Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.2 Practical Case

2.2.1. An op amp with an open loop voltage gain of 80dB and poles at 10^5Hz , 10^6 Hz and 2×10^6 Hz is said to be compensated to be stable for unity β. Assume that op amp incorporates an amplifier circuit equivalent to Fig.2.2.1 with C_1=150pF ,C_2=5pF and g_m=40mA/V and that f_{p1} is caused by input circuit and f_{p2} by the output circuit of this amplifier.Find the required value of compensating miller capacitance and the new frequency of the output pole

Solution:
The analysis of the circuit yields the transfer function

$$\frac{V_0}{I_i} = \frac{\left(sC_f - g_m\right)R_1R_2}{1 + s\left[P\right] + s^2\left[Q\right]} \tag{2.2.1.1}$$

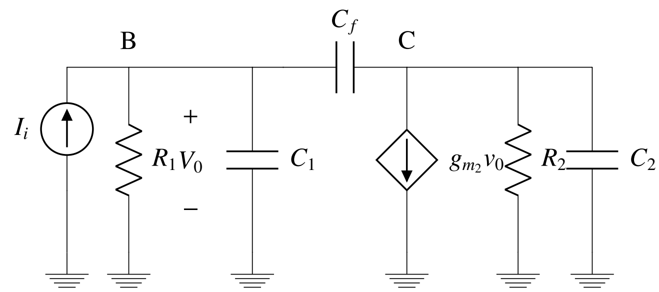

Fig. 2.2.1: Equivalent amplifier circuit

where

$$P = C_1R_1 + C_2R_2 + C_f\left(g_mR_1R_2 + R_1 + R_2\right) \tag{2.2.1.2}$$

$$Q = \left(C_1C_2 + C_f\left(C_1 + C_2\right)\right)R_1R_2 \tag{2.2.1.3}$$

The zero is usually at a much higher frequency so neglecting its effect ,the denominator of the transfer function can be written in the form

$$D(s) = \left(1 + \frac{s}{\omega'_{p1}}\right)\left(1 + \frac{s}{\omega'_{p2}}\right) \tag{2.2.1.4}$$

Here ω'_{p1} and ω'_{p2} are the new frequencies of the two poles and one of the pole will be dominant

$$\omega'_{p1} < \omega'_{p2} \tag{2.2.1.5}$$

Thus

$$D(s) \approx 1 + \frac{s}{\omega'_{p1}} + \frac{s^2}{\omega'_{p1}\omega'_{p2}} \tag{2.2.1.6}$$

Equating the coefficient of s in the Eq. (2.2.1.1) and Eq. (2.2.1.6) we get

$$\omega'_{p1} = \frac{1}{C_1R_1 + C_2R_2 + C_f\left(g_mR_1R_2 + R_1 + R_2\right)} \tag{2.2.1.7}$$

This can be approximated to

$$\omega'_{p1} = \frac{1}{g_mR_1R_2C_f} \tag{2.2.1.8}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

In order to obtain the value of ω'_{p2} we equate the coefficient of s^2 in Eq. (2.2.1.1) and Eq. (2.2.1.6) and use the value of Eq. (2.2.1.8)

$$\omega'_{p2} = \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)} \tag{2.2.1.9}$$

2.2.2. Find the value of G

$$80 = 20 \log (G) \tag{2.2.2.1}$$
$$G = 10^4 \tag{2.2.2.2}$$

2.2.3. Find the values of R_1 and R_2

Solution: The pole f_{p1} is caused by input circuit and f_{p2} by the output circuit.

$$f_{p1} = \frac{1}{2\pi R_1 C_1} \tag{2.2.3.1}$$
$$f_{p2} = \frac{1}{2\pi R_2 C_2} \tag{2.2.3.2}$$

finding the values of R_1 and R_2

$$R_1 = \frac{1}{2\pi (150 \times 10^{-12}) (10^5)} = 10.61 k\Omega \tag{2.2.3.3}$$

$$R_2 = \frac{1}{2\pi (5 \times 10^{-12}) (10^6)} = 31.8 k\Omega \tag{2.2.3.4}$$

Assuming that the pole f_{p2} will move to a frequency f'_{p2}. This requires the modified first pole to be located at

$$f'_{p1} = \frac{f_{p3}}{G} \tag{2.2.3.5}$$
$$= \frac{2 \times 10^6}{10^4} = 200 Hz \tag{2.2.3.6}$$

2.2.4. Find the value of Miller capacitance

Solution: Compensating miller capacitance is From Eq.(2.2.1.8) we get

$$C_f = \frac{1}{2\pi g_m R_1 R_2 f'_{p1}} \tag{2.2.4.1}$$

$$C_f = \frac{1}{2\pi (40 \times 10^{-3}) \left(\frac{10^5}{3\pi}\right) \left(\frac{10^5}{\pi}\right) (200)} \tag{2.2.4.2}$$

$$C_f = 58.9 pF \tag{2.2.4.3}$$

2.2.5. Find the frequency of new output pole

Solution: The new frequency of the output

Parameter	Value
C_1	$150pF$
C_2	$5pF$
R_1	$10.61k\Omega$
R_2	$31.8k\Omega$
C_f	$58.9pF$
g_m	$40mA/V$
f_{p1}	$10^5 Hz$
f_{p2}	$10^6 Hz$
f_{p3}	$2 \times 10^6 Hz$
f'_{p1}	$200Hz$
f'_{p2}	$37.95MHz$

