Oscillator

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Contents

For the circuit shown in Fig. 1.1, find the loop gain $L(s) = G(s)H(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

1. Draw the equivalent control system representation for the circuit in Fig. 1.1 as well as the small signal model.
   Solution: See Figs. 1.2 and

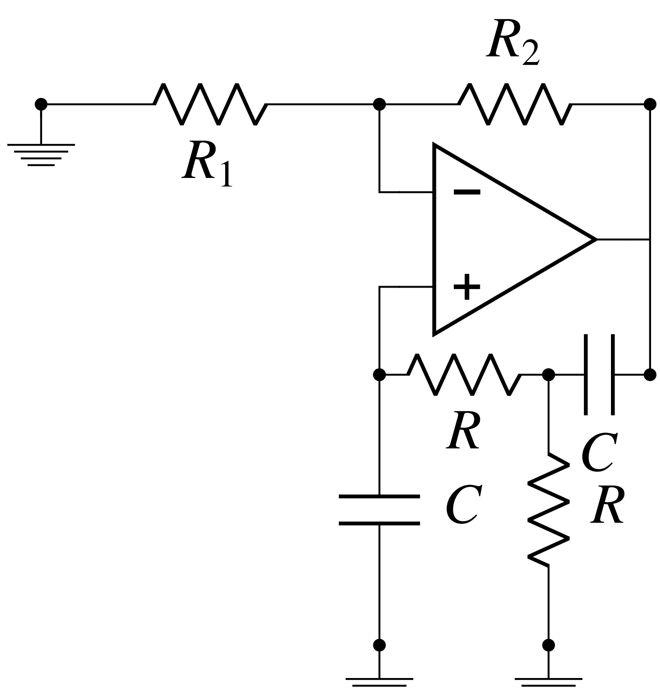

Fig. 1.1

   Solution: See Fig. 1.2. Oscillators do not include input signal.
2. Find the open loop gain G.
   Solution: Let the closed loop gain, open-loop gain of op-amp connected in non-inverting

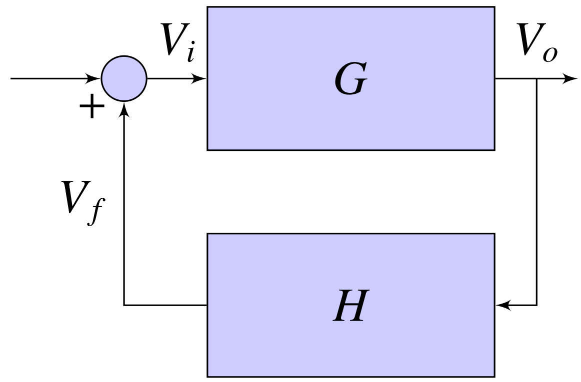

Fig. 1.2

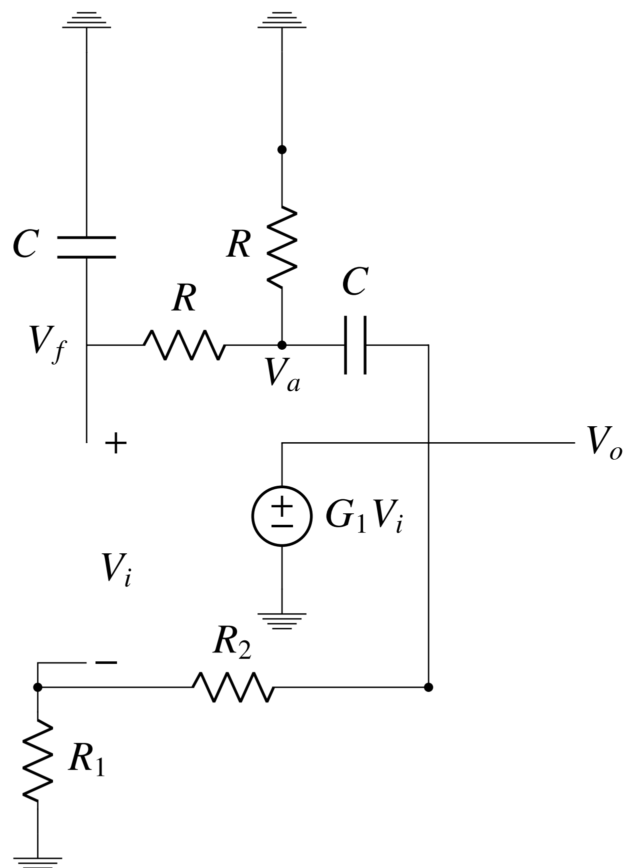

Fig. 1.3

configuration be T_1 and G_1 respectively. From

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Table **??**

$$T_1 = \frac{G_1 (R_1 + R_2)}{(R_1 + R_2) + G_1 R_1} \tag{2.1}$$

$$T_1 = \frac{(R_1 + R_2)}{(R_1 + R_2)/G_1 + R_1} \tag{2.2}$$

Assuming $G_1 \to \infty$

$$T_1 = 1 + \frac{R_2}{R_1} \tag{2.3}$$

The open loop gain of the circuit shown in Fig. 1.1 is equal to the closed loop gain of an op-amp connected in non-inverting configuration.

