Oscillator

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For the circuit shown in Fig. 1.1, find the loop gain $L(s) = G(s)H(s)$, $L(\jmath\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

1. Draw the equivalent control system representation for the circuit in Fig. 1.1 as well as the small signal model.
   Solution: See Figs. 1.2, 1.3 and 1.4. Oscillators do not include input signal.

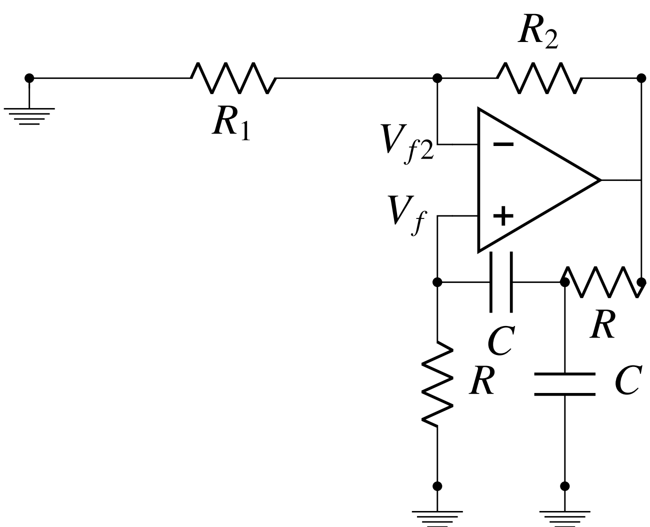

Fig. 1.1

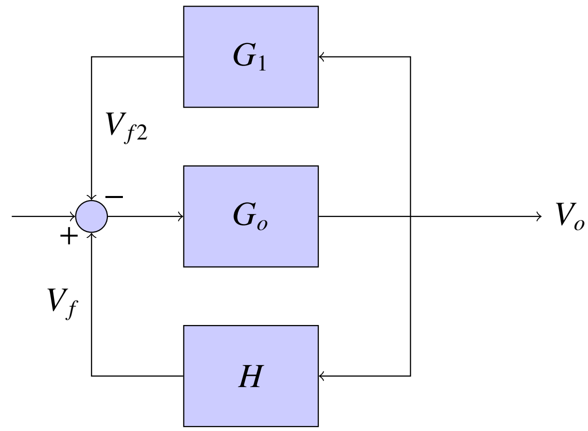

Fig. 1.2: Block diagram

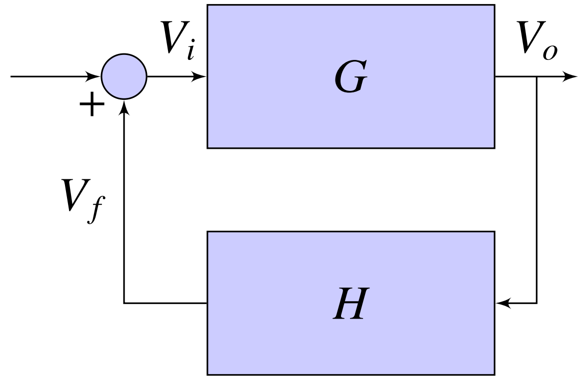

Fig. 1.3: Simplified equivalent block diagram

2. Draw the block diagram and circuit diagram for H.
   Solution: See Figs. 2.5 and 2.6.
3. Find H.
   Solution: In Fig. 2.6, let I_o be the current flowing from V_o. Then

$$I_o = \frac{V_o}{R + \frac{1}{sC} \parallel \left(\frac{1}{sC} + R\right)} \tag{3.1}$$

Using current division,

$$V_f = I_o \frac{\frac{1}{sC}}{\frac{1}{sC} + \left(\frac{1}{sC} + R\right)} \times R \tag{3.2}$$

From (3.1) and (3.2),

$$\frac{V_f}{V_o} = \frac{\frac{1}{sC}}{\frac{1}{sC} + \left(\frac{1}{sC} + R\right)} \times R \tag{3.3}$$

$$\times \frac{1}{R + \frac{1}{sC} \parallel \left(\frac{1}{sC} + R\right)} \tag{3.4}$$

