Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

1.1 Introduction

1.1.1. Fig. 1.1.1.1 shows a non-inverting op-amp configuration with parameters described in Table 1.1.1. Draw the equivalent control system.

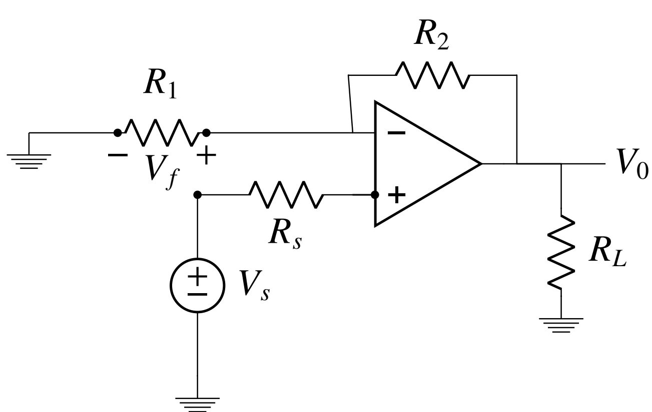

Fig. 1.1.1.1

Solution: See Fig. 1.1.1.2

Parameter	Value
input resistance	∞
output resistance	0
Input voltage	V_s
Output Voltage	V_o
Feeding resistance	R_1
Feedback resistance	R_2
Source resistance	R_s
load resistance	R_L

TABLE 1.1.1

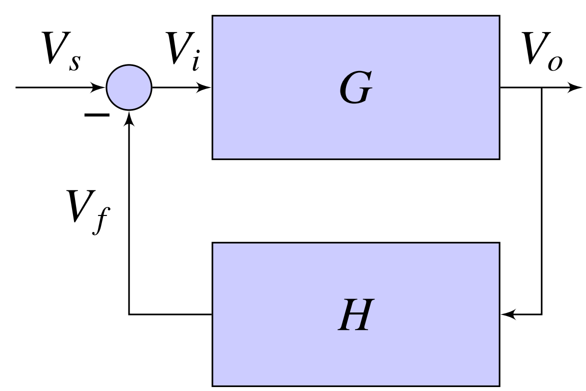

Fig. 1.1.1.2

1.1.2. Draw the small signal model for Fig. 1.1.1.1.
Solution: The equivalent circuit of the amplifier is in Fig. 1.1.2

1.1.3. Assuming that the operational amplifier has infinite input resistance and zero output resistance, find the *feedback factor H*.
Solution: From Fig. 1.1.2,

$$V_0 = GV_i \tag{1.1.3.1}$$

$$V_i = V_s - V_f \tag{1.1.3.2}$$

$$V_f = \frac{R_1}{R_1 + R_2} V_o \tag{1.1.3.3}$$

assuming that the current through R_s is very

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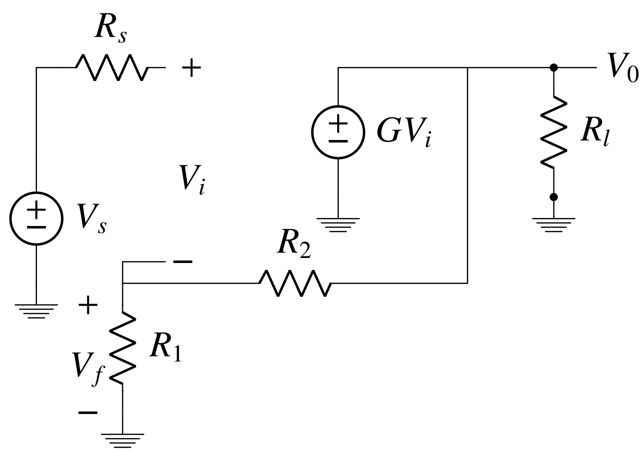

Fig. 1.1.2

small. Thus,

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \tag{1.1.3.4}$$

1.1.4. Obtain the closed loop gain T and summarize your results through a Table.
Solution: Table 1.1.4 provides a summary.

