Oscillator

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For the circuit shown in Fig. 1.1, find the loop gain $L(s) = G(s)H(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

1. Draw the equivalent control system representation for the circuit in Fig. 1.1 as well as the small signal model.
   Solution: See Figs. 1.2, 1.3 and 1.4. Oscillators do not include input signal.

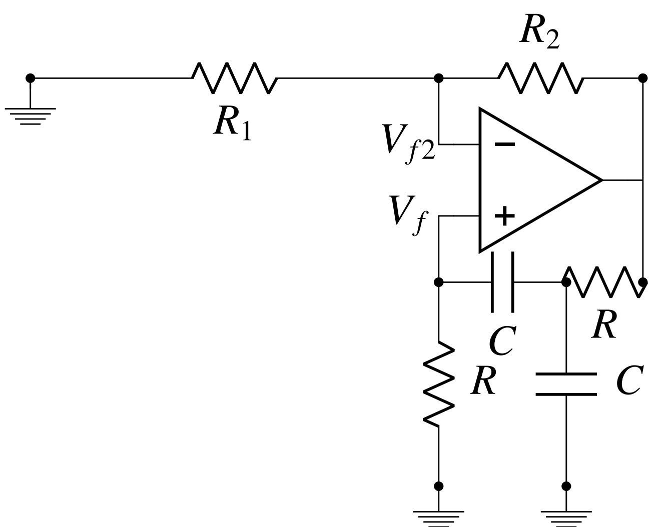

Fig. 1.1

2. Draw the block diagram and circuit diagram for H.
   Solution: See Figs. 2.5 and 2.6.
3. Find H.
   Solution: In Fig. 2.6, let I_o be the current flowing from V_o. Then

$$I_o = \frac{V_o}{R + \frac{1}{sC} \parallel \left(\frac{1}{sC} + R\right)} \tag{3.1}$$

Using current division,

$$V_f = I_o \frac{\frac{1}{sC}}{\frac{1}{sC} + \left(\frac{1}{sC} + R\right)} \times R \tag{3.2}$$

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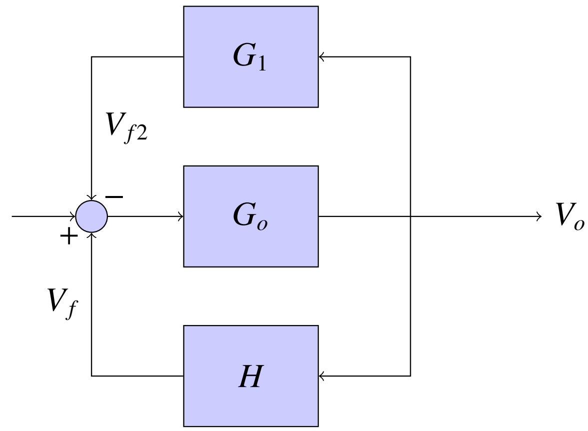

Fig. 1.2: Block diagram

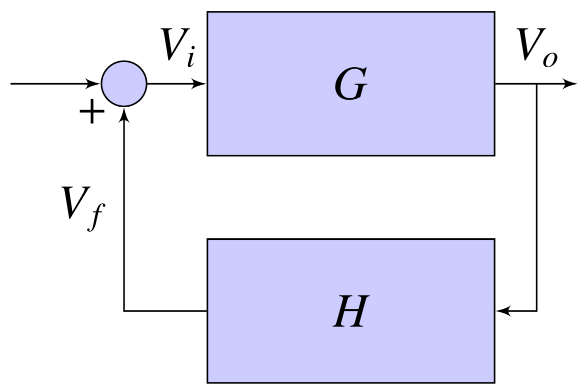

Fig. 1.3: Simplified equivalent block diagram

From (3.1) and (3.2),

$$\frac{V_f}{V_o} = \frac{\frac{1}{sC}}{\frac{1}{sC} + \left(\frac{1}{sC} + R\right)} \times R \tag{3.3}$$

$$\times \frac{1}{R + \frac{1}{sC} \parallel \left(\frac{1}{sC} + R\right)} \tag{3.4}$$

