Phase Margin

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Consider an op amp having a single pole open loop response $G_0 = 10^5$ and $f_p = 10$ Hz. Let the OPAMP be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100

(a) A manufacturing error introducing a second pole at 10 kHz. Find the frequency at which $|GH| = 1$ and the corresponding phase margin.
(b) For what values of H is the phase margin greater than 45°?

1. Find the transfer function of the two pole OPAMP.
Solution: For a two-pole amplifier open loop transfer function is

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} \tag{1.1}$$

Poles are at $f_1 = 10$ and $f_2 = 10^4$

$$G(f) = \frac{G_0}{\left(1 + \jmath\frac{f}{f_1}\right)\left(1 + \jmath\frac{f}{f_2}\right)} \tag{1.2}$$

$$= \frac{10^5}{\left(1 + \jmath\frac{f}{10}\right)\left(1 + \jmath\frac{f}{10^4}\right)} \tag{1.3}$$

2. Find the feedback H.
Solution: Since the closed loop gain

$$|T| = 100 \tag{2.1}$$

and for nominal low frequency $|GH| \gg 1$,

$$H \approx \frac{1}{|T|} = 0.01 \tag{2.2}$$

3. Find the PM and the crossover frequency.
Solution: From (1.3) and (2.2)

$$|GH| = 1 \tag{3.1}$$

$$\implies \frac{10^3}{\left(\sqrt{1 + \frac{f^2}{100}}\right)\left(\sqrt{1 + \frac{f^2}{10^8}}\right)} = 1 \tag{3.2}$$

$$\text{or } f_{180} = 7.8615\,kHz. \tag{3.3}$$

using the following python code.

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codes/ee18btech11034/ee18btech11034.py
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From (1.3), $\because \angle H = 0°$,

$$\angle G(f)H(f) = \angle G(f) \tag{3.4}$$

$$- \tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) \tag{3.5}$$

$$\implies PM = 180° + \angle G(f_{180}) \tag{3.6}$$

$$= 180° - 128.1° = 51.9° \tag{3.7}$$

4. Verify your result using a Bode plot.
Solution: The following code generates Fig. 4

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codes/ee18btech11034/ee18btech11034_1.py
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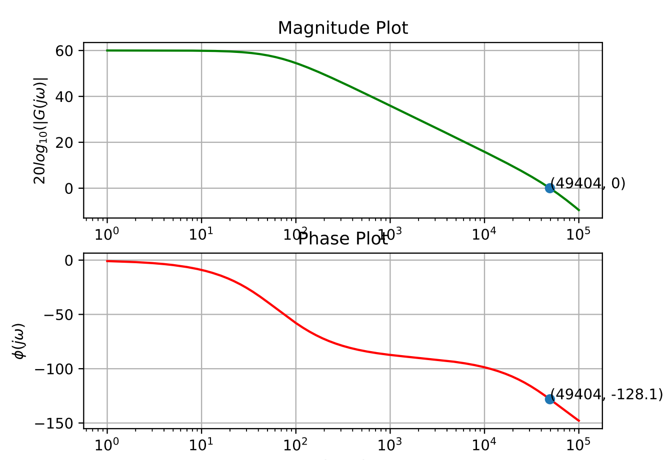

Fig. 4

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5. Realise the above system with $PM = 51.9°$ using a feedback circuit.
Solution:

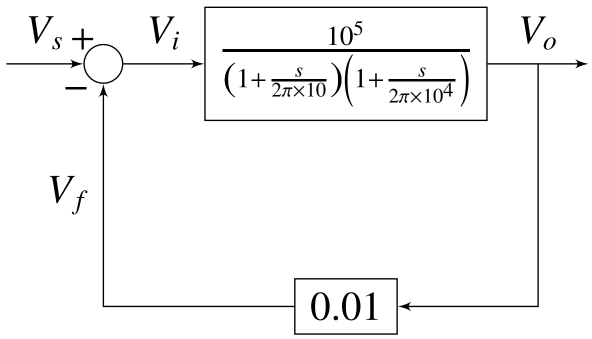

Fig. 5

The transfer function of OPAMP is

$$G(s) = \frac{10^5}{\left(1+\frac{s}{2\pi\times10}\right)\left(1+\frac{s}{2\pi\times10^4}\right)} \tag{5.1}$$

