

Control Systems

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Abstract—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/feedback/codes>

1 FEEDBACK VOLTAGE AMPLIFIER: SERIES-SHUNT

1.1. Fig. 1.1.1 shows a non-inverting op-amp configuration with parameters described in Table 1.1. Draw the equivalent control system.

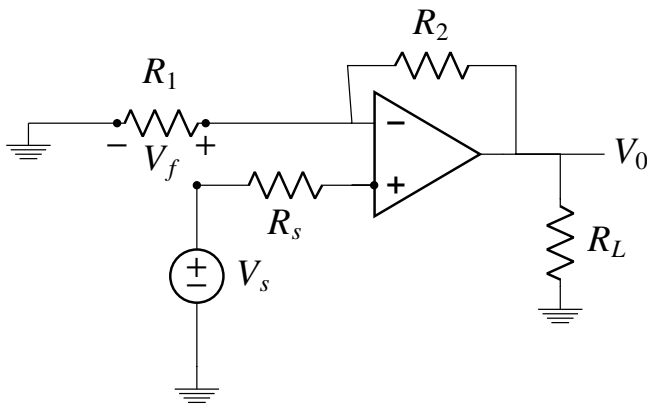


Fig. 1.1.1

Solution: See Fig. 1.1.2

1.2. Draw the small signal model for Fig. 1.1.1.

Solution: The equivalent circuit of the amplifier is in Fig. 1.2

Parameter	Value
input resistance	∞
output resistance	0
Input voltage	V_s
Output Voltage	V_o
Feeding resistance	R_1
Feedback resistance	R_2
Source resistance	R_s
load resistance	R_L

TABLE 1.1

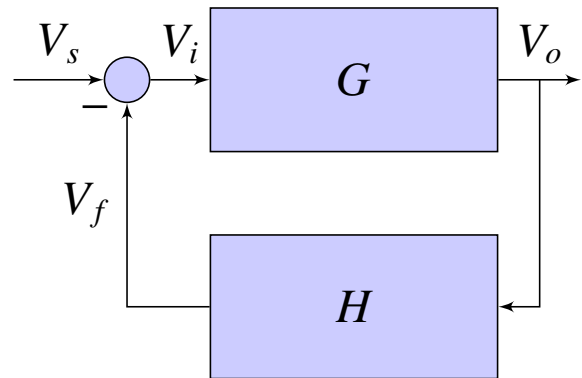


Fig. 1.1.2

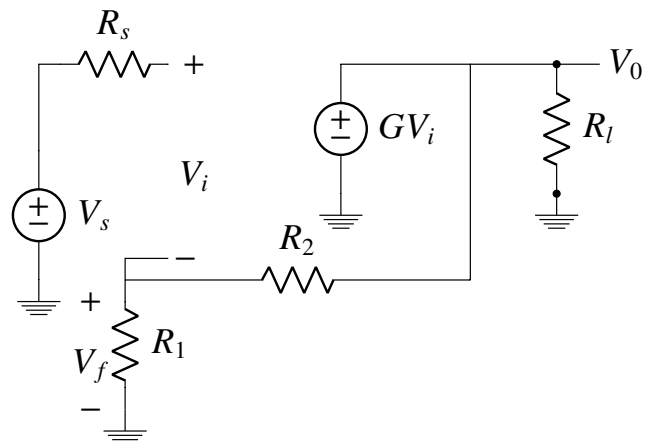


Fig. 1.2

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1.3. Assuming that the operational amplifier has

infinite input resistance and zero output resistance, find the *feedback factor* H .

Solution: From Fig. 1.2,

$$V_o = GV_i \quad (1.3.1)$$

$$V_i = V_s - V_f \quad (1.3.2)$$

$$V_f = \frac{R_1}{R_1 + R_2} V_o \quad (1.3.3)$$

assuming that the current through R_s is very small. Thus,

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \quad (1.3.4)$$

1.4. Obtain the closed loop gain T and summarize your results through a Table.

Solution: Table 1.4 provides a summary.

$$T = \frac{V_o}{V_i} = \frac{G}{1 + GH} \quad (1.4.1)$$

$$= \frac{G(R_1 + R_2)}{(R_1 + R_2) + GR_1} \quad (1.4.2)$$

Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1 + R_2}$
Loop gain	GH	$G \frac{R_1}{R_1 + R_2}$
Amount of feedback	$1 + GH$	$1 + \frac{GR_1}{R_1 + R_2}$
Closed loop gain	$\frac{G}{1 + GH}$	$\frac{G(R_1 + R_2)}{R_1 + R_2 + GR_1}$

