Oscillator

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1. Consider the quadrature-oscillator circuit given Fig. 1.1 without the limiter. Let the resistance R_f be equal to $\frac{2R}{1+\Delta}$ where $\Delta << 1$. Show that the poles of the characteristic equation are in the right-half s plane and given by s $\approx \frac{1}{CR}(\frac{\Delta}{4} \pm j)$

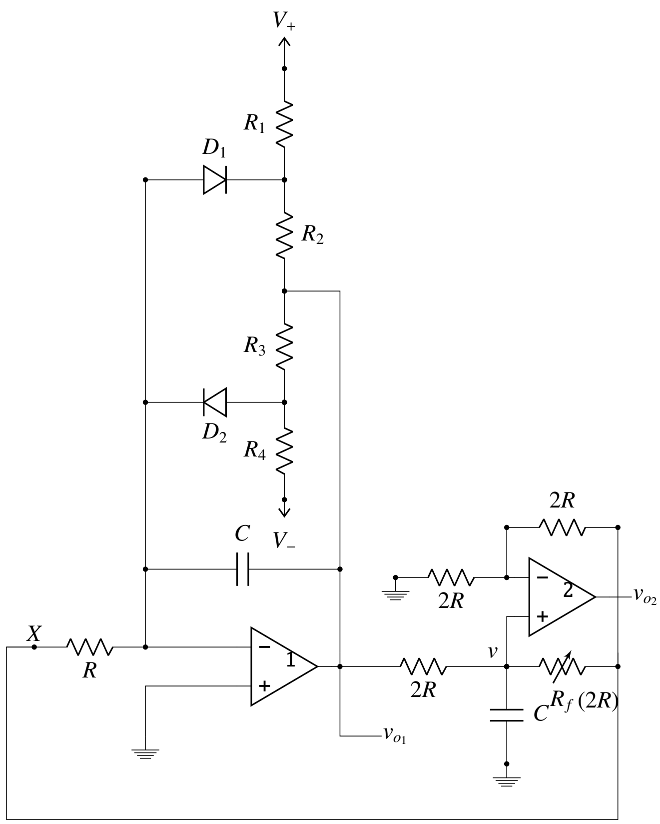

Fig. 1.1

2. And the Equivalent circuit at the input of op-amp 2 is given in Fig 2.2
3. **Solution:** Find the open loop gain.
   Consider the general open loop block diagram as shown in Fig 3.3

$$G = \frac{v_o}{v_i} \tag{3.1}$$

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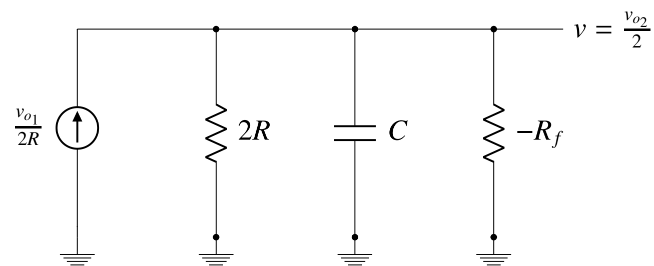

Fig. 2.2

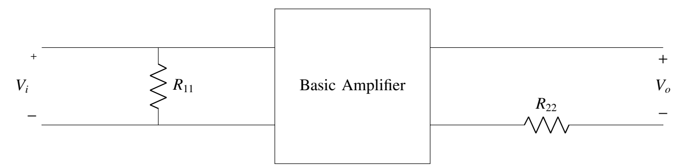

Fig. 3.3: Open Loop Block diagram

4. Equivalent circuit diagram for Fig 3.3 is shown in 4.4

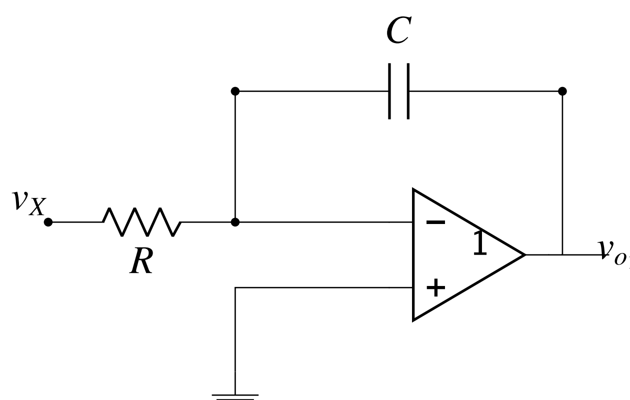

Fig. 4.4: Equivalent Circuit for open loop block diagram

When we consider the circuit without the limiter and break the loop at X, The expression for open loop gain is

