Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

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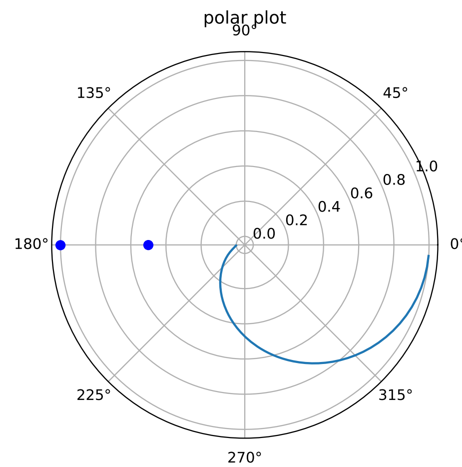

Fig. 1.1.1

1 Frequency Response Analysis

1.1 Polar Plot

1.1.1. Sketch the Polar Plot for

$$G(s) = \frac{1}{(1+s)(1+2s)} \tag{1.1.1}$$

Solution: The following code generates Fig. 1.1.1

```
codes/ee18btech11012.py
```

The polar plot is to the right of $(-1, 0)$. Hence the closed loop system is stable.

1.1.2. Sketch the polar plot of

$$G(s) = \frac{1}{(s^2)(s+1)(s+2)}. \tag{1.1.2}$$

Solution: Substituting $s = \jmath\omega$ in (1.1.2), Now the magnitude will be

$$r = |G(\jmath\omega)| = \frac{1}{(\omega^2)(\sqrt{1+\omega^2})(\sqrt{1+4\omega^2})} \tag{1.1.3}$$

$$\theta = \angle G(\jmath\omega) = -\tan^{-1}(0) - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{1.1.4}$$

$$= 180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{1.1.5}$$

The polar plot is the (r, θ) plot for $\omega \in (0, \infty)$. The following python code generates the polar plot in Fig. 1.1.2

codes/ee18btech11028.py

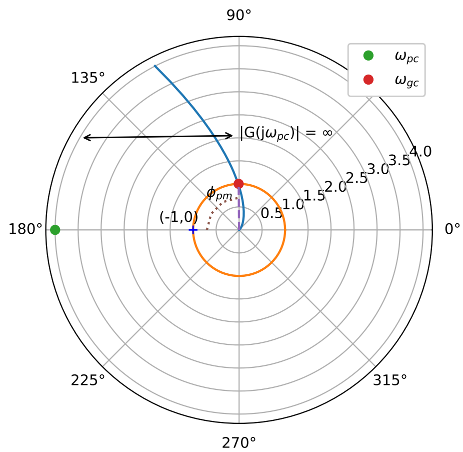

Fig. 1.1.2

The location of $(-1, 0)$ with respect to the polar plot provides information regarding the stability of the system.

- If $(-1, 0)$ is not enclosed, then it is stable.
- If $(-1, 0)$ is enclosed by polar plot then it is unstable.
- If $(-1, 0)$ is on the polar plot then it is marginally stable

In Fig. 1.1.2, the point $(-1, 0)$ is enclosed by the polar plot, which implies system is not stable. The polar plot also provides info on the GM and PM, which can then be used for determining the stability of the system.

- If the $GM > 1 \cap PM > 0$, then the control system is **stable**.
- If the $GM = 1 \cap PM = 0$, then the control system is **marginally stable**.
- If the $GM < 1 \cup PM < 0$, then the control system is **unstable**.

Therefore, our system is unstable $\because GM < 1 \cap PM < 0$.

1.1.3. Sketch the Polar Plot of

$$G(s) = \frac{1}{s(1 + s^2)} \tag{1.1.6}$$

Solution: From (1.1.6),

$$G(\jmath\omega) = \frac{1}{\jmath\omega(1 - \omega^2)} \tag{1.1.7}$$

$$|G(\jmath\omega)| = \frac{1}{|\omega(1 - \omega^2)|} \tag{1.1.8}$$

$$\angle G(\jmath\omega) = \begin{cases} \frac{\pi}{2} & \omega > 1 \\ -\frac{\pi}{2} & 0 < \omega < 1 \end{cases} \tag{1.1.9}$$

The corresponding polar plot is generated in Fig. 1.1.3 using

codes/ee18btech11023.py

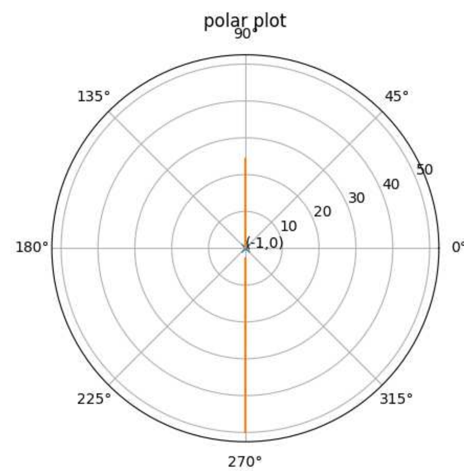

Fig. 1.1.3

In Fig. 1.1.3, (–1,0) is exactly on the polar plot. Hence, the system is marginally stable.

1.1.4. Sketch the Polar Plot of

$$G(s) = \frac{\left(1 + \frac{s}{29}\right)(1 + 0.0025s)}{(s^3)(1 + 0.005s)(1 + 0.001s)} \tag{1.1.10}$$

Solution: The following code generates the polar plot in Fig. 2.4.1

codes/ee18btech11029.py

- The polar plots use open loop transfer function to determine the stability and hence reference point is shifted to $(-1, 0)$
- If $(-1, 0)$ is left of the polar plot or $(-1, 0)$ is not enclosed, then it is stable
- If $(-1, 0)$ is on right side of the polar plot or $(-1, 0)$ is enclosed by polar plot then it is unstable.

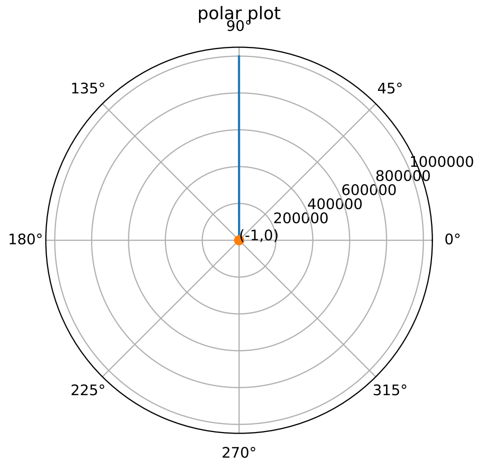

Fig. 1.1.4

- If $(-1, 0)$ is on the polar plot then it is marginally stable

In Fig. 2.4.1, $(-1, 0)$ is on the polar plot so the system is marginally stable.

1.1.5. Plot the polar plot of

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}. \tag{1.1.11}$$

Solution:

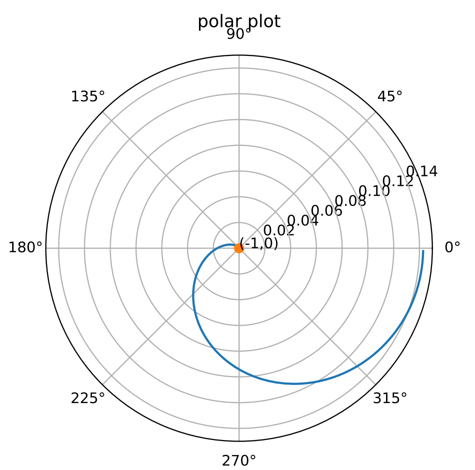

Fig. 1.1.5

The following python code generates the polar plot in Fig. 1.1.5

```
codes/ee18btech11033.py
```

∵ (-1,0) is on the right side of the polar plot, the system is unstable.

1.1.6. Plot the polar plot of

$$G(s) = \frac{100(s+5)}{s(s+3)(s^2+4)}. \tag{1.1.12}$$

Solution: The following python code generates the polar plot in Fig. 1.1.6

```
codes/ee18btech11042.py
```

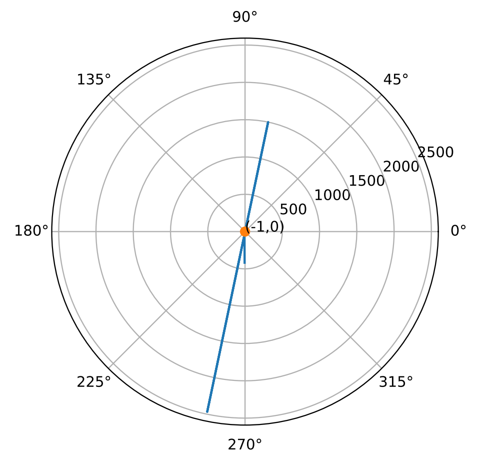

Fig. 1.1.6

Since (-1,0) is on the polar plot, the above system is marginally stable.

1.2 Direct and Inverse Polar Plot

Sketch the direct polar plot for a unity feedback system with open loop transfer function

$$G(s) = \frac{1}{s(1+s)^2} \tag{1.2.1}$$

Solution: The polar plot is obtained by plotting (r, ϕ)

$$r = |H(\jmath\omega)||G(\jmath\omega)| \tag{1.2.2}$$

$$\phi = \angle H(\jmath\omega)G(\jmath\omega), 0 < \omega < \infty \tag{1.2.3}$$

The following code plots the polar plot in Fig. 1.2.1

```
codes/ee18btech11002/polarplot.py
```

Sketch the inverse polar plot for (1.2.1)
Solution: The above code plots the polar plot in Fig. 1.2.2 by plotting $\left(\frac{1}{r}r, -\phi\right)$

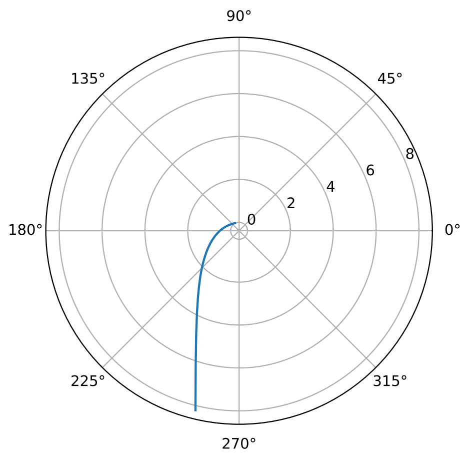

Fig. 1.2.1: Polar Plot

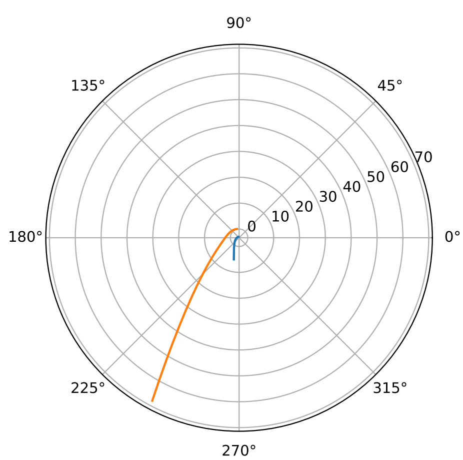

Fig. 1.2.2: Inverse Polar Plot

1.3 Bode Plot

1.3.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain margin and the phase margin.

$$G(s) = \frac{10}{s(1+0.5s)(1+.01s)} \tag{1.3.1}$$

Solution: The Bode magnitude and phase plot are available in Fig. 1.3.1 and generated by

```
codes/ee18btech11048.py
```

The pole-zero locations are available in Table 1.3.1.

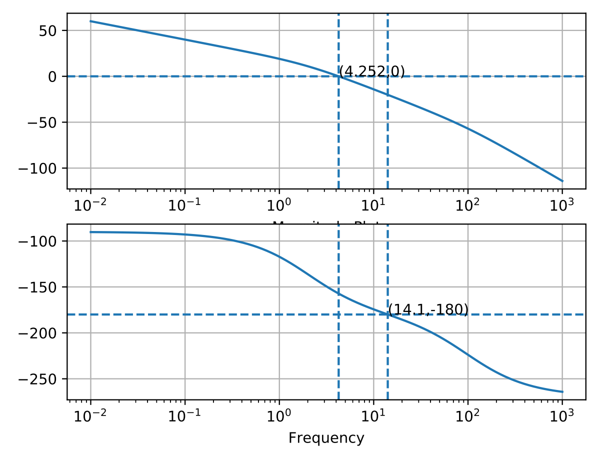

Fig. 1.3.1: Graphs

Zeros	**Poles**
-	0
	-2
	-100

TABLE 1.3.1: Zeros and Poles

The *Gain* and *Phase* of (1.3.2) are

$$\left|G(\jmath\omega)\right| = \frac{100}{\omega\sqrt{(0.5\omega)^2+1}\sqrt{(0.01\omega)^2+1}} \tag{1.3.2}$$

$$\angle G(\jmath\omega) = \tan^{-1}(0) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{100}\right) \tag{1.3.3}$$

Hence,

$$\left|G\left(\jmath\omega_{gc}\right)\right| = 1 \tag{1.3.4}$$
$$\implies \omega_{gc} = 4.25 \tag{1.3.5}$$
$$\angle G\left(\jmath\omega_{gc}\right) = -157.2 \tag{1.3.6}$$
$$\implies PM = 22.8 \tag{1.3.7}$$

Similarly,

$$\angle G\left(\jmath\omega_{pc}\right) = -180^\circ \tag{1.3.8}$$
$$\implies \omega_{pc} = 14.1 \tag{1.3.9}$$
$$\implies -\left|G\left(\jmath\omega_{pc}\right)\right| = -20.2dB \tag{1.3.10}$$
$$\implies GM = 20.2dB \tag{1.3.11}$$

1.3.2. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} \tag{1.3.12}$$

Also compute gain margin and phase margin .
Solution: From (1.3.12), we have

$$G(\jmath\omega) = \frac{75(1+0.2\jmath\omega)}{\jmath\omega\left((\jmath\omega)^2+16\jmath\omega+100\right)} \tag{1.3.13}$$

poles = 0 , -8-6j , -8+6j
zeros = -5
Gain and phase plots are shown in Fig. 1.3.2

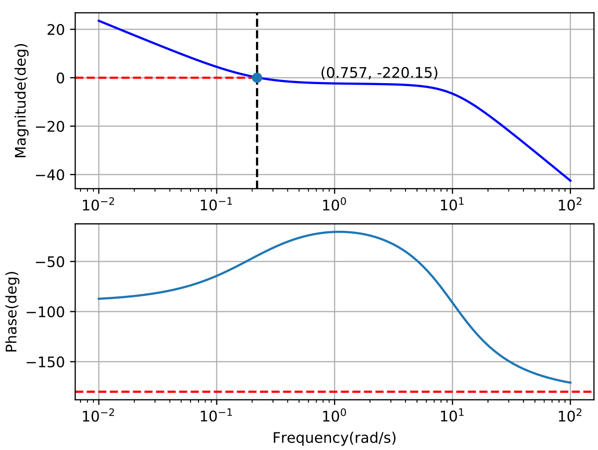

Fig. 1.3.2: a

The following code plots Fig. 1.3.2

```
codes/ee18btech11049.py
```

Solving

$$\left|G(\jmath\omega)\right| = \frac{75\sqrt{\omega^2+25}}{\omega\sqrt{(\omega+6)^2+64}\sqrt{(\omega-6)^2+64}} = 1, \tag{1.3.14}$$

or from Fig. 1.3.2, the gain crossover frequency

$$\implies \omega_{gc} = 0.757 \tag{1.3.15}$$

$$\angle G\left(\jmath\omega_{gc}\right) = -88.3 \tag{1.3.16}$$

$$\implies PM = 91.7 \tag{1.3.17}$$

Solution: From Fig. 1.3.2 ,we can say that phase never crosses $-180°$. So , the gain margin is *infinite*. Which means we can add any gain, and the equivalent closed loop system never becomes unstable.