On further simplification we get,

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \tag{3.5}$$

4. Find R_{11} and R_{22} from Fig. 2.6.

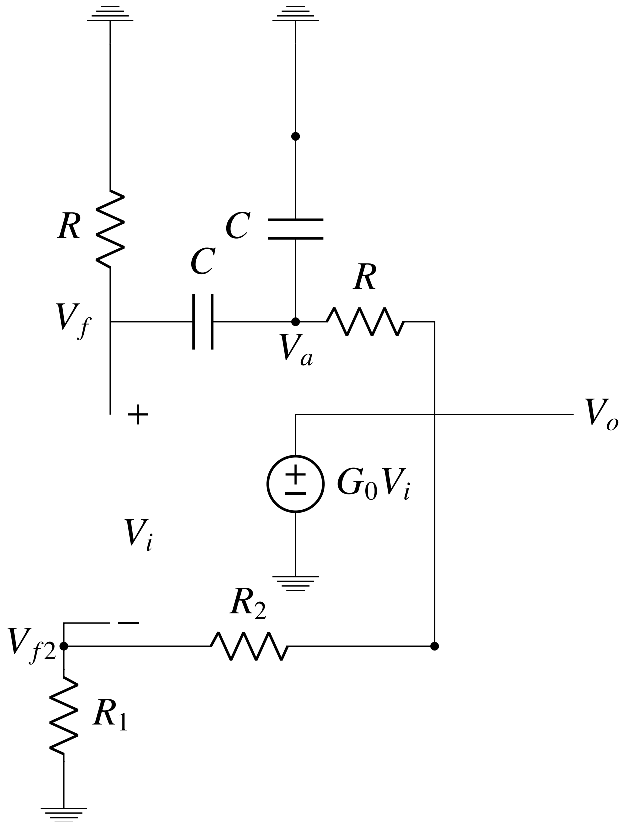

Fig. 1.4

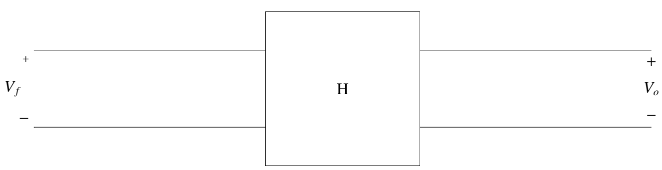

Fig. 2.5: Feedback block diagram

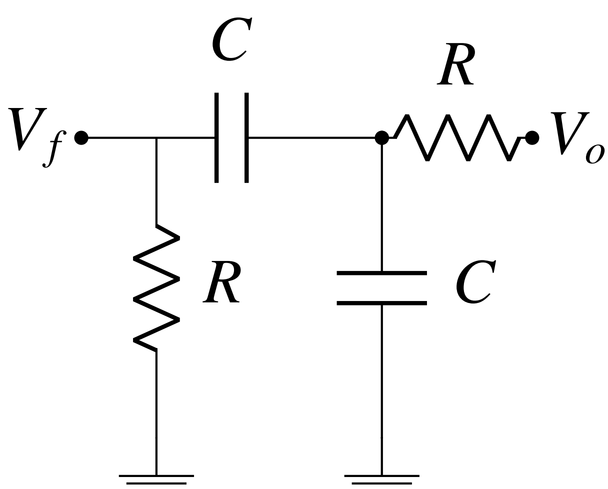

Fig. 2.6: Feedback circuit

Solution: Shorting V_o to ground,

$$R_{11} = R \parallel \left(\frac{1}{sC} + \frac{1}{sC} \parallel R\right) \tag{4.1}$$

Shorting V_f to ground,

$$R_{22} = \frac{1}{2sC} + R \tag{4.2}$$

5. Draw the block diagram and circuit diagram for G.
Solution: See Figs. 5.1 for the block diagram and Figs. 5.2 for the circuit diagram.

