Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

1.1. Fig. 1.1.1 shows a non-inverting op-amp configuration with parameters described in Table 1.1. Draw the equivalent control system.

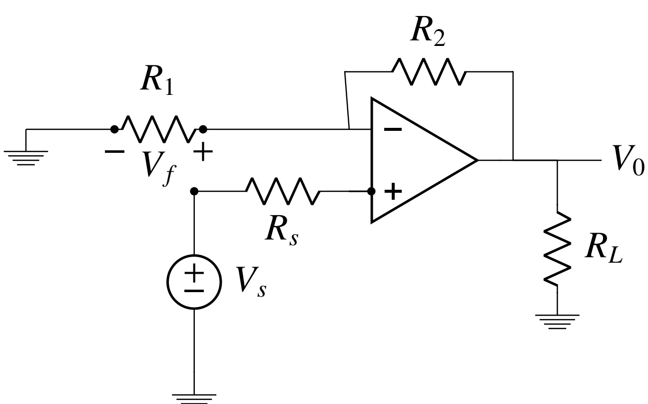

Fig. 1.1.1

Solution: See Fig. 1.1.2

Parameter	Value
input resistance	∞
output resistance	**0**
Input voltage	V_s
Output Voltage	V_o
Feeding resistance	R_1
Feedback resistance	R_2
Source resistance	R_s
load resistance	R_L

TABLE 1.1

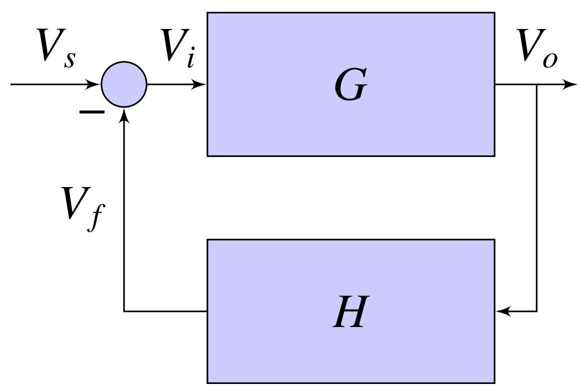

Fig. 1.1.2

1.2. Draw the small signal model for Fig. 1.1.1.
Solution: The equivalent circuit of the amplifier is in Fig. 1.2

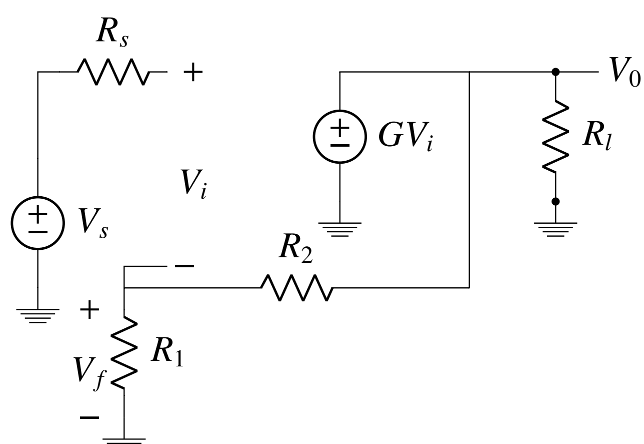

Fig. 1.2

1.3. Assuming that the operational amplifier has infinite input resistance and zero output resistance, find the *feedback factor* H.
Solution: From Fig. 1.2,

$$V_0 = GV_i \tag{1.3.1}$$

$$V_i = V_s - V_f \tag{1.3.2}$$

$$V_f = \frac{R_1}{R_1 + R_2} V_o \tag{1.3.3}$$

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assuming that the current through R_s is very small. Thus,

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \tag{1.3.4}$$

1.4. Obtain the closed loop gain T and summarize your results through a Table.
Solution: Table 1.4 provides a summary.

$$T = \frac{V_0}{V_i} = \frac{G}{1 + GH} \tag{1.4.1}$$
$$= \frac{G(R_1 + R_2)}{(R_1 + R_2) + GR_1} \tag{1.4.2}$$

Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1+R_2}$
Loop gain	GH	$G\frac{R_1}{R_1+R_2}$
Amount of feedback	1+GH	$1 + \frac{GR_1}{R_1+R_2}$
Closed loop gain	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$

TABLE 1.4

1.5. Find the condition under which closed loop gain T is almost entirely determined by the feedback network.
Solution: If

$$GH \gg 1, \tag{1.5.1}$$
$$T \approx \frac{1}{H} = 1 + \frac{R_2}{R_1} \tag{1.5.2}$$

1.6. If

$$G = 10^4 \tag{1.6.1}$$
$$T = 10, \tag{1.6.2}$$

find H.
Solution: From Table 1.4

$$T = \frac{G}{1 + GH} = 10 \tag{1.6.3}$$
$$\implies H = 0.0999 \tag{1.6.4}$$

1.7. *Gain Desensitivity:* If G decreases by 20%,what is the corresponding decrease in T? Comment.
Solution: From From Table 1.4, Given

$$T = \frac{G}{1 + GH} \tag{1.7.1}$$
$$\implies dT = \frac{dG}{(1 + GH)^2} \tag{1.7.2}$$
$$\implies \frac{dT}{T} = \frac{1}{1 + GH}\frac{dG}{G} \tag{1.7.3}$$

From the information available so far,

$$dG = 20\%, G = 10^4, H = 0.0999 \implies \frac{dT}{T} = 0.025\% \tag{1.7.4}$$

using the following code.

```
codes/ee18btech11005/ee18btech11005.py
```

Thus the closed loop gain is almost invariant to a relatively large (20%) variation in the open loop gain G. This is known as gain desensitivity.