$$T = \frac{V_0}{V_i} = \frac{G}{1+GH} \tag{1.1.4.1}$$

$$= \frac{G(R_1 + R_2)}{(R_1 + R_2) + GR_1} \tag{1.1.4.2}$$

Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1+R_2}$
Loop gain	GH	$G\frac{R_1}{R_1+R_2}$
Amount of feedback	1+GH	$1 + \frac{GR_1}{R_1+R_2}$
Closed loop gain	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$

TABLE 1.1.4

1.1.5. Find the condition under which closed loop gain T is almost entirely determined by the feedback network.
Solution: If

$$GH \gg 1, \tag{1.1.5.1}$$

$$T \approx \frac{1}{H} = 1 + \frac{R_2}{R_1} \tag{1.1.5.2}$$

1.1.6. If

$$G = 10^4 \tag{1.1.6.1}$$

$$T = 10, \tag{1.1.6.2}$$

find H.
Solution: From Table 1.1.4

$$T = \frac{G}{1+GH} = 10 \tag{1.1.6.3}$$

$$\implies H = 0.0999 \tag{1.1.6.4}$$

1.1.7. *Gain Desensitivity:* If G decreases by 20%,what is the corresponding decrease in T? Comment.
Solution: From From Table 1.1.4, Given

$$T = \frac{G}{1+GH} \tag{1.1.7.1}$$

$$\implies dT = \frac{dG}{(1+GH)^2} \tag{1.1.7.2}$$

$$\implies \frac{dT}{T} = \frac{1}{1+GH}\frac{dG}{G} \tag{1.1.7.3}$$

From the information available so far,

$$dG = 20\%, G = 10^4, H = 0.0999 \implies \frac{dT}{T} = 0.025\% \tag{1.1.7.4}$$

using the following code.

```
codes/ee18btech11005/ee18btech11005.py
```

Thus the closed loop gain is almost invariant to a relatively large (20%) variation in the open loop gain G. This is known as gain desensitivity.

2 Feedback Current Amplifier: Shunt-Series

2.1 Introduction

2.1.1. Draw the equivalent control system for the feedback current amplifier shown in 2.1.1.4
Solution: See Fig. 2.1.1.5.

2.1.2. For the feedback current amplifier shown in 2.1.1.4, draw the Small-Signal Model. Neglect the Early effect in Q_1 and Q_2.
Solution: See Fig. 2.1.2.

