

Hartley Oscillator

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1. Find the frequency of oscillation for given hartley circuit and also find condition on g_m .
Below is the figure, Fig 1

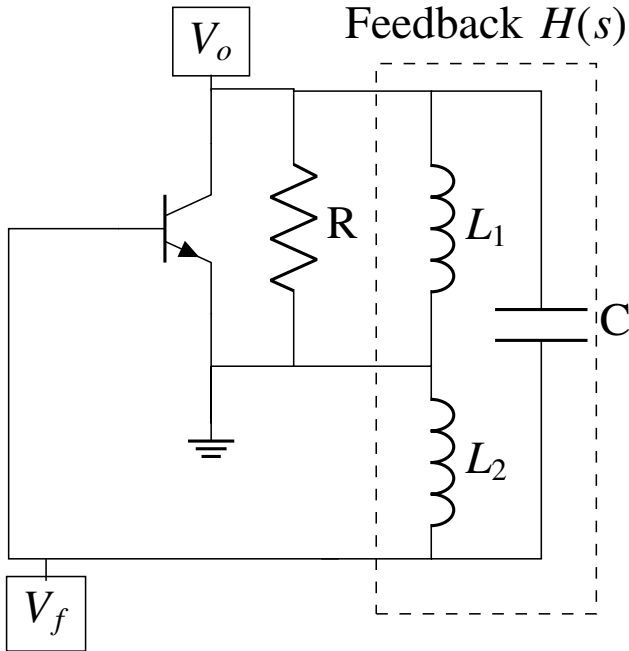


Fig. 1: Hartley oscillator

Solution: We will first draw an equivalent circuit for the above circuit.

To draw an equivalent block diagram we will draw small signal model for transistor.¹ And, its block diagram is as follows:

Here $G(s)$ is the amplification gain, and $H(s)$ is the feedback gain.
and $G(s)$ is given by, $\frac{V_o}{V_f}$

$$V_o = I(sL_1 \parallel R) \quad (1.1)$$

$$I = i_1 + g_m V_f \quad (1.2)$$

$$i_1 = \frac{V_\pi}{sL_2} \quad (1.3)$$

$$(1.4)$$

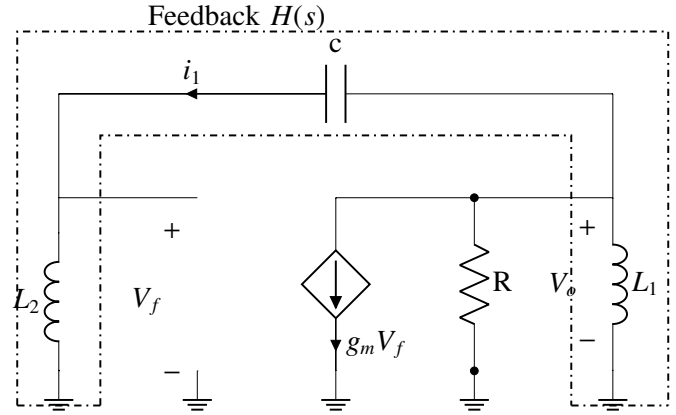


Fig. 1: Small signal model

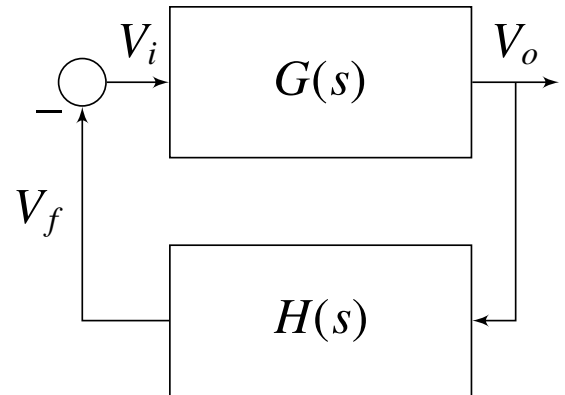


Fig. 1: Block diagram

Solving these equations we get,

$$\frac{V_o}{V_f} = G(s) = \left(g_m + \frac{1}{sL_2} \right) \left(\frac{RsL_1}{R + sL_1} \right) \quad (1.5)$$

Now, solving for $H(s)$

From the small signal model feedback, we know that $H(s)$ is output/input,

Where,

Output is V_f

Input is V_o

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$$H(s) = \frac{V_f}{V_o} \quad (1.6)$$

$$V_o = V_f + i_1 \times \frac{1}{sC} \quad (1.7)$$

$$i_1 = \frac{V_f}{sL_2} \quad (1.8)$$

$$(1.9)$$

Solving,

$$H(s) = \left(\frac{s^2 CL_2}{s^2 CL_2 + 1} \right) \quad (1.10)$$

Characteristic equation is given by:

$$1 + G(s)H(s) = 0 \quad (1.11)$$

Substituting the values and simplifying, we get

$$s^3(g_m CL_1 L_2 + CL_1 L_2) + s^2(RCL_1 + RCL_2) + sL_1 + R = 0 \quad (1.12)$$

Now, for it to oscillate, roots of the equation should lie on imaginary axis, therefore $j\omega$ should be a solution