On further simplification we get,

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \tag{3.5}$$

4. Find R_{11} and R_{22} from Fig. 2.6.

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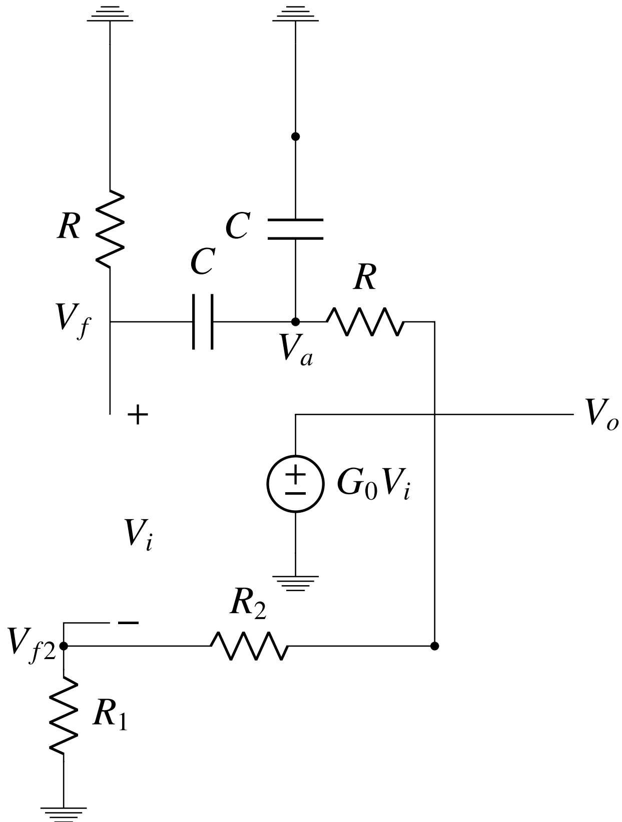

Fig. 1.4

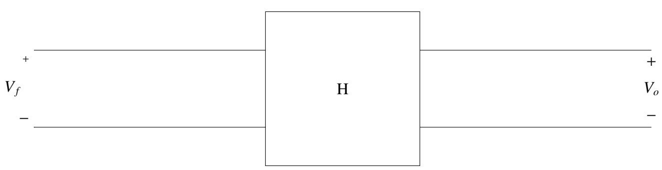

Fig. 2.5: Feedback block diagram

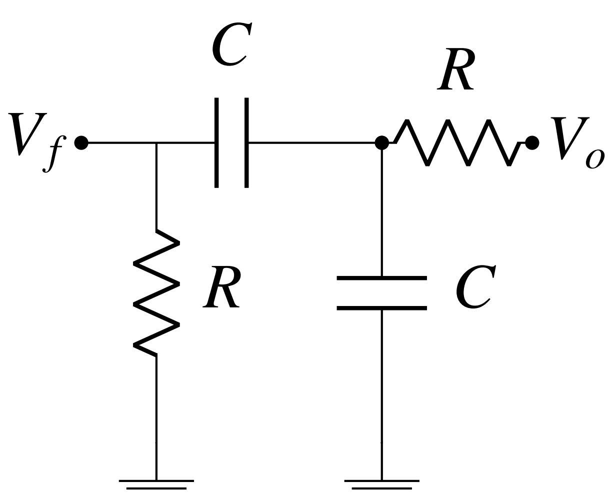

Fig. 2.6: Feedback circuit

Solution: Shorting V_o to ground,

$$R_{11} = R \parallel \left(\frac{1}{sC} + \frac{1}{sC} \parallel R\right) \tag{4.1}$$

Shorting V_f to ground,

$$R_{22} = \frac{1}{2sC} + R \tag{4.2}$$

5. Draw the block diagram and circuit diagram for G.
Solution: See Figs. 5.1 for the block diagram and Figs. 5.2 for the circuit diagram.