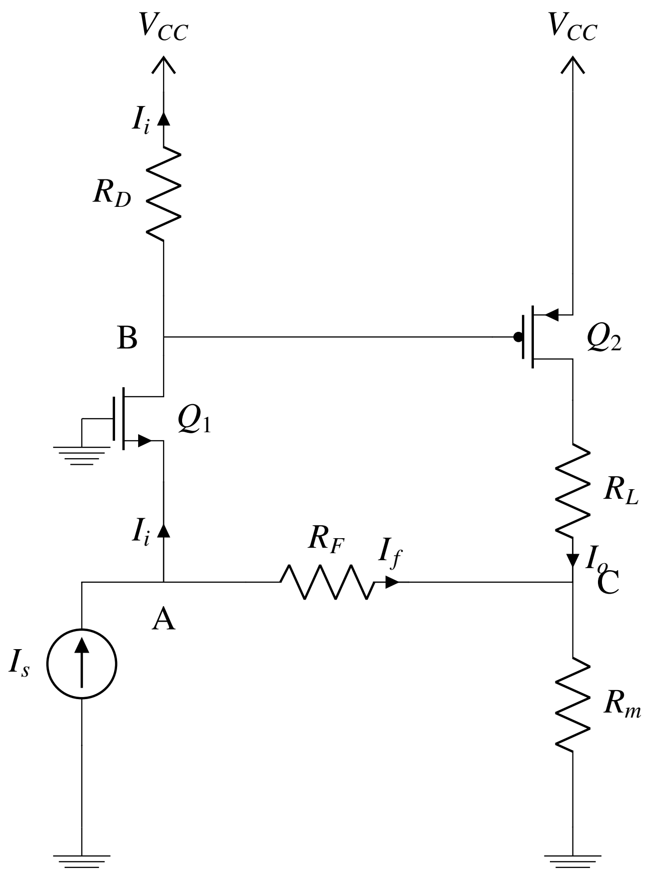

Fig. 2.1.1.4

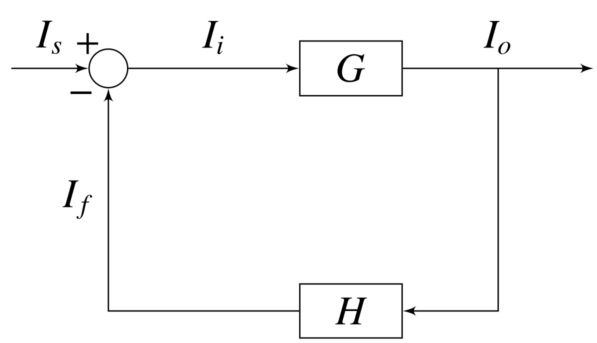

Fig. 2.1.1.5

While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

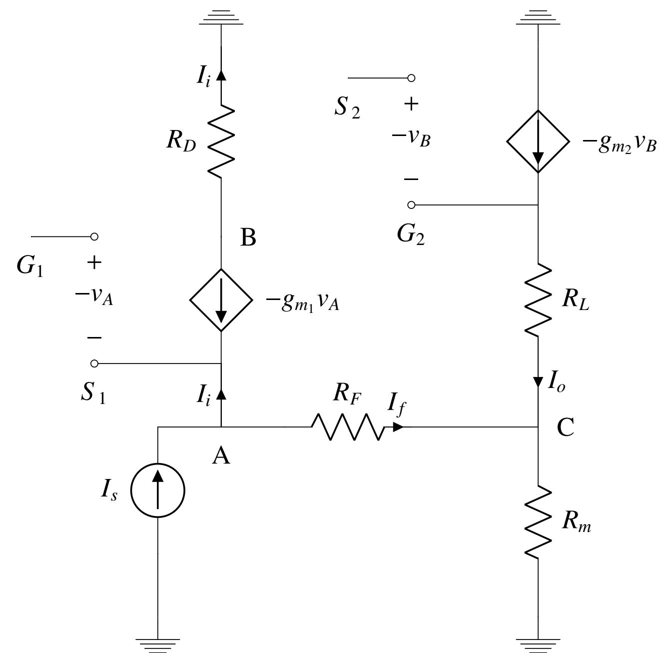

Fig. 2.1.2: Small Signal Model

2.1.3. Describe how the given circuit is a Negetive Feedback Current Amplifier.
Solution: For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the p-channel device Q_2 , its increase will cause I_o , the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed for the initial change in I_s, verifying that the feedback is indeed negative.

2.1.4. Find the Expression for the Open-Loop Gain $G = \frac{I_o}{I_i}$, from the Small-Signal Model. in Fig. 2.1.2.
Solution: In Small-Signal Model,

$$v_B = I_i R_D \tag{2.1.4.1}$$

$$v_{gs_2} = v_B = I_i R_D \tag{2.1.4.2}$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.4.3}$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.4}$$

2.1.5. Find the Expression of the Feedback Factor $H = \frac{I_f}{I_o}$, from Small-Signal Model.
Solution: I_o is fed to a current divider formed by R_M and R_F. R_F is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it leaving a small current as input to Amplifier. Hence the voltage at point 'A' is very small and is considered, $v_A \simeq 0$. So R_F and R_M are parallel and Voltage Drop across them is same.

$$(I_o + I_f) R_M \simeq -I_f R_F \tag{2.1.5.1}$$

$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.2}$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.3}$$

2.1.6. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$.
Solution: From (2.1.4.4) and (2.1.5.3),

$$T = \frac{I_o}{I_s} = \frac{G}{1 + GH} \tag{2.1.6.1}$$

$$= -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.2}$$

$$\implies T = -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.3}$$