TABLE 1.4

1.5. Find the condition under which closed loop gain T is almost entirely determined by the feedback network.

Solution: If

$$GH \gg 1, \quad (1.5.1)$$

$$T \approx \frac{1}{H} = 1 + \frac{R_2}{R_1} \quad (1.5.2)$$

1.6. If

$$G = 10^4 \quad (1.6.1)$$

$$T = 10, \quad (1.6.2)$$

find H .

Solution: From Table 1.4

$$T = \frac{G}{1 + GH} = 10 \quad (1.6.3)$$

$$\Rightarrow H = 0.0999 \quad (1.6.4)$$

1.7. *Gain Desensitivity:* If G decreases by 20%, what is the corresponding decrease in T ? Comment.

Solution: From Table 1.4, Given

$$T = \frac{G}{1 + GH} \quad (1.7.1)$$

$$\Rightarrow dT = \frac{dG}{(1 + GH)^2} \quad (1.7.2)$$

$$\Rightarrow \frac{dT}{T} = \frac{1}{1 + GH} \frac{dG}{G} \quad (1.7.3)$$

From the information available so far,

$$dG = 20\%, G = 10^4, H = 0.0999 \Rightarrow \frac{dT}{T} = 0.025\% \quad (1.7.4)$$

using the following code.

```
codes/ee18btech11005/ee18btech11005.py
```

Thus the closed loop gain is almost invariant to a relatively large (20%) variation in the open loop gain G . This is known as gain desensitivity.

2 FEEDBACK CURRENT AMPLIFIER: SHUNT-SERIES

2.1. Draw the equivalent control system for the feedback current amplifier shown in 2.1.4

Solution: See Fig. 2.1.5.

2.2. For the feedback current amplifier shown in 2.1.4, draw the Small-Signal Model. Neglect the Early effect in Q_1 and Q_2 .

Solution: See Fig. 2.2.

While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal parameters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m V_{gs}$ flowing from Drain to Source. Whereas