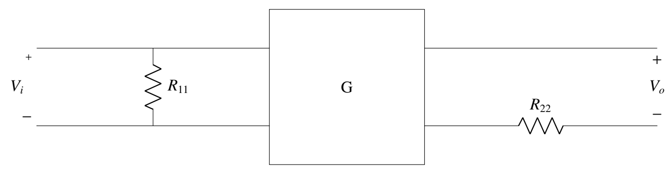

Fig. 5.1: Open loop block diagram

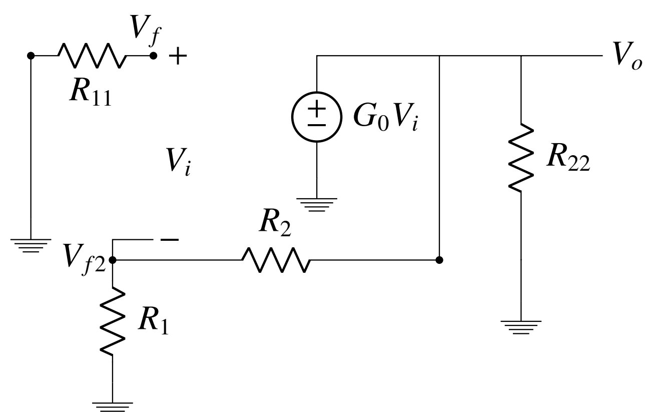

Fig. 5.2: Open loop circuit diagram

6. Find G.
Solution: From Fig. 5.2,

$$V_{f_2} = \left(\frac{R_1}{R_1 + R_2}\right) V_o \tag{6.1}$$

From Fig. 1.2,

$$G_1 = \frac{V_{f_2}}{V_o} \tag{6.2}$$

$$= \frac{R_1}{R_1 + R_2} \tag{6.3}$$

From Fig.1.2, G_1 is the negative feedback factor and G_0 is the gain of the op-amp.

Therefore, equivalent G is given by

$$G = \frac{G_0}{1 + G_0 G_1} \tag{6.4}$$

$$= \frac{1}{\frac{1}{G_0} + G_1} \tag{6.5}$$

On substituting $G_0 \to \infty$

$$G \approx \frac{1}{G_1} \tag{6.6}$$

$$G = \frac{R_1 + R_2}{R_1} \tag{6.7}$$

$$\implies G = 1 + \frac{R_2}{R_1} \tag{6.8}$$

7. Find the loop gain $L(s)$.
Solution: From (6.8) and (3.5),

$$L(s) = G(s) H(s) \tag{7.1}$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}} \right) \tag{7.2}$$

8. Find the closed loop gain $T(s)$.
Solution: From Fig. 1.3,

$$T(s) = \frac{G}{1 - GH(s)} = \frac{G}{1 - L(s)} \tag{8.1}$$

$$\implies \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 - \left(\frac{1+\frac{R_2}{R_1}}{3+sRC+\frac{1}{sRC}}\right)} \tag{8.2}$$

9. Find the conditions for oscillation.
Solution: For oscillations to start,
- $T(s)$ should have imaginary poles.
- $L(0) \geq 1$

For $T(s)$ to have imaginary poles,

$$\operatorname{Im}\{L(\jmath\omega)\} = 0 \tag{9.1}$$

$$\implies L(\jmath\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)} \right) \tag{9.2}$$

From (7.2),

$$L(\jmath\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)} \right) \tag{9.3}$$

$$\implies j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \tag{9.4}$$

$$\text{or, } \omega = \frac{1}{RC} \tag{9.5}$$

Also, from equation (7.2)

$$L(0) \geq 1 \tag{9.6}$$

$$= \left(\frac{1 + \frac{R_2}{R_1}}{3 + j(0)} \right) \geq 1 \tag{9.7}$$

$$\implies \frac{R_2}{R_1} \geq 2 \tag{9.8}$$

The following code plots the step response of the system. This, in fact is the output of Fig. 1.1.

```
codes/es17btech11002/es17btech11002.py
```

