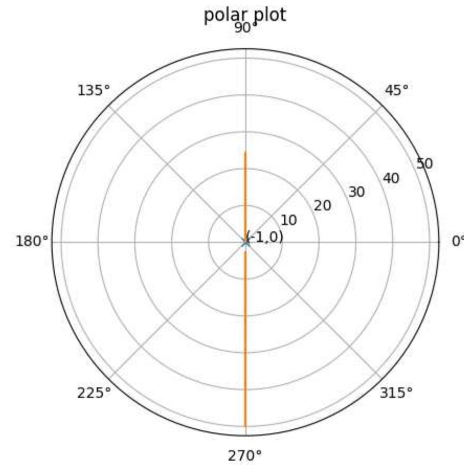

Fig. 1.1.3

In Fig. 1.1.3, (–1,0) is exactly on the polar plot. Hence, the system is marginally stable.

1.1.4. Sketch the Polar Plot of

$$G(s) = \frac{\left(1+\frac{s}{29}\right)(1+0.0025s)}{(s^3)(1+0.005s)(1+0.001s)} \tag{1.1.10}$$

Solution: The following code generates the polar plot in Fig. 2.5.1

```
codes/ee18btech11029.py
```

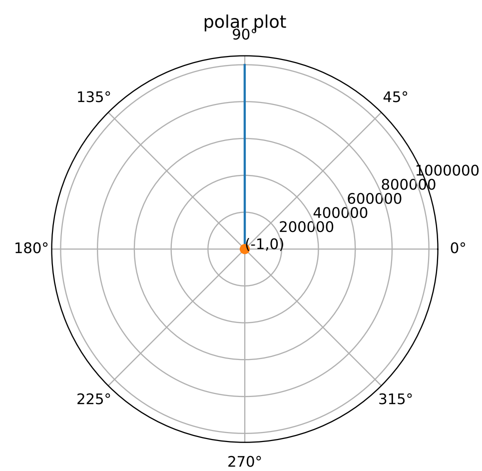

Fig. 1.1.4

- The polar plots use open loop transfer function to determine the stability and hence reference point is shifted to $(-1, 0)$
- If $(-1, 0)$ is left of the polar plot or $(-1, 0)$ is not enclosed, then it is stable

- If $(-1, 0)$ is on right side of the polar plot or $(-1, 0)$ is enclosed by polar plot then it is unstable.
- If $(-1, 0)$ is on the polar plot then it is marginally stable

In Fig. 2.5.1, $(-1, 0)$ is on the polar plot so the system is marginally stable.

1.1.5. Plot the polar plot of

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}. \tag{1.1.11}$$

Solution:

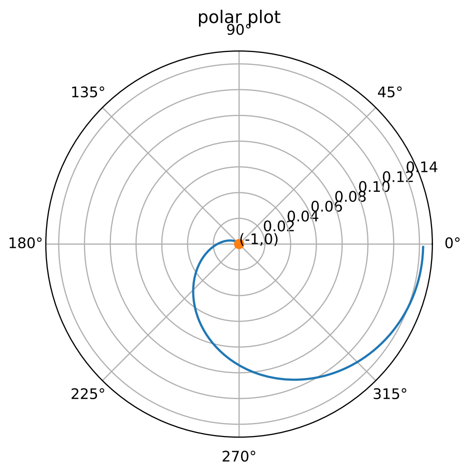

Fig. 1.1.5

The following python code generates the polar plot in Fig. 1.1.5

codes/ee18btech11033.py

$\because$ (-1,0) is on the right side of the polar plot, the system is unstable.

1.1.6. Plot the polar plot of

$$G(s) = \frac{100(s+5)}{s(s+3)(s^2+4)}. \tag{1.1.12}$$

Solution: The following python code generates the polar plot in Fig. 1.1.6

codes/ee18btech11042.py

Since (-1,0) is on the polar plot, the above system is marginally stable.

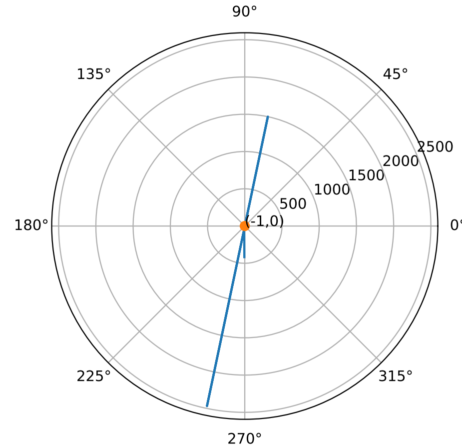

Fig. 1.1.6

1.2 Direct and Inverse Polar Plot

Sketch the direct polar plot for a unity feedback system with open loop transfer function

$$G(s) = \frac{1}{s(1+s)^2} \tag{1.2.1}$$

Solution: The polar plot is obtained by plotting (r, ϕ)

$$r = |H(\jmath\omega)||G(\jmath\omega)| \tag{1.2.2}$$

$$\phi = \angle H(\jmath\omega)G(\jmath\omega), 0 < \omega < \infty \tag{1.2.3}$$

