Control Systems

Varum SM*

Consider the positive-feedback circuit shown in Figure: 1.

1) Find the loop transmission L(s) and the characteristic equation
2) Find the expressions for resulting pole frequency ω_o and Q factor?
3) Sketch a Pole-Zero plot for varying K. For what value of K do the poles coincide? For what value of K does the response becomes maximally flat? For what value of K does the circuit oscillate?

Assume that the amplifier has frequency-independent gain, infinite input impedance, and zero output impedance.

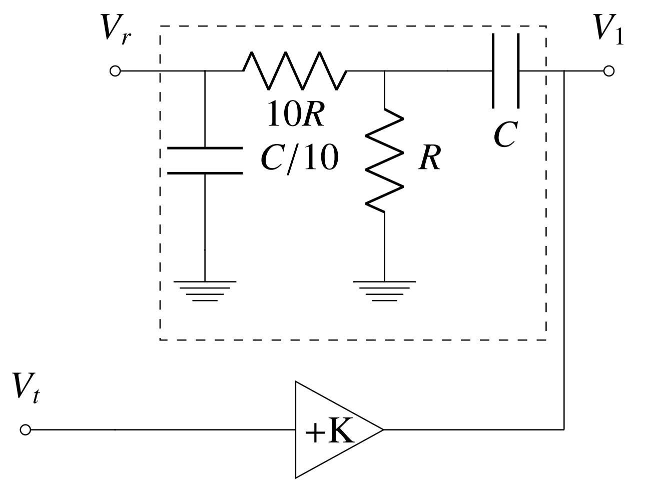

Fig. 1

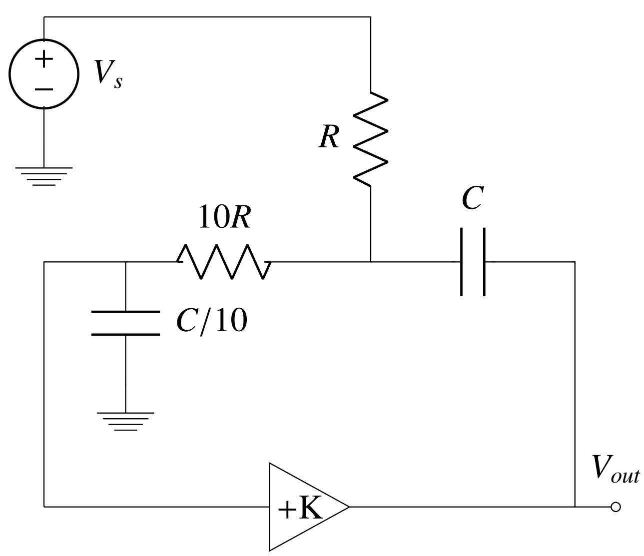

Fig. 3: Positive Feedback Circuit

1. **Solution:** To obtain the loop transmission L(s),
   - Short-circuit the signal source V_s.
   - Break the loop at the Amplifier input.
   - Then apply a test voltage V_t and find the returned voltage V_r, as indicated in Figure: 1.

The loop transmission is given by

$$L(s) = -\frac{V_r(s)}{V_t(s)} = -KT(s) \tag{1.1}$$

where T(s) is the transfer function of the two-port RC network shown inside the broken-line box in Figure: 1.

$$T(s) = \frac{V_r(s)}{V_1(s)} \tag{1.2}$$

Applying KCL at nodes present in the RC network yields

$$T(s) = \frac{s(\frac{1}{CR})}{s^2 + s(\frac{2.1}{CR}) + (\frac{1}{CR})^2} \tag{1.3}$$

Substituting T(s) in Eq: 1.1

$$L(s) = \frac{-s(\frac{K}{CR})}{s^2 + s(\frac{2.1}{CR}) + (\frac{1}{CR})^2} \tag{1.4}$$

The characteristic equation is

$$1 + L(s) = 0 \tag{1.5}$$

$$s^2 + s(\frac{2.1-K}{CR}) + (\frac{1}{CR})^2 = 0 \tag{1.6}$$

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The standard characteristic equation of a second order network can be written as

$$s^2 + \frac{\omega_o}{Q}s + \omega_o^2 = 0 \tag{1.7}$$

ω_o is called pole frequency , Q is called pole Qfactor. By comparing the Eq:1.6 with the standard characteristic equation Eq:1.7

$$\omega_o = \frac{1}{RC}; Q = \frac{1}{2.1 - K} \tag{1.8}$$

Closed Loop gain

$$T = \frac{s(\frac{-K}{RC})}{s^2 + s(\frac{2.1-K}{CR}) + (\frac{1}{CR})^2} \tag{1.9}$$