6. For the feedback gain H
Solution:
Choose a resistance network such that

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1+R_2} \approx 0.01 \tag{6.1}$$

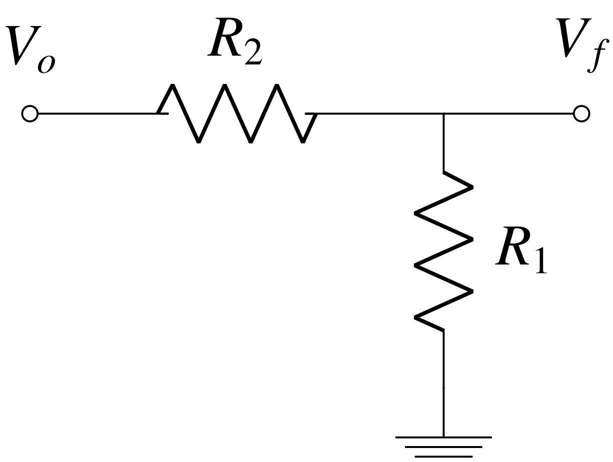

Fig. 6

Choose R_1 and R_2 as

$$R_1 = 10\Omega \tag{6.2}$$
$$R_2 = 990\Omega \tag{6.3}$$

$$H = \frac{R_1}{R_1+R_2} = \frac{10}{10+990} = 0.01 \tag{6.4}$$

7. Feedback Circuit for $PM = 51.9°$
Solution:

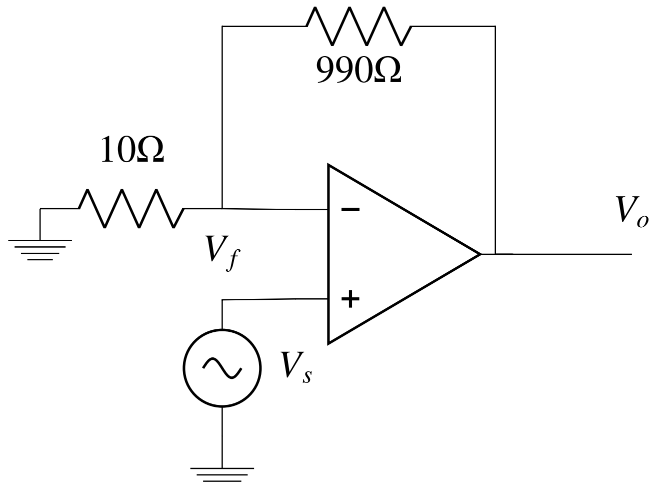

Fig. 7

8. Verification using spice circuit simulation
Solution: For $H = 0.01$ the closed loop response is

$$|T| \approx \frac{1}{H} = 100 \tag{8.1}$$

The following is the netlist file for spice

spice/ee18btech11034/ee18btech11034_1.net

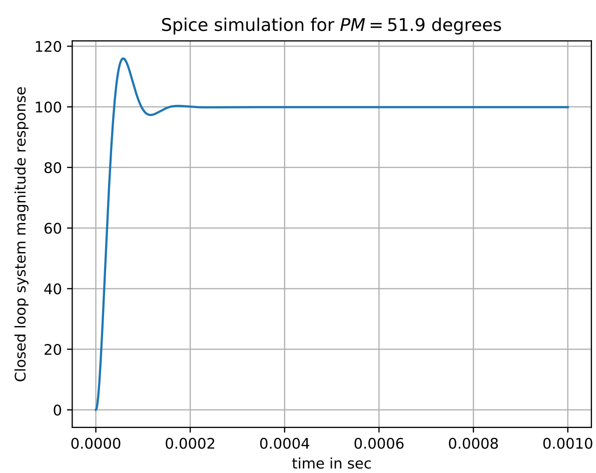

Fig. 8

The following python code plots the closed loop step response verses time

spice/ee18btech11034/ ee18btech11034_spice_result1.py

9. Verification of step response through python
Solution: For H = 0.01 step response through python

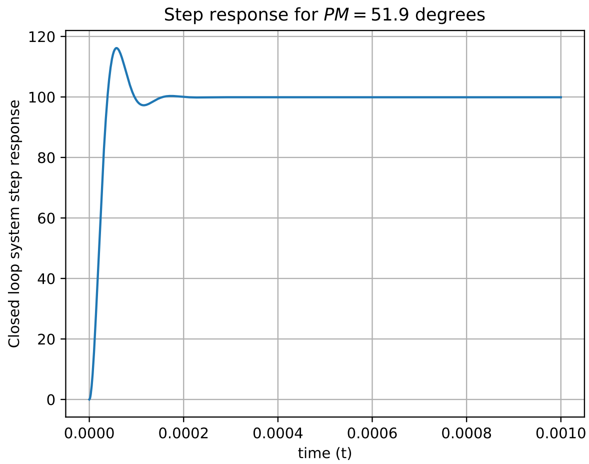

Fig. 9

The following python code is for the above verification

spice/ee18btech11034/ ee18btech11034_spice_verify1.py

From Fig.8 and Fig.9 we can see the circuit simulation output and python output are same for step signal input.