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.1.1. Draw the equivalent control system for the feedback current amplifier shown in 2.1.1.1
Solution: See Fig. 2.1.1.2.

2.1.2. For the feedback current amplifier shown in 2.1.1.1, draw the Small-Signal Model. Neglect the Early effect in Q_1 and Q_2.
Solution: See Fig. 2.1.2.
While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

2.1.3. Write all the node/loop equations using KCL/KVL.

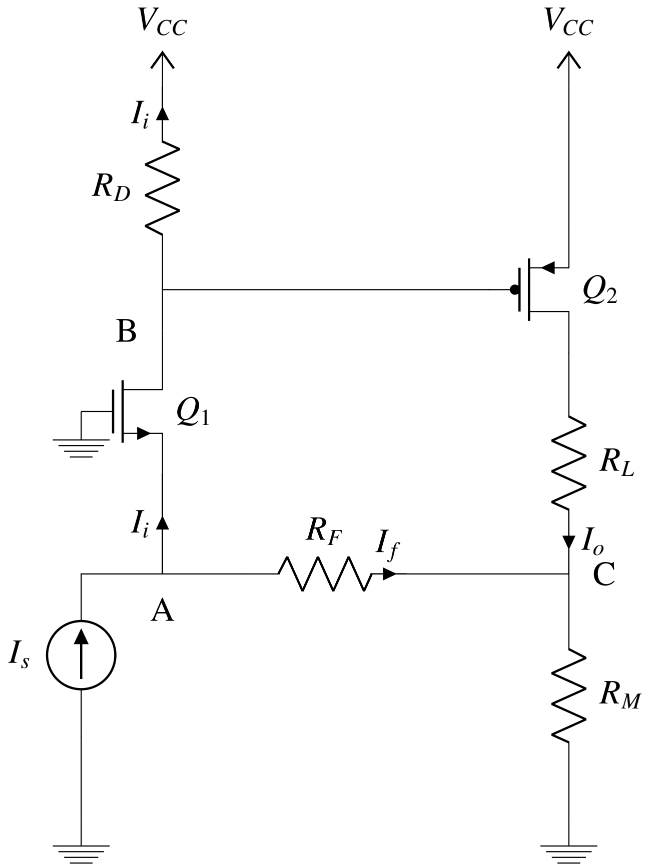

Fig. 2.1.1.1

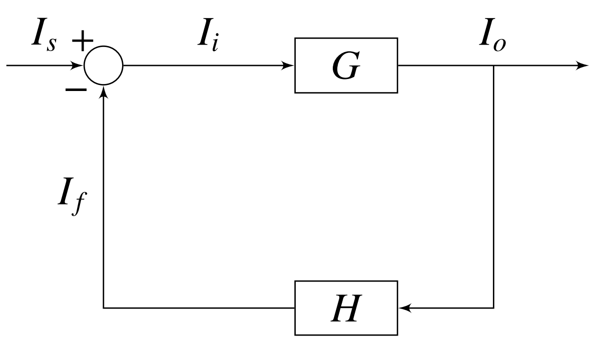

Fig. 2.1.1.2

Solution: From Figs. 2.1.1.1 and 2.1.2,

$$I_i = \frac{v_B}{R_D} \tag{2.1.3.1}$$

$$I_o = -g_{m_2} v_B \tag{2.1.3.2}$$

$$v_C - v_A = -I_f R_F \tag{2.1.3.3}$$

$$v_C = \left(I_o + I_f\right) R_M \tag{2.1.3.4}$$

$$I_i = g_{m_1} v_A \tag{2.1.3.5}$$

2.1.4. Find the Expression for the Open-Loop Gain G.

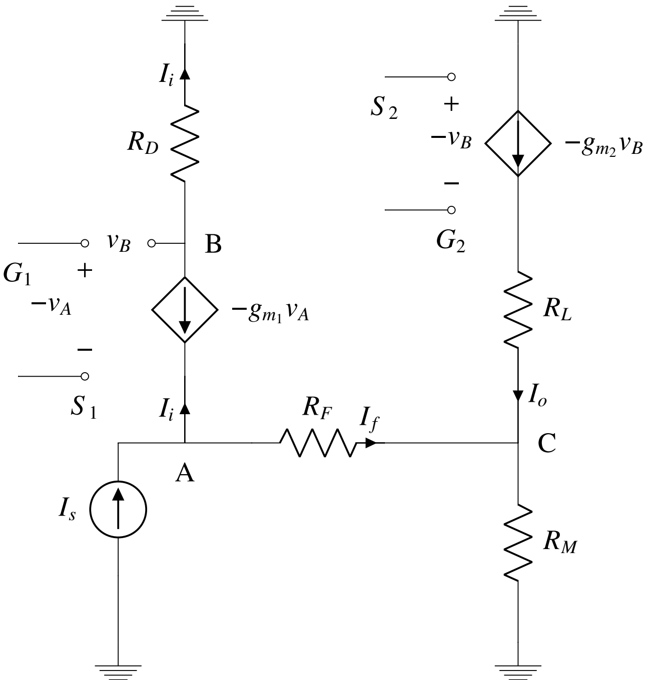

Fig. 2.1.2: Small Signal Model

Solution: From (2.1.3.1) and (2.1.3.2),

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.1}$$

2.1.5. Find the Expression of the Feedback Factor H.
Solution:

$$H = \frac{I_f}{I_o}, \tag{2.1.5.1}$$

From (2.1.3.3) and (2.1.3.4),

$$\left(I_o + I_f\right) R_M - v_A = -I_f R_F \tag{2.1.5.2}$$

$$\implies \left(I_o + I_f\right) R_M + \frac{I_i}{g_{m_1}} = -I_f R_F \tag{2.1.5.3}$$