1.3.3. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)} \tag{1.3.18}$$

Also compute gain margin and phase margin .
Solution: Substituting $s = \jmath\omega$ in (3.7.1.1) and assuming K =1,

$$G(\jmath\omega) = \frac{(\jmath\omega)^2}{(1+0.2\jmath\omega)(1+0.02\jmath\omega)} \tag{1.3.19}$$

The corner frequencies are

$$\omega_{c1} = 1/0.2 = 5 \tag{1.3.20}$$

$$\omega_{c2} = 1/0.02 = 50 \tag{1.3.21}$$

$$20\log\left|G(\jmath\omega)\right| = 20\log\left|(\jmath\omega)^2\right| - 20\log\left|(1+0.2\jmath\omega)\right| - 20\log\left|(1+0.02\jmath\omega)\right| \tag{1.3.22}$$

The various values of G($\jmath\omega$) are available in Table 3.7.1, in the increasing order of their corner frequencies also slope contributed by each term and the change in slope at the corner frequency. The pase

TERM	**Corner Freq**	**Slope**	**Slope change**
$(\jmath\omega)^2$	$--$	+40	$--$
$\frac{1}{1+\jmath 0.2}$	$\omega_{c1} = \frac{1}{0.2}$	-20	40-20=20
$\frac{1}{1+\jmath 0.02}$	$\omega_{c2} = \frac{1}{0.02}$	-20	20-20=0

TABLE 1.3.2: Magnitude

$$\phi = \angle G(\jmath\omega) = 180° - tan^{-1}(0.2\omega) - tan^{-1}(0.02\omega) \tag{1.3.23}$$

The phase angle of G($\jmath\omega$) are calculated for various value of ω in Table 1.3.3. The magnitude and phase plot are generated in Fig. 1.3.3 using the following python code

```
codes/es17btech11002.py
```

$\because$ the gain crossover frequency is 2 and the corresponding gain At $\omega = 2$ is 13dB,

$$20\log K = -13db \tag{1.3.24}$$

$$\implies K = 0.65 \tag{1.3.25}$$

ω	$\tan^{-1}(0.2\omega)$	$\tan^{-1}(0.02\omega)$	$\phi = \angle G(j\omega)$
0.5	5.7	0.6	174
1	11.3	1.1	168
2	21.8	2.3	156
5	45	5.7	130
10	63.4	11.3	106
50	84.3	45	50

TABLE 1.3.3: Phase

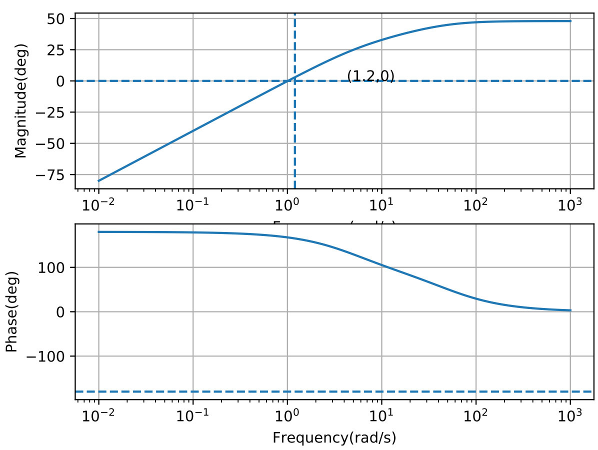

Fig. 1.3.3: Graphs

Solving (1.3.19) or from Fig. 1.3.3, the gain crossover frequency,

$$\omega_{gc} = 1.2 \tag{1.3.26}$$

$$\implies PM = 344.8 \tag{1.3.27}$$

From Fig. 1.3.3 ,we can say that phase never crosses $-180°$. So, the gain margin is *infinite*. Which means we can add any gain, and the equivalent closed loop system never goes unstable.

1.3.4. Sketch the Bode magnitude and phase plots for

$$G(s) = \frac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)} \tag{1.3.28}$$

Also compute the gain margin and phase margin.

Solution:

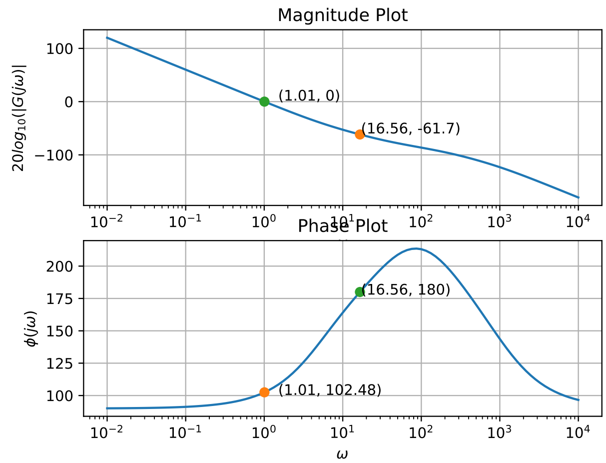

Fig. 1.3.4: Bode plot

From fig. 1.3.4,

$$\omega_{gc} = 16.55, \text{ Gain Margin} = -61.7dB \tag{1.3.29}$$

$$\omega_{pc} = 1, \text{ Phase Margin} = -77.52^0 \tag{1.3.30}$$

The program for plotting bode plot and finding phase margin and gain margin -

```
codes/ee18btech11039.py
```

1.3.5. Sketch the bode magnitude and phase plots for the closed loop (negative feedback) system given by:

$$G(s) = \frac{100(s+2)(s+4)}{s^2 - 3s + 10} \tag{1.3.31}$$

$$H(s) = \frac{1}{s} \tag{1.3.32}$$

Solution: The system can be represented as: The closed loop transfer function of the system is given by:

$$G_m(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s).H(s)} \tag{1.3.33}$$

$$= \frac{100s(s+2)(s+4)}{s^3 + 97s^2 + 610s + 800} \tag{1.3.34}$$

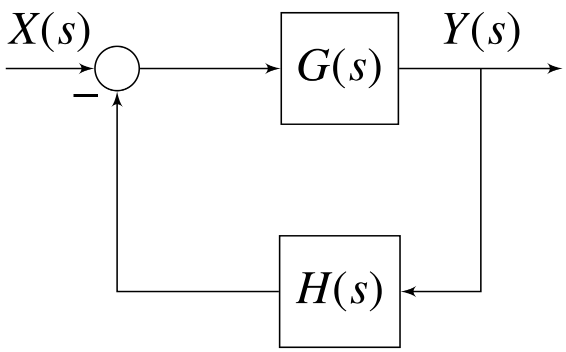

Fig. 1.3.5: Block diagram for the system

Evaluate at $s = \jmath\omega$:

$$G_m(\jmath\omega) = \frac{100\jmath\omega(\jmath\omega+2)(\jmath\omega+4)}{(\jmath\omega)^3 + 97(\jmath\omega)^2 + 610(\jmath\omega) + 800} \tag{1.3.35}$$

$$= \frac{-600\omega^2 + \jmath\left(800\omega - 100\omega^3\right)}{800 - 97\omega^2 + \jmath\left(610\omega - \omega^3\right)} \tag{1.3.36}$$

From (1.3.36):

$$\left|G_m(\jmath\omega)\right| = \frac{\sqrt{\left(600\omega^2\right)^2 + \left(800\omega - 100\omega^3\right)^2}}{\sqrt{\left(800 - 97\omega^2\right)^2 + \left(610\omega - \omega^3\right)^2}} \tag{1.3.37}$$

$$\underline{/G_m}(\jmath\omega) = \tan^{-1}\left(\frac{\omega^2 - 8}{6\omega}\right) - \tan^{-1}\left(\frac{610\omega - \omega^3}{800 - 97\omega^2}\right) \tag{1.3.38}$$

The following code plots the bode magnitude and phase plots in Fig. 1.3.6:

```
codes/ee18btech11045/
    ee18btech11045_bode1.py
```

$$G(\jmath\omega)H(\jmath\omega) = \left(\frac{100(\jmath\omega+2)(\jmath\omega+4)}{(\jmath\omega)^2 - 3\jmath\omega + 10}\right)\left(\frac{1}{\jmath\omega}\right) \tag{1.3.39}$$

$$= \frac{100\left(-\omega^2 + 8 + \jmath 6\omega\right)}{3\omega^2 + \jmath\left(10\omega - \omega^3\right)} \tag{1.3.40}$$

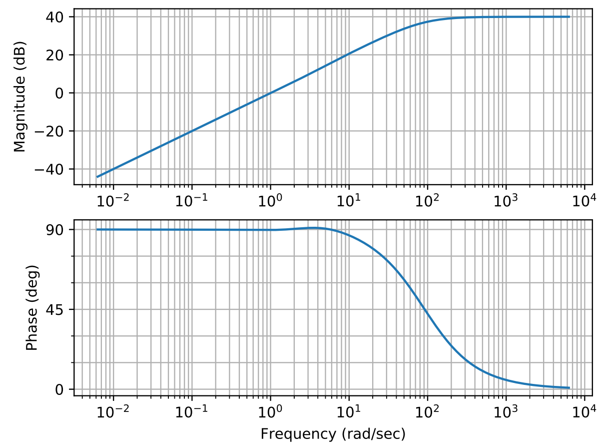

Fig. 1.3.6: Bode plot for $G_m(\jmath\omega)$

Using (1.3.40)

$$\left|G(\jmath\omega)H(\jmath\omega)\right| = \frac{100\sqrt{\left(8-\omega^2\right)^2 + (6\omega)^2}}{\sqrt{\left(3\omega^2\right)^2 + \left(10\omega - \omega^3\right)^2}} \tag{1.3.41}$$

$$\underline{/G}(\jmath\omega)H(\jmath\omega) = \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) - \tan^{-1}\left(\frac{10-\omega^2}{3\omega}\right) \tag{1.3.42}$$

At the phase crossover frequency ω_{pc}:

$$\left|\underline{/G}(\jmath\omega)H(\jmath\omega)\right| = 180 \tag{1.3.43}$$

$$\implies \tan^{-1}\left(\frac{6\omega_{pc}}{8-\omega_{pc}^2}\right) - \tan^{-1}\left(\frac{10-\omega_{pc}^2}{3\omega_{pc}}\right) = 180 \tag{1.3.44}$$

Solving the above equation:

$$\frac{6\omega_{pc}}{8-\omega_{pc}^2} = \frac{10-\omega_{pc}^2}{3\omega_{pc}} \tag{1.3.45}$$

$$\implies \omega_{pc} = 5.8 rad/sec \tag{1.3.46}$$

$$\left|G(\jmath\omega)H(\jmath\omega)\right|_{\omega=\omega_{pc}} = 28.1 dB \tag{1.3.47}$$

Gain Margin GM :

$$GM = 0 - \left|G(\jmath\omega) H(\jmath\omega)\right|_{\omega=\omega_{pc}} dB \quad (1.3.48)$$
$$= -28.1dB \quad (1.3.49)$$

At the gain crossover frequency ω_{gc}:

$$\left|G(\jmath\omega) H(\jmath\omega)\right|_{\omega=\omega_{gc}} = 1 \quad (1.3.50)$$

From (1.3.41),

$$10^4\left(\left(8-\omega^2\right)^2 + (6\omega)^2\right) = 9\omega^4 + \left(10\omega - \omega^3\right)^2 \quad (1.3.51)$$

$$\implies \omega_{gc} = 100.15 rad/sec \quad (1.3.52)$$

Substitute ω_{gc} in (1.3.42):

$$\angle G(\jmath\omega) H(\jmath\omega)_{\omega=\omega_{gc}} = 265^\circ \quad (1.3.53)$$

Phase Margin PM:

$$PM = 180^\circ - \angle G(\jmath\omega) H(\jmath\omega)_{\omega=\omega_{gc}} \quad (1.3.54)$$
$$= 180^\circ - 265^\circ = -85^\circ \quad (1.3.55)$$

The following code is used to verify the gain and phase margins:

```
codes/ee18btech11045/
    ee18btech11045_bode2.py
```

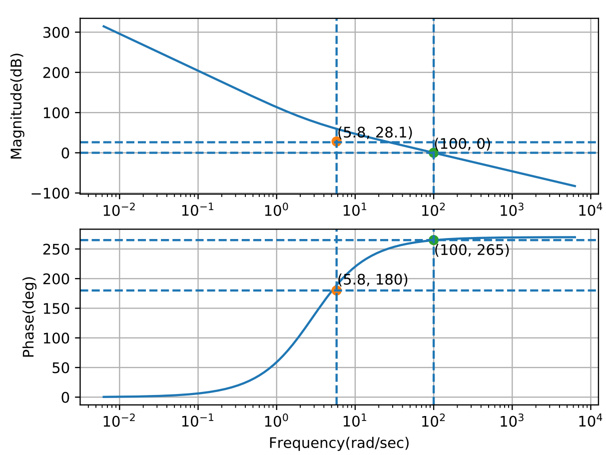

Fig. 1.3.7: Bode plot for $G(\jmath\omega)H(\jmath\omega)$

As both the Gain Margin (GM) and Phase Margin (PM) are found to be negative, the system is unstable.

1.4 Transient Response from Bode Plot

1.4.1. Consider the following transfer functions as open-loop transfer functions in two different unity feedback(negative) systems.

$$G(s) = \frac{50(s+3)(s+5)}{s(s+2)(s+4)(s+6)} \quad (1.4.1.1)$$

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} \quad (1.4.1.2)$$

Estimate transient response of these systems from their respective bode plots.

Solution:

a) The dominant pole approximation is used to characterize higher order systems because it is difficult to characterize and analyse systems with order greater than 3.

b) Consider a transfer function.

$$H(s) = K\frac{\alpha\beta}{(s+\alpha)(s+\beta)} \quad (1.4.1.3)$$

It has two poles $-\alpha$ and $-\beta$. If the magnitude of β is very large compared to α (typically if $\frac{|\beta|}{|\alpha|} > 5$) we can approximate for the transfer function assuming s is sufficiently small compared to β as follows.

$$H(s) = K_2\left(\frac{1}{s+\alpha}\right) \quad (1.4.1.4)$$

Note that the value of $H(0)$ should be unchanged for the exact and approximate transfer functions.This is necessary to ensure that the final value of the step response is unchanged.

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (1.4.1.5)$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sU(s)H(s) = H(0) \quad (1.4.1.6)$$

In order to acheive this we adjust the gain value of the approximated transfer function by equating $H(0)$ values.

$$\implies H(s) = K\frac{\alpha}{(s+\alpha)} \quad (1.4.1.7)$$

c) In terms of poles, the pole closer to the origin is considered as the dominating pole.As considered above,the magnitude of α is small therefore the time constant $\frac{1}{\alpha}$ will be high and reaches equilibrium slowly and vice versa in case of β.Therefore,this approximation assumes that the slowest part of the system dominates the response.The faster parts of the system are ignored.

d) Complex poles along with real poles : In this case the dominant pole(s) can be determined by comparing only the real parts.If the real part of the complex conjugate poles is greater in magnitude than the real pole, the two complex conjugate poles the dominant poles.

e) If the transfer function has zeros along with poles,we have to consider the fact that pole and zero cancel out each other if their respective magnitudes are comparable.