Substituting that, we get

$$(R - \omega^2(RC(L_1 + L_2))) + j(\omega L_1 - \omega^3(g_m R + 1)CL_1 L_2) = 0 \quad (1.13)$$

Equating Real part to 0

$$\omega^2(RC(L_1 + L_2)) = R \quad (1.14)$$

$$\omega = \frac{1}{\sqrt{C(L_1 + L_2)}} \quad (1.15)$$

Equating Imaginary part to 0

$$g_m R + 1 = \frac{C(L_1 + L_2)}{CL_2} \quad (1.16)$$

$$g_m R = \frac{L_1}{L_2} \quad (1.17)$$

Therefore to have stable oscillations, we need $g_m R \geq \frac{L_1}{L_2}$

Simulation

For simulation more elaborate circuit of

Hartley oscillator was used, i.e. more passive components like capacitors and resistors, so as the the oscillations don't die out quickly, and a voltage source so as the oscillations start.

Below is the circuit which was used 1

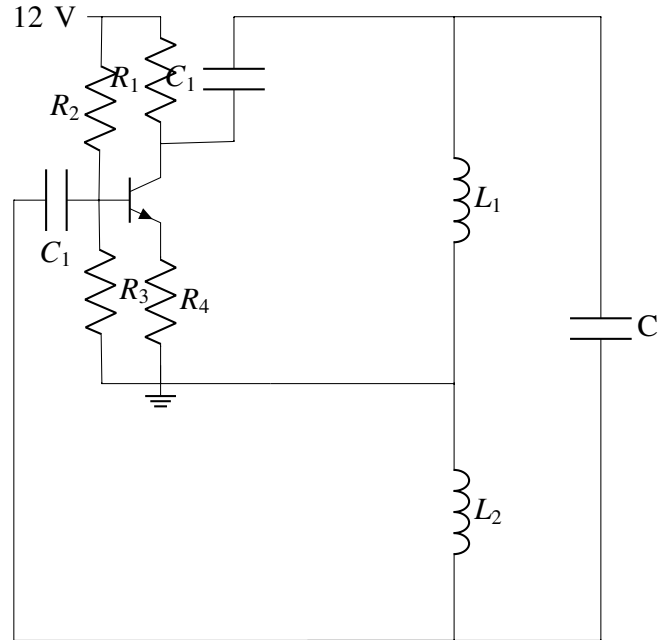


Fig. 1: Simulation circuit

Table for parameter values taken Verifying

Parameter	Value
R_1	1.2k Ω
R_2	50.5k Ω
R_3	10.5k Ω
R_4	298 Ω
C_1	22 μF
C_2	22 μF
C	1.1 μF
L_1	1mH
L_2	1mH

TABLE 1

the output:

Plot generated from transfer function, taking impulse response

?? Taking an equivalent R

$$R = L_1 \text{ and } g_m R = \frac{L_1}{L_2}$$

Code for generating impulse response

```
codes/ee18btech11019_1.py
```

Actual expected frequency is:

$$= \frac{1}{2\pi \sqrt{C(L_1 + L_2)}} \quad (1.18)$$

$$= 3394 \text{ Hz} \quad (1.19)$$

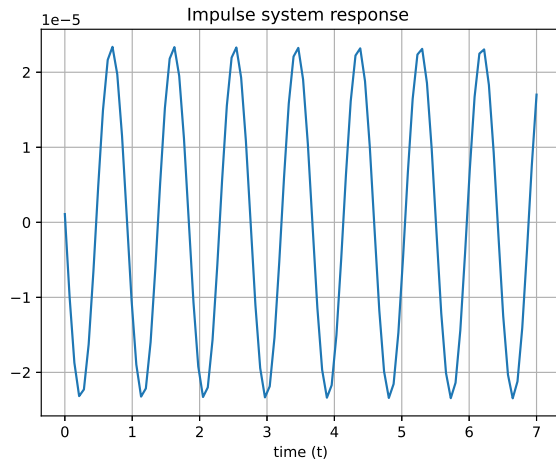


Fig. 1: Output when taken from transfer function

Actual simulation results,

Running ngspice netlist file, we produce dat file. From that data we get plot and frequency from python script found in

```
spice/ee18btech11019_2.py
```

Plot:

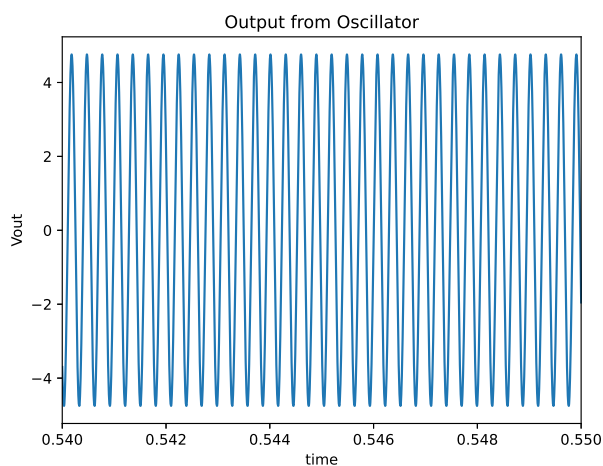


Fig. 1: Simulation result

Frequency obtained is **3384 Hz**