Oscillator

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For the circuit shown in Fig. 1.1, find the loop gain $L(s) = G(s)H(s)$, $L(\jmath\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

1. Draw the equivalent control system representation for the circuit in Fig. 1.1 as well as the small signal model.
   Solution: See Figs. 1.2, 1.3 and 1.4. Oscillators do not include input signal.

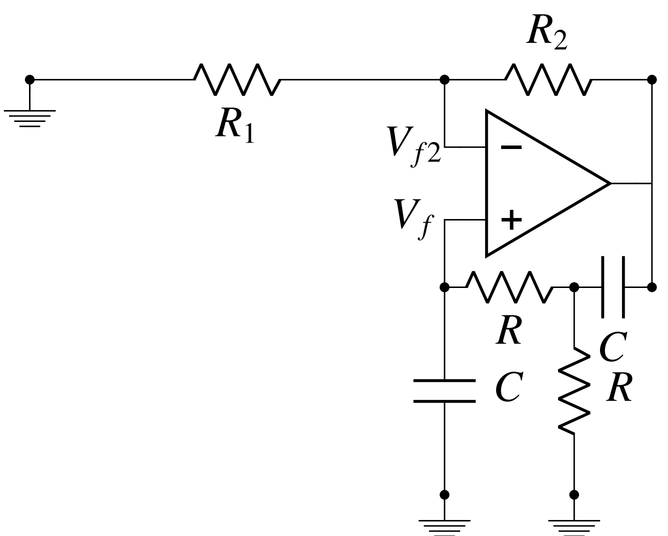

Fig. 1.1

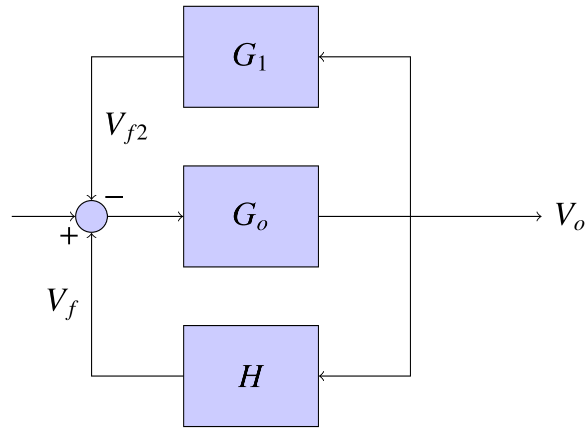

Fig. 1.2: Block diagram

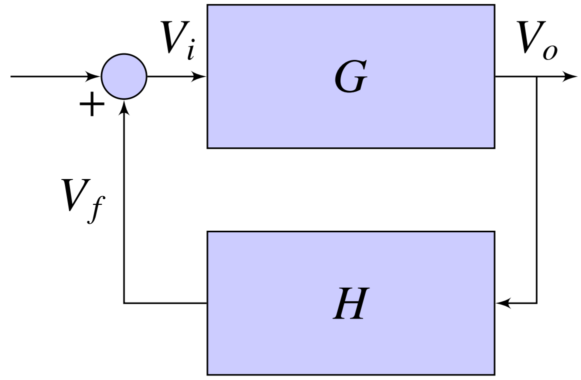

Fig. 1.3: Simplified equivalent block diagram

2. Draw the block diagram and circuit diagram for H.
   Solution: See Figs. 2.5 and 2.6.
3. Find H.
   Solution: In Fig. 2.6, let I_o be the current flowing from V_o. Then

$$I_o = \frac{V_o}{\frac{1}{sC} + R \parallel \left(R + \frac{1}{sC}\right)} \tag{3.1}$$

Using current division,

$$V_f = I_o \frac{R}{R + \left(R + \frac{1}{sC}\right)} \times \frac{1}{sC} \tag{3.2}$$

From (3.1) and (3.2),

$$\frac{V_f}{V_o} = \frac{R}{R + \left(R + \frac{1}{sC}\right)} \times \frac{1}{sC} \times \frac{1}{\frac{1}{sC} + R \parallel \left(R + \frac{1}{sC}\right)} \tag{3.3}$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \tag{3.4}$$

after simplification.