1.4.2. Find the closed loop transfer function of a negative unity feedback system given open loop transfer function $G(s)$.
Solution:

$$T(s) = \frac{G(s)}{1 + G(s)} \tag{1.4.2.1}$$

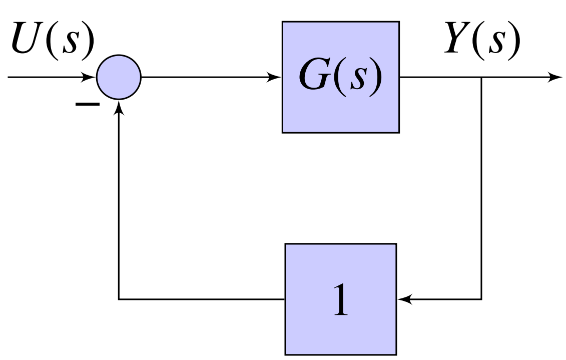

Fig. 1.4.1

1.4.3. Find the approximate transfer function for the open loop transfer function.

$$G(s) = \frac{50(s+3)(s+5)}{s(s+2)(s+4)(s+6)} \tag{1.4.3.1}$$

Solution: Using equation(1.4.2.1)

$$T(s) = \frac{50(s^2+8s+15)}{s^4+12s^3+94s^2+448s+750} \tag{1.4.3.2}$$

The following code gives the poles and zeros of the transfer function.

```
codes/ee18btech11047/ee18btech11047_1.py
```

Poles	Zeros
$p_1 = -5.14$	$z_1 = -5$
$p_2 = -3.09$	$z_2 = -3$
$p_3 = -1.87 + 6.60j$	
$p_4 = -1.87 - 6.60j$	

TABLE 1.4.1

The real poles (p_1, p_2) and zeros (z_1, z_2) cancel out each other as mentioned above.So, we are left with the two conjugate poles.The approximated transfer function is

$$T_1(s) = \frac{K_1}{(s-p_3)(s-p_4)} \tag{1.4.3.3}$$

$$T(0) = T_1(0) \tag{1.4.3.4}$$

$$\implies K_1 = p_3 p_4 \tag{1.4.3.5}$$

$$T_1(s) = \frac{47.09}{s^2+3.74s+47.09} \tag{1.4.3.6}$$

1.4.4. Estimate the transient response of the obtained second order system using the respective bode plot.
Solution: The following code generates the bode plot for open loop transfer function.

```
codes/ee18btech11047/ee18btech11047_2.py
```

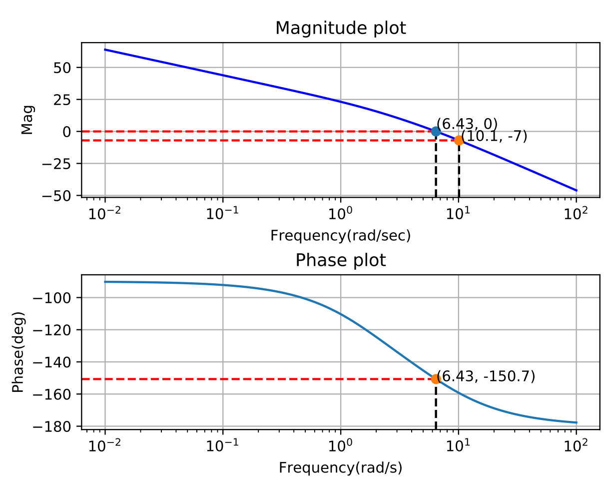

Fig. 1.4.2: 1

The phase margin is

$$\phi_M = 180^\circ - 150.7^\circ \implies \phi_M = 29.3^\circ \tag{1.4.4.1}$$

The closed-loop bandwith, ω_{BW}(-3 dB frequency), equals the frequency at which the open-loop magnitude response is around -7 dB.

$$\omega_{BW} = 10.1 rad/sec \tag{1.4.4.2}$$

Damping ratio: Substitute ϕ_M value from equation (1.4.4.1)

$$\phi_M = tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^2}}}\right) \tag{1.4.4.3}$$

$$\implies \zeta = 0.34 \tag{1.4.4.4}$$

Settling time: Substitute ω_{BW} value from equation(1.4.4.2) and ζ

$$T_s = \frac{4}{\omega_{BW}\zeta}\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \tag{1.4.4.5}$$

$$\implies T_s = 1.65 sec \tag{1.4.4.6}$$

Peak time:

$$T_p = \frac{\pi\zeta T_s}{4\sqrt{1-\zeta^2}} \tag{1.4.4.7}$$

$$\implies T_p = 0.325 sec \tag{1.4.4.8}$$

Percent overshoot:

$$\%OS = 100e^{-(\frac{\zeta\pi}{\sqrt{1-\zeta^2}})} \tag{1.4.4.9}$$

$$\implies \%OS = 35.1\% \tag{1.4.4.10}$$

Note that the answers will be approximate due to the dominant pole approximation.The following code generates the step response of the system.

```
codes/ee18btech11047/ee18btech11047_3.py
```

1.4.5. Find the approximate transfer function for the open loop transfer function

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} \tag{1.4.5.1}$$

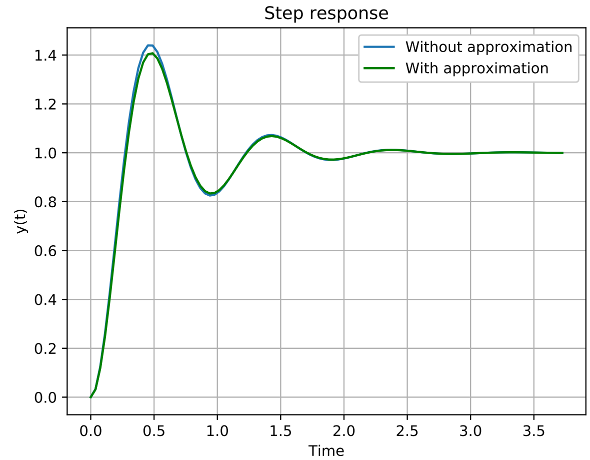

Fig. 1.4.3: 2

Solution: Using equation (1.4.2.1)

$$T(s) = \frac{75(1+0.2s)}{s^3+16s^2+115s+75} \tag{1.4.5.2}$$

The following code gives the poles and zeros of the transfer function.

```
codes/ee18btech11047/ee18btech11047_4.py
```

Poles	Zeros
$p_1 = -0.72$	$z_1 = -5$
$p_2 = -7.64 + 6.75j$	
$p_3 = -7.63 - 6.75j$	

TABLE 1.4.2

The real part of the complex conjugate poles is comparable with the zero z_1 of the transfer function.So,they cancel out each other.The approximated transfer function is of first order.

$$T_2(s) = \frac{K_2}{(s-p_1)} \tag{1.4.5.3}$$

$$T(0) = T_2(s) \tag{1.4.5.4}$$

$$\implies K_2 = p_1 \tag{1.4.5.5}$$

$$T_2(s) = \frac{0.72}{s+0.72} \tag{1.4.5.6}$$

1.4.6. Estimate the transient response of the obtained first order system.
Solution: Time constant: The time constant

is the time taken by the step response to rise to 63% of it's final value.

$$T = \frac{1}{|pole|} \tag{1.4.6.1}$$

$$T = \frac{1}{0.72} = 1.388sec \tag{1.4.6.2}$$

Rise time: Rise time is the time for the waveform to go from 0.1 to 0.9 of it's final value.

$$T_r = \frac{2.2}{|pole|} \tag{1.4.6.3}$$

$$T_r = \frac{2.2}{0.72} = 3.05sec \tag{1.4.6.4}$$

Settling time: Settling time is defined as the time for the response to reach and stay within, 2% of its final value.

$$T_s = \frac{4}{|pole|} \tag{1.4.6.5}$$

$$T_s = \frac{4}{0.72} = 5.55sec \tag{1.4.6.6}$$

The following code plots the step response of the system.

codes/ee18btech11047/ee18btech11047_5.py

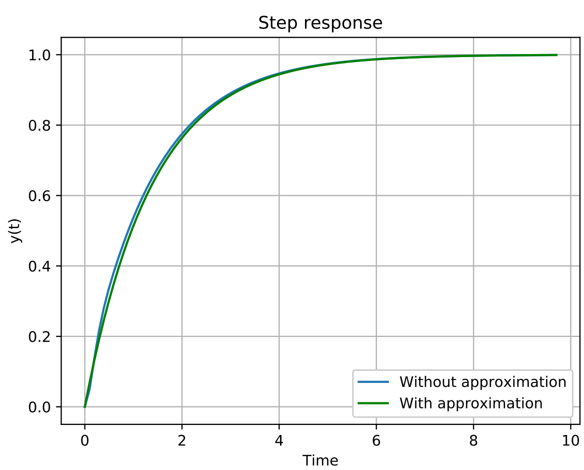

Fig. 1.4.4

2 Stability in Frequency Domain

2.1 Nyquist Criterion

2.1.1. Using Nyquist Criterion, find out whether this system is stable or not.

$$G(s) = \frac{50}{s(s+3)(s+6)} \tag{2.1.1.1}$$

$$H(s) = 1. \tag{2.1.1.2}$$

Nyquist Stability:

$$N = Z - P \tag{2.1.1.3}$$

where Z is number of unstable poles of closed loop transfer function, P is number of unstable poles of open loop transfer function. and N is number of clockwise encirclement of $-1 + j0$. Closed Loop Transfer Function:

$$T(s) = \frac{50}{s^3 + 9s^2 + 18s + 50} \tag{2.1.1.4}$$

$$Z = 0, P = 0 \tag{2.1.1.5}$$

$$N = 0 \tag{2.1.1.6}$$

Thus, system is stable, which can be verified from Nyquist Plot in Fig 2.1.1

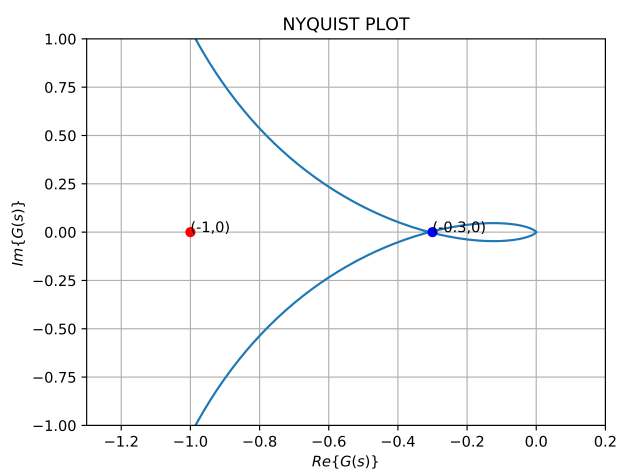

Fig. 2.1.1

The following code generates Fig 2.1.1

codes/ee18btech11050_1.py

2.1.2. Using Nyquist criterion find the range of K for which closed loop system is stable.

$$G(s) = \frac{K}{s(s+6)} \tag{2.1.2.1}$$

$$H(s) = \frac{1}{s+9} \tag{2.1.2.2}$$

Solution: The system flow can be described as,

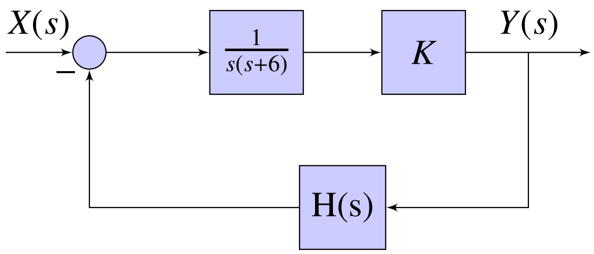

Fig. 2.1.2

$$G_1(s) = \frac{1}{s(s+6)}. \tag{2.1.2.3}$$

For Nyquist plot,

$$\text{Im}\left\{G_1(\jmath\omega)H(\jmath\omega)\right\} = \frac{-(54-\omega^2)}{(\omega)(\omega^2+56)(\omega^2+81)} \tag{2.1.2.4}$$

$$\text{Re}\left\{G_1(\jmath\omega)H(\jmath\omega)\right\} = \frac{-15\omega}{(\omega)(\omega^2+56)(\omega^2+81)} \tag{2.1.2.5}$$

From (2.1.2.4) and (2.1.2.5)

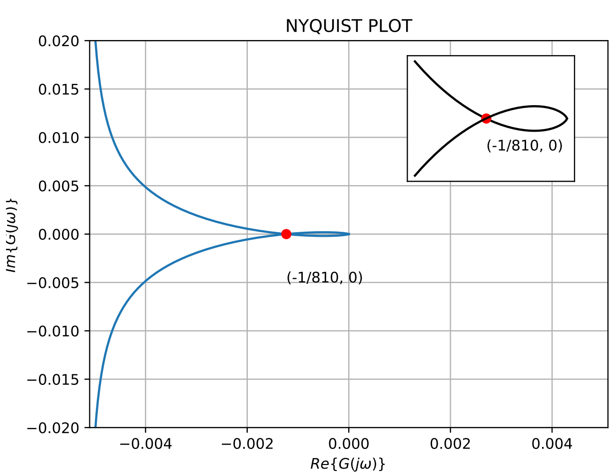

Fig. 2.1.3: Nyquist plot for $G_1(s)H(s)$

Nyquist Stability Criterion:

$$N = Z - P \tag{2.1.2.6}$$

where Z is # unstable poles of closed loop transfer function, P is # unstable poles of open loop transfer function and N is # clockwise encirclement of $(-1/K, 0)$.

For stable system,

$$Z = 0 \tag{2.1.2.7}$$

From (2.1.2.2) and (2.1.2.3),

$$P = 0 \tag{2.1.2.8}$$
$$\implies N = 0 \tag{2.1.2.9}$$

Since, there is a zero at origin, an infinite radius half circle will enclose the right hand side of end points of the Nyquist plot. So for (2.1.2.9),

$$\implies \frac{-1}{K} < \frac{-1}{810} \implies K < 810 \tag{2.1.2.10}$$

And also,

$$K > 0 \tag{2.1.2.11}$$
$$\implies 0 < K < 810 \tag{2.1.2.12}$$

The following python code generates Fig. 2.1.3

```
codes/ee18btech11028_1.py
```

2.1.3. Using Nyquist criterion, find out whether the system below is stable or not

$$G(s) = \frac{41}{s^2(s+3)} \tag{2.1.3.1}$$

$$H(s) = (s+4) \tag{2.1.3.2}$$

Solution: According to the Nyquist criteria the number of unstable closed-loop poles (Z) is equal to the number of unstable open-loop poles (P) plus the number of clockwise encirclements (N) of the point (-1,j0) of the Nyquist plot of G(s)H(s), i.e

$$Z = N + P \tag{2.1.3.3}$$

Open loop transfer function :

$$G(s)H(s) = \frac{41(s+4)}{s^2(s+3)} \tag{2.1.3.4}$$

Closed loop transfer function:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{41}{s^3+3s^2+41s+164} \tag{2.1.3.5}$$

In Fig.2.1.4 it can be seen that there is a

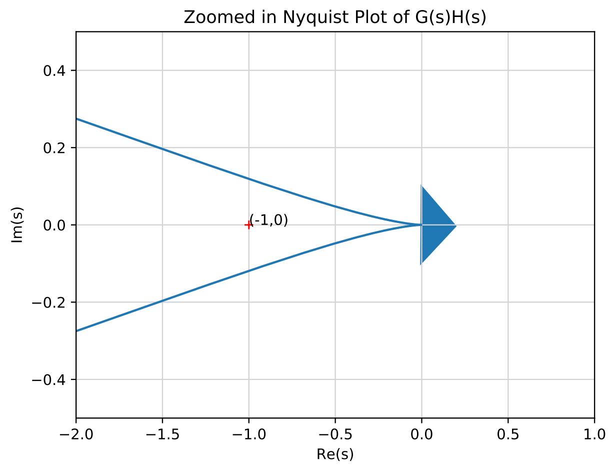

Fig. 2.1.4

clockwise encirclement around (-1+0j). As the open loop transfer function has zero pole of multiplicity 2, therefore it should be assumed that the phasor travels 2 times clock-wise along a semicircle of infinite radius.