$$G = T_1 \tag{2.4}$$

$$\implies G = 1 + \frac{R_2}{R_1} \tag{2.5}$$

3. Find the feedback factor H.
Solution: The small signal model is shown in Fig. 1.3 Applying KCL at node V_f

$$\frac{V_f - 0}{\frac{1}{sC}} + \frac{V_f - V_a}{R} = 0 \tag{3.1}$$

$$V_f\left(sC + \frac{1}{R}\right) = \frac{V_a}{R} \tag{3.2}$$

$$V_a = V_f (sRC + 1) \tag{3.3}$$

Applying KCL at node V_a

$$\frac{V_a - V_f}{R} + \frac{V_a - 0}{R} + \frac{V_a - V_o}{\frac{1}{sC}} = 0 \tag{3.4}$$

$$V_a\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \tag{3.5}$$

Substitute V_a value from equation(3.3)

$$V_f(sRC + 1)\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \tag{3.6}$$

$$V_f\left(3 + sRC + \frac{1}{sRC}\right) = V_o \tag{3.7}$$

The feedback factor H is given by

$$H = \frac{V_f}{V_o} \tag{3.8}$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \tag{3.9}$$

4. Find the loop gain L(s).
Solution: The transfer function of the equivalent positive feedback circuit in Fig. 1.2 is

$$T = \frac{G}{1 - GH} \tag{4.1}$$

Therefore, loop gain is given by

$$L = GH \tag{4.2}$$

From equations (2.5) and (3.9)

$$L(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3 + sRC + \frac{1}{sRC}}\right) \tag{4.3}$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}}\right) \tag{4.4}$$

5. Find the loop gain in terms of $j\omega$.
Solution: Substitute $s = j\omega$ in equation (4.4)

$$L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\omega RC + \frac{1}{j\omega RC}}\right) \tag{5.1}$$

$$\implies L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}\right) \tag{5.2}$$

6. Find the frequency for zero loop phase.
Solution: The frequency at which loop phase will be zero (i.e. loop gain will be a real number).To obtain the required frequency, equate the imaginary part of the loop gain $L(j\omega)$ to zero.

$$j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \tag{6.1}$$

$$\omega^2 = \frac{1}{(RC)^2} \tag{6.2}$$

$$\implies \omega = \frac{1}{RC} \tag{6.3}$$

7. Find R_2/R_1 for oscillation.
Solution: For oscillations to start,
- the imaginary part of the loop gain should become zero.
- the loop gain must be at least equal to unity.

From equation (5.2)

$$\left(\frac{1+\frac{R_2}{R_1}}{3+j(0)}\right) \geq 1 \tag{7.1}$$

$$1+\frac{R_2}{R_1} \geq 3 \tag{7.2}$$

$$\implies \frac{R_2}{R_1} \geq 2 \tag{7.3}$$

8. Draw the block diagram and circuit diagram for H.
Solution: See figs 8.4 and 8.5 .From Fig.

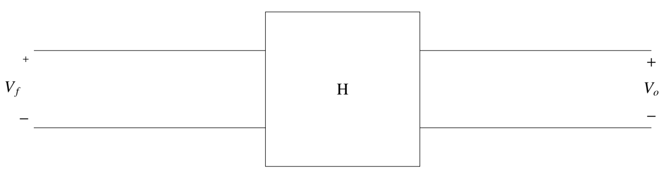

Fig. 8.4: Feedback block diagram

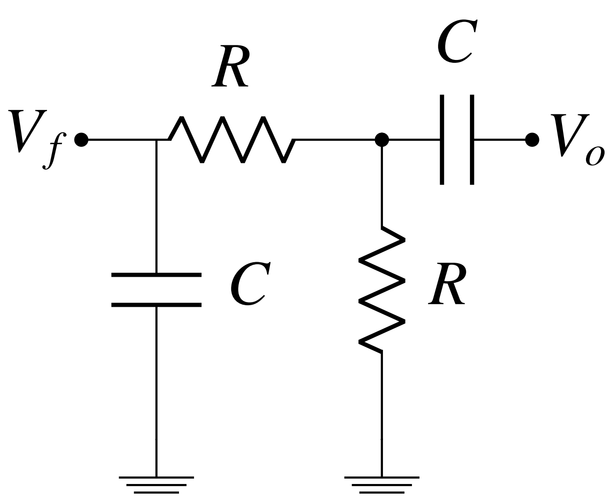

Fig. 8.5: Feedback circuit

8.5,the analysis is same as problem 3

$$\frac{V_f}{V_o} = \frac{1}{\left(3+sRC+\frac{1}{sRC}\right)} \tag{8.1}$$

$$\implies H = \frac{1}{\left(3+sRC+\frac{1}{sRC}\right)} \tag{8.2}$$

9. Find the input and output resistances of the feedback network.
Solution: To find the input resistance R_{11} short the output node V_o to ground.

$$R_{11} = Z||(R+(R||Z)) \tag{9.1}$$

where $Z = \frac{1}{sC}$ is the impedance of the capacitor.