from (2.1.3.5). Dividing by I_o,

$$\implies (1 + H) R_M + \frac{1}{g_{m_1} G} = -H R_F \tag{2.1.5.4}$$

upon substituting from and . Simplifying further, we obtain

$$\implies H = \frac{\frac{1}{g_{m_1} g_{m_2} R_D} - R_M}{R_F + R_M} \tag{2.1.5.5}$$

$$\approx -\frac{R_M}{R_F + R_M} \tag{2.1.5.6}$$

for $R_M \gg \frac{1}{g_{m_1} g_{m_2} R_D}$.

2.1.6. Find the Expression for the Closed-Loop Gain

$T = \frac{I_o}{I_s}$.
Solution: From (2.1.5) and (2.1.5.6),

$$T = \frac{I_o}{I_s} = \frac{G}{1 + GH} \tag{2.1.6.1}$$

$$= -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.2}$$

2.2 Practical Case

2.2.1. Draw the Block Diagram and Circuit Diagram for H.
Solution: The Block Diagram is available in Fig. 2.2.1.1

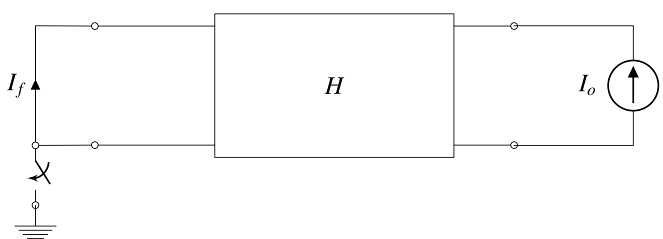

Fig. 2.2.1.1: Feedback Block Diagram

and the corresponding circuit diagram in Fig. 2.2.1.2

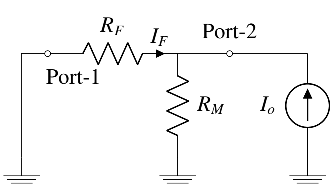

Fig. 2.2.1.2: Feedback Network

2.2.2. Find H from Fig. 2.2.1.2.
Solution: Using current division,

$$\frac{I_f}{I_o} = -\frac{R_M}{R_F + R_M} \tag{2.2.2.1}$$

$$\implies H = -\frac{R_M}{R_F + R_M} \tag{2.2.2.2}$$

2.2.3. Find R_{11} and R_{22} of Feedback Network where R_{11} is input resistance through Port-1 and R_{22} is Input Resistance through Port-2.
Solution: R_{11} is calculated by opening the current source at Port-2. Hence,