N=2, P=0

$$\implies Z = 2 \tag{2.1.3.6}$$

Therefore, The system T(s) is unstable as it has two poles on the right side of the s plane. The following code generates the nyquist plot

```
codes/ee18btech11041.py
```

2.1.4. Using Nyquist criterion, find out whether the following is stable or not.

$$G(s) = \frac{100(s+5)}{s(s^2+4)(s+3)} \tag{2.1.4.1}$$

$$H(s) = 1 \tag{2.1.4.2}$$

Solution: Open loop transfer function (oltf):

$$G(s)H(s) = \frac{100(s+5)}{s(s^2+4)(s+3)} \tag{2.1.4.3}$$

Closed loop transfer function (cltf):

$$\frac{G(s)}{1+G(s)H(s)} \tag{2.1.4.4}$$

Nyquist Stability Criterion can be expressed as:

$$Z = N + P \tag{2.1.4.5}$$

where:

- Z = zeros of $1 + G(s)H(s)$ in RHS of s-plane
- N = number of encirclement of critical point $1{+}0j$ in the clockwise direction.
- P = poles of G(s)H(s) in RHS of s-plane.

The pole-zero plot of equation (2.1.4.3) is fig. (2.1.5) which gives **P = 0**.

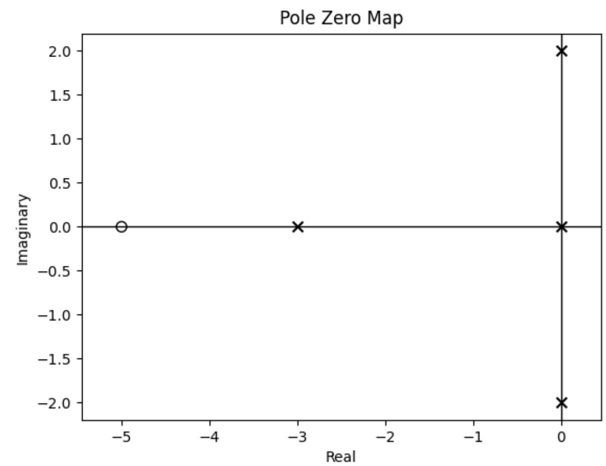

Fig. 2.1.5

- Since the multiplicity of zero pole is 1 (fig.2.1.5), it should be assumed that the phasor travels one time clockwise along a semicircle of infinite radius.
- Same applies for poles at $2j$ and -$2j$.
- Fig. (2.1.6) shows a schematic, the dotted lines are infinite radii semi-circles.
- The point -1+0j is not encircled by the nyquist plot (fig. 2.1.8).

From the nyquist plot (fig. 2.1.8), -1+$0j$ is not encircled by the plot. So from above points, the only clockwise encirclement is considered due to the mentioned poles (zero, $2j$ and -$2j$) with multiplicity of 1.
Therefore, **N=2**
Substituting values of P = 0 and N = 2 in equation (2.1.4.5):

$$\implies Z = 2 \tag{2.1.4.6}$$

This is verified using pole zero plot of 1+G(s)H(s) (fig. 2.1.9). Two zeroes on RHS of s-plane i.e. Z=2.
Since $Z \neq 0$, **the closed loop system is unstable**.
The **open loop system is stable** as there are **no poles on RHS** of s-plane.

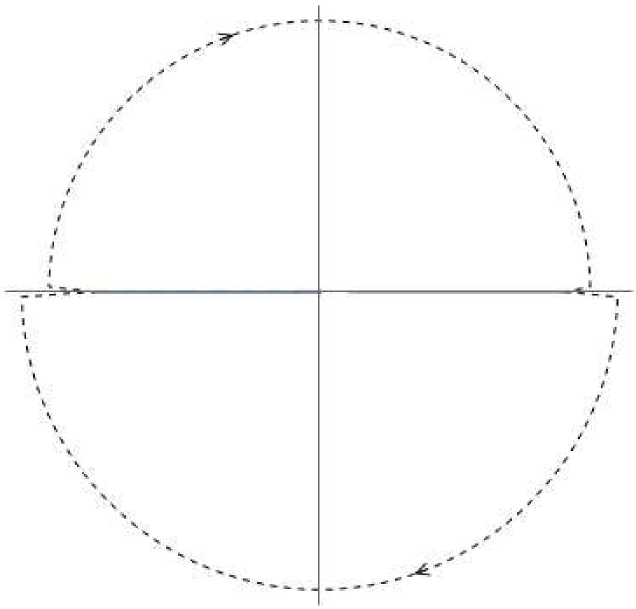

Fig. 2.1.6

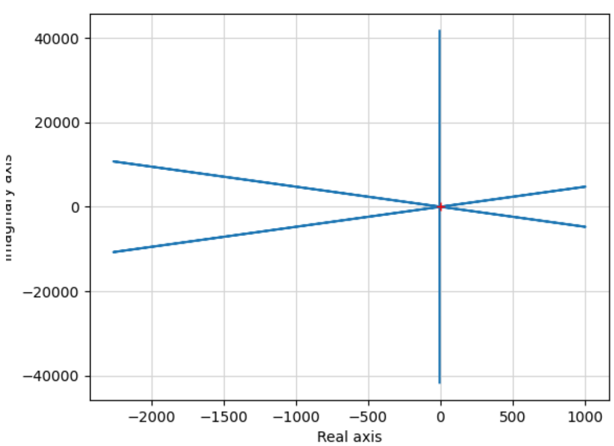

Fig. 2.1.7: Nyquist plot of G(s)H(s)

The following code plots the pole zero plot and the nyquist plot.

```
codes/ee18btech11025.py
```

2.1.5. Using Nyquist criterion, find out whether the system below is stable or not.

$$G(s) = \frac{20}{s(s+1)}, H(s) = \frac{s+3}{s+4} \qquad (2.1.5.1)$$

Solution: The following python code generates the Nyquist plot in Fig.2.1.10.

```
codes/ee18btech11011.py
```

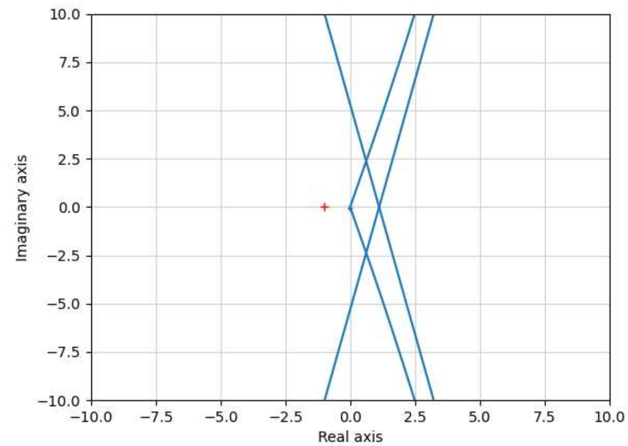

Fig. 2.1.8: Zoomed in

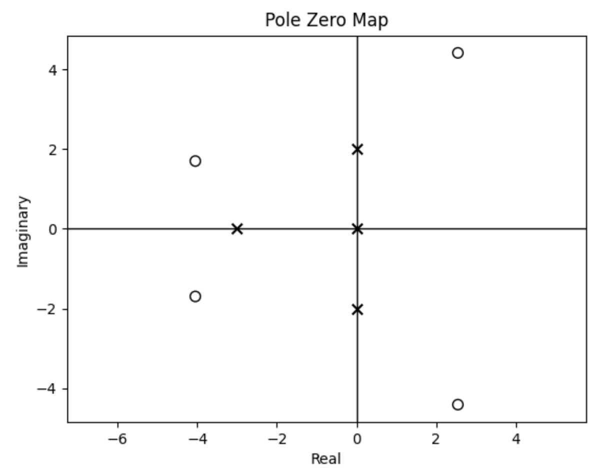

Fig. 2.1.9

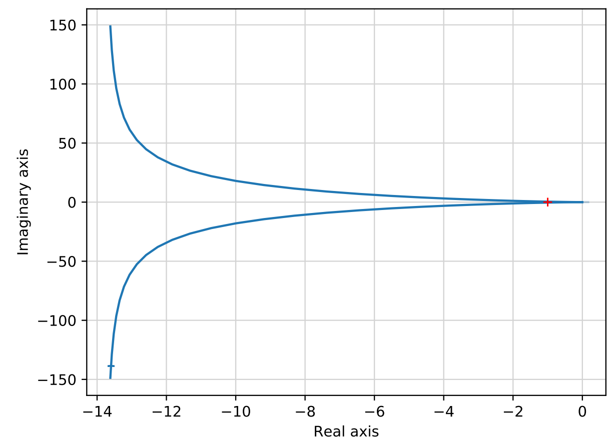

Fig. 2.1.10: Nyquist Plot

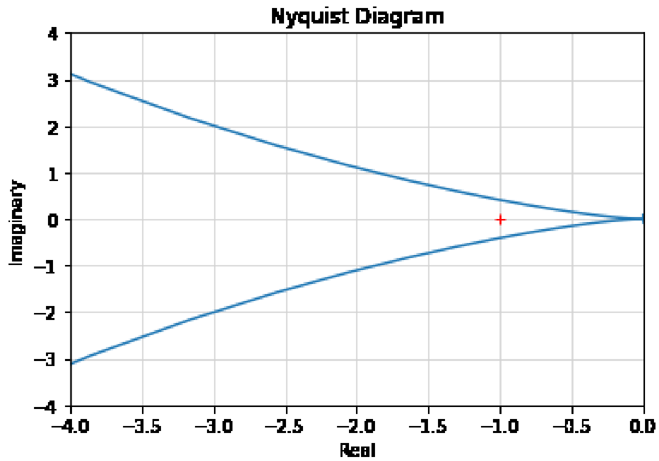

Fig. 2.1.11: Zoomed image

The closed loop system the transfer function will be =

$$\frac{G(s)}{1+G(s)H(s)} \tag{2.1.5.2}$$

$$\implies G(s)H(s) = \frac{20(s+3)}{s(s+1)(s+4)} \tag{2.1.5.3}$$

So it has 3 open-loop poles 0,-1 and -4, therefore P=0.Further we know that N = Z-P , now we know Z = Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane.To find the poles we can use the following Routh Hurwitz python code.Using this we get Z = 0.

```
codes/ee18btech11011_1.py
```

$$P = 0, Z = 0 \tag{2.1.5.4}$$

$$\implies N = 0 \tag{2.1.5.5}$$

This can also be seen from the Fig. 2.1.10 that the encirclement is counter-clockwise not clockwise.Hence the system is stable.

2.1.6. Using Nyquist criterion, find out the range of K for which the closed loop system will be stable.

$$G(s) = \frac{K(s+2)(s+4)}{s^2-3s+10}; H(s) = \frac{1}{s} \tag{2.1.6.1}$$

Solution: The system flow can be described by Fig. 2.1.12 From (2.1.6.1),

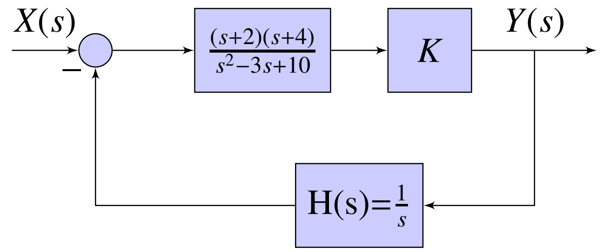

Fig. 2.1.12

$$G(s)H(s) = \frac{K(s+2)(s+4)}{s(s^2-3s+10)} \tag{2.1.6.2}$$

$$G(\jmath\omega)H(\jmath\omega) = \frac{K(j\omega+2)(j\omega+4)}{j\omega((10-\omega^2)-3j\omega)} \tag{2.1.6.3}$$

$$\text{Re}\left\{G(\jmath\omega)H(\jmath\omega)\right\} = \frac{K(84\omega^2-9\omega^4)}{\omega^6-11\omega^4+100\omega^2} \tag{2.1.6.4}$$

$$\text{Im}\left\{G(\jmath\omega)H(\jmath\omega)\right\} = \frac{K(-\omega^5+36\omega^3-80\omega)}{\omega^6-11\omega^4+100\omega^2} \tag{2.1.6.5}$$

The Nyquist plot is a graph of Re $\{G(\jmath\omega)H(\jmath\omega)\}$ vs Im $\{G(\jmath\omega)H(\jmath\omega)\}$. Let's take K =1 and draw the nyquist plot .

The following python code generates the Nyquist plot in Fig. 2.1.13

```
codes/ee18btech11016.py
```

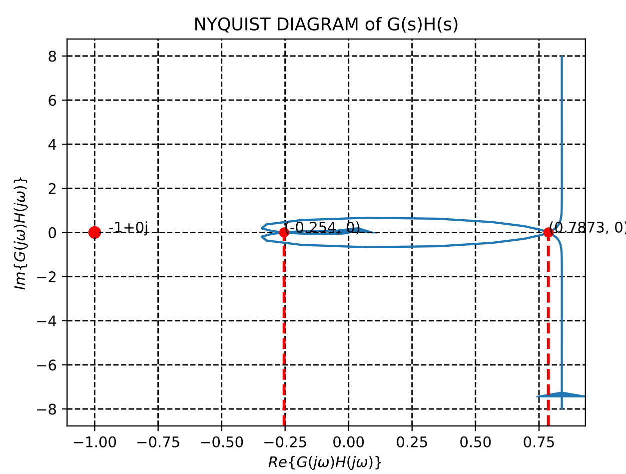

Fig. 2.1.13

Note that this nyquist plot is plotted when K=1.

Nyquist criterion-For the stable system :

$$Z = P + N = 0, \tag{2.1.6.6}$$

Variable	Description
Z	Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane
N	No of encirclements of $G(s)H(s)$ about -1+j0 in the Nyquist plot
P	Poles of $G(s)H(s)$ in right half of s plane

TABLE 2.1.1

Since from the equation (2.1.6.2), P = 2 as the number of poles on right hand side of s-plane is equal to 2 .So, for Z to be equal to 0 ,we have to choose the range of K such that N should be equal to -2.
If we consider the Nyquist plot with K term i.e. of equation (2.1.6.2) , then it will cut x-axis at x = -0.254K , x = 0 and at x = 0.7873K (as we have nyquist graph at K=1, now we just need to multiply the intersected coordinates on x-axis by K).

So, we have to make sure that $(-1+\ j0)$ should be included in between x = -0.254K to x = 0, because then only N = -2 (as the no. of encirclements are 2 in anticlockwise direction in this case so N=-2)

$$-0.254K < -1 < 0 \tag{2.1.6.7}$$

So,

$$K > \frac{1}{0.254} \tag{2.1.6.8}$$

i.e.

$$K > 3.937 \tag{2.1.6.9}$$

Hence K > 3.937 ensures that the system is stable as no. of poles on the right hand side of s-plane (in this case) is 0.

2.2 Nyquist Criterion

2.2.1. Using Nyquist criterion, find out the range of K for which the closed loop system will be stable.

$$G(s) = \frac{K}{(s+1)(s+3)}$$
$$H(s) = \frac{1}{(s+5)(s+7)} \tag{2.2.1.1}$$

The system flow can be described by Fig. 2.2.1
From (2.2.1.1),

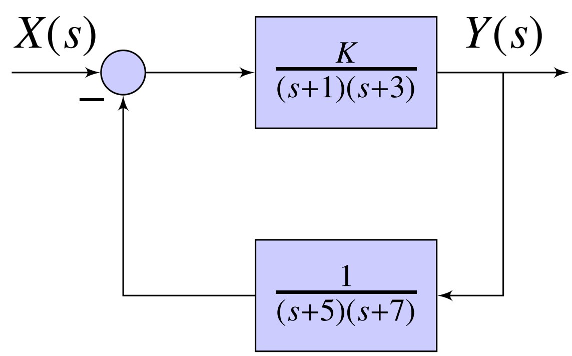

Fig. 2.2.1

$$L(s) = G(s)H(s)$$
$$= \frac{K}{(s+1)(s+3)(s+5)(s+7)} \tag{2.2.1.2}$$

$$L(j\omega) = G(\jmath\omega)H(\jmath\omega)$$
$$= \frac{K}{(j\omega+1)(j\omega+3)(j\omega+5)(j\omega+7)} \tag{2.2.1.3}$$

The Nyquist plot is a graph of $\text{Re}\{L(jw)\}$ vs $\text{Im}\{L(j\omega)\}$. Let's take K=1 and draw the nyquist plot.