The following code plots the polar plot in Fig. 1.2.1

codes/ee18btech11002/polarplot.py

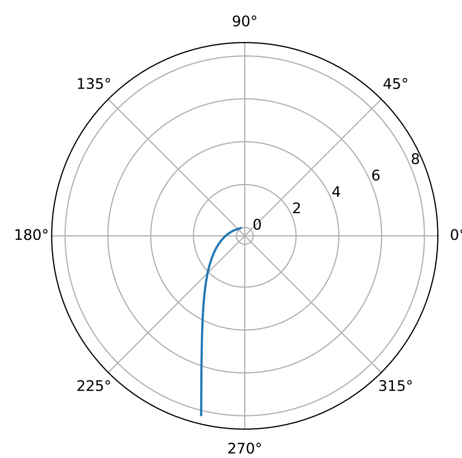

Fig. 1.2.1: Polar Plot

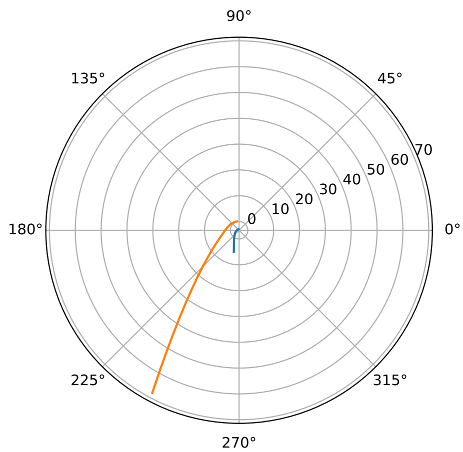

Fig. 1.2.2: Inverse Polar Plot

Sketch the inverse polar plot for (1.2.1)
Solution: The above code plots the polar plot in Fig. 1.2.2 by plotting $\left(\frac{1}{r}r, -\phi\right)$

1.3 Bode Plot

1.3.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain margin and the phase margin.

$$G(s) = \frac{10}{s(1+0.5s)(1+.01s)} \tag{1.3.1}$$

Solution: The Bode magnitude and phase plot are available in Fig. 1.3.1 and generated by

```
codes/ee18btech11048.py
```

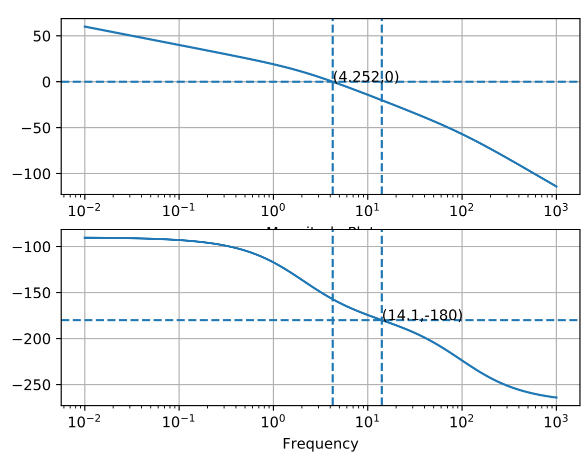

Fig. 1.3.1: Graphs

The pole-zero locations are available in Table 1.3.1.

Zeros	Poles
-	0
	-2
	-100

TABLE 1.3.1: Zeros and Poles

The *Gain* and *Phase* of (1.3.2) are

$$\left|G(\jmath\omega)\right| = \frac{100}{\omega\sqrt{(0.5\omega)^2+1}\sqrt{(0.01\omega)^2+1}} \tag{1.3.2}$$

$$\angle G(\jmath\omega) = \tan^{-1}(0) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{100}\right) \tag{1.3.3}$$

Hence,

$$\left|G\left(\jmath\omega_{gc}\right)\right| = 1 \tag{1.3.4}$$

$$\implies \omega_{gc} = 4.25 \tag{1.3.5}$$

$$\angle G\left(\jmath\omega_{gc}\right) = -157.2 \tag{1.3.6}$$

$$\implies PM = 22.8 \tag{1.3.7}$$

Similarly,

$$\angle G\left(\jmath\omega_{pc}\right) = -180^\circ \tag{1.3.8}$$

$$\implies \omega_{pc} = 14.1 \tag{1.3.9}$$

$$\implies -\left|G\left(\jmath\omega_{pc}\right)\right| = -20.2dB \tag{1.3.10}$$

$$\implies GM = 20.2dB \tag{1.3.11}$$

1.3.2. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} \tag{1.3.12}$$

Also compute gain margin and phase margin .
Solution: From (1.3.12), we have

$$G(\jmath\omega) = \frac{75(1+0.2\jmath\omega)}{\jmath\omega\left((\jmath\omega)^2+16\jmath\omega+100\right)} \tag{1.3.13}$$

poles = 0 , -8-6j , -8+6j
zeros = -5
Gain and phase plots are shown in Fig. 1.3.2
The following code plots Fig. 1.3.2

