Frequency Compensation

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Parameter	Value
C_1	$150pF$
C_2	$5pF$
g_m	$40mA/V$
f_1	10^5Hz
f_2	10^6Hz
f_3	2×10^6Hz

TABLE 0: Uncompensated System.

An op amp with an open loop voltage gain of 80dB and poles at 10^5Hz , 10^6 Hz and 2×10^6 Hz is said to be compensated to be stable for unity β. Assume that op amp incorporates an amplifier circuit equivalent to Fig. 0 with input parameters in Table 0

where f_1 is caused by input circuit and f_2 by the output circuit of this amplifier. Find the required value of compensating miller capacitance C_f and the new frequency of the output pole

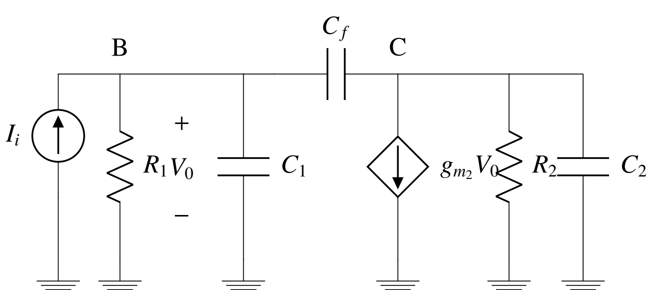

Fig. 0: Equivalent amplifier circuit

1 Without Compensation: $C_f = 0$

1.1. Find the gain of the opamp $G(s)$ and draw the block diagram of the unity feedback system.

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Solution: The transfer function of the opamp is

$$G(s) = \frac{10^4}{\left(1+\frac{s}{2\pi\times10^5}\right)\left(1+\frac{s}{2\pi\times10^6}\right)\left(1+\frac{s}{2\pi\times2\times10^6}\right)} \tag{1.1.1}$$

The block diagram is available in Fig. 1.1

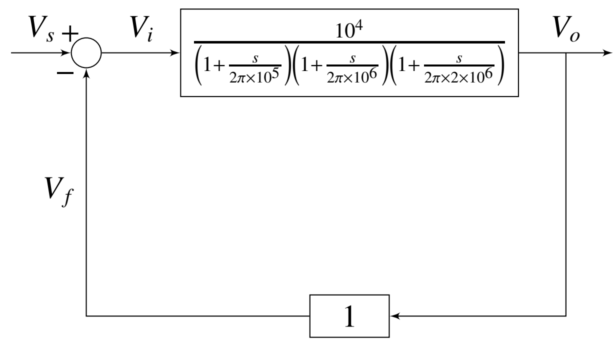

Fig. 1.1

1.2. Design the feedback circuit
Solution: See Fig. 1.2

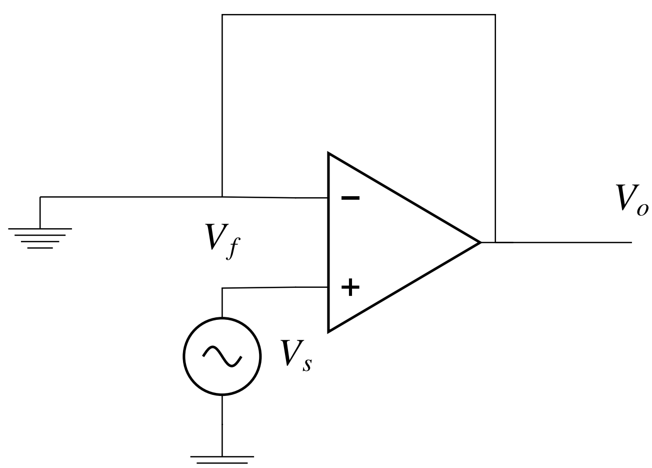

Fig. 1.2

1.3. Find $\frac{V_o}{I_i}$ in Fig. 0 for $C_f = 0$.
Solution:

$$\frac{V_0}{I_i} = \frac{-g_m R_1 R_2}{1 + sP + s^2Q} \tag{1.3.1}$$

Parameter	Value
R_1	$10.61k\Omega$
R_2	$31.8k\Omega$

TABLE 1.5: Resistance values in Fig. 0

where

$$P = C_1R_1 + C_2R_2 \tag{1.3.2}$$
$$Q = C_1C_2R_1R_2 \tag{1.3.3}$$

1.4. Let ω_1, ω_2 be the poles in (1.3.1). Find ω_1 and ω_2.
Solution: From (1.3.1),

$$1 + sP + s^2Q = \left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right) \tag{1.4.1}$$
$$P = \implies \frac{1}{\omega_1} + \frac{1}{\omega_2} \tag{1.4.2}$$
$$Q = \frac{1}{\omega_1\omega_2} = Q \tag{1.4.3}$$

