Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 PID Controller

1.1 Introduction

1.1.1. Tabulate the transfer functions of a PID controller and its variants.
Solution: See Table 1.1.1.

Controller	Gain
PID	$K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$
PD	$K_p(1 + T_d s)$
PI	$K_p\left(1 + \frac{1}{T_i s}\right)$

TABLE 1.1.1

1.1.2. For a unity Feedback system

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{1.1.2.1}$$

Design a PD Controller with $K_v = 2$ and Phase Margin 30°

Solution: The gain after cascading the PD Controller with G(s) is

$$G_c(s) = \frac{K_p(1 + T_d s)K}{s(s+2)(s+4)(s+6)} \tag{1.1.2.2}$$

Choosing $K_p = 1$ in ,

$$K_v = \lim_{s\to 0} sG_c(s) = 2 \tag{1.1.2.3}$$

$$\implies K = 96 \tag{1.1.2.4}$$

For Phase Margin 30°, at Gain Crossover Frequency ω,

$$\tan^{-1}(T_d\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = -60° \tag{1.1.2.5}$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_d^2\omega^2 + 1}}{\omega\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \tag{1.1.2.6}$$

By Hit and Trial, one of the best combinations is

$$\omega = 4 \tag{1.1.2.7}$$

$$T_d = 1.884 \tag{1.1.2.8}$$

We get a Phase Margin of 30.31°

1.1.3. Verify using a Python Plot
Solution: The following code plots Fig. 1.1.3

```
codes/ee18btech11021/EE18BTECH11021_3.
    py
```

1.1.4. Design a PI Controller with $K_v = \infty$ and Phase Margin 30°
Solution: From Table 1.1.1, the open loop gain in this case is

$$G_1(s) = \frac{K_p\left(1 + \frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \tag{1.1.4.1}$$

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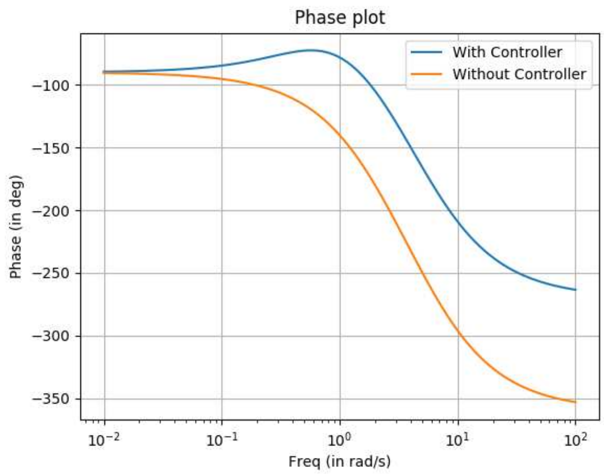

Fig. 1.1.3

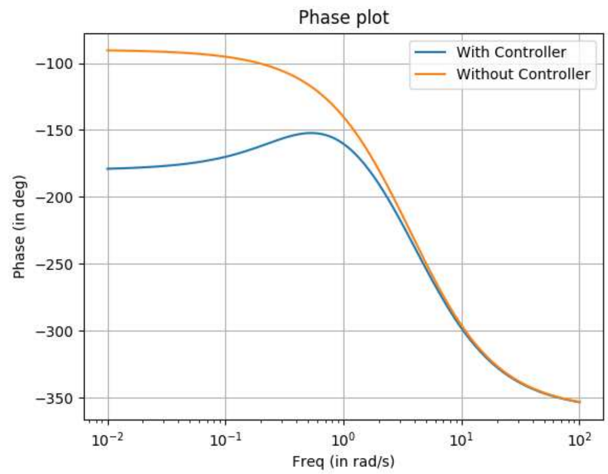

Fig. 1.1.5

Choose $K_pK = 96$. Then

$$G_1(s) = \frac{96(T_is+1)}{T_is^2(s+2)(s+4)(s+6)} \quad (1.1.4.2)$$

For Phase Margin 30°, at Gain Crossover Frequency ω

$$\tan^{-1}(T_i\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \quad (1.1.4.3)$$

and

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_i^2\omega^2+1}}{T_i^2\omega^2\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \quad (1.1.4.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 0.75 \quad (1.1.4.5)$$
$$T_i = 2.713 \quad (1.1.4.6)$$

We get a Phase Margin of 25.53°

1.1.5. Verify using a Python Plot

Solution: The following code plots Fig. 1.1.5.

```
codes/ee18btech11021/EE18BTECH11021_4.
    py
```