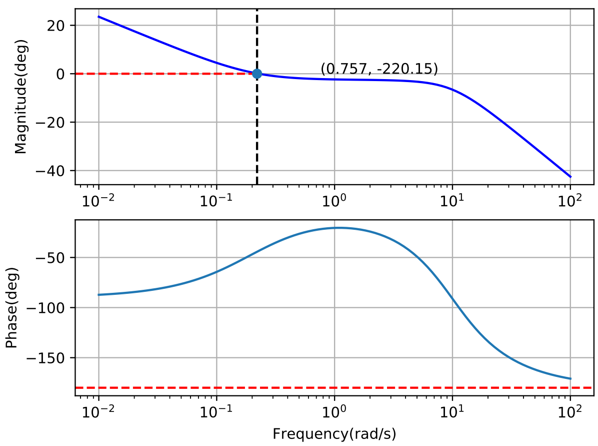

Fig. 1.3.2: a

codes/ee18btech11049.py

Solving

$$\left|G(\jmath\omega)\right| = \frac{75\sqrt{\omega^2+25}}{\omega\sqrt{(\omega+6)^2+64}\sqrt{(\omega-6)^2+64}} = 1, \tag{1.3.14}$$

or from Fig. 1.3.2, the gain crossover frequency

$$\implies \omega_{gc} = 0.757 \tag{1.3.15}$$

$$\angle G\left(\jmath\omega_{gc}\right) = -88.3 \tag{1.3.16}$$

$$\implies PM = 91.7 \tag{1.3.17}$$

Solution: From Fig. 1.3.2 ,we can say that phase never crosses $-180°$. So , the gain margin is *infinite*. Which means we can add any gain, and the equivalent closed loop system never becomes unstable.

1.3.3. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)} \tag{1.3.18}$$

Also compute gain margin and phase margin .
Solution: Substituting $s = \jmath\omega$ in (1.3.18) and assuming K =1,

$$G(\jmath\omega) = \frac{(\jmath\omega)^2}{(1+0.2\jmath\omega)(1+0.02\jmath\omega)} \tag{1.3.19}$$

The corner frequencies are

$$\omega_{c1} = 1/0.2 = 5 \tag{1.3.20}$$

$$\omega_{c2} = 1/0.02 = 50 \tag{1.3.21}$$

$$20\log\left|G(\jmath\omega)\right| = 20\log\left|(\jmath\omega)^2\right| - 20\log\left|(1+0.2\jmath\omega)\right| - 20\log\left|(1+0.02\jmath\omega)\right| \tag{1.3.22}$$

The various values of G($\jmath\omega$) are available in Table 1.3.2, in the increasing order of their corner frequencies also slope contributed by each term and the change in slope at the corner frequency. The pase

TERM	**Corner Freq**	**Slope**	**Slope change**
$(\jmath\omega)^2$	−−	+40	−−
$\frac{1}{1+\jmath 0.2}$	$\omega_{c1} = \frac{1}{0.2}$	-20	40-20=20
$\frac{1}{1+\jmath 0.02}$	$\omega_{c2} = \frac{1}{0.02}$	-20	20-20=0

TABLE 1.3.2: Magnitude

$$\phi = \angle G(\jmath\omega) = 180° - tan^{-1}(0.2\omega) - tan^{-1}(0.02\omega) \tag{1.3.23}$$

The phase angle of G($\jmath\omega$) are calculated for various value of ω in Table 1.3.3. The magnitude and phase plot are generated in Fig. 1.3.3 using the following python code

ω	$\tan^{-1}(0.2\omega)$	$\tan^{-1}(0.02\omega)$	$\phi = \angle G(j\omega)$
0.5	5.7	0.6	174
1	11.3	1.1	168
2	21.8	2.3	156
5	45	5.7	130
10	63.4	11.3	106
50	84.3	45	50

TABLE 1.3.3: Phase

codes/es17btech11002.py

$\because$ the gain crossover frequency is 2 and the corresponding gain At $\omega = 2$ is 13dB,

$$20\log K = -13db \tag{1.3.24}$$

$$\implies K = 0.65 \tag{1.3.25}$$

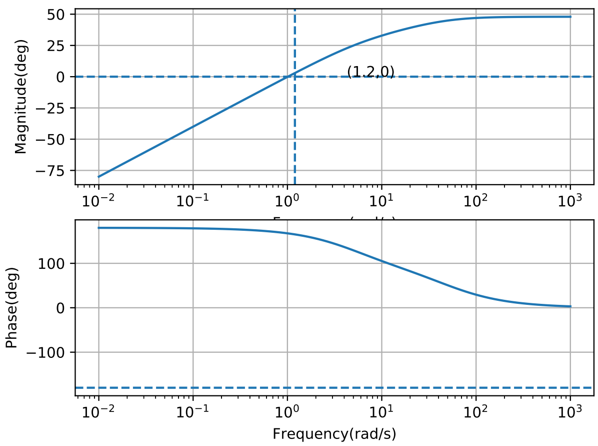

Fig. 1.3.3: Graphs

Solving (1.3.19) or from Fig. 1.3.3, the gain crossover frequency,

$$\omega_{gc} = 1.2 \tag{1.3.26}$$

$$\implies PM = 344.8 \tag{1.3.27}$$

From Fig. 1.3.3 ,we can say that phase never crosses $-180°$. So, the gain margin is *infinite*. Which means we can add any gain, and the equivalent closed loop system never goes unstable.