yielding

$$\omega_1 = \frac{1}{R_1C_1}$$
$$\omega_2 = \frac{1}{R_2C_2} \tag{1.4.4}$$

1.5. Find the values of R_1 and R_2
Solution: From (1.4.4)

$$R_1 = \frac{1}{2\pi C_1 f_1} \tag{1.5.1}$$
$$R_2 = \frac{1}{2\pi C_2 f_2} \tag{1.5.2}$$

which can be computed using the parameters in Table 2.3 and are listed in Table 1.5

1.6. Investigate the stability of the closed loop system in Fig. 1.1 by finding the step response of Fig. 1.2 using spice.
Solution: The following netlist simulates the uncompensated system

```
codes/ee18btech11029/spice/
    ee18btech11029_2.net
```

The following code plots the output of the uncompensated system generated by the above netlist in Fig.1.6

```
codes/ee18btech11029/spice/
    ee18btech11029_2.py
```

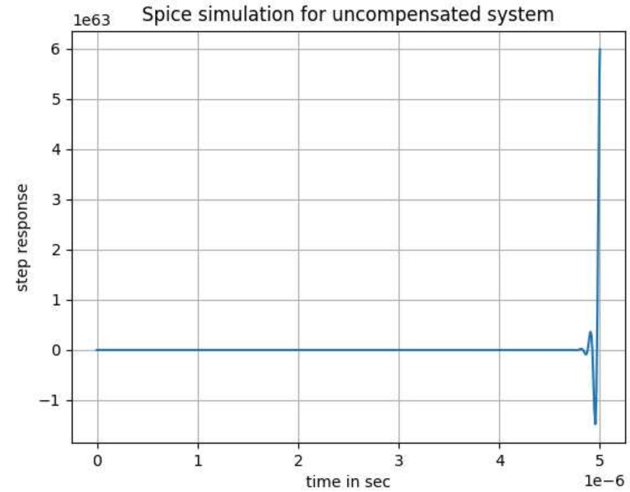

Fig. 1.6: Step response of Uncompensated system

2 With Compensation: $C_f \neq 0$

2.1. Find $\frac{V_o}{I_i}$ in Fig. 0 for $C_f \neq 0$.
Solution:

$$\frac{V_0}{I_i} = \frac{\left(sC_f - g_m\right)R_1R_2}{1 + sP + s^2Q} \tag{2.1.1}$$

where

$$P = C_1R_1 + C_2R_2 + C_f\left(g_mR_1R_2 + R_1 + R_2\right) \tag{2.1.2}$$
$$Q = \left(C_1C_2 + C_f\left(C_1 + C_2\right)\right)R_1R_2 \tag{2.1.3}$$

2.2. Let ω_1, ω_2 be the poles in (2.1.1). Find ω_1 and ω_2.
Solution:

$$1 + sP + s^2Q = \left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right) \tag{2.2.1}$$
$$\implies \frac{1}{\omega_1} + \frac{1}{\omega_2} = P \tag{2.2.2}$$
$$\text{or, } \frac{1}{\omega_1} \approx P \because (\omega_1 < \omega_2) \tag{2.2.3}$$

From (2.1.2), assuming that $g_m \gg 1$,

$$P \approx C_f g_m R_1R_2 \tag{2.2.4}$$
$$\implies \omega_1 = \frac{1}{P} = \frac{1}{C_f g_m R_1R_2} \tag{2.2.5}$$

From (2.2.1),

Parameter	Value
C_f	$58.9pF$
f_1	$200Hz$
f_2	$37.95MHz$

TABLE 2.4: Compensated System

$$Q = \frac{1}{\omega_1 \omega_2} \tag{2.2.6}$$

$$\implies \omega_2 = \frac{1}{Q\omega_1} = \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)} \tag{2.2.7}$$

upon substituting from (2.1.3) and (2.2.5).

2.3. For the compensated system, let

$$f_1 = \frac{f_3}{G} \tag{2.3.1}$$

$$= \frac{2 \times 10^6}{10^4} = 200Hz \tag{2.3.2}$$

Find the value of Miller capacitance C_f.
Solution: From (2.2.5),

$$C_f = \frac{1}{2\pi g_m R_1 R_2 f_1} \tag{2.3.3}$$

$$\implies C_f = 58.9pF \tag{2.3.4}$$

upon substituting the values from Tables and 1.5.