$$G = \frac{v_{o_1}}{v_x} = -\frac{1}{sCR} \tag{4.1}$$

5. Consider the general block diagram for Feedback network in Fig 5.5

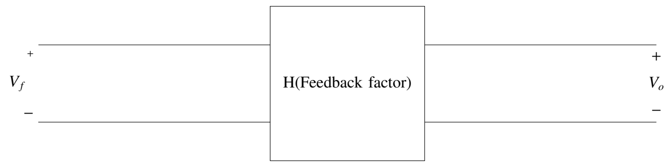

Fig. 5.5

$$H = \frac{V_f}{V_o} \tag{5.1}$$

6. The equivalent Circuit is shown in Fig 6.6

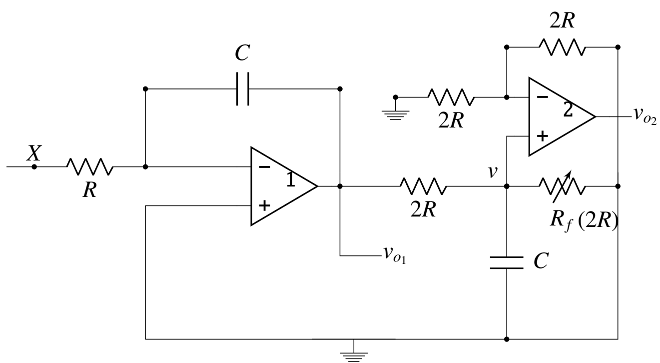

Fig. 6.6

$$H = \frac{v_{o_2}}{v_{o_1}} \tag{6.1}$$

7. Draw the equivalent control system representation for the circuit in Fig. 6.6 which is given in 7.7

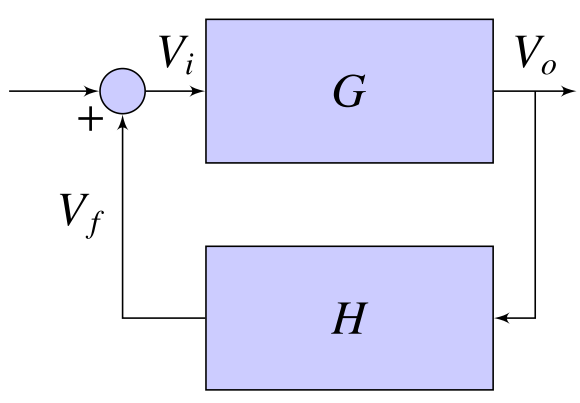

Fig. 7.7: Simplified equivalent block diagram

8. The Quadrature oscillator is based on the second integrator.
   As an active filter, the loop is damped to locate the poles in the left half of the s-plane. In the quadrature oscillator, the op amp 1 is connected as an inverting miller integrator with a limiter in the feedback for controlling the amplitude. The op amp 2 is connected as a non-inverting integrator.
   Consider the part of the circuit with op-amp 2 shown in Fig 8.8

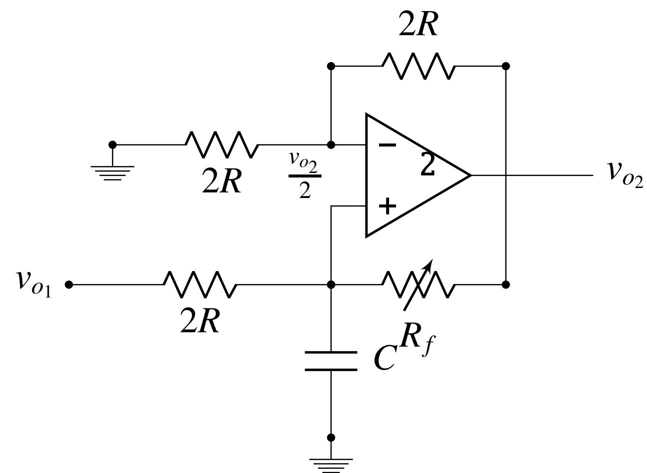

Fig. 8.8

9. In 8.8.
   Use voltage division principle to write the expression of the fraction of its input voltage.

$$v_+ = v_- = \left(\frac{v_{o_2}}{2R + 2R}\right)(2R) = \frac{v_{o_2}}{2} \tag{9.1}$$