$$R_{11} = R_F + R_M \tag{2.2.3.1}$$

While calculating R_{22}, Port-1 should be shorted. Hence,

$$R_{22} = R_F || R_M \tag{2.2.3.2}$$

$$= \frac{R_F R_M}{R_F + R_M} \tag{2.2.3.3}$$

2.2.4. Draw the block diagram and circuit diagram for calculating G.
Solution: See Figs. 2.2.4.1 and 2.2.4.2

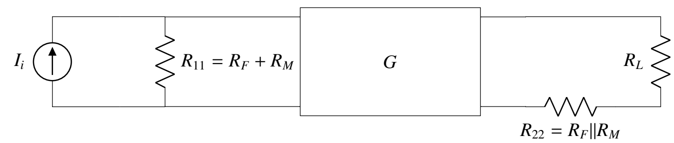

Fig. 2.2.4.1: Open-Loop Block Diagram

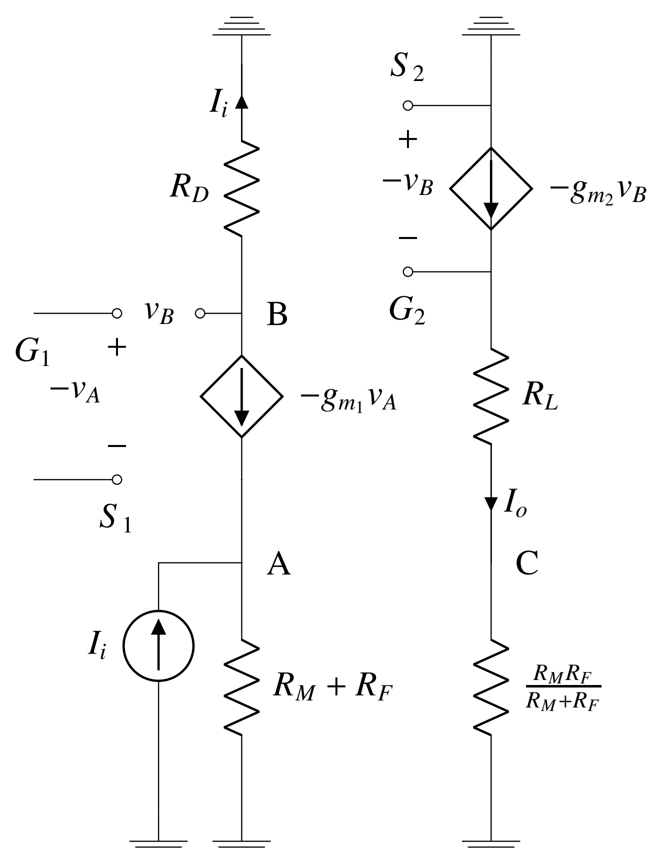

Fig. 2.2.4.2: Open-Loop Network

2.2.5. Find G.
Solution: The analysis is the same as Problem 2.1.4.

3 Feedback Current Amplifier: Example

3.1. Consider a Feedback Current Amplifier formed by cascading an Inverting Opamp μ with a

MOSFET (NMOS) as shown in Fig. 3.1.1. The output current is the Drain Current of the NMOS. Assume that Opamp has an input resistance R_{id}, an Open Circuit Voltage Gain μ, and an output resistance r_{o1}. Express this as a control system.