2.4. Find f_2.
Solution: From (2.2.7),

$$f_2 = \frac{g_m C_f}{2\pi \left[C_1 C_2 + C_f (C_1 + C_2)\right]} \tag{2.4.1}$$

$$f_2 = 37.95MHz \tag{2.4.2}$$

All unknown parameters for the compensated system are listed in Table 2.4

2.5. Find $G(s)$ for the compensated system and sketch the block diagram.
Solution: The transfer function of the opamp is

$$G(s) = \frac{10^4}{\left(1 + \frac{s}{2\pi \times 200}\right)\left(1 + \frac{s}{2\pi \times 37.95 \times 10^6}\right)\left(1 + \frac{s}{2\pi \times 2 \times 10^6}\right)} \tag{2.5.1}$$

and the block diagram is available in Fig. 2.5

2.6. Through Bode plots, show how C_f affects $G(s)$
Solution:

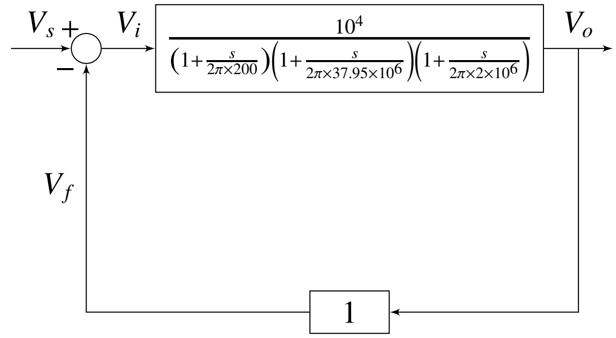

Fig. 2.5: Compensated system

- C_f downshifts the first pole by factor of $\frac{10^5}{200} = 500$
- C_f upshifts the second pole by factor of $\frac{37.95 \times 10^6}{10^6} = 37.95$

This is visible in Fig. 2.6 plotted using

```
codes/ee18btech11029/ee18btech11029_1.py
```

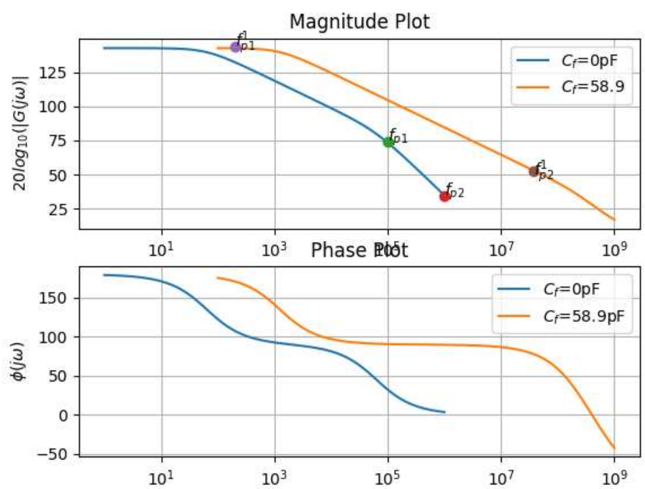

Fig. 2.6

2.7. Show that the compensated system is stable by plotting the step response. **Solution:** The following netlist simulates the compensated system

```
codes/ee18btech11029/spice/
    ee18btech11029_3.net
```

The following code plots the output of the compensated system generated by the above netlist in Fig.2.7

```
codes/ee18btech11029/spice/
    ee18btech11029_3.py
```

Instructions for simulations are given in

codes/ee18btech11029/spice/README.md

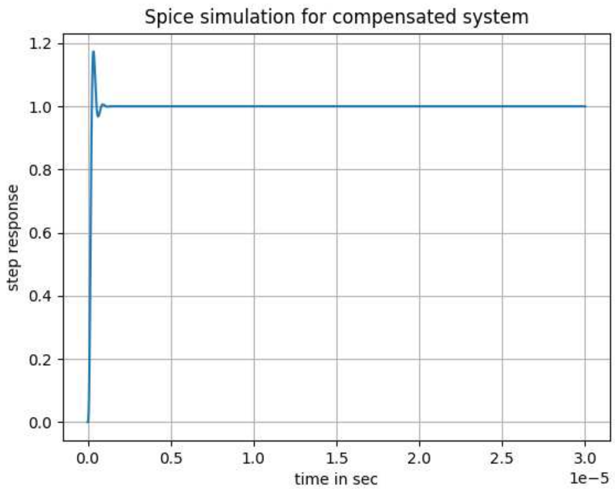

Fig. 2.7: Step response of compensated system