1.1.6. Design a PID Controller with $K_v = \infty$ and Phase Margin 30°

Solution:

$$G_1(s) = \frac{K_p\left(1+T_ds+\frac{1}{T_is}\right)K}{s(s+2)(s+4)(s+6)} \quad (1.1.6.1)$$

Choose $K_pK = 96$. The open loop gain is

$$G_1(s) = \frac{96(T_iT_ds^2+T_is+1)}{T_is^2(s+2)(s+4)(s+6)} \quad (1.1.6.2)$$

For Phase Margin 30°, at Gain Crossover Frequency ω,

$$\tan^{-1}\left(\frac{T_i\omega}{1-TiT_dw^2}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \quad (1.1.6.3)$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{(1-TiT_d\omega^2)^2+T_i^2}}{T_i^2\omega^2\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \quad (1.1.6.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 1 \quad (1.1.6.5)$$
$$T_i = 1.738 \quad (1.1.6.6)$$
$$T_d = 0.4 \quad (1.1.6.7)$$

We get a Phase Margin of 30°

1.1.7. Verify using a Python Plot

Solution: The following code plots Fig. 1.1.7

codes/ee18btech11021/EE18BTECH11021_5.py

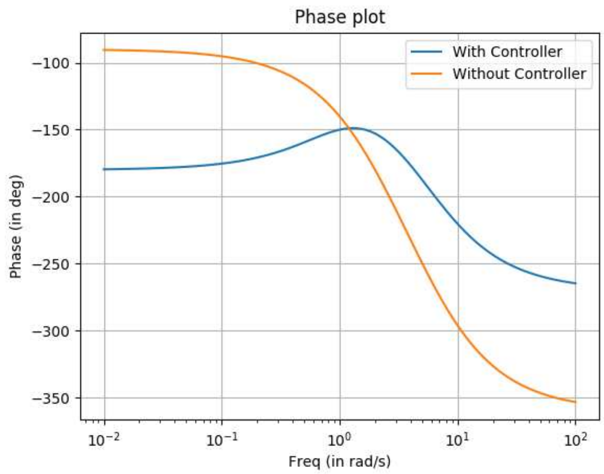

Fig. 1.1.7

2 Polar Plot

2.1 Introduction

2.1.1. Sketch the Polar Plot of

$$G(s) = \frac{\left(1 + \frac{s}{29}\right)(1 + 0.0025s)}{(s^3)(1 + 0.005s)(1 + 0.001s)} \tag{2.1.1.1}$$

Solution: The following code generates the polar plot in Fig. 2.1.1

codes/ee18btech11029.py

- The polar plots use open loop transfer function to determine the stability and hence reference point is shifted to $(-1, 0)$
- If $(-1, 0)$ is left of the polar plot or $(-1, 0)$ is not enclosed, then it is stable
- If $(-1, 0)$ is on right side of the polar plot or $(-1, 0)$ is enclosed by polar plot then it is unstable.
- If $(-1, 0)$ is on the polar plot then it is marginally stable

In Fig. 2.1.1, $(-1, 0)$ is on the polar plot so the system is marginally stable.

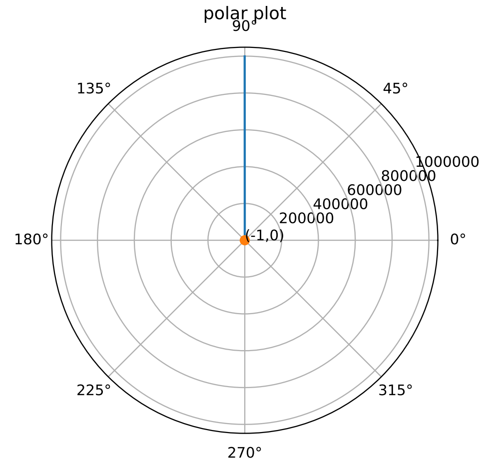

Fig. 2.1.1

2.2 Example

2.2.1. Sketch the Polar Plot of

$$G(s) = \frac{1}{s(1 + s^2)} \tag{2.2.1.1}$$

Solution: From (2.2.1.1),

$$G(\jmath\omega) = \frac{1}{\jmath\omega(1 - \omega^2)} \tag{2.2.1.2}$$

$$|G(\jmath\omega)| = \frac{1}{|\omega(1 - \omega^2)|} \tag{2.2.1.3}$$

$$\angle G(\jmath\omega) = \begin{cases} \frac{\pi}{2} & \omega > 1 \\ -\frac{\pi}{2} & 0 < \omega < 1 \end{cases} \tag{2.2.1.4}$$

The corresponding polar plot is generated in Fig. 2.2.1 using

codes/ee18btech11023.py

In Fig. 2.2.1, (–1,0) is exactly on the polar plot. Hence, the system is marginally stable.