2 Stability in Frequency Domain

2.1 Nyquist Criterion

2.1.1. Using Nyquist criterion find the range of K for which closed loop system is stable.

$$G(s) = \frac{K}{s(s+6)} \tag{2.1.1.1}$$

$$H(s) = \frac{1}{s+9} \tag{2.1.1.2}$$

Solution: The system flow can be described as,

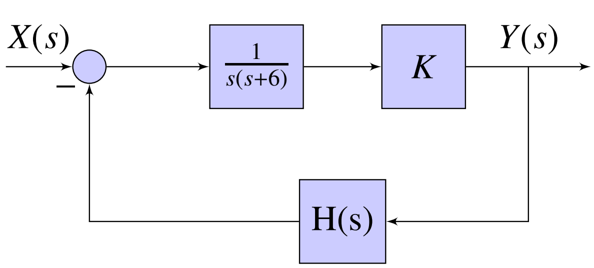

Fig. 2.1.1

$$G_1(s) = \frac{1}{s(s+6)}. \tag{2.1.1.3}$$

For Nyquist plot,

$$\text{Im}\{G_1(\jmath\omega)H(\jmath\omega)\} = \frac{-(54-\omega^2)}{(\omega)(\omega^2+56)(\omega^2+81)} \tag{2.1.1.4}$$

$$\text{Re}\{G_1(\jmath\omega)H(\jmath\omega)\} = \frac{-15\omega}{(\omega)(\omega^2+56)(\omega^2+81)} \tag{2.1.1.5}$$

From (2.1.1.4) and (2.1.1.5)

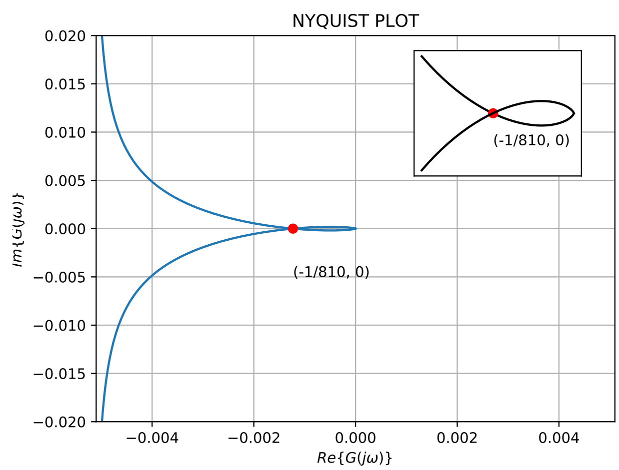

Fig. 2.1.2: Nyquist plot for $G_1(s)H(s)$

Nyquist Stability Criterion:

$$N = Z - P \tag{2.1.1.6}$$

where Z is # unstable poles of closed loop transfer function, P is # unstable poles of open loop transfer function and N is # clockwise encirclement of $(-1/K, 0)$.
For stable system,

$$Z = 0 \tag{2.1.1.7}$$

From (2.1.1.2) and (2.1.1.3),

$$P = 0 \tag{2.1.1.8}$$

$$\implies N = 0 \tag{2.1.1.9}$$

Since, there is a zero at origin, an infinite radius half circle will enclose the right hand side of end points of the Nyquist plot. So for (2.1.1.9),

$$\implies \frac{-1}{K} < \frac{-1}{810} \implies K < 810 \quad (2.1.1.10)$$

And also,

$$K > 0 \quad (2.1.1.11)$$

$$\implies 0 < K < 810 \quad (2.1.1.12)$$

The following python code generates Fig. 2.1.2

```
codes/ee18btech11028_1.py
```

2.1.2. Using Nyquist criterion, find out the range of K for which the closed loop system will be stable.

$$G(s) = \frac{K(s+2)(s+4)}{s^2-3s+10}; H(s) = \frac{1}{s} \quad (2.1.2.1)$$

Solution: The system flow can be described by Fig. 2.1.3 From (2.1.2.1),

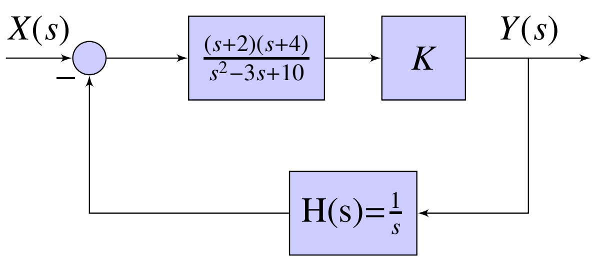

Fig. 2.1.3

$$G(s)H(s) = \frac{K(s+2)(s+4)}{s(s^2-3s+10)} \quad (2.1.2.2)$$

$$G(\jmath\omega)H(\jmath\omega) = \frac{K(j\omega+2)(j\omega+4)}{j\omega((10-\omega^2)-3j\omega)} \quad (2.1.2.3)$$

$$\text{Re}\{G(\jmath\omega)H(\jmath\omega)\} = \frac{K(84\omega^2-9\omega^4)}{\omega^6-11\omega^4+100\omega^2} \quad (2.1.2.4)$$

$$\text{Im}\{G(\jmath\omega)H(\jmath\omega)\} = \frac{K(-\omega^5+36\omega^3-80\omega)}{\omega^6-11\omega^4+100\omega^2} \quad (2.1.2.5)$$

The Nyquist plot is a graph of $\text{Re}\{G(\jmath\omega)H(\jmath\omega)\}$ vs $\text{Im}\{G(\jmath\omega)H(\jmath\omega)\}$. Let's take K =1 and draw the nyquist plot .