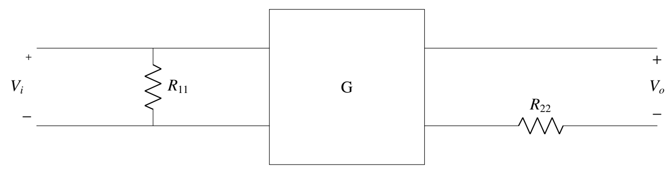

Fig. 5.1: Open loop block diagram

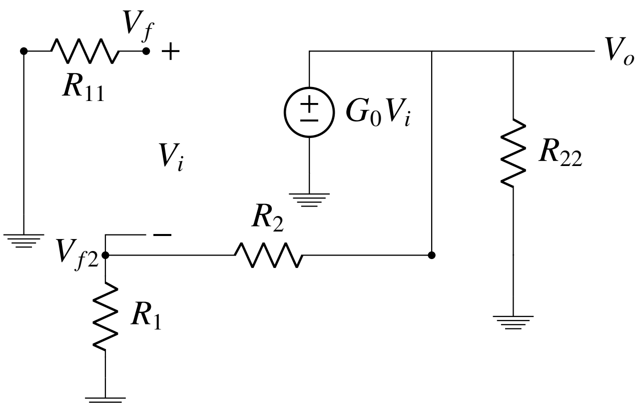

Fig. 5.2: Open loop circuit diagram

6. Find G.
Solution: From Fig. 5.2,

$$V_{f_2} = \left(\frac{R_1}{R_1 + R_2}\right) V_o \tag{6.1}$$

From Fig. 1.2,

$$G_1 = \frac{V_{f_2}}{V_o} \tag{6.2}$$

$$= \frac{R_1}{R_1 + R_2} \tag{6.3}$$

From Fig.1.2, G_1 is the negative feedback factor and G_0 is the gain of the op-amp.

Therefore, equivalent G is given by

$$G = \frac{G_0}{1 + G_0 G_1} \tag{6.4}$$

$$= \frac{1}{\frac{1}{G_0} + G_1} \tag{6.5}$$

On substituting $G_0 \to \infty$

$$G \approx \frac{1}{G_1} \tag{6.6}$$

$$G = \frac{R_1 + R_2}{R_1} \tag{6.7}$$

$$\implies G = 1 + \frac{R_2}{R_1} \tag{6.8}$$

7. Find the loop gain $L(s)$.
Solution: From (6.8) and (3.5),

$$L(s) = G(s) H(s) \tag{7.1}$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}} \right) \tag{7.2}$$

8. Find the closed loop gain $T(s)$.
Solution: From Fig. 1.3,

$$T(s) = \frac{G}{1 - GH(s)} = \frac{G}{1 - L(s)} \tag{8.1}$$

$$\implies \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 - \left(\frac{1+\frac{R_2}{R_1}}{3+sRC+\frac{1}{sRC}}\right)} \tag{8.2}$$

9. Find the conditions for oscillation.
Solution: For oscillations to start,
- $T(s)$ should have imaginary poles.
- $L(0) \geq 1$

For $T(s)$ to have imaginary poles,

$$\text{Im}\{L(j\omega)\} = 0 \tag{9.1}$$

$$\implies L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)} \right) \tag{9.2}$$

From (7.2),

$$L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)} \right) \tag{9.3}$$

$$\implies j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \tag{9.4}$$

$$\text{or, } \omega = \frac{1}{RC} \tag{9.5}$$

Also, from equation (7.2)

$$L(0) \geq 1 \tag{9.6}$$

$$= \left(\frac{1 + \frac{R_2}{R_1}}{3 + j(0)} \right) \geq 1 \tag{9.7}$$

$$\implies \frac{R_2}{R_1} \geq 2 \tag{9.8}$$

10. Find the Amplitude and frequency for some arbitrary R, C values given in Table 10.

Parameter	Value
R	250Ω
C	$1mF$
R_2	$2k\Omega$
R_1	$1k\Omega$

TABLE 10

Solution: The following python code plots the impulse response of the system Fig 10.1. This, in fact is the output of Fig. 1.1.

```
codes/es17btech11002/es17btech11002_imp1.
    py
```