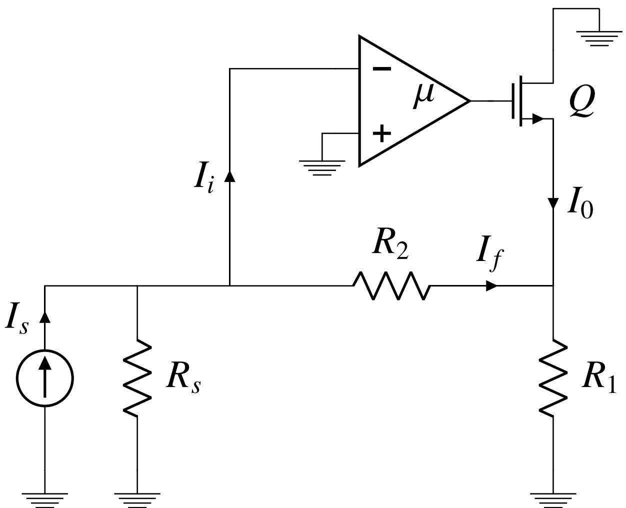

Fig. 3.1.1: Complete Circuit

Solution: See Fig. 3.1.2

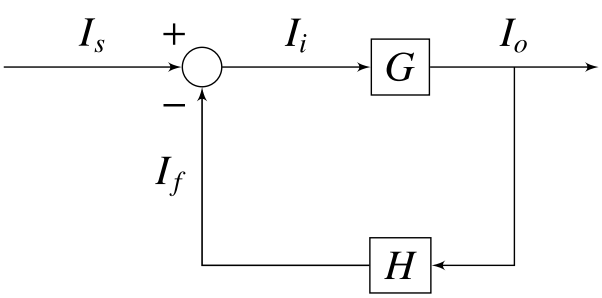

Fig. 3.1.2: Block Diagram

3.2. Represent Fig. 3.1.1 using a Small Signal Equivalent Model.
Solution: See Fig. 3.2

3.3. Find G.
Solution: From Fig. 3.2 we have the following

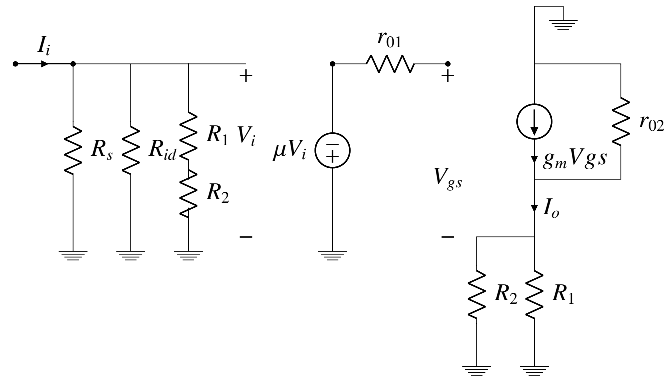

Fig. 3.2: Small Signal Model

equations

$$V_g = -\mu V_i \tag{3.3.1}$$

$$\frac{V_i}{R_{id}} = I_i \tag{3.3.2}$$

$$I_s = \frac{V_i}{R_s} + I_f + I_i \tag{3.3.3}$$

$$I_f = \frac{V_i - V_s}{R_2} \tag{3.3.4}$$

$$\frac{V_g}{R_2} = I_f + I_o \tag{3.3.5}$$

$$I_o = g_m\left(V_g - V_s\right) + \frac{V_s}{r_{o2}} \tag{3.3.6}$$

$$G = \frac{I_o}{I_i} \tag{3.3.7}$$

$$H = \frac{I_f}{I_o} \tag{3.3.8}$$

$$R_i = R_s \| R_{id} \| (R_1 + R_2) \tag{3.3.9}$$

where R_i is the resistance seen by the current source I_s and R_{id} is the internal resistance of the OPAMP.

$$V_i = I_s R_i \tag{3.3.10}$$

$$I_i = I_s \frac{R_s \parallel (R_1 + R_2)}{R_s + R_{id} + R_1 + R_2} \tag{3.3.11}$$

for small values of I_f.

$$I_o = -\mu V_i \frac{1}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \tag{3.3.12}$$

$$G = \frac{I_o}{I_i} = -\mu \frac{R_i}{1/g_m + (R_1||R_2||r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1||R_2)} \tag{3.3.13}$$

We use the approximation

$$1/g_m \ll (R_1||R_2||r_{o2}) \tag{3.3.14}$$

This is because the $\frac{1}{g_m}$ is in order of few Ωs but, R_1, R_2 and r_{o2} are in order of kΩs

$$G = -\mu \frac{R_i}{R_1||R_2} \tag{3.3.15}$$

$$R_o = r_{o2} + (R_1||R_2) + (g_m r_{o2})(R_1||R_2) \tag{3.3.16}$$

$$\implies R_o \simeq g_m r_{o2} (R_1||R_2) \tag{3.3.17}$$

3.4. Find expression for Loop Gain H
Solution:

$$H = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \tag{3.4.1}$$

3.5. If loop gain is large, find approximate expression for closed loop gain T
Solution: Given,

$$GH \gg 1 \tag{3.5.1}$$

$$T = \frac{G}{1 + GH} \simeq \frac{1}{H} \tag{3.5.2}$$

$$T \simeq \frac{1}{H} = -\left(1 + \frac{R_2}{R_1}\right) \tag{3.5.3}$$

3.6. Give expressions for GH, T, R_{if}, R_{in}, R_{of}, R_{out}
Solution:

$$GH = \mu \frac{R_i}{\frac{1}{g_m} + (R_1||R_2||r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1||R_2)} \frac{R_1}{R_1 + R_2} \tag{3.6.1}$$

Once again, using the approximation,

$$\implies GH \simeq \mu \frac{R_i}{R_1||R_2} \frac{R_1}{R_1 + R_2} = \mu \frac{R_i}{R_2} \tag{3.6.2}$$

For Input Resistance,

$$R_{if} = R_i/(1 + GH) \tag{3.6.3}$$

$$\implies \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_2} \tag{3.6.4}$$

$$\implies R_{if} = R_i||\frac{R_2}{\mu} \tag{3.6.5}$$

Substituting the value of R_i,

$$R_{if} = R_s||R_{id}||(R_1 + R_2)||\frac{R_2}{\mu} \tag{3.6.6}$$

$$R_{if} = R_s||R_{in} \tag{3.6.7}$$

$$\implies R_{in} = R_{id}||(R_1 + R_2)||\frac{R_2}{\mu} \tag{3.6.8}$$

$$R_{in} \simeq \frac{R_2}{\mu} \tag{3.6.9}$$

For Output Resistance,

$$R_{of} = R_o(1 + GH) \simeq GHR_o \tag{3.6.10}$$

$$R_{of} \simeq \mu(\frac{R_i}{R_2})(g_m r_{o2})(R_1||R_2) \tag{3.6.11}$$

$$R_{out} = R_{of} = \mu \frac{R_i}{R_1 + R_2}(g_m r_{o2})R_1 \tag{3.6.12}$$

3.7. Given the following values

Parameter	Value
μ	1000
R_s	∞
R_{id}	∞
r_{o1}	$1k\Omega$
R_1	$10k\Omega$
R_2	$90k\Omega$
g_m	$5mA/V$
r_{o2}	$20k\Omega$