2.3 Example

2.3.1. Sketch the Polar Plot for

$$G(s) = \frac{1}{(1 + s)(1 + 2s)} \tag{2.3.1.1}$$

Solution: The following code generates Fig. 2.3.1

codes/ee18btech11012.py

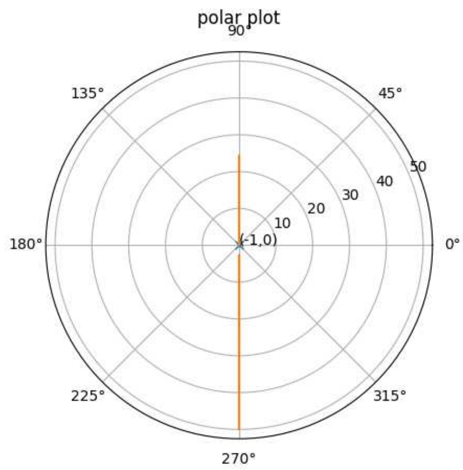

Fig. 2.2.1

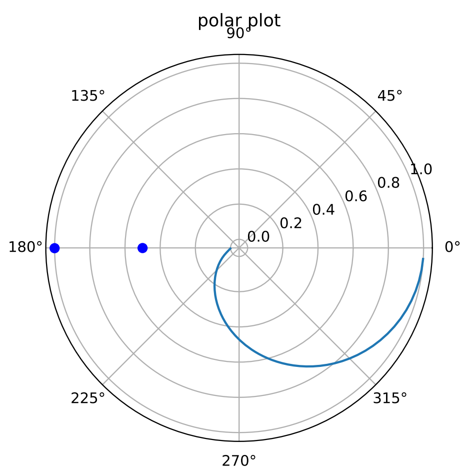

Fig. 2.3.1

The polar plot is to the right of $(-1, 0)$. Hence the closed loop system is stable.

2.4 *Example*

2.1. Sketch the direct polar plot for a unity feedback system with open loop transfer function

$$G(s) = \frac{1}{s(1+s)^2} \tag{2.1.1}$$

Solution: The polar plot is obtained by plotting (r, ϕ)

$$r = |H(\jmath\omega)||G(\jmath\omega)| \tag{2.1.2}$$

$$\phi = \angle H(\jmath\omega)G(\jmath\omega), 0 < \omega < \infty \tag{2.1.3}$$

The following code plots the polar plot in Fig. 2.1

```
codes/ee18btech11002/polarplot.py
```

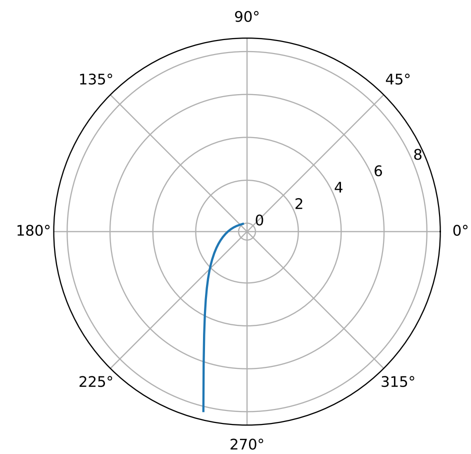

Fig. 2.1: Polar Plot

2.2. Sketch the inverse polar plot for (2.1.1)
Solution: The above code plots the polar plot in Fig. 2.2 by plotting $\left(\frac{1}{r}r, -\phi\right)$

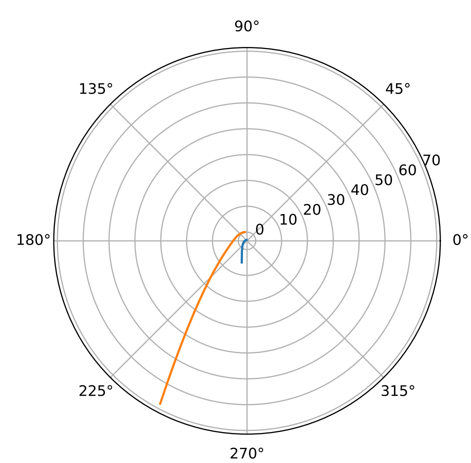

Fig. 2.2: Inverse Polar Plot