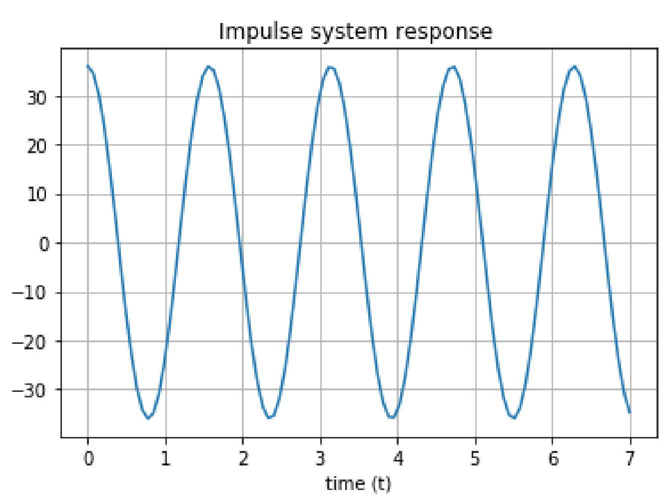

Fig. 10.1

The following python code plots the step response of the system. Fig.10.2.

```
codes/es17btech11002/es17btech11002_step1.py
```

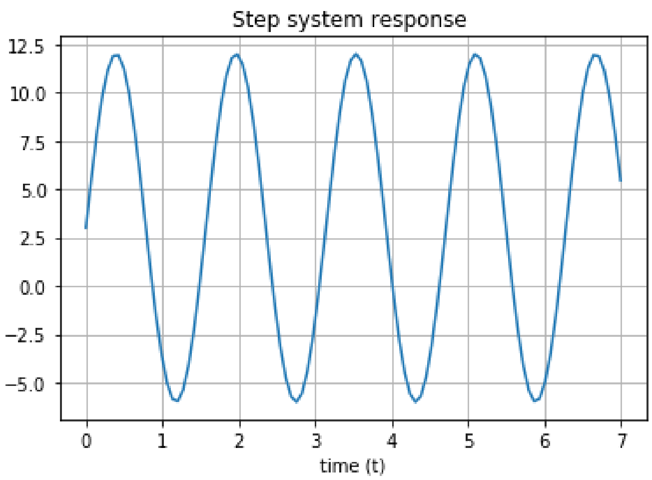

Fig. 10.2

Amplitude: From Fig. 10.2 V $(peak-peak)$ is

$$V_{p-p} = 11.929 - (-5.957) = 17.886 \quad (10.1)$$

$$V_{max} = \frac{V_{p-p}}{2} = 8.943 \quad (10.2)$$

Frequency: From equation (9.5)

$$\omega = \frac{1}{RC} = 4rad/sec \quad (10.3)$$

$$f = \frac{\omega}{2\pi} = 0.636Hz \quad (10.4)$$

11. Verify the frequency using spice simulation.
Solution: The following readme file provides necessary instructions to simulate the circuit in spice.

```
codes/es17btech11002/spice2/README
```

The following netlist simulates the given circuit.

```
codes/es17btech11002/spice2/es17btech11002.net
```

The following code plots the output from the oscillator spice simulation which is shown in Fig. 11.1.

```
codes/es17btech11002/spice2/es17btech11002_2_spice.py
```

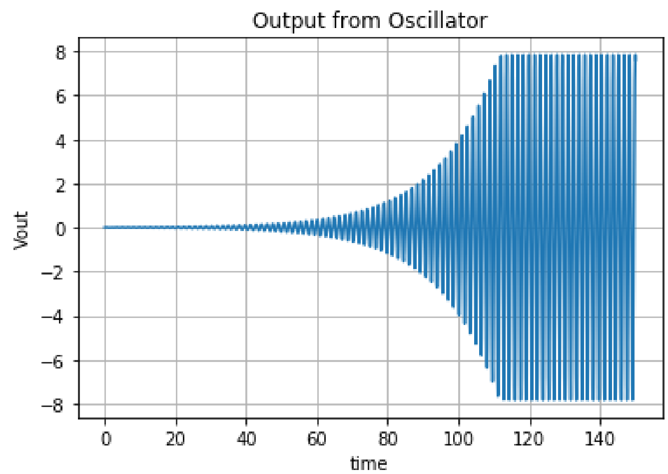

Fig. 11.1

The following code plots a part of the spice output from which we can observe a clear sinusoidal output shown in Fig. 11.2.

```
codes/es17btech11002/spice2/es17btech11002_spice2.py
```