4. Find R_{11} and R_{22} from Fig. 2.6.

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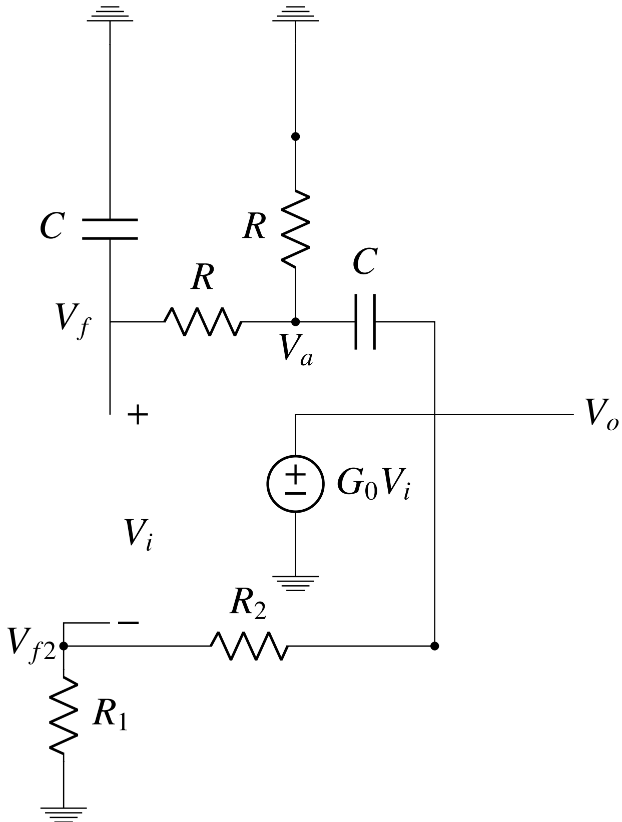

Fig. 1.4

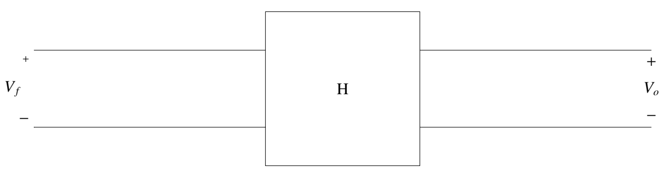

Fig. 2.5: Feedback block diagram

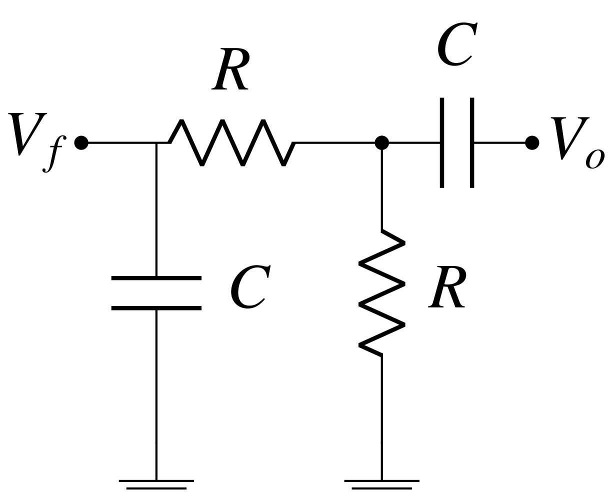

Fig. 2.6: Feedback circuit

Solution: Shorting V_o to ground,

$$R_{11} = \frac{1}{sC} \parallel \left(R + R \parallel \frac{1}{sC}\right) \tag{4.1}$$