The following python code generates the Nyquist plot in Fig. 2.1.4

```
codes/ee18btech11016.py
```

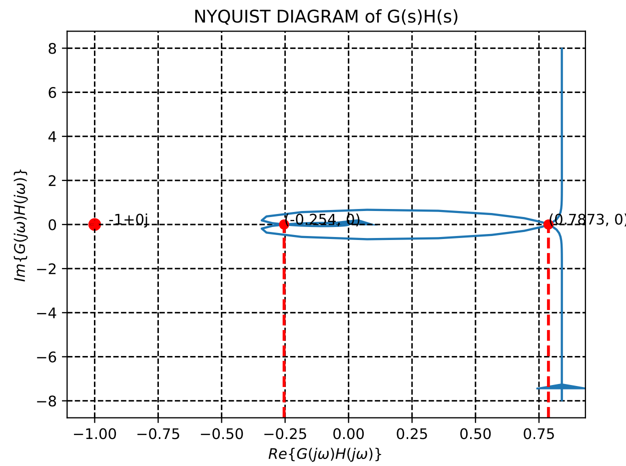

Fig. 2.1.4

Note that this nyquist plot is plotted when K=1.

Nyquist criterion-For the stable system :

$$Z = P + N = 0, \quad (2.1.2.6)$$

Variable	Description
Z	Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane
N	No of encirclements of $G(s)H(s)$ about -1+j0 in the Nyquist plot
P	Poles of $G(s)H(s)$ in right half of s plane

TABLE 2.1.1

Since from the equation (2.1.2.2), P = 2 as the number of poles on right hand side of s-plane is equal to 2 .So, for Z to be equal to 0 ,we have to choose the range of K such that N should be equal to -2.
If we consider the Nyquist plot with K term i.e. of equation (2.1.2.2) , then it will cut x-axis at x = -0.254K , x = 0 and at x = 0.7873K (as we have nyquist graph at K=1, now we just need to multiply the intersected

coordinates on x-axis by K).

So, we have to make sure that $(-1+\ j0)$ should be included in between x = -0.254K to x = 0, because then only N = -2 (as the no. of encirclements are 2 in anticlockwise direction in this case so N=-2)

$$-0.254K < -1 < 0 \tag{2.1.2.7}$$

So,

$$K > \frac{1}{0.254} \tag{2.1.2.8}$$

i.e.

$$K > 3.937 \tag{2.1.2.9}$$

Hence K > 3.937 ensures that the system is stable as no. of poles on the right hand side of s-plane (in this case) is 0.

2.2

2.3

2.4

2.5

2.5.1. In the block diagram Fig.2.5.1

$$G(s) = \frac{K}{(s+4)(s+5)} \tag{2.5.1.1}$$

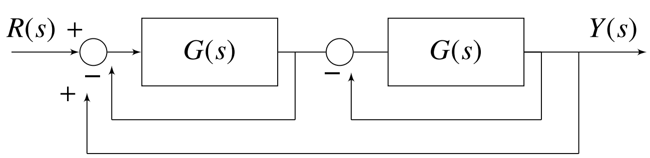

Fig. 2.5.1

2.5.2. Find the range of K for stability by Nyquist criterion

Solution:

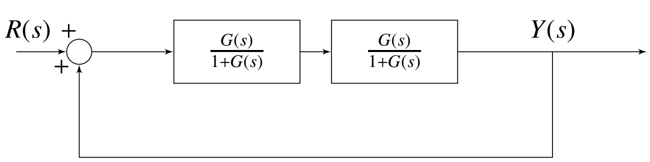

Fig. 2.5.2

The open loop transfer function from Fig.2.5.2

$$T(s) = \left(\frac{\frac{K}{(s+4)(s+5)}}{1+\frac{K}{(s+4)(s+5)}}\right)^2 \tag{2.5.2.1}$$

$$T(\jmath\omega) = \left(\frac{\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}{1+\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}\right)^2 \tag{2.5.2.2}$$

- Since it is connected in positive feedback the transfer function cuts at $(1, \jmath0)$

$$\implies \mathrm{Re}\{T(\jmath\omega)\} = 1 \tag{2.5.2.3}$$

$$\implies \mathrm{Im}\{T(\jmath\omega)\} = 0 \tag{2.5.2.4}$$

$$\left(\frac{\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}{1+\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}\right)^2 = 1+\jmath0 \tag{2.5.2.5}$$

$$(\jmath\omega+4)(\jmath\omega+5) + 2K = 0 \tag{2.5.2.6}$$

$$-\omega^2 + 9\jmath\omega + 20 + 2K = 0 \tag{2.5.2.7}$$

From (2.5.2.4)

$$20 + 2K = 0 \tag{2.5.2.8}$$

$$\implies K = -10 \tag{2.5.2.9}$$

The minimum value of stability for the system to be stable is

$$K_{min} > -10 \tag{2.5.2.10}$$

The range of K for which the system is stable is

$$-10 < K < \infty \tag{2.5.2.11}$$

2.5.3. From the table.2.5.1, Stability criterion for K is N+P=Z

2.5.4. Verify the Nyquist plots by

codes/ee18btech11029_1.py

2.5.5. Verify the result using Routh-Hurwitz criterion

K	P	N	Z	Description
-10	0	0	0	System is marginally stable
-9	0	0	0	System is stable
-11	0	1	1	System is unstable