10. Apply KCL at the non- inverting terminal of the op amp in Fig 8.8

$$\frac{\frac{v_{o_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{o_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{o_2}}{2} - v_{o_2}}{R_f} = 0 \tag{10.1}$$

$$\frac{v_{o_2} - 2v_{o_1}}{4R} + sC\frac{v_{o_2}}{2} + \frac{v_{o_2} - 2v_{o_2}}{2R_f} = 0 \tag{10.2}$$

$$\frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{R_f} = 0 \tag{10.3}$$

11. Substitute $R_f = \frac{2R}{1+\Delta}$ and find the feedback factor H

$$\frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{2R}(1 + \Delta) = 0 \tag{11.1}$$

$$v_{o_2}\left(\frac{1}{2R} + sC - \frac{1+\Delta}{2R}\right) = \frac{v_{o_1}}{R} \tag{11.2}$$

$$v_{o_2}(1 + 2sRC - 1 - \Delta) = 2v_{o_1} \tag{11.3}$$

Simplifying further,

$$\frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \tag{11.4}$$

$$H = \frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \tag{11.5}$$

12. The transfer function of the equivalent positive feedback circuit in Fig. 8.8 is

$$T = \frac{G}{1 - GH} \tag{12.1}$$

Therefore, loop gain is given by

$$L = GH \tag{12.2}$$

From (4.1) and (11.5)

$$L(s) = \frac{-1}{sCR}\frac{1}{sCR - \frac{\Delta}{2}} \tag{12.3}$$

$$L(s) = \frac{1}{-s^2C^2R^2 + \frac{sCR\Delta}{2}} \tag{12.4}$$

Consider the characteristic equation of the transfer function (12.1),

$$1 - L(s) = 0 \tag{12.5}$$

$$L(s) = 1 \tag{12.6}$$

$$-s^2C^2R^2 + \frac{sCR\Delta}{2} = 1 \tag{12.7}$$

$$\left(C^2R^2\right)s^2 + \left(-\frac{CR\Delta}{2}\right)s + 1 = 0 \tag{12.8}$$

13. Write the expression for roots of a general quadratic equation

$$s_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{13.1}$$

Substitute $a = C^2R^2$, $b = -\frac{CR\Delta}{2}$, $c = 1$ in (13.1),

$$s_p = \frac{-\left(-\frac{CR\Delta}{2}\right) \pm \sqrt{\left(-\frac{CR\Delta}{2}\right)^2 - 4\left(C^2R^2\right)(1)}}{2C^2R^2} \tag{13.2}$$

$$= \frac{RC\left(\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}\right)}{2C^2R^2} \tag{13.3}$$

$$= \frac{\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}}{2RC} \tag{13.4}$$

$$= \frac{\frac{\Delta}{2} \pm 2j\sqrt{1 - \left(\frac{\Delta}{4}\right)^2}}{2RC} \tag{13.5}$$

As $\Delta << 1$,

$$\left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{\frac{1}{2}} = 1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2 \tag{13.6}$$

$$s_p = \frac{\frac{\Delta}{2} \pm 2j\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{13.7}$$

$$s_p = \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{13.8}$$

From (13.8),

$$Re\left(s_p\right) > 0 \tag{13.9}$$

Hence, the poles of the characteristic equation are in the right half of the s plane. As $\Delta << 1$, higher order terms are neglected.

$$s_p = \frac{\frac{\Delta}{2} \pm 2j}{2RC} \tag{13.10}$$

$$s_p = \frac{\frac{\Delta}{4} \pm j}{RC} \tag{13.11}$$

14. Find the frequency for arbitrary R,C values as given in Table 14

Parameter	**Value**
R	$5k\Omega$
C	$10\mu F$
Δ	0.1
$R_f = \frac{2R}{1+\Delta}$	9090.9

TABLE 14

The loop will oscillate at frequency ω_o, given by

$$\omega_o = \frac{1}{RC} \tag{14.1}$$

From Table 14,

$$\omega_o = 20rad/s \tag{14.2}$$

$$f = \frac{\omega_o}{2\pi} = 3.184Hz \tag{14.3}$$

From (12.1),

$$T = \frac{-SCR + \frac{\Delta}{2}}{s^2C^2R^2 - \frac{sCR\Delta}{2} + 1} \tag{14.4}$$