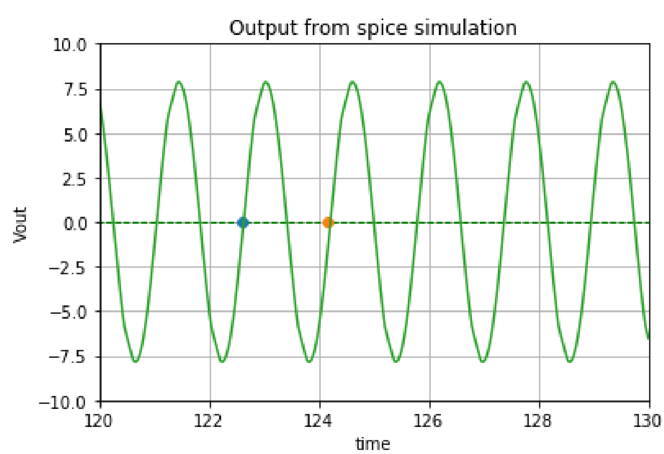

Fig. 11.2

Amplitude: From Fig. 11.2 V(peak-peak) is

$$V_{p-p} = 7.86 - (-7.86) = 15.72 \quad (11.1)$$

$$V_{max} = \frac{V_{p-p}}{2} = 7.86. \quad (11.2)$$

Frequency: time period is calculated by any two end points of one cycle,

$$T = 124.150 - (122.160) = 1.61sec \quad (11.3)$$

$$f = \frac{1}{T} = 0.64Hz \quad (11.4)$$

Hence,the frequency is verified through the spice simulation.

Parameter	Value
R	10Ω
C	$0.01mF$
R_1	1000Ω
R_2	2030Ω

TABLE 11

12. Now find the Amplitude and frequency for some arbitrary R, C values given in Table 11.

Solution: The following python code plots the step response of the system. Fig. 12.1.

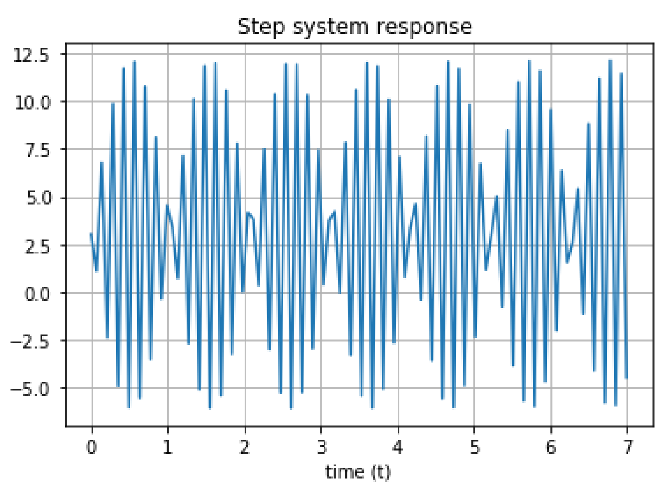

Fig. 12.1

codes/es17btech11002/es17btech11002_step2.py

The following python code plots the impulse response of the system Fig. 12.2.

codes/es17btech11002/es17btech11002_imp2.py

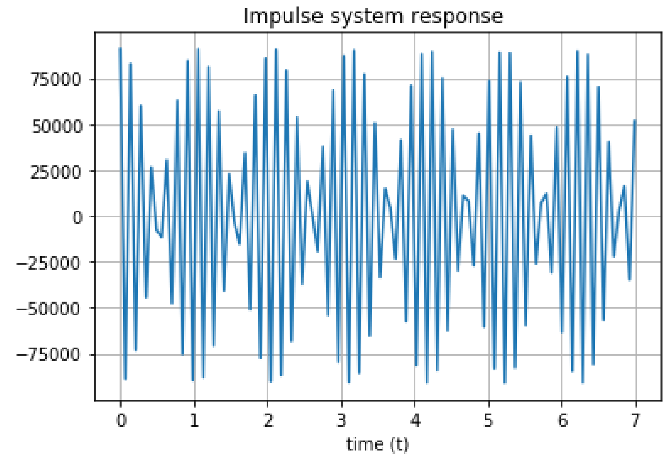

Fig. 12.2

Frequency: From equation (9.5)