TABLE 3.7

Find numerical value of R_i and use it to find the value of G
Solution: Using the given numerical values on the previously obtained equations, we obtain:

$$R_i = \infty||\infty||(10 + 90) = 100k\Omega \tag{3.7.1}$$

$$G = -1000 \frac{100}{10||90} = -11.11 \times 10^3 \tag{3.7.2}$$

3.8. Check the validity of the approximation that we use to neglect $1/g_m$
Solution:

$$1/g_m = 0.2k\Omega \ll (10\|90\|20)k\Omega = 6.2k\Omega \tag{3.8.1}$$

Hence, we can see that our approximation is valid

3.9. Find the value of feedback gain H and open loop gain GH
Solution:

$$H = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1 \tag{3.9.1}$$

$$GH = 1111 \gg 1 \tag{3.9.2}$$

3.10. Find the approximate value of closed loop gain T
Solution:

$$T \simeq \frac{1}{H} = -\frac{1}{0.1} = -10 \tag{3.10.1}$$

3.11. Find the values of R_{in} and R_{out}
Solution:

$$R_{in} = \frac{R_2}{\mu} = \frac{90k\Omega}{1000} = 90\Omega \tag{3.11.1}$$

$$R_o = g_m r_{o2}(R_1\|R_2) = 5 \times 20(10\|90) = 900k\Omega \tag{3.11.2}$$

$$R_{out} = (1 + GH)R_o = 1112 \times 900 \simeq 1000M\Omega \tag{3.11.3}$$

Parameter	Value
R_i	$100k\Omega$
$1/g_m$	200Ω
G	-1.11×10^4
H	-0.1
GH	1111
T	-10
R_{in}	90Ω
R_o	$900k\Omega$
R_{out}	$1000M\Omega$

TABLE 3.11

3.12. Verify the above calculations using a Python code.
Solution:

```
codes/ee18btech11021/ee18btech11021_calc.py
```

4 Feedback Transconductance Amplifier: Series-Series

4.1. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in Fig. 4.1.1 with values of various parameters given in Table 4.1. Draw the equivalent block diagram.

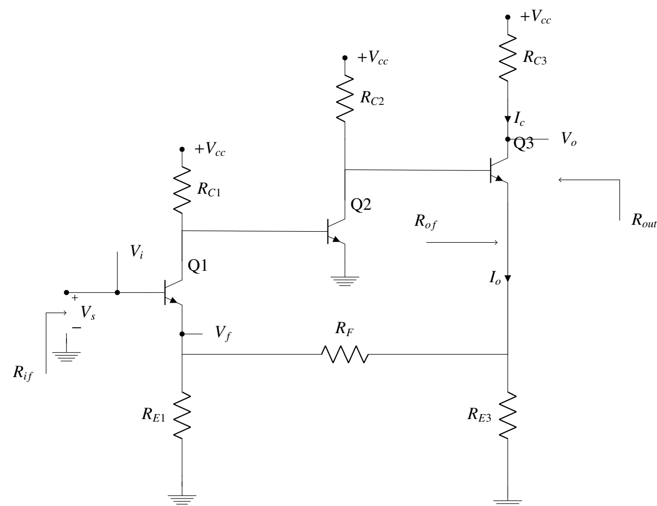

Fig. 4.1.1

Solution: The block diagram is available in Fig. 4.1.2

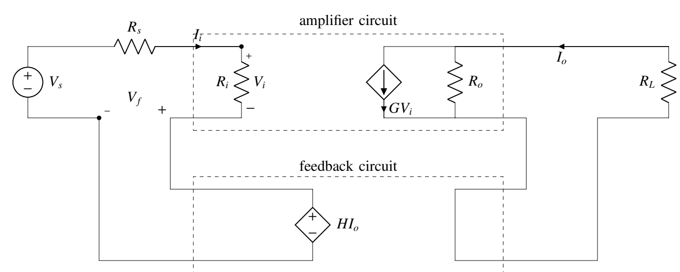

Fig. 4.1.2: Feedback Transconductance Amplifier

4.2. Draw the block diagram and equivalent circuit for H for Fig. 4.1.2.
Solution: Fig. 4.2.1 gives the required block diagram

$$H = \frac{V_f}{I_o}\Big|_{I_1=0} \tag{4.2.1}$$

and the equivalent H circuit is available in Fig. 4.2.2.