TABLE 2.2.3

pole from Eq. (2.2.1.9)

$$f'_{p2} = \frac{g_m C_f}{2\pi \left[C_1 C_2 + C_f (C_1 + C_2)\right]} \tag{2.2.5.1}$$

$$f'_{p2} = \frac{\left(40 \times 10^{-3}\right)\left(58.9 \times 10^{-12}\right)}{2\pi (9.8 \times 10^{-21})} \tag{2.2.5.2}$$

$$f'_{p2} = 37.95 MHz \tag{2.2.5.3}$$

2.2.6. Verify using bode plots

Solution: The presence of C_f has the effects

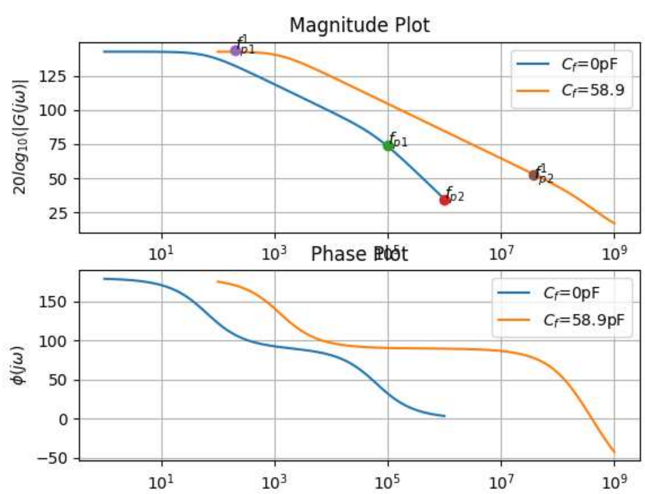

Fig. 2.2.6

- It downshifts the first pole by factor of $\frac{10^5}{200} = 500$

- It upshifts the second pole by factor of $\frac{37.95\times10^6}{10^6} = 37.95$

Verify the above plot by

```
codes/ee18btech11029/ee18btech11029_1.py
```

2.2.7. Design the circuit
Solution:

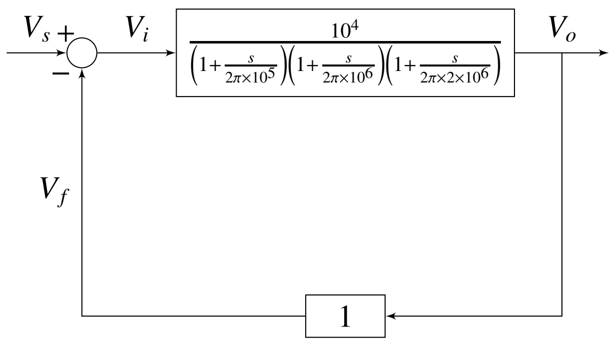

Fig. 2.2.7

The transfer function of the opamp is

$$G(s) = \frac{10^4}{\left(1 + \frac{s}{2\pi\times10^5}\right)\left(1 + \frac{s}{2\pi\times10^6}\right)\left(1 + \frac{s}{2\pi\times2\times10^6}\right)} \tag{2.2.7.1}$$

2.2.8. For feedback gain H
Solution: The value of the feedback gain is 1,So just place a wire between the input and the output terminal

$$H = \frac{V_f}{V_o} = 1 \tag{2.2.8.1}$$

2.2.9. Design the feedback circuit

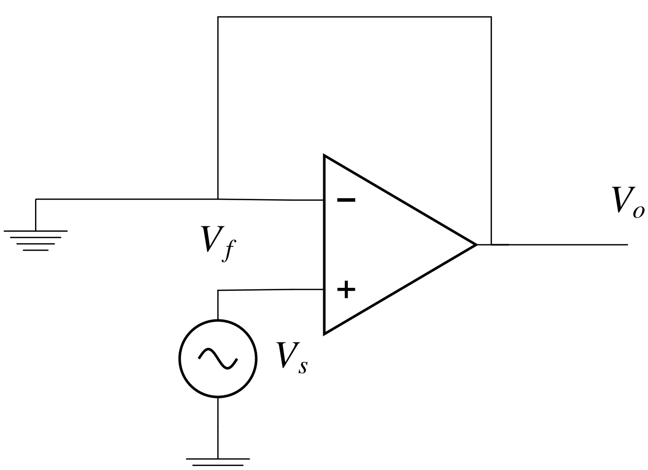

Fig. 2.2.9