The following python code generates the Nyquist plot.

```
/codes/es17btech11015.py
```

The Fig. 2.2.2 shows the Nyquist plot for K = 1
Nyquist criterion-For the stable system :

$$Z = P + N = 0, \tag{2.2.1.4}$$

where,
Z = Poles of $\frac{G(s)}{1+G(s)H(s)}$ in right half of s plane

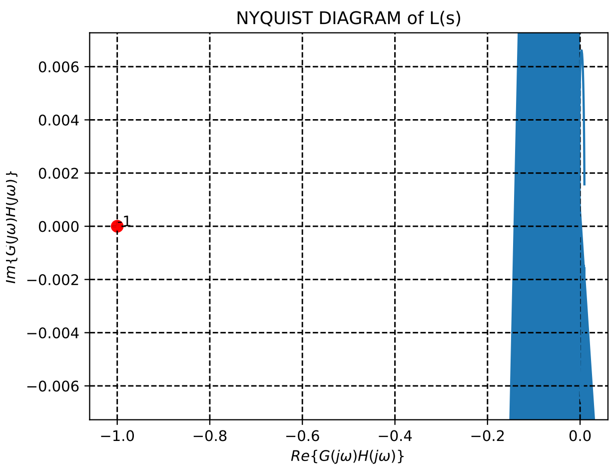

Fig. 2.2.2

P = Poles of $G(s)H(s)$ in right half of s plane

N = No. of encirclements of $G(s)H(s)$ about -1 in the Nyquist plot

Since from the equation (2.2.1.2), P = 0

So, for Z to be equal to 0 ,we have to choose the range of K such that N is equal to 0. From the figure 2.2.2, we can observe that the plot is not cutting the x-axis. If we consider the Nyquist plot with K term even then the plot won't cut the x-axis.
So, N = 0 irrespective of K.
Therefore, the system is stable for

$$-\infty < K < \infty \tag{2.2.1.5}$$

2.3

2.4 Nyquist and Routh-Hurwitz

2.4.1. In the block diagram Fig.2.4.1

$$G(s) = \frac{K}{(s+4)(s+5)} \tag{2.4.1.1}$$

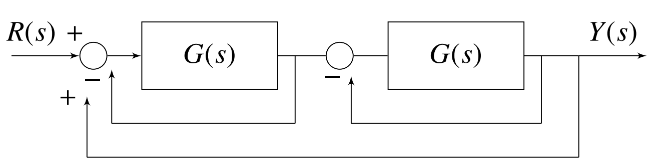

Fig. 2.4.1

2.4.2. Find the range of K for stability by Nyquist criterion
Solution:

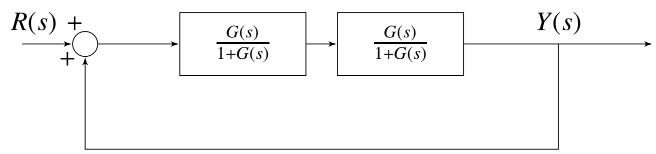

Fig. 2.4.2

The open loop transfer function from Fig.2.4.2

$$T(s) = \left(\frac{\frac{K}{(s+4)(s+5)}}{1+\frac{K}{(s+4)(s+5)}}\right)^2 \tag{2.4.2.1}$$

$$T(\jmath\omega) = \left(\frac{\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}{1+\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}\right)^2 \tag{2.4.2.2}$$

- Since it is connected in positive feedback the transfer function cuts at $(1, \jmath 0)$

$$\implies \text{Re}\{T(\jmath\omega)\} = 1 \tag{2.4.2.3}$$
$$\implies \text{Im}\{T(\jmath\omega)\} = 0 \tag{2.4.2.4}$$

$$\left(\frac{\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}{1+\frac{K}{(\jmath\omega+4)(\jmath\omega+5)}}\right)^2 = 1 + \jmath 0 \tag{2.4.2.5}$$

$$(\jmath\omega+4)(\jmath\omega+5) + 2K = 0 \tag{2.4.2.6}$$

$$-\omega^2 + 9\jmath\omega + 20 + 2K = 0 \tag{2.4.2.7}$$

From (2.4.2.4)

$$20 + 2K = 0 \tag{2.4.2.8}$$
$$\implies K = -10 \tag{2.4.2.9}$$

The minimum value of stability for the system to be stable is

$$K_{min} > -10 \tag{2.4.2.10}$$

The range of K for which the system is stable is

$$-10 < K < \infty \tag{2.4.2.11}$$

2.4.3. From the table.2.4.1, Stability criterion for K is N+P=Z

K	P	N	Z	Description
-10	0	0	0	System is marginally stable
-9	0	0	0	System is stable
-11	0	1	1	System is unstable

TABLE 2.4.1

2.4.4. Verify the Nyquist plots by

```
codes/ee18btech11029_1.py
```

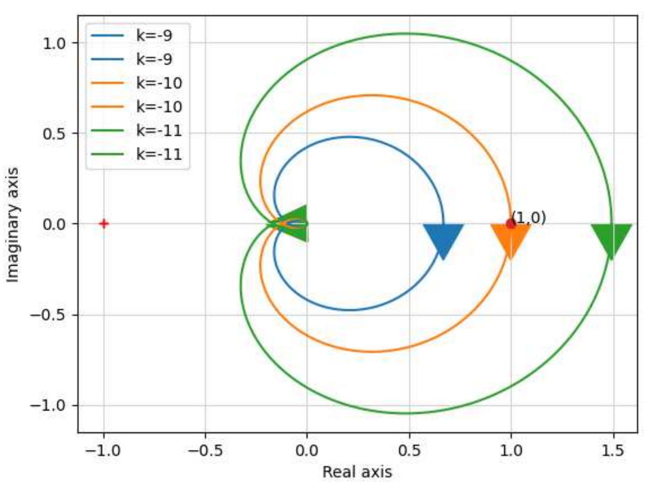

Fig. 2.4.3: Nyquist Plot

2.4.5. Verify the result using Routh-Hurwitz criterion
Solution: The characteristic equation is

$$1 - T(s) = 0 \tag{2.4.5.1}$$

$$1 - \left(\frac{\frac{K}{(s+4)(s+5)}}{1 + \frac{K}{(s+4)(s+5)}}\right)^2 = 0 \tag{2.4.5.2}$$

$$1 + 2\left(\frac{K}{(s+4)(s+5)}\right) = 0 \tag{2.4.5.3}$$

$$s^2 + 9s + 20 + 2K = 0 \tag{2.4.5.4}$$

$$\begin{vmatrix} s^2 \\ s^1 \\ s^0 \end{vmatrix}\begin{vmatrix} 1 & 20+2K \\ 9 & 0 \\ 20+2K & 0 \end{vmatrix} \tag{2.4.5.5}$$

For a system to be stable it should not have any sign changes

$$20 + 2K > 0 \tag{2.4.5.6}$$

This is valid for all positive values of K but the minimum value of K is

$$K > -10 \tag{2.4.5.7}$$

So the range of K for stability is

$$-10 < K < \infty \tag{2.4.5.8}$$

2.4.6. Verify the result by

```
codes/ee18btech11029_2.py
```

2.5 Nyquist

Consider the system shown in Fig. 2.5.1 below. Sketch the nyquist plot of the system when

1) $G_c(s) = 1$
2) $G_c(s) = 1 + \frac{1}{s}$

and determine the maximum value of K for stability. Take

$$G(s) = \frac{K}{s(1+s)(1+4s)} \tag{2.5.1}$$

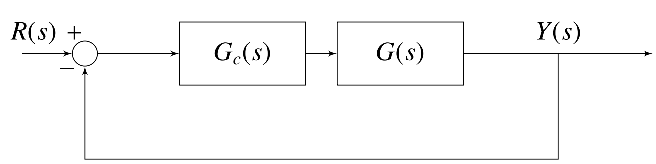

Fig. 2.5.1

Solution: For $G_c(s) = 1$,

The open loop transfer function is

$$G_c(s)G(s) = \frac{K}{s(1+s)(1+4s)} \tag{2.5.2}$$

$$G_c(\jmath\omega)\,G(\jmath\omega) = \frac{K}{\jmath\omega\,(1+\jmath\omega)\,(1+4\jmath\omega)} \tag{2.5.3}$$
$$= \frac{K}{\jmath\omega\,(1-4\omega^2+5\jmath\omega)} \tag{2.5.4}$$
$$= \frac{K\left(-5\omega-\jmath\left(1-4\omega^2\right)\right)}{\omega\left((1-4\omega^2)^2+25\omega^2\right)} \tag{2.5.5}$$

The maximum K for stability is where the nyquist plot of open loop transfer function cuts the coordinate $(-1, \jmath 0)$

$$\implies \text{Re}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = -1 \tag{2.5.6}$$
$$\implies \text{Im}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = 0 \tag{2.5.7}$$

$$\implies \text{Re}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = \frac{-5K\omega}{\omega\left((1-4\omega^2)^2+25\omega^2\right)} \tag{2.5.8}$$

$$\implies \text{Im}\left\{G(\jmath\omega)\,G_c(\jmath\omega)\right\} = \frac{-K\left(1-4\omega^2\right)}{\omega\left((1-4\omega^2)^2+25\omega^2\right)} \tag{2.5.9}$$

From (2.5.9) and (2.5.7)

$$1-4\omega^2 = 0 \implies \omega = \frac{1}{2} \tag{2.5.10}$$

From (2.5.8),(2.5.6) and substituting $\omega = \frac{1}{2}$

$$\frac{-5K\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{25}{4}\right)} = -1 \implies K = \frac{5}{4} = 1.25 \tag{2.5.11}$$

For $K < 0$ the system with negative feedback is unstable the range of K is

$$0 < K < \frac{5}{4} \tag{2.5.12}$$

Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 2.5.2 The following code gives the nyquist plot

codes/ee18btech11034/ee18btech11034_1.py

Stability Criterian for K

$$N + P = Z \tag{2.5.13}$$

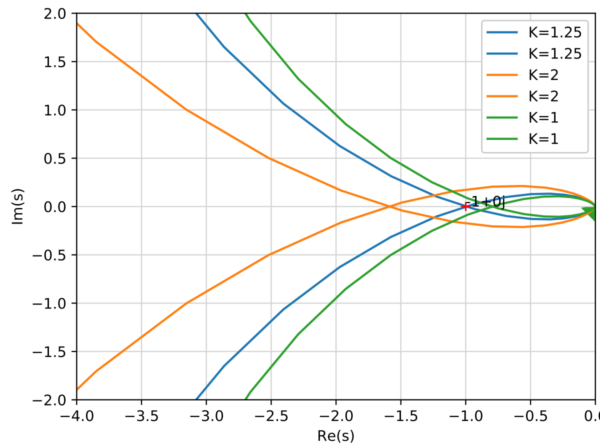

Fig. 2.5.2

K	P	N	Z	Description
1.25	0	0	0	System is marginally stable
2	0	1	1	System is unstable
1	0	0	0	System is stable

TABLE 2.5.1

From the Fig.2.5.2

$$K_{max} = \frac{5}{4} \tag{2.5.14}$$

Solution: For $G_c(s) = \frac{1+s}{s}$,

the open loop transfer function is

$$G_c(s)\,G(s) = \frac{K(s+1)}{s^2(1+s)(1+4s)} \tag{2.5.15}$$

$$G_c(s)\,G(s) = \frac{K}{s^2(1+4s)} \tag{2.5.16}$$

$$G_c(\jmath\omega)\,G(\jmath\omega) = \frac{K}{(\jmath\omega)^2(1+4\jmath\omega)} \tag{2.5.17}$$
$$= \frac{\frac{-K}{\omega^2}(1-4\jmath\omega)}{1+16\omega^2} \tag{2.5.18}$$

From (2.5.7)

$$\implies \operatorname{Im}\{G(\jmath\omega)G_c(\jmath\omega)\} = \frac{4K}{\omega(1+16\omega^2)} = 0 \tag{2.5.19}$$

This is possible when

$$K = 0 \tag{2.5.20}$$

The system is unstable for both

$$K < 0 \tag{2.5.21}$$

$$K > 0 \tag{2.5.22}$$

Sketching the Nyquist plot for $G(s)G_c(s)$ in Fig. 2.5.3 The following code gives the nyquist plot

```
codes/ee18btech11034/ee18btech11034_2.py
```

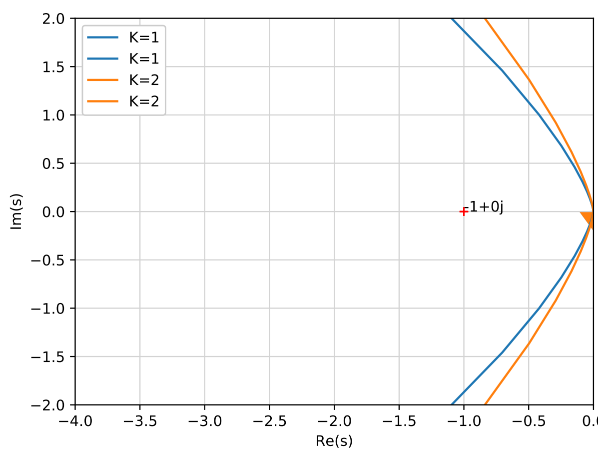

Fig. 2.5.3

From (2.5.13)

K	P	N	Z	Description
1	0	1	1	System is unstable
2	0	1	1	System is unstable

TABLE 2.5.2

From (2.5.20) K_{max} must be 0 which is not possible. Hence the system is unstable for all real K

2.6 *Nyquist*

Sketch the Nyquist plot for a closed loop system having open-loop transfer function

$$G(s)H(s) = \frac{2e^{-s\tau}}{s(1+s)(1+0.5s)} \tag{2.6.1}$$

Determine the maximum value of τ for the system to be stable.

Solution: From (2.6.1),

$$\implies \operatorname{Re}\{G(\jmath\omega)H(\jmath\omega)\} = -4\left[\frac{3\omega^2\cos(\omega\tau) - \left(\omega^3 - 2\omega\right)\sin(\omega\tau)}{(3\omega^2)^2 + (\omega^3 - 2\omega)^2}\right] \tag{2.6.2}$$

$$\implies \operatorname{Im}\{G(\jmath\omega)H(\jmath\omega)\} = 4\left[\frac{\left(\omega^3 - 2\omega\right)\cos(\omega\tau) + 3\omega^2\sin(\omega\tau)}{(3\omega^2)^2 + (\omega^3 - 2\omega)^2}\right] \tag{2.6.3}$$

Determining the stability of closed loop transfer function using Nyquist stability Criterion.

$$Z = P + N \tag{2.6.4}$$

Poles of open loop transfer function are on left half of s-plane.Therefore,$P = 0$
To ensure that the system is stable N should be 0
For maximum value of τ for stability ,the nyquist plot cuts the real axis at -1+j0.

$$G(s)H(s) = -1 + j0 \tag{2.6.5}$$

$$\operatorname{Im}\{G(\jmath\omega)H(\jmath\omega)\} = 0 \tag{2.6.6}$$

$$\operatorname{Re}\{G(\jmath\omega)H(\jmath\omega)\} = -1 \tag{2.6.7}$$

From (2.6.3) and (2.6.6)

$$\implies \tan(\omega\tau) = \frac{-\left(\omega^3 - 2\omega\right)}{3\omega^2} \tag{2.6.8}$$

From (2.6.2) and (2.6.7) and substituting $\tan(\omega\tau) = \frac{-(\omega^3-2\omega)}{3\omega^2}$

$$\implies \omega^6 + 5\omega^4 + 4\omega^2 - 16 = 0 \tag{2.6.9}$$

Solving (2.6.9) graphically.