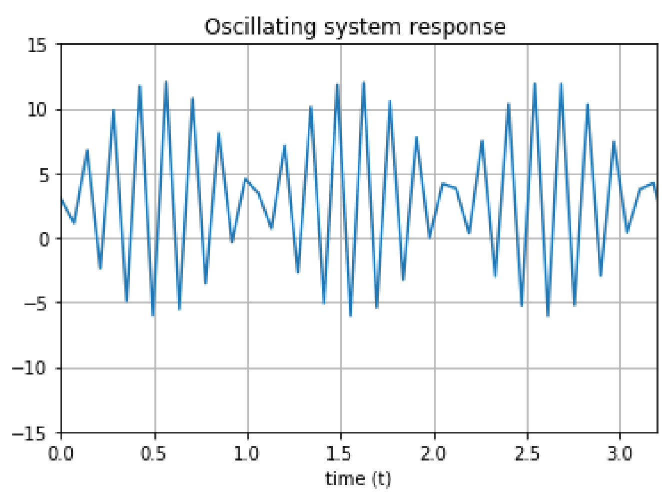

Fig. 9

10. Find the frequency for some arbitrary R, C values given in Table 10.

Parameter	Value
R	10Ω
C	$0.01mF$
R_1	1000Ω
R_2	2030Ω

TABLE 10

Solution: Frequency: From equation (9.5)

$$\omega = \frac{1}{RC} = 10000 rad/sec \tag{10.1}$$

$$f = \frac{\omega}{2\pi} = 1.57kHz \tag{10.2}$$

11. Verify the frequency using spice simulation.
Solution: The following readme file provides necessary instructions to simulate the circuit in spice.

```
codes/es17btech11002/spice/README
```

The following netlist simulates the given circuit.

```
codes/es17btech11002/spice/es17btech11002.
    net
```

The following code plots the output from the oscillator spice simulation which is shown in Fig. 11.1.

```
codes/es17btech11002/spice/
    es17btech11002_spice.py
```

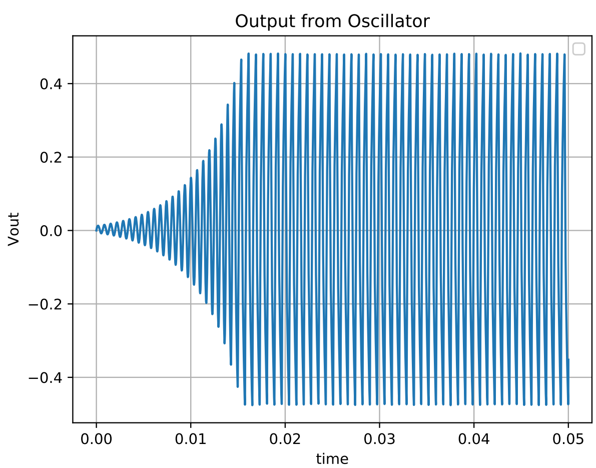

Fig. 11.1

The following code plots a part of the spice output from which we can observe a clear sinusoidal output shown in Fig. 11.2.

```
codes/es17btech11002/spice/
    es17btech11002_spice2.py
```

Amplitude: From Fig. 11.2 V(peak-peak) is

$$V_{p-p} = 0.47 - (-0.47) = 0.94V \tag{11.1}$$

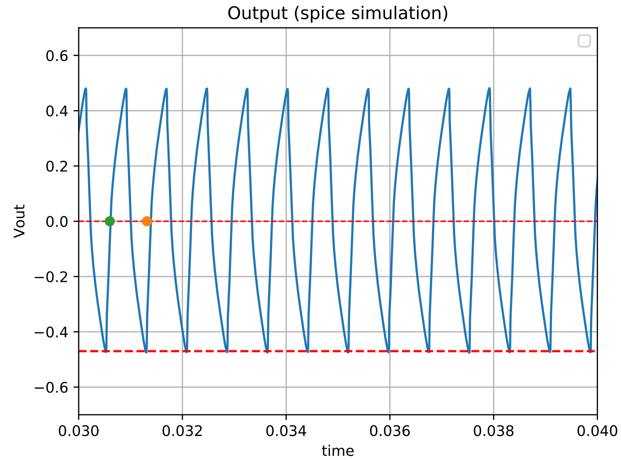

Fig. 11.2

$$V_{max} = \frac{V_{p-p}}{2} = 0.47. \tag{11.2}$$

Frequency: time period is calculated by any two end points of one cycle,

$$T = 0.0313164 - (0.03060) = 0.7164ms \tag{11.3}$$

$$f = \frac{1}{T} = 1.39kHz \tag{11.4}$$

Hence,the frequency is verified through the spice simulation.