TABLE 2.5.1

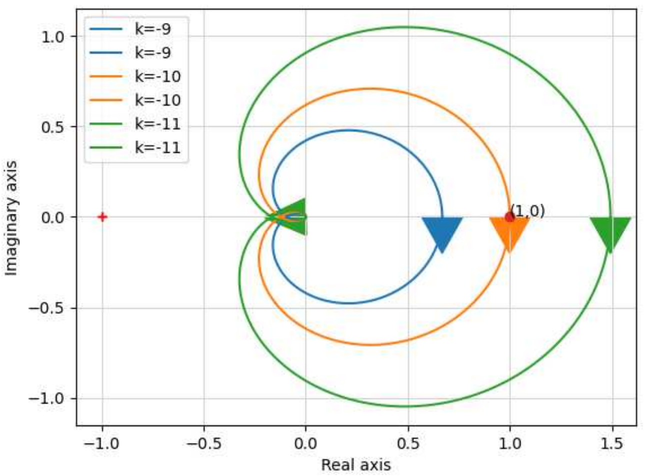

Fig. 2.5.3: Nyquist Plot

Solution: The characteristic equation is

$$1 - T(s) = 0 \tag{2.5.5.1}$$

$$1 - \left(\frac{\frac{K}{(s+4)(s+5)}}{1 + \frac{K}{(s+4)(s+5)}}\right)^2 = 0 \tag{2.5.5.2}$$

$$1 + 2\left(\frac{K}{(s+4)(s+5)}\right) = 0 \tag{2.5.5.3}$$

$$s^2 + 9s + 20 + 2K = 0 \tag{2.5.5.4}$$

$$\begin{vmatrix} s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & 20 + 2K \\ 9 & 0 \\ 20 + 2K & 0 \end{vmatrix} \tag{2.5.5.5}$$

For a system to be stable it should not have any sign changes

$$20 + 2K > 0 \tag{2.5.5.6}$$

This is valid for all positive values of K but the minimum value of K is

$$K > -10 \tag{2.5.5.7}$$

So the range of K for stability is

$$-10 < K < \infty \tag{2.5.5.8}$$

2.5.6. Verify the result by

```
codes/ee18btech11029_2.py
```

2.6

Consider the system shown in Fig. 2.6.1 below. Sketch the nyquist plot of the system when

1) $G_c(s) = 1$
2) $G_c(s) = 1 + \frac{1}{s}$

and determine the maximum value of K for stability. Take

$$G(s) = \frac{K}{s(1+s)(1+4s)} \tag{2.6.1}$$

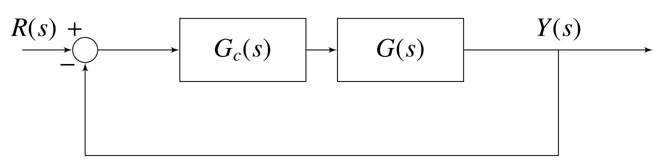

Fig. 2.6.1

Solution: For $G_c(s) = 1$,

The open loop transfer function is

$$G_c(s)\,G(s) = \frac{K}{s(1+s)(1+4s)} \tag{2.6.2}$$

$$G_c(\jmath\omega)\,G(\jmath\omega) = \frac{K}{\jmath\omega(1+\jmath\omega)(1+4\jmath\omega)} \tag{2.6.3}$$

$$= \frac{K}{\jmath\omega(1-4\omega^2+5\jmath\omega)} \tag{2.6.4}$$

$$= \frac{K\left(-5\omega - \jmath\left(1-4\omega^2\right)\right)}{\omega\left(\left(1-4\omega^2\right)^2 + 25\omega^2\right)} \tag{2.6.5}$$

The maximum K for stability is where the nyquist plot of open loop transfer function cuts the coordinate $(-1, \jmath 0)$

$$\implies \mathrm{Re}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = -1 \tag{2.6.6}$$

$$\implies \mathrm{Im}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = 0 \tag{2.6.7}$$

$$\implies \operatorname{Re}\left\{G(\jmath\omega)G_c(\jmath\omega)\right\} = \frac{-5K\omega}{\omega\left((1-4\omega^2)^2+25\omega^2\right)} \tag{2.6.8}$$

$$\implies \operatorname{Im}\left\{G(\jmath\omega)G_c(\jmath\omega)\right\} = \frac{-K\left(1-4\omega^2\right)}{\omega\left((1-4\omega^2)^2+25\omega^2\right)} \tag{2.6.9}$$

From (2.6.9) and (2.6.7)

$$1-4\omega^2 = 0 \implies \omega = \frac{1}{2} \tag{2.6.10}$$

From (2.6.8),(2.6.6) and substituting $\omega = \frac{1}{2}$

$$\frac{-5K\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{25}{4}\right)} = -1 \implies K = \frac{5}{4} = 1.25 \tag{2.6.11}$$

For $K < 0$ the system with negative feedback is unstable the range of K is

$$0 < K < \frac{5}{4} \tag{2.6.12}$$

Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 2.6.2 The following code gives the nyquist plot

```
codes/ee18btech11034/ee18btech11034_1.py
```