Python code for the above plot is

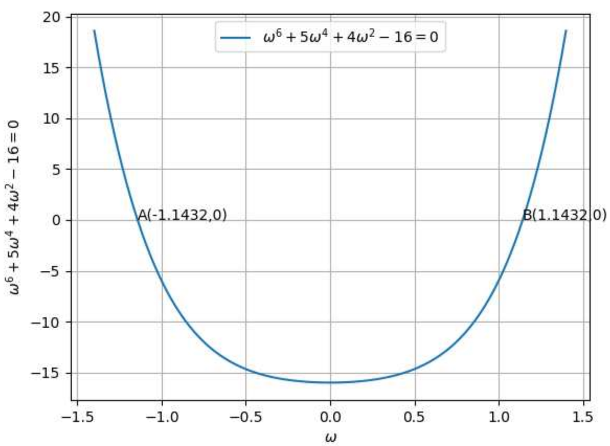

Fig. 2.6.1

```
codes/ee18btech11035_1.py
```

ω = 1.1432,-1.1432 (As, ω is positive)
Therefore,ω = 1.1432

Substituting ω in (2.6.8)

$$\tan(1.1432\tau) = 0.2021 \tag{2.6.10}$$
$$\tau = 0.1744 \tag{2.6.11}$$

The following python code generates the Nyquist plot.

```
codes/ee18btech11035_2.py
```

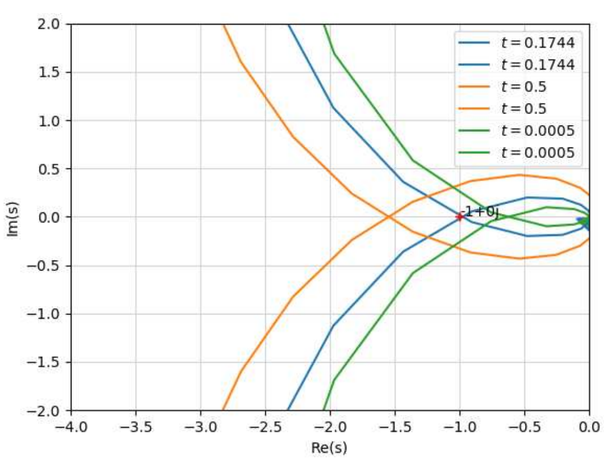

Fig. 2.6.2: Nyquist plot for variable τ

From the above figure (2.6.2) $\tau \leq 0.1744$ for a stable system.

τ	**P**	**N**	**Z**	**Description**
0.1744	0	1	1	System is Marginally stable
0.5	0	0	0	System is unstable
0.0005	0	0	0	System is stable

TABLE 2.6.1

Therefore,τ_{max} = 0.1744
.

3 Design in Frequency Domain

3.1

3.2

3.3 Lag Compensator

3.3.1. Given the unity feedback system, with

$$G(S) = \frac{K(s+10)(s+11)}{s(s+3)(s+6)(s+9)} \tag{3.3.1.1}$$

Use frequency response method to design a lag compensator to yield Kv = 1000 and peak overshoot of 15% .Use second order approximation.
Solution: Fig. 3.3.1 models the equivalent of compensated closed loop system.

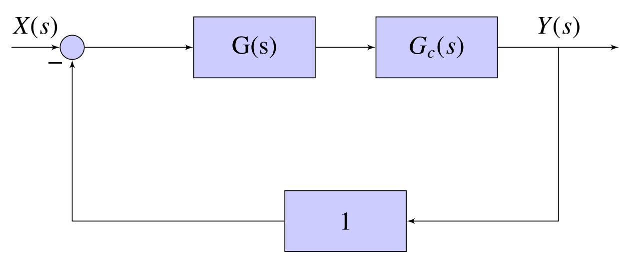

Fig. 3.3.1

Velocity constant

$$K_v = \lim_{t\to 0} sG(s) \tag{3.3.1.2}$$

$$\lim_{t\to 0} s\frac{K(s+10)(s+11)}{s(s+3)(s+6)(s+9)} = 1000 \tag{3.3.1.3}$$

$$\implies K = 1473 \tag{3.3.1.4}$$

Bode plot of G(s) for the value of k

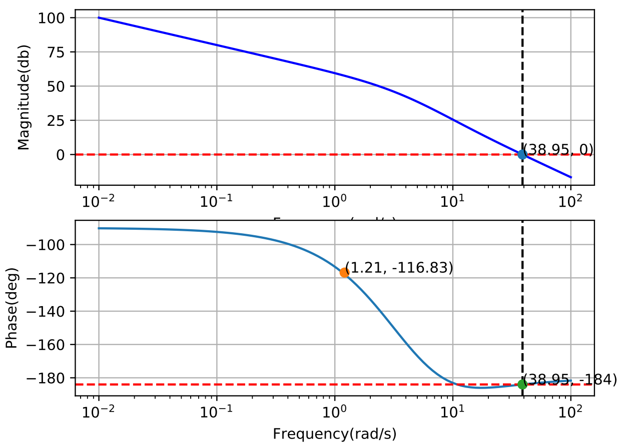

Fig. 3.3.2

The following code verifies the result.

```
codes/ee18btech11030/ee18btech11030.py
```

Relation between %OS and Damping ratio

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{(\pi)^2 + (\ln(\%OS/100))^2}} \tag{3.3.1.5}$$

$$\implies \zeta = 0.517 \tag{3.3.1.6}$$

Phase Margin for a Damping ratio is given by Eq (3.3.1.7)

$$\phi_m = 90^\circ - \arctan(\frac{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}{2\zeta} \tag{3.3.1.7}$$

$$\implies \phi_m = 53.17^\circ \tag{3.3.1.8}$$

From Fig : 3.3.2 of uncompensated system with K = 1473

$$\phi_m = -4^\circ, \omega_{PM} = 38.95 rad/sec \tag{3.3.1.9}$$

$$\phi_{ml} \text{ lies in } (5,12) \tag{3.3.1.10}$$

$$\phi_{total} = 53.17^\circ - (-4^\circ) + \phi ml \tag{3.3.1.11}$$

$$\phi_{total} = 63.17^\circ \tag{3.3.1.12}$$

Note : Adding 6 degrees phase angle to compensate the phase angle contribution of the lag compensator.

From Figure 3.3.2

$$\phi_{total} = 63.17^\circ \text{ at, } \omega_{PM} = 1.21 rad/sec \tag{3.3.1.13}$$

At this Phase Margin frequency, the magnitude plot must go through 0 dB. But The magnitude of uncompensated system at 1.21 rad/sec is 57.55 dB = 754.2

3.3.2. Designing Lag Compensator Gc(s)

Solution: General lag compensator

$$G_c(s) = \left(\frac{s + \frac{1}{T}}{s + \frac{1}{T\alpha}}\right) \quad \alpha > 1 \tag{3.3.2.1}$$

- First draw the high-frequency asymptote at -57.55 dB.So that magnitude at 1.21 rad/sec becomes 0 dB.
- Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency, or 0.121 rad/sec.
- Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a -20 dB/decade line until 0 dB is reached.That intersection gives the lower break frequency.
- The lower break frequency is found to be 0.0001604 rad/sec.
- The compensator must have a dc gain of unity to retain the value of Kv that we have already designed by setting K = 1473.
- Gain in the lag compensator = 0.001326

$$Gain = \frac{0.0001604}{0.121} = 0.001326 \tag{3.3.2.2}$$

Hence the lag compensator transfer function is

$$G_c(s) = \frac{0.001326(s + 0.121)}{s + 0.0001604} \tag{3.3.2.3}$$

3.3.3. Verifying Lag Compensator using Plots

Solution: Magnitude and Phase plot

The following code

```
codes/ee18btech11030/ee18btech11030_1.py
```

Specification	**Propsed**	**Actual**
OS%	15%	15.16%
K_v	1000	1000.47

TABLE 3.3.1: Comparing the Proposed and Actual results

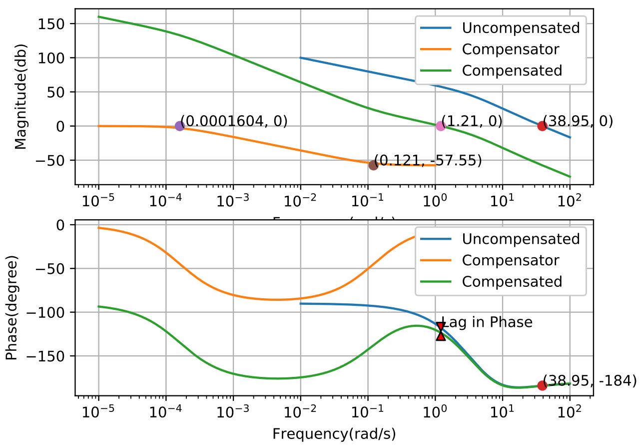

Fig. 3.3.3

3.3.4. Verifying in time domain
Solution: Time response for a unit step function

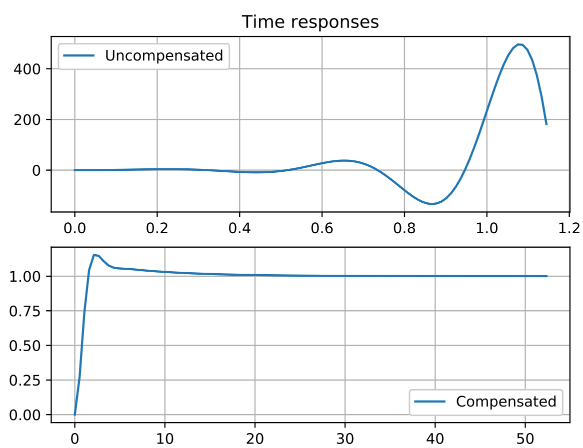

Fig. 3.3.4

The following code

codes/ee18btech11030/ee18btech11030_2.py

3.4 Lead Compensator

3.4.1. For a unity feedback system shown in Fig. 1

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{3.4.1.1}$$

Design a lead compensator to yield a $K_v = 2$ and a phase margin of 30°.
Solution: For unity feedback we have Velocity error constant (K_v)

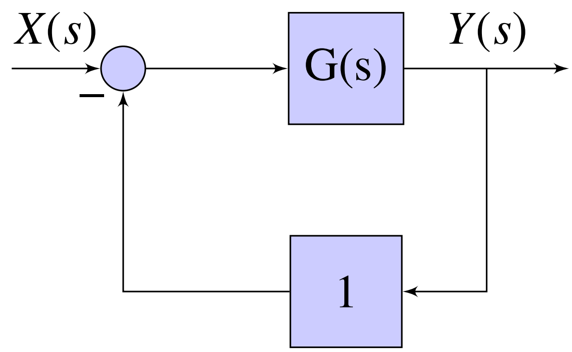

Fig. 3.4.1

$$K_v = \lim_{s\to 0} sG(s) \tag{3.4.1.2}$$

$$\lim_{s\to 0}\left(\frac{K}{(2+s)(4+s)(6+s)}\right) = 2 \tag{3.4.1.3}$$

$$\implies K = 96 \tag{3.4.1.4}$$

Check the phase margin and gain crossover frequency by running the following code

codes/ee18btech11036_1.py

- The Phase margin: 19.76°
- Gain Crossover Frequency:1.469 rad/sec

The Bode plot of system is as shown,

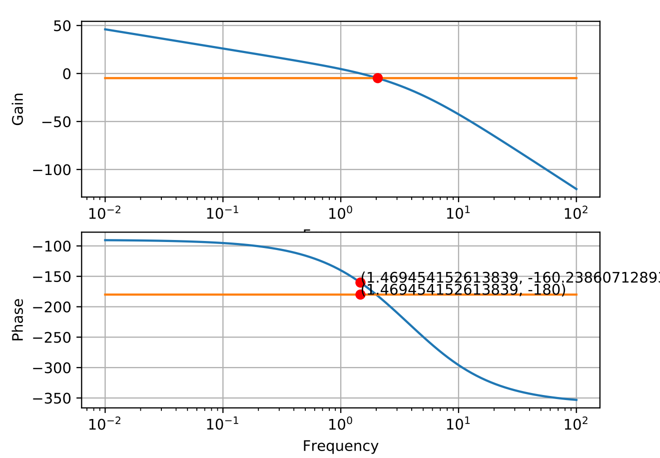

Fig. 3.4.2

Therefor amount of phase to be added: 30-19.76=10.24
The circuit of lead compensator is given by

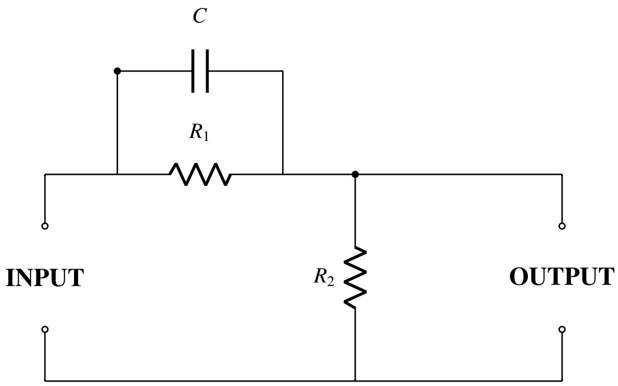

Fig. 3.4.3

Transfer function:

$$C(s) = \beta\left(\frac{1 + j\tau\omega}{1 + j\beta\tau\omega}\right) \tag{3.4.1.5}$$

$$\beta = \left(\frac{R_2}{R_1 + R_2}\right) \tag{3.4.1.6}$$

$$\tau = R_1 C \tag{3.4.1.7}$$

Find the values of β and τ

Solution: The maximum phase lead compensated by a lead compensator is given by

$$\phi = \sin^{-1}\frac{1-\beta}{1+\beta} \tag{3.4.1.8}$$

at

$$\omega = \frac{1}{\sqrt{\beta}\tau} \tag{3.4.1.9}$$

Now we know that from Gain crossover frequency

$$\omega = 1.469 rad/sec \tag{3.4.1.10}$$

and the phase margin to be added:

$$\phi = 10.24^\circ \tag{3.4.1.11}$$

But to compensate for the added magnitude of lead compensator, a correction factor of $10^\circ - 20^\circ$ is added.Hence

$$\phi = 30.24^\circ \implies \beta = 0.33 \tag{3.4.1.12}$$

From the bode plot ω is chosen at which gain of original system is

$$-20\log\left(1/\sqrt{\beta}\right) = -4.81 \tag{3.4.1.13}$$

Find the plot using the following code

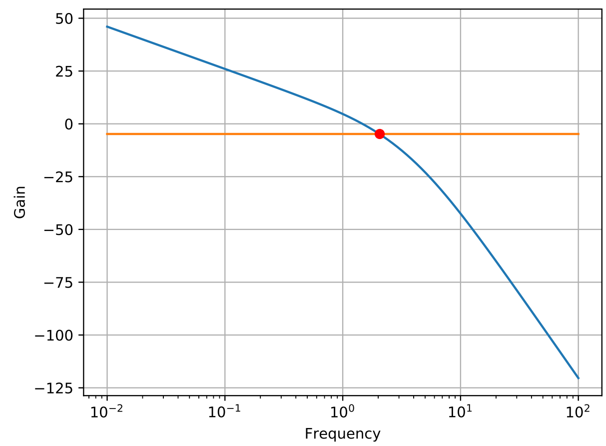

Fig. 3.4.4

```
codes/ee18btech11036_4.py
```

From plot ω=2.009 rad/sec
Solving equations 3.4.1.8 and 3.4.1.9:

$$\tau = 0.828 \tag{3.4.1.14}$$

$$\beta = 0.33 \tag{3.4.1.15}$$

$$\tag{3.4.1.16}$$

New Transfer Function:

$$G(s) = \frac{96\,(1 + 0.828s)}{(s)\,(2 + s)\,(4 + s)\,(6 + s)\,(1 + 0.273s)} \tag{3.4.1.17}$$

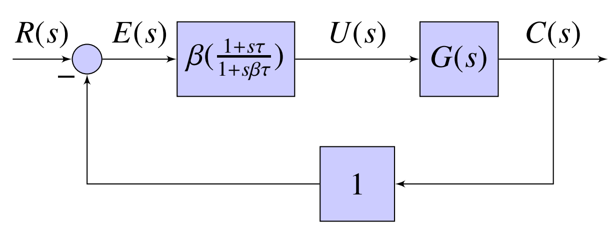

Fig. 3.4.5

Verify your results from the following code:

```
codes/ee18btech11036_2.py
```

- The Phase margin: 29.269°
- The Gain Crossover Frequency: 2.02 rad/sec

The Bode plot is as shown,

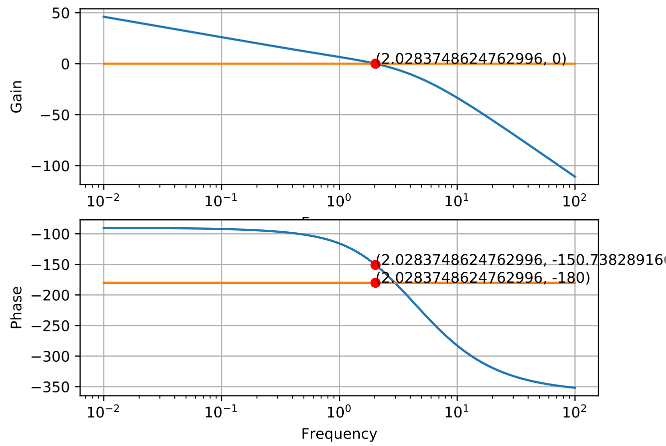

Fig. 3.4.6

3.4.2. For a unity feedback system

$$G(s)=\frac{K}{(s)(s+2)(s+4)(s+6)} \qquad (3.4.2.1)$$

Design a lag compensator to yield a $K_v = 2$ and Phase Margin of 30° **Solution:** Fig.3.4.7 models the equivalent of compensated closed loop system.