$$\implies R_{11} = \left(\frac{1}{sC}\Big|\Big|\left(R+R\Big|\Big|\frac{1}{sC}\right)\right) \tag{9.2}$$

To find the output resistance R_{22} short the input node V_f to ground.

$$R_{22} = Z+(R||R) \tag{9.3}$$

$$\implies R_{22} = \frac{1}{sC}+\frac{R}{2} \tag{9.4}$$

10. Draw the block diagram and circuit diagram for G.
Solution: See figs 10.6 and 10.7.From Fig.

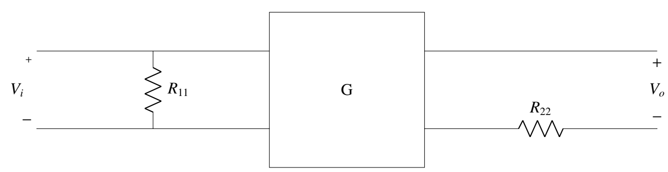

Fig. 10.6: Open loop block diagram

10.7 using same analysis as problem 2

$$G = \frac{V_o}{V_i} \tag{10.1}$$

$$G = \frac{R_1+R_2}{R_1} \tag{10.2}$$

$$\implies G = 1+\frac{R_2}{R_1} \tag{10.3}$$

Hence verified with equation (2.5).

11. Find the amplitude and frequency for some arbitrary values given in Table 11.
Solution: From equation (2.5)

Parameter	Value
R	250Ω
C	$1mF$
R_2	$2k\Omega$
R_1	$1k\Omega$

TABLE 11

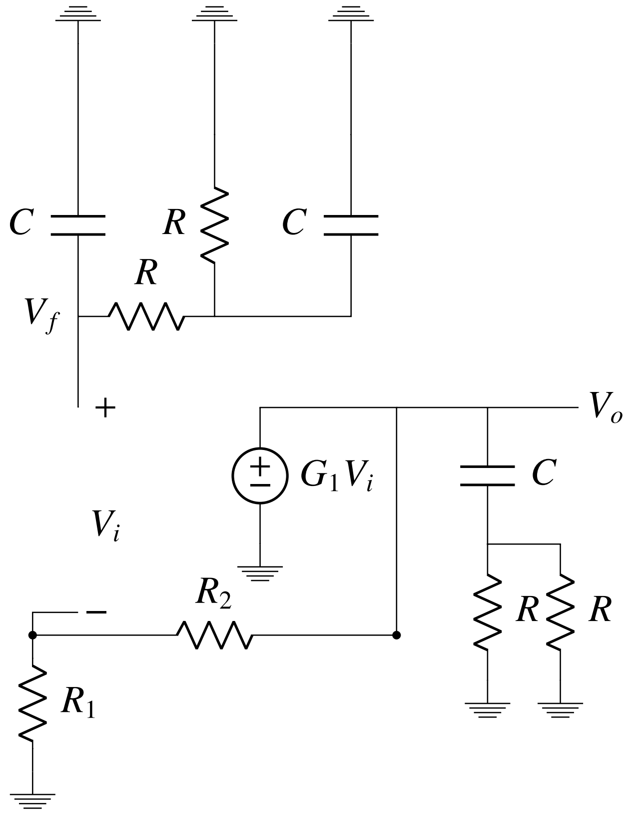

Fig. 10.7: Open loop circuit diagram

$$G = 1 + \frac{R_2}{R_1} = 3 \tag{11.1}$$

From equation (3.9)

$$H = \frac{1}{3 + 0.25s + \frac{1}{0.25s}} \tag{11.2}$$

From equation (4.1)

$$T = \frac{3\left(0.0625s^2 + 0.75s + 1\right)}{0.0625s^2 + 1} \tag{11.3}$$

The following code plots the oscillating response of the system.

```
codes/ee18btech11047/ee18btech11047.py
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Amplitude: From Fig. 11 V(peak-peak) is

$$V_{p-p} = 18.12 \tag{11.4}$$

$$V_{max} = \frac{V_{p-p}}{2} = 9.06 \tag{11.5}$$

Frequency: From equation (6.3)

$$\omega = \frac{1}{RC} = 4rad/sec \tag{11.6}$$

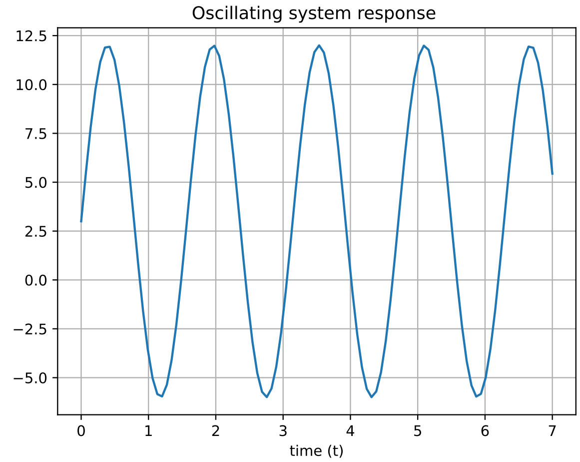

Fig. 11

$$f = \frac{\omega}{2\pi} = 0.636Hz \tag{11.7}$$