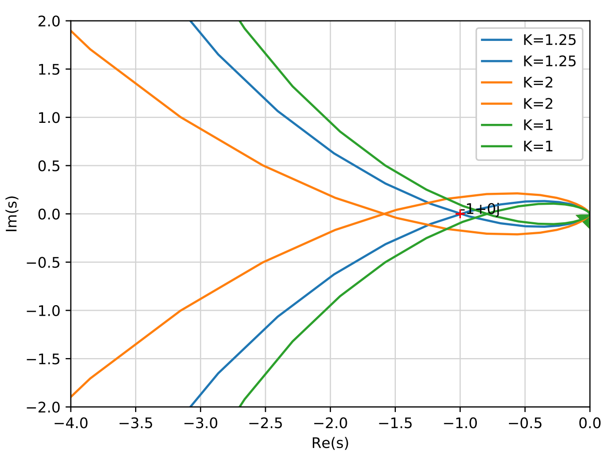

Fig. 2.6.2

Stability Criterian for K

$$N + P = Z \tag{2.6.13}$$

K	P	N	Z	Description
1.25	0	0	0	System is marginally stable
2	0	1	1	System is unstable
1	0	0	0	System is stable

TABLE 2.6.1

From the Fig.2.6.2

$$K_{max} = \frac{5}{4} \tag{2.6.14}$$

Solution: For $G_c(s) = \frac{1+s}{s}$,
the open loop transfer function is

$$G_c(s)G(s) = \frac{K(s+1)}{s^2(1+s)(1+4s)} \tag{2.6.15}$$

$$G_c(s)G(s) = \frac{K}{s^2(1+4s)} \tag{2.6.16}$$

$$G_c(\jmath\omega)G(\jmath\omega) = \frac{K}{(\jmath\omega)^2(1+4\jmath\omega)} \tag{2.6.17}$$

$$= \frac{\frac{-K}{\omega^2}(1-4\jmath\omega)}{1+16\omega^2} \tag{2.6.18}$$

From (2.6.7)

$$\implies \operatorname{Im}\left\{G(\jmath\omega)G_c(\jmath\omega)\right\} = \frac{4K}{\omega(1+16\omega^2)} = 0 \tag{2.6.19}$$

This is possible when

$$K = 0 \tag{2.6.20}$$

The system is unstable for both

$$K < 0 \tag{2.6.21}$$

$$K > 0 \tag{2.6.22}$$

Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 2.6.3 The following code gives the nyquist plot

```
codes/ee18btech11034/ee18btech11034_2.py
```

From (2.6.13)

From (2.6.20) K_{max} must be 0 which is not

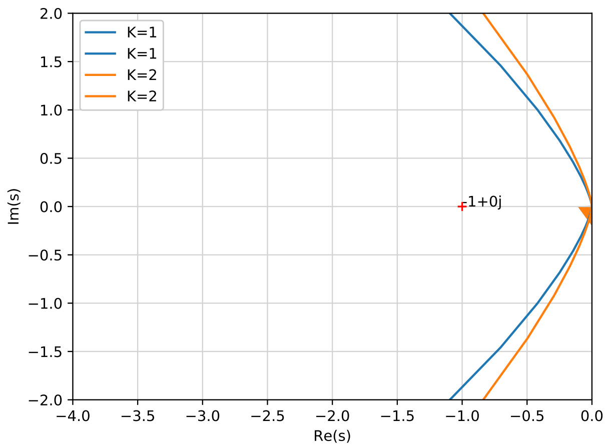

Fig. 2.6.3

K	P	N	Z	Description
1	0	1	1	System is unstable
2	0	1	1	System is unstable

TABLE 2.6.2

possible. Hence the system is unstable for all real K

3 Design in Frequency Domain

4 PID Controller Design

4.1 Introduction

4.1.1. Tabulate the transfer functions of a PID controller and its variants.
Solution: See Table 4.1.1.

Controller	Gain
PID	$K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$
PD	$K_p(1 + T_d s)$
PI	$K_p\left(1 + \frac{1}{T_i s}\right)$

TABLE 4.1.1

4.1.2. For a unity Feedback system

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{4.1.2.1}$$

Design a PD Controller with $K_v = 2$ and Phase Margin 30°

Solution: The gain after cascading the PD Controller with G(s) is

$$G_c(s) = \frac{K_p(1+T_d s)K}{s(s+2)(s+4)(s+6)} \tag{4.1.2.2}$$

Choosing $K_p = 1$ in ,

$$K_v = \lim_{s\to 0} sG_c(s) = 2 \tag{4.1.2.3}$$

$$\implies K = 96 \tag{4.1.2.4}$$

For Phase Margin 30°, at Gain Crossover Frequency ω,

$$\tan^{-1}(T_d\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = -60^\circ \tag{4.1.2.5}$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_d^2\omega^2+1}}{\omega\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \tag{4.1.2.6}$$

By Hit and Trial, one of the best combinations is

$$\omega = 4 \tag{4.1.2.7}$$

$$T_d = 1.884 \tag{4.1.2.8}$$

We get a Phase Margin of 30.31°

4.1.3. Verify using a Python Plot
Solution: The following code plots Fig. 4.1.1

```
codes/ee18btech11021/EE18BTECH11021_3.
    py
```