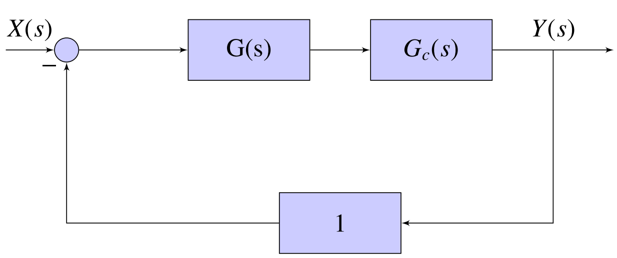

Fig. 3.4.7

Static Velocity Error Constant K_v is the steady-state error of a system for a unit-ramp input i.e.,

$$K_v = \lim_{s\to 0} sG(s)G_c(s) \qquad (3.4.2.2)$$

Therefore,

$$K_v = \lim_{s\to 0} s\frac{K}{s(s+2)(s+4)(s+6)}\frac{Ts+1}{\beta Ts+1}$$
$$\implies 2 = \frac{K}{(0+2)(0+4)(0+6)}\frac{T(0)+1}{\beta T(0)+1}$$
$$\therefore K = 96 \qquad (3.4.2.3)$$

$$G(s)=\frac{96}{s(s+2)(s+4)(s+6)} \qquad (3.4.2.4)$$

Substituting $s = \jmath\omega$ in (3.4.2.4),

$$G(\jmath\omega) = \frac{96}{(j\omega)(j\omega+2)(j\omega+4)(j\omega+6)} \qquad (3.4.2.5)$$

$$\left|G(\jmath\omega)\right| = \frac{|96|}{\omega\sqrt{4+\omega^2}\sqrt{16+\omega^2}\sqrt{36+\omega^2}} \qquad (3.4.2.6)$$

$$\angle G(\jmath\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) \qquad (3.4.2.7)$$

The standard Transfer equation of Lag Compensator and its Phase and Gain

$$G_c(s) = \frac{Ts+1}{\beta Ts+1} \qquad (3.4.2.8)$$

$$|G_c(s)| = \frac{1}{\beta}\frac{1+\left(\frac{\omega}{T}\right)^2}{1+\left(\frac{\omega}{\beta T}\right)^2} \qquad (3.4.2.9)$$

$$\angle G_c(s) = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T) \qquad (3.4.2.10)$$

Where $\beta > 1$.
It can be approximated that for $\omega > \frac{1}{T}$

$$|G_c(s)| = \frac{1}{\beta} \qquad (3.4.2.11)$$

and Phase to be very small($< 12^\circ$).
The Phase Margin(PM) of the Transfer function G(s)
From (3.4.2.6) and (3.4.2.6)
At Gain Crossover,

$$|G(s)| = 1 \qquad (3.4.2.12)$$
$$\implies \omega_{gc} = 1.47 rad/sec \qquad (3.4.2.13)$$
$$\implies \angle G\left(\jmath\omega_{gc}\right) = -160.26^\circ \qquad (3.4.2.14)$$

$$PM = 180^\circ + \angle G\left(\jmath\omega_{gc}\right) \qquad (3.4.2.15)$$
$$\implies PM = 19.74^\circ \qquad (3.4.2.16)$$

The following are the Bode plots of uncompensated system
The code for Bode plots of uncompensated system

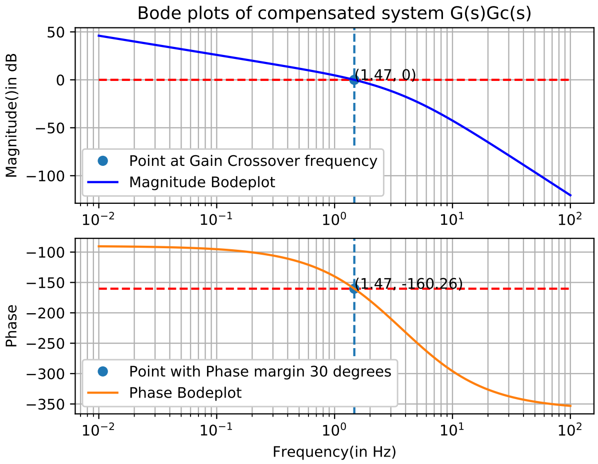

Fig. 3.4.8

codes/ee18btech11046_1.py

The lag compansator form is given in (3.4.2.8), Let

$$G'(s) = G(s)\,G_c(s) \tag{3.4.2.17}$$

The $PM = 30°$ when $\angle G'(\jmath\omega) = -150°$
Since the addition of compensator reduces Gain of system, thereby reducing Gain Crossover frequency which increases Phase Margin(PM) of system.
Since, Compansator also has small negative phase(say ϵ),let $\epsilon = 5°$.i.e, $\angle G_c(s) = 5$

$$\angle G'(s) = \angle G(s) + \angle G_c(s) \tag{3.4.2.18}$$

$$\implies -150° = \angle G(s) - 5° \tag{3.4.2.19}$$

$$\implies \angle G(s) = -145° \tag{3.4.2.20}$$

The value of ω where $\angle G(s) = -145°$ is

$$\angle G(s) = -145° \tag{3.4.2.21}$$

$$\implies \omega_{req} = 1.10953 rad/sec \tag{3.4.2.22}$$

The value $\frac{1}{T}$ is exactly 2 octaves below ω_{req} obtained in (3.4.2.22)

$$\frac{1}{T} = \frac{\omega_{req}}{4} \tag{3.4.2.23}$$

$$\implies T = 3.605 \tag{3.4.2.24}$$

Now we should take β such that Gain Crossover frequency occurs at ω_{req} i.e., to make $\left|G'(\jmath\omega)\right| = 1$
From (3.4.2.11),

$$\left|G'(\jmath\omega_{gc})\right| = \left|G(\jmath\omega_{gc})\right|\left|G_c(\jmath\omega_{gc})\right| = 1 \tag{3.4.2.25}$$

$$\implies 1.4936 \times \frac{1}{\beta} = 1 \tag{3.4.2.26}$$

$$\implies \beta = 1.4936 \tag{3.4.2.27}$$

Substituting values of T and β obtained from(3.4.2.24) and(3.4.2.27) in (3.4.2.8) The required Compensator Transfer is

$$G_c(s) = \frac{3.605s + 1}{5.384s + 1} \tag{3.4.2.28}$$

The following are the Bode plots of compensated system

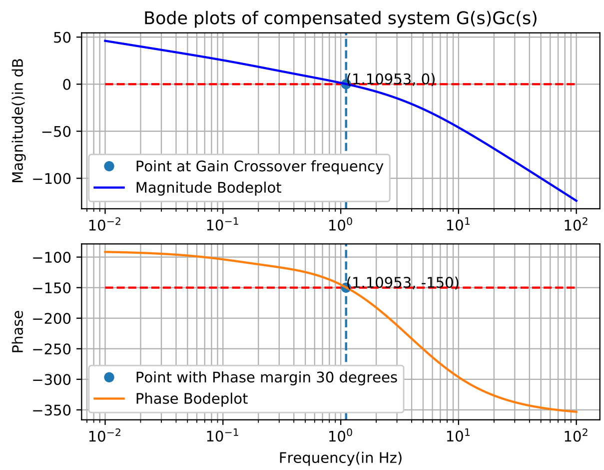

Fig. 3.4.9

The code for Bode plots of compensated system

codes/ee18btech11046_2.py

3.5 *Compensator for Overshoot*

3.1. Given the unity feedback system of Fig. 3.5.1 , with

$$G(s) = \frac{K}{s(s+5)(s+20)} \tag{3.1.1}$$

The uncompensated system has about 55% peak overshoot and a peak time if 0.5 seconds when $K_v = 10$. Use frequency response technique to design a lead compensator to reduce the percent overshoot to 10% , while keeping the peak time and steady state error about the same or less. Consider second order approximations.

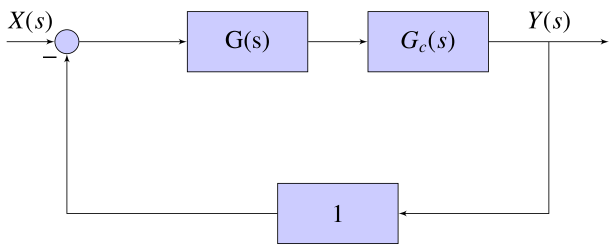

Fig. 3.5.1

3.2. **Solution:**

$$K_v = \lim_{s\to 0} sG(s) = 10 \tag{3.2.1}$$

$$\implies K = 1000 \tag{3.2.2}$$

The bode plot for G(s)is as follows :

$$G(s) = \frac{1000}{s(s+5)(s+20)} \tag{3.2.3}$$

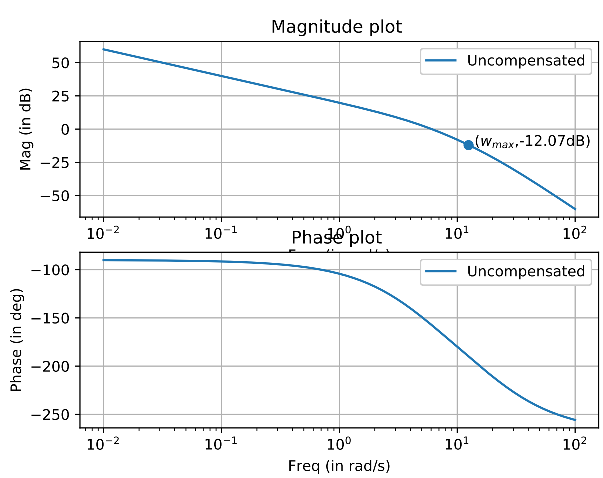

Fig. 3.5.2: G(s) Bode Plot

$$\zeta = \frac{-\ln\left(\frac{OS\%}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\frac{OS\%}{100}\right)\right)^2}} \tag{3.2.4}$$

$$PhaseMargin = \phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}\right) \tag{3.2.5}$$

The following code computes the above quantities.

```
codes/ee18btech11026/ee18btech11026_1.py
```

Specifications	Actual	Expected
OS%	55%	10%
ζ	0.186	0.591
ϕ_m	21.16°	58.59°
T_p	0.5	<= 0.5
K_v	10	<= 10

TABLE 3.5.1: Table of Specifications

The required additional phase contribution by the compensator will be:

$$\phi_{max} = 58.9 - 21.16 + correctionfactor \tag{3.2.6}$$

$$CorrectionFactor = 25^\circ \tag{3.2.7}$$

$$\phi_{max} = 62^\circ \tag{3.2.8}$$

Note : Since we know that the lead network will also increase the phase-margin frequency, we add a correction factor to compensate for the lower uncompensated system's phase angle.Choosing the correction factor is a trail and error procedure so as to reach our expected specifications.
The gain compensator's T.F will be of the form:

$$G_c(s) = \frac{1}{\beta}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{T\beta}}\right) \tag{3.2.9}$$

This form of T.F does not influence the steady state error.
Important Relations to find T and β:

$$\phi_{max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} \tag{3.2.10}$$

The Compensator's magnitude at the phase margin frequency ω_{max}

$$|G_c(j\omega_{max})| = \frac{1}{\sqrt{\beta}} \tag{3.2.11}$$

$$T = \frac{1}{\omega_{max}\sqrt{\beta}} \tag{3.2.12}$$

Using the above formulae :

$$\beta = 0.062 \tag{3.2.13}$$

$$|G_c(j\omega_{max})| = 12.07dB \tag{3.2.14}$$

If we select ω_{max} to be the new phase-margin frequency, the uncompensated system's magnitude at this frequency must be -12.07 dB to yield a 0 dB crossover at ω_{max} for the compensated system.
From the bode plot of the un-compensated system, find ω_{max} where the magnitude is -12.07 dB. This becomes our new phase-margin frequency.

$$\omega_{max} = 12.5rad/sec \tag{3.2.15}$$

$$T = 0.321 \tag{3.2.16}$$

The Compensator's T.F is as follows :

$$G_c(s) = 16.13\left(\frac{s + 3.115}{s + 50.25}\right) \tag{3.2.17}$$

The open loop T.F for the compensated system is :

$$G(s).G_c(s) = 16130\left(\frac{(s + 3.115)}{s(s + 50.25)(s + 5)(s + 20)}\right) \tag{3.2.18}$$

3.3. **Verification :** We could observe the affect of the lead-phase compensator from the phase plots.