From Table 14,

$$T = \frac{-0.05s + 0.05}{0.0025s^2 - 0.0025s + 1} \tag{14.5}$$

The following code plots the oscillating response of the system as shown in 14.9

```
codes/es17btech11009/es17btech11009_1_1.
    py
```

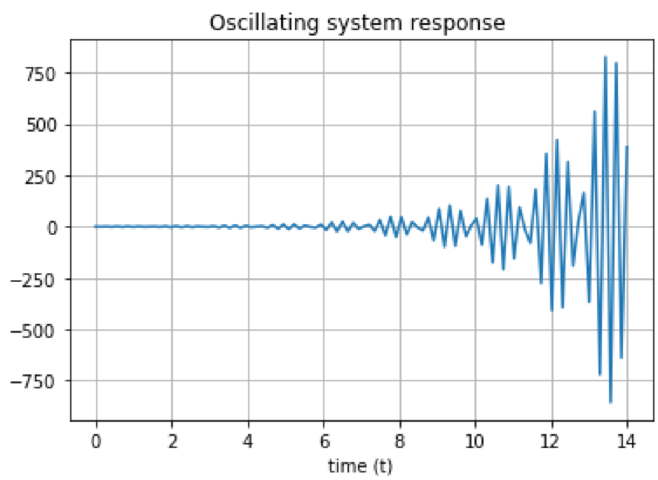

Fig. 14.9

15. Simulate the circuit Fig 8.8 using spice simulators and plot the generated output using python script.
Find the netlist for the simulated circuit here:

```
spice/es17btech11009.net
```

Python code used for generating the output:

```
codes/es17btech11009/es17btech11009_spice.
    py
```

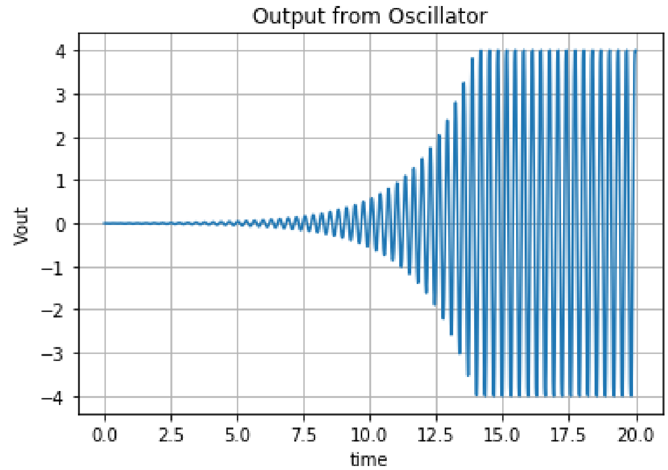

Fig. 15.10

16. Consider part of the spice simulation and the following code plots the part of the output as shown in Fig 16.11

```
codes/es17btech11009/
    es17btech11009_spice1.py
```

From the Fig 15.10, Time period of oscillation

$$T = 15.3738 - 15.0569 \tag{16.1}$$

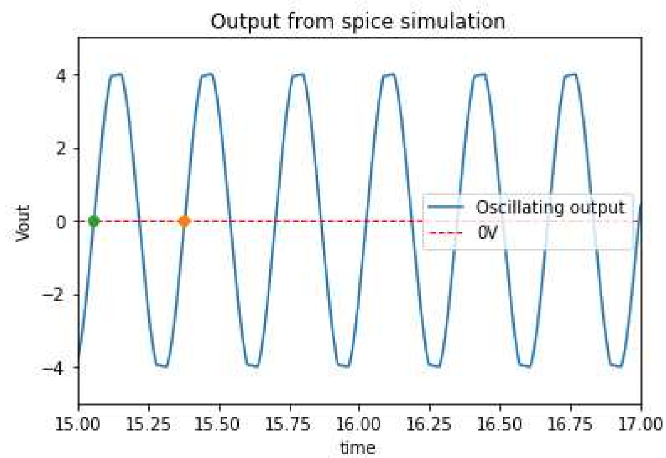

Fig. 16.11

$$f = \frac{1}{T} = 3.155Hz \tag{16.2}$$

$$\omega_o = 2\pi f = 19.8rad/s \tag{16.3}$$

Hence the frequency calculated from the formulae and the plot are approximately same.