10. Find H such that $PM = 45°$.
Solution: From (3.4), assuming constant H,

$$\angle G(f_{180}) = 45^\circ - 180^\circ = -135^\circ \tag{10.1}$$

$$\implies -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) = -135^\circ \tag{10.2}$$

$$\implies \frac{\frac{f}{10} + \frac{f}{10^4}}{1 - \frac{f^2}{10^5}} = -1 \tag{10.3}$$

$$\text{or, } f_{180} \approx 10\,kHz \tag{10.4}$$

From (1.3),

$$\because |G(f_{180})H| = 1, \tag{10.5}$$

$$\frac{\left(10^5\right)H}{\left(\sqrt{1 + \frac{10^8}{100}}\right)\left(\sqrt{1 + \frac{10^8}{10^8}}\right)} = 1 \tag{10.6}$$

$$\implies H = 1.414 \times 10^{-2} \tag{10.7}$$

$$\text{or, } H_{max} = 1.414 \times 10^{-2} \tag{10.8}$$

which is the value of H for which $PM > 45°$.

11. Verify the above using a Bode plot.
Solution: The following code plots Fig. 11.

codes/ee18btech11034/ee18btech11034_2.py

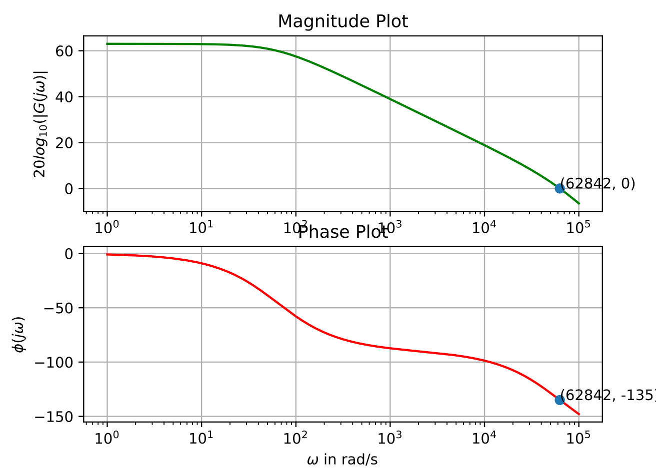

Fig. 11

12. Realise the above system with $PM = 45°$ using a feedback circuit.
Solution:

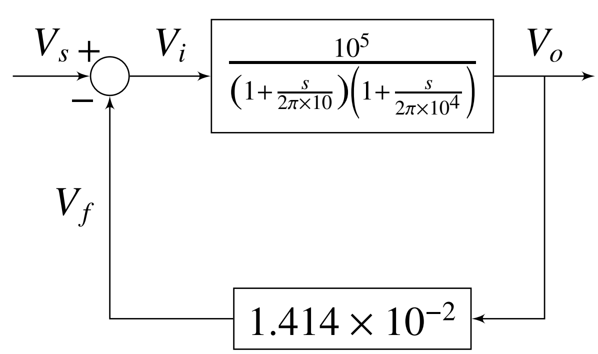

Fig. 12

13. For the feedback gain H
Solution:

$$R_1 = 10\Omega \tag{13.1}$$

$$R_2 = 700\Omega \tag{13.2}$$

$$H = \frac{R_1}{R_1 + R_2} \implies \frac{10}{10 + 700} \approx 1.41 \times 10^{-2} \tag{13.3}$$