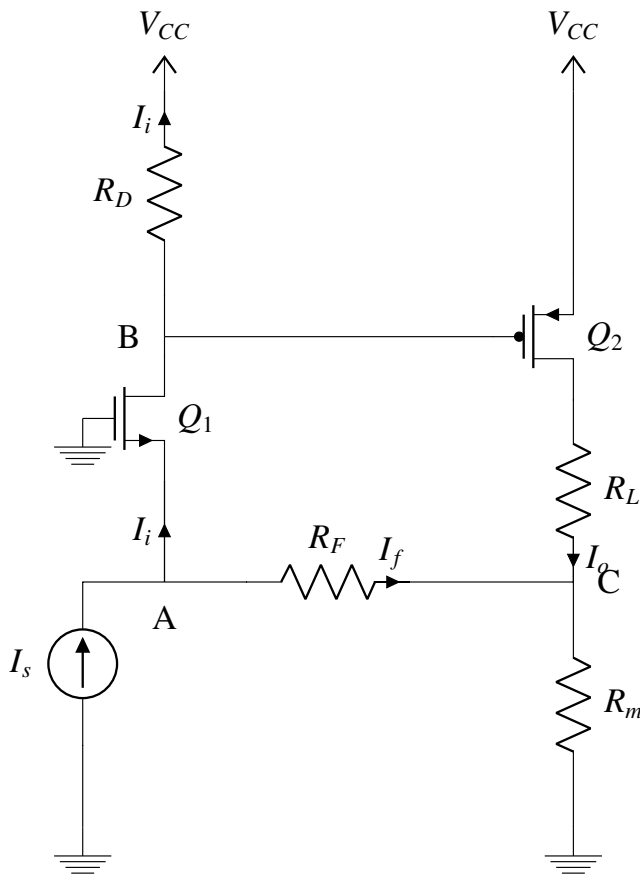


Fig. 2.1.4

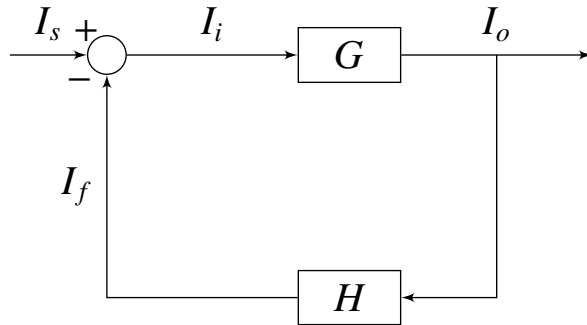


Fig. 2.1.5

a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

2.3. Write all the node/loop equations using KCL/KVL.

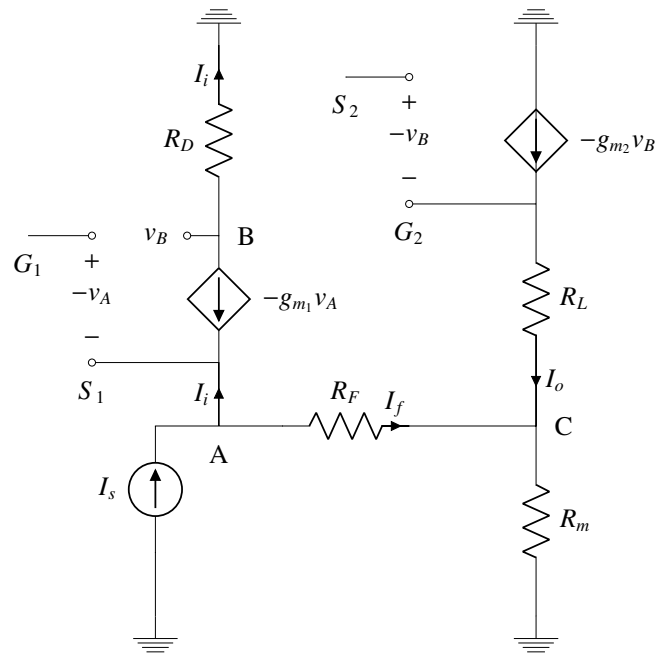


Fig. 2.2: Small Signal Model

Solution: From Figs. 2.1.4 and 2.2,

$$I_i = \frac{v_B}{R_D} \quad (2.3.1)$$

$$I_o = -g_{m2} v_B \quad (2.3.2)$$

$$v_C + v_B = -I_o R_L \quad (2.3.3)$$

$$v_C = (I_o - I_f) R_M \quad (2.3.4)$$

2.4. Find the Expression for the Open-Loop Gain G .

Solution: From (2.3.1) and (2.3.2),

$$G = \frac{I_o}{I_i} = -g_{m2} R_D \quad (2.4.1)$$

2.5. Find the Expression of the Feedback Factor H .

Solution:

$$H = \frac{I_f}{I_o}, \quad (2.5.1)$$

From (2.3.3) and (2.3.4),

$$(I_o + I_f) R_M + v_B = -I_o R_L \quad (2.5.2)$$

$$\Rightarrow (I_o + I_f) R_M + I_i R_D = -I_o R_L \quad (2.5.3)$$

from (2.3.1)

$$\Rightarrow (1 + H) R_M + \frac{R_D}{G} = -R_L \quad (2.5.4)$$

dividing by I_o

$$\Rightarrow H = -1 - \frac{1}{R_M} \left(R_L - \frac{1}{g_{m2}} \right) \quad (2.5.5)$$

upon substituting from (2.4.1) and simplifying. I_o is fed to a current divider formed by R_M and R_F . R_F is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it leaving a small current as input to Amplifier. Hence the voltage at point 'A' is very small and is considered, $v_A \simeq 0$. So R_F and R_M are parallel and Voltage Drop across them is same.

$$(I_o + I_f)R_M \simeq -I_f R_F \quad (2.5.6)$$

$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \quad (2.5.7)$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \quad (2.5.8)$$

2.6. Find the Expression for the Closed-Loop Gain
 $T = \frac{I_o}{I_s}$.

Solution: From (??) and (2.5.8),

$$T = \frac{I_o}{I_s} = \frac{G}{1 + GH} \quad (2.6.1)$$

$$= -\frac{g_{m2}R_D}{1 + g_{m2}R_D / \left(1 + \frac{R_F}{R_M}\right)} \quad (2.6.2)$$

$$\Rightarrow T = -\frac{g_{m2}R_D}{1 + g_{m2}R_D / \left(1 + \frac{R_F}{R_M}\right)} \quad (2.6.3)$$