Parameter	Value
R_{C1}	$9k\Omega$
R_{E1}	100Ω
R_{C2}	$5k\Omega$
R_F	640Ω
R_{E2}	100Ω
R_{C3}	600Ω
h_{fe}	**100**
r_o	$\infty\Omega$
I_{C1}	**0.6mA**
I_{C2}	**1mA**
I_{C3}	**4mA**
r_{e1}	41.7Ω
$r_{\pi 2}$	$2.5k\Omega$
$\alpha 1$	**0.99**
g_{m2}	**40mA/V**
r_{e3}	6.25Ω
r_{o3}	$25k\Omega$
$r_{\pi 3}$	625Ω

TABLE 4.1: parameters

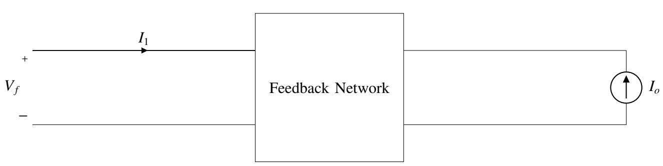

Fig. 4.2.1: Feedback circuit block diagram

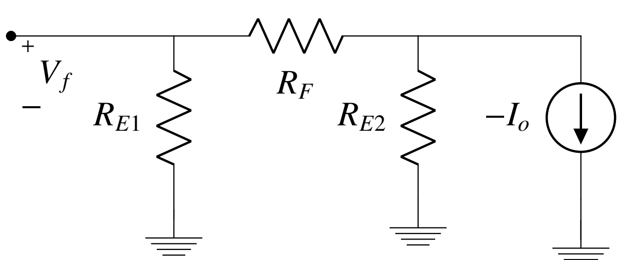

Fig. 4.2.2: H circuit

4.3. Find the feedback Factor H
Solution: From Fig. 4.2.2,

$$H = \frac{V_f}{I_0} = \frac{R_{E1}R_{E2}}{R_{E2} + R_F + R_{E1}} \tag{4.3.1}$$

4.4. Find R_{11} and R_{22} from Figs. 4.4 and 4.2.2

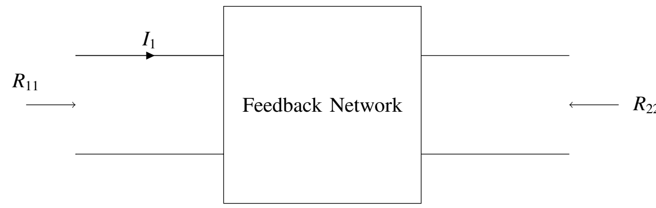

Fig. 4.4: feedback network

Solution:

$$R_{11} = R_{E1}||(R_F + R_{E2}) \tag{4.4.1}$$

$$R_{22} = R_{E2}||(R_F + R_{E1}) \tag{4.4.2}$$

4.5. Draw the block diagram and equivalent circuit for G.
Solution: The required block diagram is available in Fig. 4.5 and the equivalent circuit in

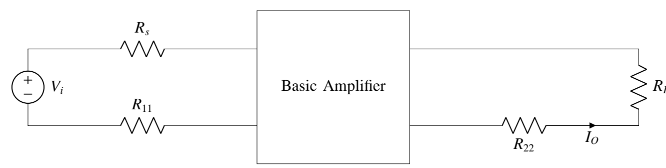

Fig. 4.5: Amplifier circuit block diagram

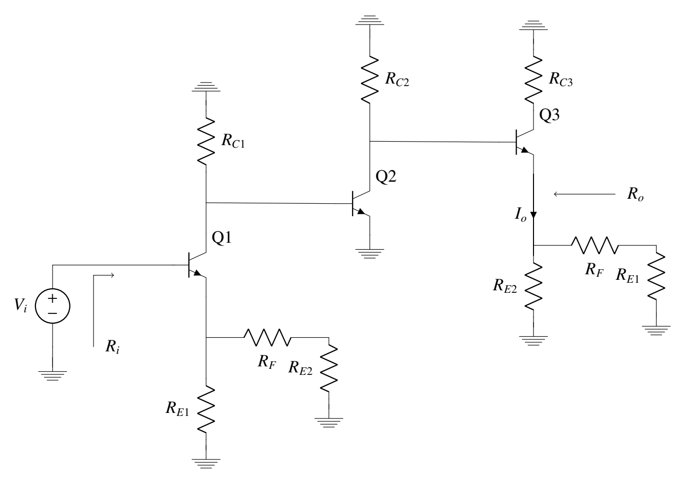

Fig. 4.5: G circuit

4.6. feedback analysis to find open loop gain G
Solution: employing equivalent amplifier block diagram fig.4.5 into circuit in fig.4.1.1, R_{11} and R_{22} are found from feedback circuit in fig.4.2.2 using rule from fig.4.4 we finally obtain

$$R_s, R_L = 0 \tag{4.6.1}$$

$$R_{11} = R_{E1}||(R_F + R_{E2}) \quad (4.6.2)$$

$$R_{22} = R_{E2}||(R_F + R_{E1}) \quad (4.6.3)$$

finally Amplifier circuit is obtained shown in fig.4.5 to find $G = \frac{I_0}{V_i}$ we determine the gain of