14. Feedback Circuit for $PM = 45°$
Solution:

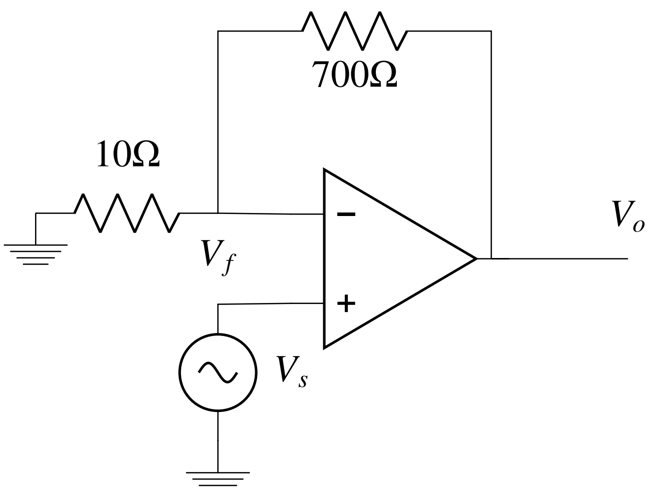

Fig. 14

15. Verification using spice circuit simulation
Solution: For $H = 0.014$ the closed loop response is

$$|T| \approx \frac{1}{H} = 70.72 \tag{15.1}$$

The following is the netlist file for spice

spice/ee18btech11034/ee18btech11034_2.net

The following python code plots the closed loop step response verses time

spice/ee18btech11034/ ee18btech11034_spice_result2.py

16. Verification of step response through python
Solution: For H = 0.014 step response through python The following python code is for the above verification

spice/ee18btech11034/ ee18btech11034_spice_verify2.py

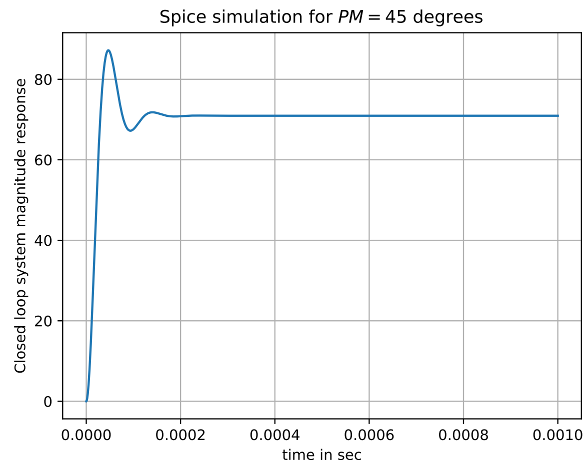

Fig. 15

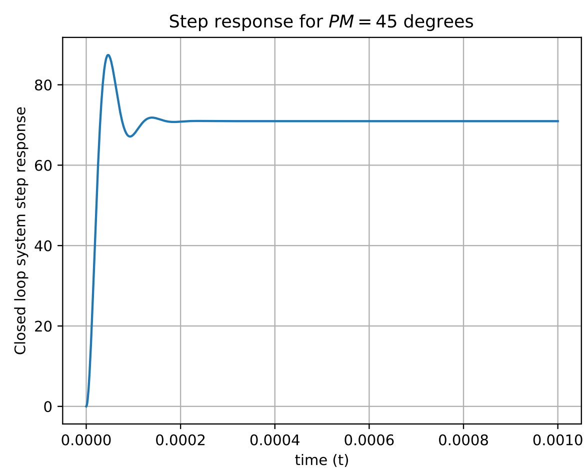

Fig. 16

From Fig.15 and Fig.16 we can see the circuit simulation output and python output are same for step signal input.
Follow the instructions in below file for running spice files

spice/ee18btech11034/README.md

17. Check for unstability
Solution: For a closed loop system to be unstable PM of GH is negative

$$PM < 0° \tag{17.1}$$

$$\implies \underline{/G(f)H(f)} < -180° \tag{17.2}$$

For the given GH

$$\underline{/G(f)H(f)} = \underline{/G(f)} \tag{17.3}$$

$$= -\tan^{-1}\left(\frac{f}{10}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) \tag{17.4}$$

At $f = \infty$

$$\underline{/G(f)} = -90^\circ - 90^\circ = -180^\circ \tag{17.5}$$

So there will be no positive f where $\underline{/G(f)} < -180^\circ$
Hence,the system is stable for any constant feedback gain H