Shorting V_f to ground,

$$R_{22} = \frac{1}{sC} + \frac{R}{2} \tag{4.2}$$

5. Draw the block diagram and circuit diagram for G.
   Solution: See Figs. 5.1 and 5.2.

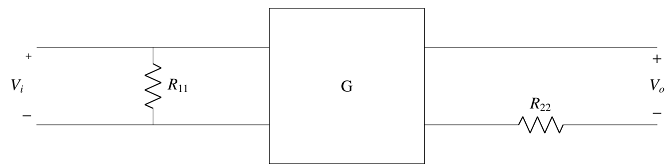

Fig. 5.1: Open loop block diagram

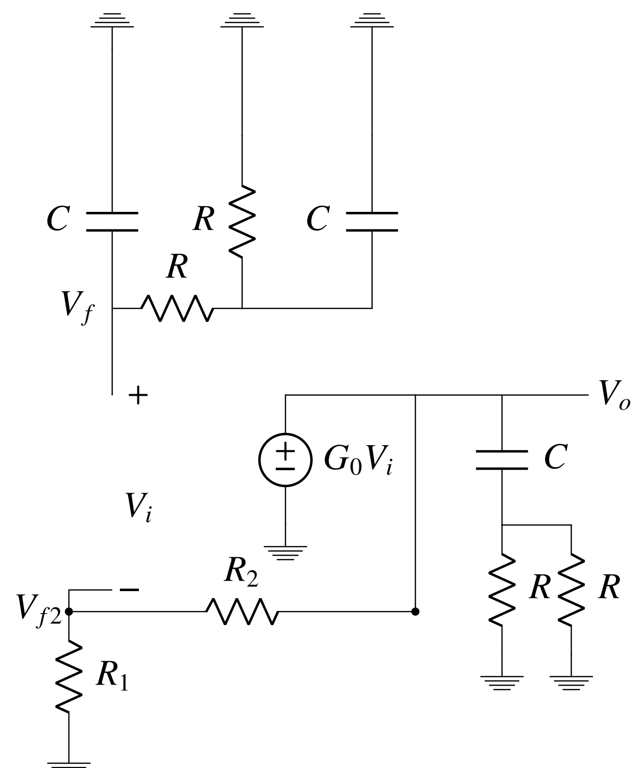

Fig. 5.2: Open loop circuit diagram

6. Find G.
   Solution: From Fig.5.2

$$V_{f_2} = \left(\frac{R_1}{R_1 + R_2}\right) V_o \tag{6.1}$$

$$G_1 = \frac{V_{f_2}}{V_o} \tag{6.2}$$

$$\implies G_1 = \frac{R_1}{R_1 + R_2} \tag{6.3}$$

From Fig. 1.2 ,G_1 is the negative feedback factor and G_o is the gain of the op-amp.Therefore,equivalent G is given by

$$G = \frac{G_o}{1 + G_o G_1} \tag{6.4}$$

$$G = \frac{1}{\frac{1}{G_o} + G_1} \tag{6.5}$$

We assumed $G_0 \to \infty$.

$$\implies G = \frac{1}{G_1} \tag{6.6}$$

From equation (6.3).

$$\implies G = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \tag{6.7}$$

Hence verified with equation (8.5).

7. Find the feedback factor H.
Solution: The small signal model is shown in Fig. 1.4 Applying KCL at node V_f

$$\frac{V_f - 0}{\frac{1}{sC}} + \frac{V_f - V_a}{R} = 0 \tag{7.1}$$

$$V_f\left(sC + \frac{1}{R}\right) = \frac{V_a}{R} \tag{7.2}$$

$$V_a = V_f\left(sRC + 1\right) \tag{7.3}$$

Applying KCL at node V_a

$$\frac{V_a - V_f}{R} + \frac{V_a - 0}{R} + \frac{V_a - V_o}{\frac{1}{sC}} = 0 \tag{7.4}$$

$$V_a\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \tag{7.5}$$

Substitute V_a value from equation(7.3)

$$V_f(sRC + 1)\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \tag{7.6}$$

$$V_f\left(3 + sRC + \frac{1}{sRC}\right) = V_o \tag{7.7}$$

The feedback factor H is given by

$$H = \frac{V_f}{V_o} \tag{7.8}$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \tag{7.9}$$