4.1.4. Design a PI Controller with $K_v = \infty$ and Phase Margin 30°
Solution: From Table 4.1.1, the open loop gain in this case is

$$G_1(s) = \frac{K_p\left(1+\frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \tag{4.1.4.1}$$

Choose $K_pK = 96$. Then

$$G_1(s) = \frac{96(T_i s+1)}{T_i s^2(s+2)(s+4)(s+6)} \tag{4.1.4.2}$$

For Phase Margin 30°, at Gain Crossover Frequency ω

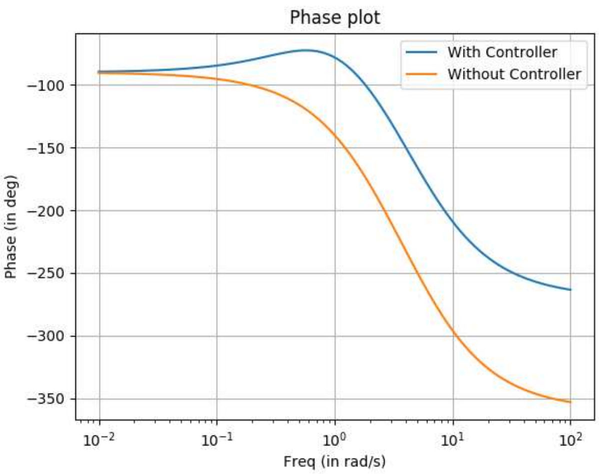

Fig. 4.1.1

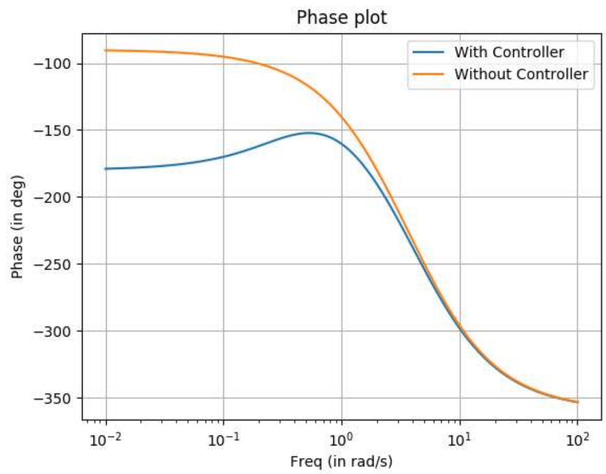

Fig. 4.1.2

$$\tan^{-1}(T_i\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \quad (4.1.4.3)$$

and

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_i^2\omega^2+1}}{T_i^2\omega^2\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \quad (4.1.4.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 0.75 \quad (4.1.4.5)$$
$$T_i = 2.713 \quad (4.1.4.6)$$

We get a Phase Margin of 25.53°

4.1.5. Verify using a Python Plot
Solution: The following code plots Fig. 4.1.2.

```
codes/ee18btech11021/EE18BTECH11021_4.
    py
```

4.1.6. Design a PID Controller with $K_v = \infty$ and Phase Margin 30°
Solution:

$$G_1(s) = \frac{K_p\left(1 + T_d s + \frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \quad (4.1.6.1)$$

Choose $K_pK = 96$. The open loop gain is

$$G_1(s) = \frac{96(T_iT_ds^2 + T_is + 1)}{T_is^2(s+2)(s+4)(s+6)} \quad (4.1.6.2)$$

For Phase Margin 30°, at Gain Crossover Frequency ω,

$$\tan^{-1}\left(\frac{T_i\omega}{1 - TiT_dw^2}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \quad (4.1.6.3)$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{(1 - TiT_d\omega^2)^2 + T_i^2}}{T_i^2\omega^2\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \quad (4.1.6.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 1 \quad (4.1.6.5)$$
$$T_i = 1.738 \quad (4.1.6.6)$$
$$T_d = 0.4 \quad (4.1.6.7)$$

We get a Phase Margin of 30°

4.1.7. Verify using a Python Plot
Solution: The following code plots Fig. 4.1.3

```
codes/ee18btech11021/EE18BTECH11021_5.
    py
```

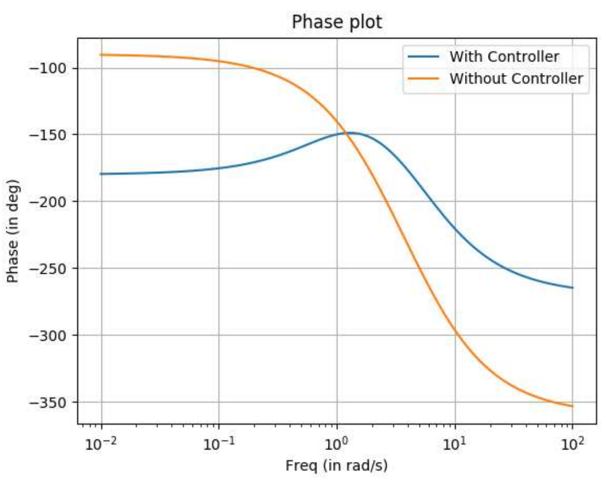
Phase plot
With Controller
Without Controller
Phase (in deg)
−100
−150
−200
−250
−300
−350
Freq (in rad/s)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Fig. 4.1.3