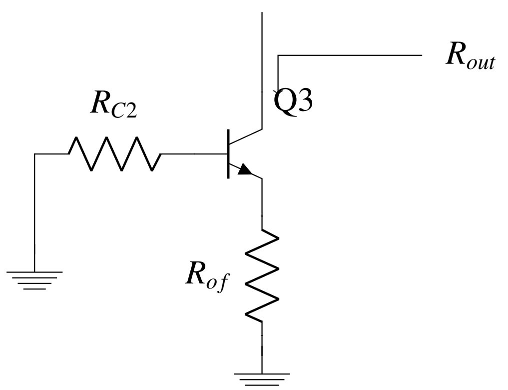

Fig. 4.6: circuit4

first stage,this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{c1}||r_{\pi2})}{r_{e1} + (R_{E1}||(R_F + R_{E2}))} \quad (4.6.4)$$

using values from 4.1

$$\frac{V_{c1}}{V_i} = -14.92V/V \quad (4.6.5)$$

Next, we determine the gain of the second stage,which can be written by inspection(noting that $V_{b2} = V_{c1}$)as

$$\frac{V_{c2}}{V_{c1}} = -g_{m2}R_{c2}||(h_{fe} + 1)[r_{e3} + (R_{E2}||(R_F + R_{E1}))] \quad (4.6.6)$$

substituting ,results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \quad (4.6.7)$$

Finally,for the third stage we can write by inspection

$$\frac{I_0}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2}||(R_F + R_{E1}))} \quad (4.6.8)$$

substituing values from 4.1 gives

$$\frac{I_0}{V_{c2}} = 10.6mA/V \quad (4.6.9)$$

combining the gains of the three stags results in

$$G = \frac{I_0}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7A/V \quad (4.6.10)$$

4.7. Find closed loop gain T and Voltage Gain V_0/V_s

Solution:

$$T = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \quad (4.7.1)$$

the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{c3}}{V_s} \approx \frac{-I_0 R_{C3}}{V_s} = -TR_{C3} \quad (4.7.2)$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \quad (4.7.3)$$

4.8. Now assume Loop gain is large and find approximate expression for closed loop gain $T = \frac{I_o}{V_s}$

Solution: When GH >>1,

$$T = \frac{I_0}{V_s} \approx \frac{1}{H} \quad (4.8.1)$$

as

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (4.8.2)$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (4.8.3)$$

thus,

$$T = \frac{1}{11.9} = 84mA/V \quad (4.8.4)$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \quad (4.8.5)$$

which we note is very close to the approximate value found in (4.7.1)

4.9. Find R_{in} and R_{out} for circuit in fig.4.1.1

Solution:

$$R_{in} = R_{if} = R_i(1 + GH) \quad (4.9.1)$$

where R_i is the input resistance of the G circuit.The value of R_i can be found from the circuit in fig.4.5 as follows:

$$R_i = (h_{fe} + 1)(r_{e1} + (R_{E1}||(R_F + R_{E2}))) = 13.65K\Omega \quad (4.9.2)$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38M\Omega \tag{4.9.3}$$

$$R_{of} = R_o(1 + GH) \tag{4.9.4}$$

where R_o can be determined to be

$$R_o = (R_{E2}||(R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe} + 1} \tag{4.9.5}$$

from values in Table 4.1, yields $R_o = 143.9\Omega$. The output resistance R_{of} of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6K\Omega \tag{4.9.6}$$

R_{out} is found by using circuit4 in fig.4.5

$$R_{out} = r_{o3} + [R_{of}||(r_{\pi 3} + R_{C2})](1 + g_{m3}r_{o3}\frac{r_{\pi 3}}{r_{\pi 3} + R_{C2}}) \tag{4.9.7}$$

$$= 25 + [35.6||(5.625)][1 + 160 \times 25\frac{0.625}{5.625}] = 2.19M\Omega \tag{4.9.8}$$

thus R_{out} is increased (from r_{o3}) but not by (1+GH)

4.10. put the obtained parameters in a table
Solution: **??** table gives us the calculated values

Parameter	**Value**
G	**20.7A/V**
H	11.9Ω
T	**83.7mA/V**
V_o/V_s	**-50.2V/V**
R_{in}	$3.38M\Omega$
R_{out}	$2.19M\Omega$
R_{of}	$35.6k\Omega$

TABLE 4.10: calculated parameters

4.11. Represent this amplifier in a control system Block Diagram
Solution: figure in fig.**??** represents our control system

4.12. write a code for doing calculations and verify the values obtained in **??**
Solution: following code does all the calculations of above equations to give parameters in **??**

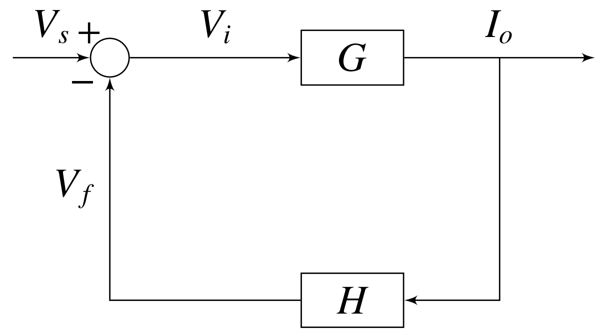

Fig. 4.11: block diagram

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codes/ee18btech11007/circuit_calc.py
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