8. Find the open loop gain G.
Solution: Let the closed loop gain, open-loop gain of op-amp connected in non-inverting configuration be T_0 and G_0 respectively. From Table **??**

$$T_0 = \frac{G_0\left(R_1 + R_2\right)}{\left(R_1 + R_2\right) + G_0 R_1} \tag{8.1}$$

$$T_0 = \frac{\left(R_1 + R_2\right)}{\left(R_1 + R_2\right)/G_0 + R_1} \tag{8.2}$$

Assuming $G_0 \to \infty$

$$T_0 = 1 + \frac{R_2}{R_1} \tag{8.3}$$

The open loop gain of the circuit shown in Fig. 1.1 is equal to the closed loop gain of an op-amp connected in non-inverting configuration.

$$G = T_0 \tag{8.4}$$

$$\implies G = 1 + \frac{R_2}{R_1} \tag{8.5}$$

9. Find the loop gain L(s).
Solution: The transfer function of the equivalent positive feedback circuit in Fig. 1.3 is

$$T = \frac{G}{1 - GH} \tag{9.1}$$

Therefore, loop gain is given by

$$L = GH \tag{9.2}$$

From equations (8.5) and (7.9)

$$L(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3 + sRC + \frac{1}{sRC}}\right) \tag{9.3}$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}}\right) \tag{9.4}$$

10. Find the loop gain in terms of $j\omega$.

Solution: Substitute $s = j\omega$ in equation (9.4)

$$L(\jmath\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\omega RC + \frac{1}{j\omega RC}}\right) \tag{10.1}$$

$$\implies L(\jmath\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}\right) \tag{10.2}$$

11. Find the frequency for zero loop phase.
Solution: The frequency at which loop phase will be zero (i.e. loop gain will be a real number).To obtain the required frequency, equate the imaginary part of the loop gain $L(j\omega)$ to zero.

$$j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \tag{11.1}$$

$$\omega^2 = \frac{1}{(RC)^2} \tag{11.2}$$

$$\implies \omega = \frac{1}{RC} \tag{11.3}$$

12. Find R_2/R_1 for oscillation.
Solution: For oscillations to start,
- the imaginary part of the loop gain should become zero.
- the loop gain must be at least equal to unity.

From equation (10.2)

$$\left(\frac{1 + \frac{R_2}{R_1}}{3 + j(0)}\right) \geq 1 \tag{12.1}$$

$$1 + \frac{R_2}{R_1} \geq 3 \tag{12.2}$$

$$\implies \frac{R_2}{R_1} \geq 2 \tag{12.3}$$

13. Find the amplitude and frequency for some arbitrary R,C values given in Table 13.
Solution: From equation (8.5)

$$G = 1 + \frac{R_2}{R_1} = 3 \tag{13.1}$$

From equation (7.9)

$$H = \frac{1}{3 + 0.25s + \frac{1}{0.25s}} \tag{13.2}$$

Parameter	Value
R	250Ω
C	$1mF$
R_2	$2k\Omega$
R_1	$1k\Omega$

TABLE 13

From equation (9.1)

$$T = \frac{3\left(0.0625s^2 + 0.75s + 1\right)}{0.0625s^2 + 1} \tag{13.3}$$

The following code plots the oscillating response of the system.

```
codes/ee18btech11047/ee18btech11047.py
```