2. Sketch the Normalised closed loop gain of above two pole feedback amplifier for various Q values
Solution:

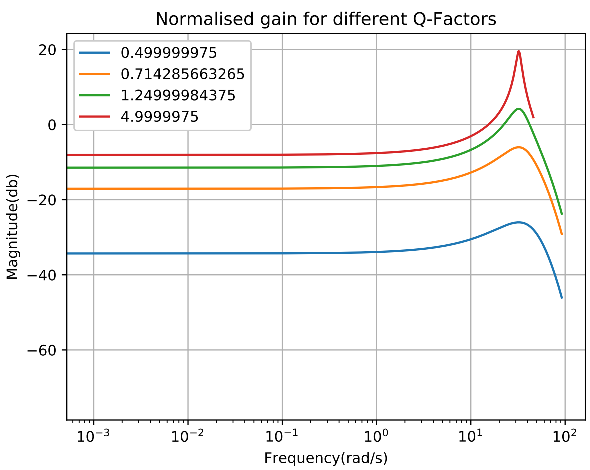

Fig. 2

The following is code for the plot

codes/ee18btech11030/ee18btech11030.py

From Figure:2 ,

- It is observed that maximally flat response is obtained when Q = 0.71
- It will be seen that response of the feedback amplifier under consideration shows almost no peaking for Q≤ 0.71

3. Sketch a Pole-Zero Plot to Eq:1.9 for a varying K
Solution: :
The following is code for the plot

codes/ee18btech11030/ee18btech11030_1.py

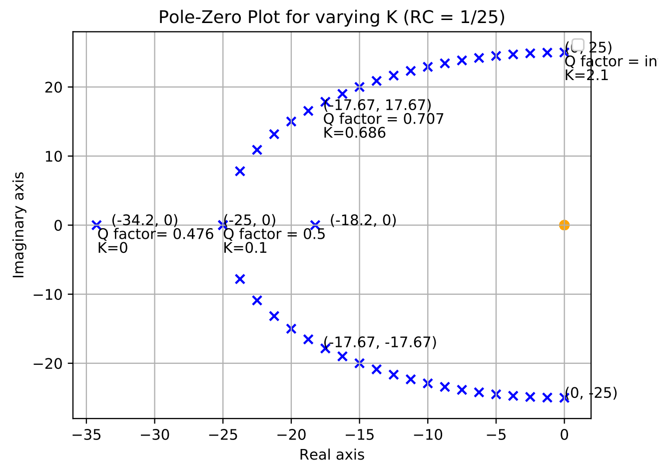

Fig. 3

From Figure : 3

- For K = 0,the poles have Q = 0.476 and therefore located on negative real axis.
- As K increases poles are brought closer together and eventually coincide at K = 0.1 and Q = 0.5
- Further increase in K results in poles becoming complex conjugate
- Maximally flat response is obtained when Q = 0.707,which results when K = 0.686.In this case poles are at 45°.
- Oscillating response is obtained when poles are completely imaginary when Q = inf which results when K = 2.1

4. Verify the response in time domain for a unit impulse input
Solution: :
The following is code for the plot

codes/ee18btech11030/ee18btech11030_2.py

Q-Factor	**K**	**Requirement**
0.5	0.1	Poles are coincident
0.707	0.686	Maximally flat response
∞	2.1	Oscillatory response

TABLE 4

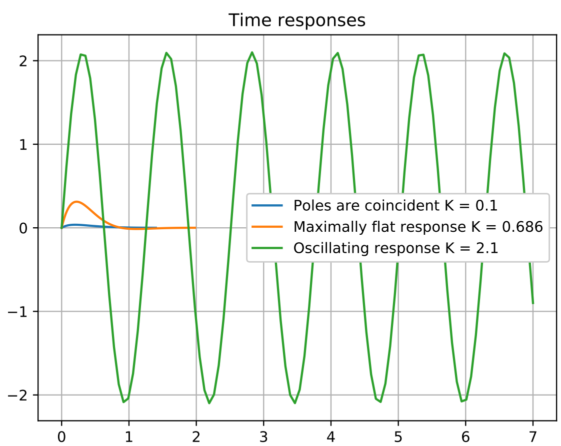
Time responses
2
1
0
−1
−2
0
1
2
3
4
5
6
7
Poles are coincident K = 0.1
Maximally flat response K = 0.686
Oscillating response K = 2.1

Fig. 4