The time responses for a unit step input in a unity feedback system with and without a compensator are as follows :

These plots are generated using the below code:

```
codes/ee18btech11026/ee18btech11026_2.py
```

3.4. **Result :** The below is the summary for the designed lead-compensator

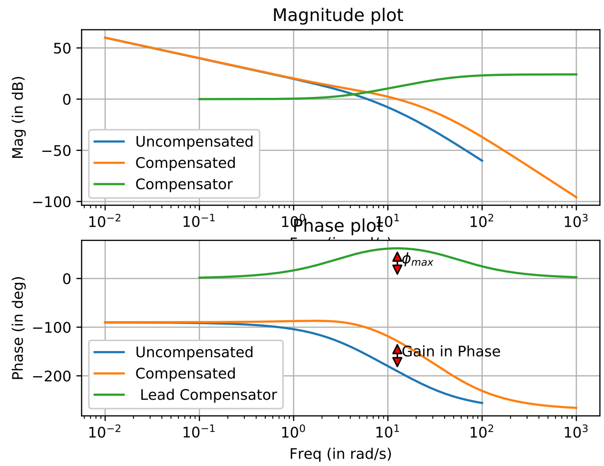

Fig. 3.5.3: Combined Bode Plots

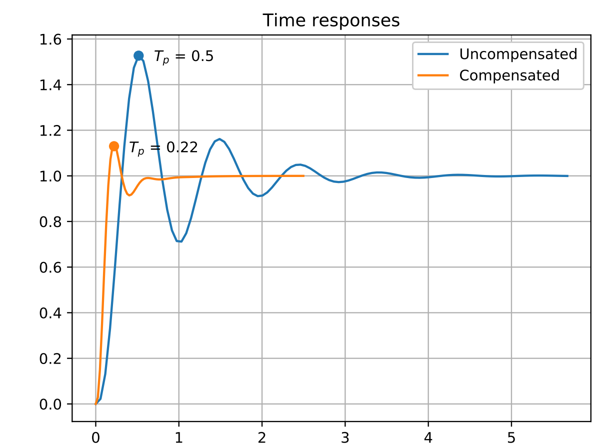

Fig. 3.5.4: Time response for a unit step input

Specifications	Expected	Proposed
OS%	10%	11%
T_p	<= 0.5	0.22
K_v	<= 10	10

TABLE 3.5.2: Comparing the desired and obtained results

3.6

3.7 Lead Compensator

3.7.1. An aircraft roll control system can be represented by a block diagram shown in Fig. 3.7.1 with G(s) in feedback system, whose error K_v

= 5. Determine K

$$G(s) = \frac{10K}{s(s+1)(s+5)} \tag{3.7.1.1}$$

The block diagram is given by Fig.3.7.1

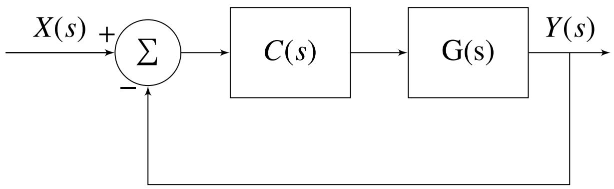

Fig. 3.7.1

For unity feedback we have Velocity error constant (K_v)

$$K_v = \lim_{s \to 0} sG(s) \tag{3.7.1.2}$$

$$\lim_{s \to 0} \left(\frac{10K}{(s+1)(s+5)} \right) = 5 \tag{3.7.1.3}$$

$$\implies K = 2.5 \tag{3.7.1.4}$$

It's Phase Margin = 3.94°
and Gain Crossover Frequency = 2.03 rad/s
Refer Fig. 3.7.2 for plot G(s).

```
codes/es17btech11002_1_new.py
```

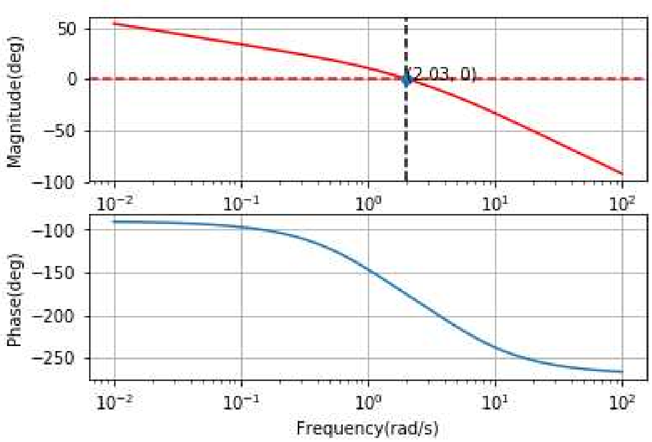

Fig. 3.7.2

Compensator required phase angle (ϕ_m) and Phase Margin Frequency $\left(\omega_{pm}\right)$,

$$\phi_m = -(180^\circ + \theta) + PM + 5 = 65^\circ \tag{3.7.1.5}$$

$$\omega_{pm} = 1.25 rad/s. \tag{3.7.1.6}$$

Attenuation factor $(\alpha\beta)$ is given by

$$\alpha = 0.5 \tag{3.7.1.7}$$

$$\beta = 20 \tag{3.7.1.8}$$

Lead and Lag Compensator Design Parameter is given in TABLE 3.7.1 And Compensator

Zeros/Poles	Parameter	Value
z_{lead}	$\omega_{pm}\sqrt{\alpha}$	0.279
p_{lead}	$\frac{z_{lead}}{\alpha}$	5.590
z_{lag}	$0.1\omega_{pm}$	0.125
p_{lag}	$\frac{z_{lag}}{\beta}$	0.00625

TABLE 3.7.1: Zeroes and Poles

obtained has transfer function

$$G_c(s) = \frac{(s+0.279)(s+0.125)}{(s+5.590)(s+0.00625)} \tag{3.7.1.9}$$

Refer Fig3.7.3 for plot $G(s)G_c(s)$.

```
codes/es17btech11002_2_new.py
```

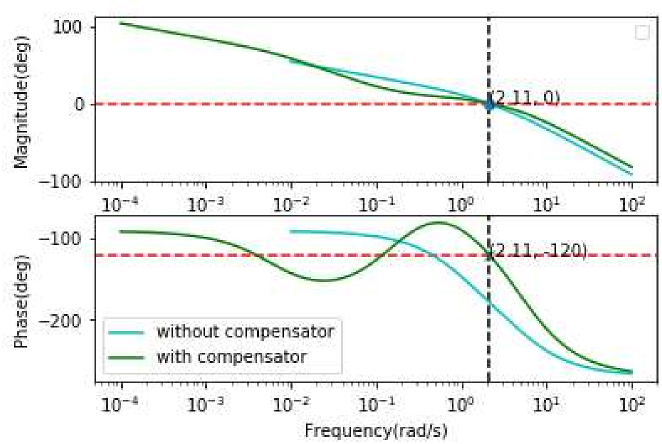

Fig. 3.7.3

NOTE : The idea of using a lead-lag network is to provide the attenuation of a phase-lag network and the lead-phase angle of a phase-lead network. This points should be noted while designing a controller, and parameters to be changed accordingly to get exact results.

3.8

3.9

3.10

4 PID Controller Design

4.1 PD

4.1. For a unity feedback system shown in Fig. 4.1.1

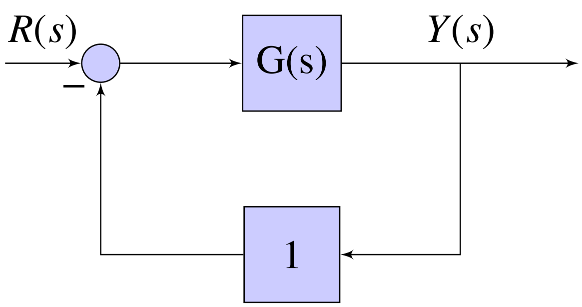

Fig. 4.1.1

$$G(s) = \frac{K}{s(s+1)} \tag{4.1.1}$$

Design a PD controller such that the phase margin is 45°and appropriate steady state error is less than or equal to $\frac{1}{15}$ units of the final output value. Further the gain crossover frequency of the system must be less than 7.5 rad/s.
Solution: Using TABLE 4.2.1 The gain after cascading the PD controller with G(s) is

$$G_c(s) = \frac{K_P(1+T_d s)K}{s(s+1)} \tag{4.1.2}$$

Type	Poles	Input	Steady State Error
Type 0	0	Step	$e_{ss} = \frac{1}{1+\lim_{s\to 0} G(s)}$
Type 1	1	Ramp	$e_{ss} = \frac{1}{\lim_{s\to 0} sG(s)}$
Type 2	2	Parabolic	$e_{ss} = \frac{1}{1+\lim_{s\to 0} s^2G(s)}$

TABLE 4.1.1: System Types and Poles at Origin

Using TABLE 4.1.1, (4.1.2) is Type 1 system.

$$e_{ss} = \frac{1}{\lim_{s\to 0} sG_c(s)} \tag{4.1.3}$$

$$e_{ss} \le \frac{1}{15} \lim_{s\to 0} sG_c(s) \tag{4.1.4}$$

$$\implies K_P K \ge \sqrt{15} \tag{4.1.5}$$

For Phase Margin 45°, at gain crossover frequency ω,

$$\tan^{-1}(T_d\omega) - \tan^{-1}(\omega) = -45° \tag{4.1.6}$$

$$|G_c(\jmath\omega)| = \frac{\sqrt{15}\sqrt{T_d^2\omega^2+1}}{\omega\sqrt{\omega^2+1}} = 1 \tag{4.1.7}$$

By Hit and Trial, one of the best combinations is

$$\omega = 2.893 \tag{4.1.8}$$

$$T_d = -0.71 \tag{4.1.9}$$

Parameters	Required	Obtained
ω	≤ 7.5	2.893
Phase Margin	45°	45°
T_d	Not Given	-0.71

TABLE 4.1.2

4.2. Verify using a Python Plot
Solution: The following code plots Fig. 4.1.2

```
codes/ee17btech11031_pd_ke.py
```

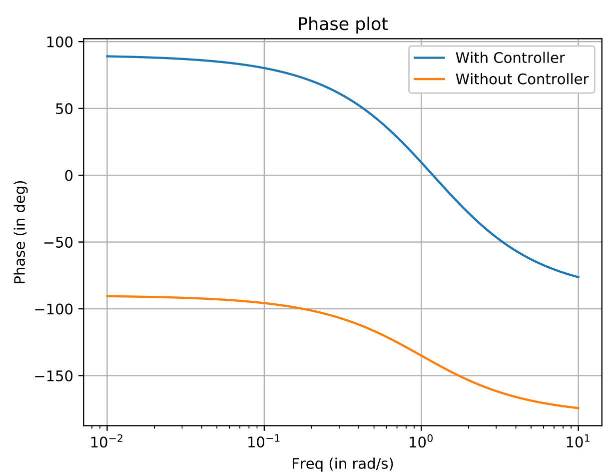

Fig. 4.1.2

4.2 PID

4.2.1. Tabulate the transfer functions of a PID controller and its variants.
Solution: See Table 4.2.1.

4.2.2. For a unity Feedback system

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{4.2.2.1}$$

Controller	Gain
PID	$K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$
PD	$K_p(1 + T_d s)$
PI	$K_p\left(1 + \frac{1}{T_i s}\right)$

TABLE 4.2.1

Design a PD Controller with $K_v = 2$ and Phase Margin 30°

Solution: The gain after cascading the PD Controller with G(s) is

$$G_c(s) = \frac{K_p(1 + T_d s)K}{s(s+2)(s+4)(s+6)} \tag{4.2.2.2}$$

Choosing $K_p = 1$ in ,

$$K_v = \lim_{s \to 0} sG_c(s) = 2 \tag{4.2.2.3}$$

$$\implies K = 96 \tag{4.2.2.4}$$

For Phase Margin 30°, at Gain Crossover Frequency ω,

$$\tan^{-1}(T_d\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = -60° \tag{4.2.2.5}$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_d^2\omega^2 + 1}}{\omega\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \tag{4.2.2.6}$$

By Hit and Trial, one of the best combinations is

$$\omega = 4 \tag{4.2.2.7}$$

$$T_d = 1.884 \tag{4.2.2.8}$$

We get a Phase Margin of 30.31°

4.2.3. Verify using a Python Plot

Solution: The following code plots Fig. 4.2.1

```
codes/ee18btech11021/EE18BTECH11021_3.py
```

4.2.4. Design a PI Controller with $K_v = \infty$ and Phase Margin 30°

Solution: From Table 4.2.1, the open loop gain in this case is

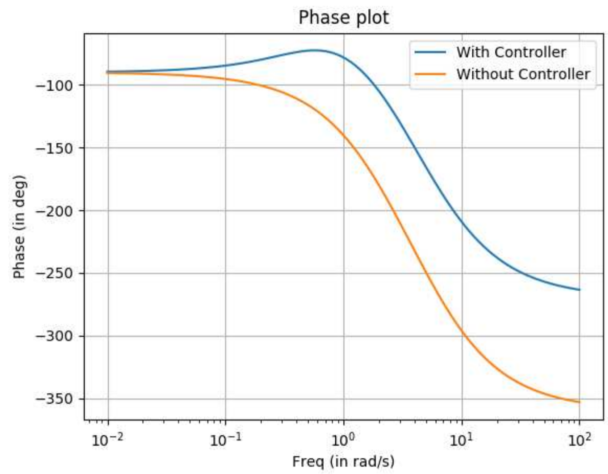

Fig. 4.2.1

$$G_1(s) = \frac{K_p\left(1 + \frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \tag{4.2.4.1}$$

Choose $K_pK = 96$. Then

$$G_1(s) = \frac{96(T_i s + 1)}{T_i s^2(s+2)(s+4)(s+6)} \tag{4.2.4.2}$$

For Phase Margin 30°, at Gain Crossover Frequency ω

$$\tan^{-1}(T_i\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \tag{4.2.4.3}$$

and

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_i^2\omega^2 + 1}}{T_i^2\omega^2\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \tag{4.2.4.4}$$

By Hit and Trial, one of the best combinations is

$$\omega = 0.75 \tag{4.2.4.5}$$

$$T_i = 2.713 \tag{4.2.4.6}$$

We get a Phase Margin of 25.53°

4.2.5. Verify using a Python Plot

Solution: The following code plots Fig. 4.2.2.

```
codes/ee18btech11021/EE18BTECH11021_4.py
```

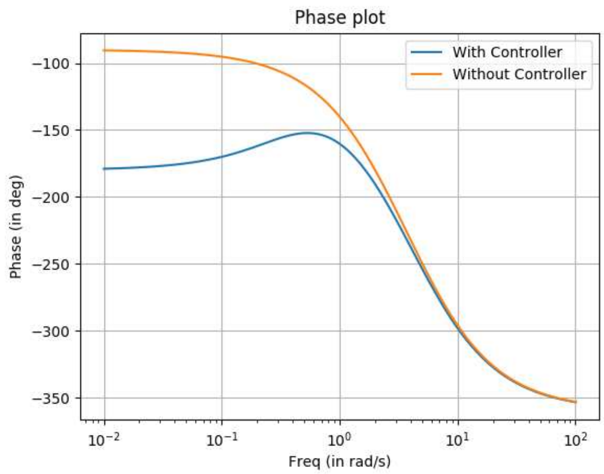

Fig. 4.2.2

4.2.6. Design a PID Controller with $K_v = \infty$ and Phase Margin 30°
Solution:

$$G_1(s) = \frac{K_p\left(1 + T_d s + \frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \qquad (4.2.6.1)$$

Choose $K_pK = 96$. The open loop gain is

$$G_1(s) = \frac{96(T_iT_d s^2 + T_i s + 1)}{T_i s^2(s+2)(s+4)(s+6)} \qquad (4.2.6.2)$$

For Phase Margin 30°, at Gain Crossover Frequency ω,

$$\tan^{-1}\left(\frac{T_i\omega}{1 - TiT_d w^2}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \qquad (4.2.6.3)$$

$$\left|G_1(\mathrm{j}\omega)\right| = \frac{96\sqrt{(1 - TiT_d\omega^2)^2 + T_i^2}}{T_i^2\omega^2\sqrt{(\omega^2+4)(\omega^2+16)(\omega^2+36)}} = 1 \qquad (4.2.6.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 1 \qquad (4.2.6.5)$$
$$T_i = 1.738 \qquad (4.2.6.6)$$
$$T_d = 0.4 \qquad (4.2.6.7)$$

We get a Phase Margin of 30°

4.2.7. Verify using a Python Plot
Solution: The following code plots Fig. 4.2.3

```
codes/ee18btech11021/EE18BTECH11021_5.py
```

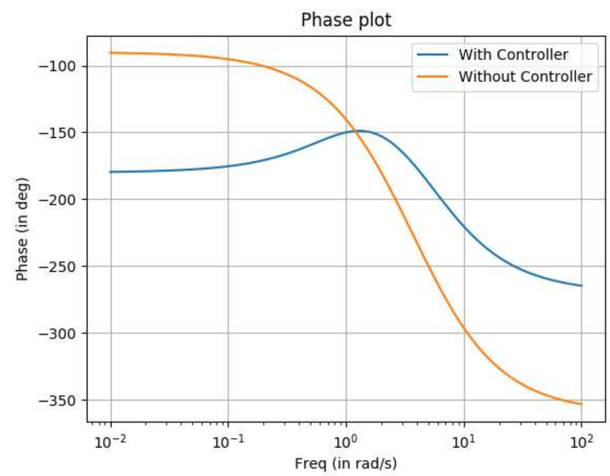

Fig. 4.2.3