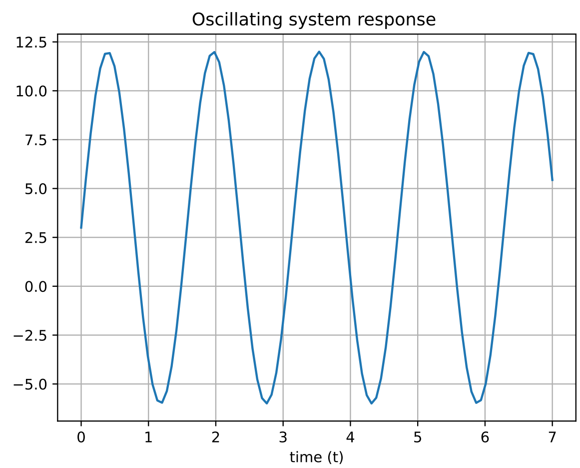

Fig. 13

Amplitude: From Fig. 13 V(peak-peak) is

$$V_{p-p} = 11.929 - (-5.957) = 17.886 \tag{13.4}$$

$$V_{max} = \frac{V_{p-p}}{2} = 8.943 \tag{13.5}$$

Frequency: From equation (11.3)

$$\omega = \frac{1}{RC} = 4rad/sec \tag{13.6}$$

$$f = \frac{\omega}{2\pi} = 0.636Hz \tag{13.7}$$

14. Verify the amplitude and frequency using spice simulation.
Solution: The following readme file provides

necessary instructions to simulate the circuit in spice.

```
codes/ee18btech11047/spice/README
```

The following netlist simulates the given circuit.

```
codes/ee18btech11047/spice/ee18btech11047.net
```

The following code plots the output from the oscillator spice simulation which is shown in Fig. 14.1.

```
codes/ee18btech11047/spice/
    ee18btech11047_spice.py
```

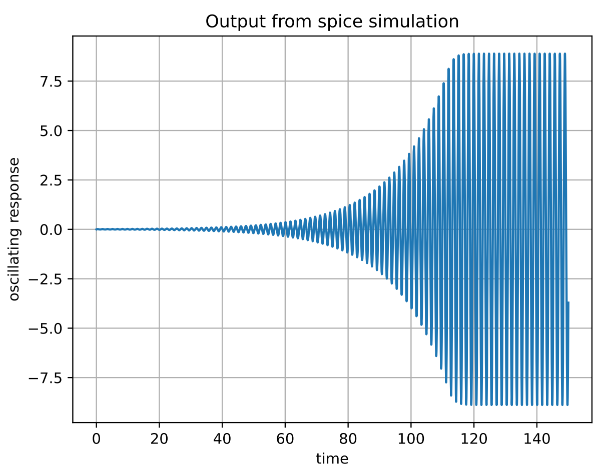

Fig. 14.1

The following code plots a part of the spice output from which we can observe a clear sinusoidal output shown in Fig. 14.2.

```
codes/ee18btech11047/spice/
    ee18btech11047_spice2.py
```

Amplitude: From Fig. 14.2 V(peak-peak) is

$$V_{p-p} = 8.89 - (-8.89) = 17.78 \tag{14.1}$$

$$V_{max} = \frac{V_{p-p}}{2} = 8.89 \tag{14.2}$$

Frequency: From Fig. 14.2 time period is calculated by any two end points of one cycle,

$$T = 120.344 - (-118.734) = 1.61 sec \tag{14.3}$$

$$f = \frac{1}{T} = 0.621 Hz \tag{14.4}$$

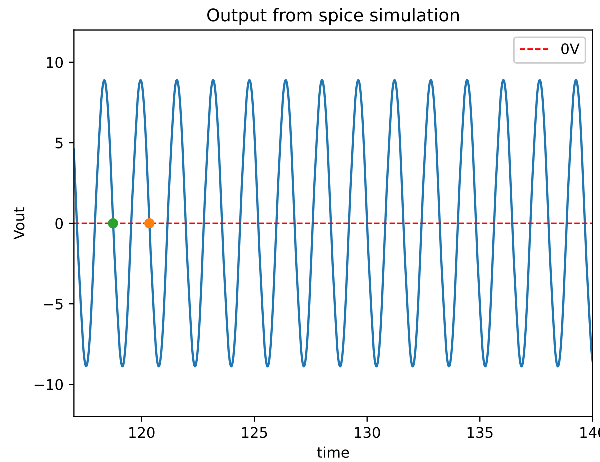

Fig. 14.2

Hence,the amplitude and frequency are verified through the spice simulation.