$$\omega = \frac{1}{RC} = 10000rad/sec \quad (12.1)$$

$$f = \frac{\omega}{2\pi} = 1.57kHz \quad (12.2)$$

13. Now again verify the frequency using spice simulation.
Solution: The following readme file provides necessary instructions to simulate the circuit in spice.

codes/es17btech11002/spice/README

The following netlist simulates the given circuit.

codes/es17btech11002/spice/es17btech11002.net

The following code plots the output from the oscillator spice simulation which is shown in Fig. 13.1.

codes/es17btech11002/spice/es17btech11002_spice.py

The following code plots a part of the spice output from Fig 13.1. which we can observe a clear sinusoidal output shown in Fig. 13.2.

codes/es17btech11002/spice/es17btech11002_spice2.py

Amplitude: From Fig. 13.2 V(peak-peak) is

$$V_{p-p} = 0.47 - (-0.47) = 0.94V \quad (13.1)$$

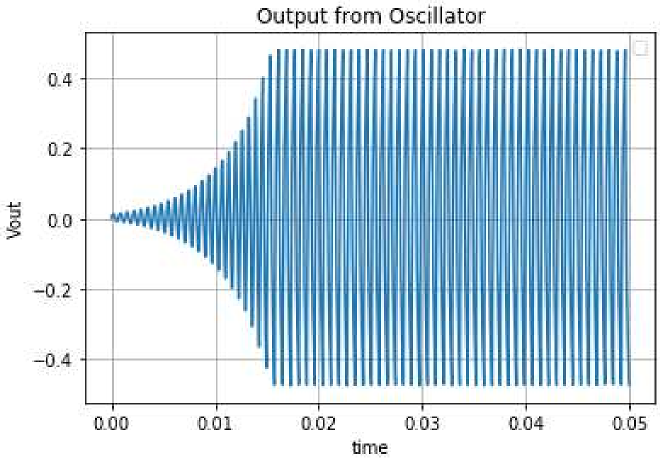

Fig. 13.1

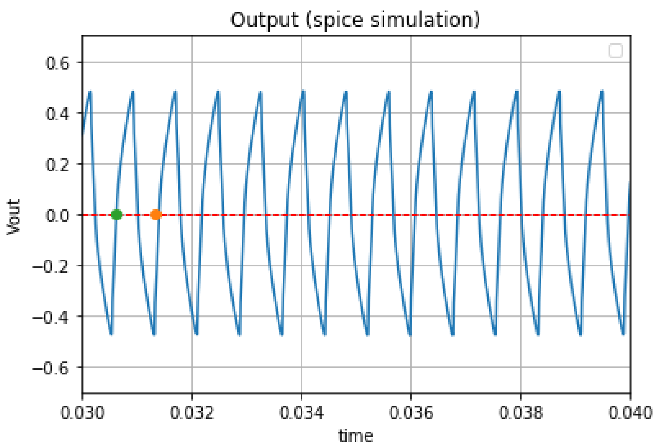

Fig. 13.2

$$V_{max} = \frac{V_{p-p}}{2} = 0.47. \tag{13.2}$$

Frequency: time period is calculated by any two end points of one cycle,

$$T = 0.0313164 - (0.03060) = 0.7164ms \tag{13.3}$$

$$f = \frac{1}{T} = 1.39kHz \tag{13.4}$$

Hence,the frequency is verified through the spice simulation.

NOTE : Generally in real life scenario oscillator generate a sinusoidal signal by consuming the thermal noise in the circuit. So in the above two experiment it can be observed that in python simulation requires an input to generates the output as there is no thermal noise, it should also be noticed that beside amplitude the frequency in the python and spice simulation are almost same.