DC Amplifier

Buereddy Varuni*

A DC amplifier has an open loop gain of 1000 and two poles, a dominant one at 1kHz and a high frequency one whose location can be controlled. It is required to connect this amplifier in a negative feedback loop that provides a DC closed loop gain of 10 and a maximally flat response.

1. Find the required value of H.
Solution: Table 1 summarises the given information. The open loop gain can be expressed as

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)} \tag{1.1}$$

$$\implies G(0) = G_0 \tag{1.2}$$

The closed loop gain

$$T(s) = \frac{G(s)}{1 + G(s)H} \tag{1.3}$$

$$\implies T(0) = \frac{G_0}{1 + G_0 H} \tag{1.4}$$

Substituting from Table 1,

$$\frac{1000}{1 + 1000H} = 10 \tag{1.5}$$

$$\implies H = 0.099 \tag{1.6}$$

Parameter	Value
dc open loop gain	1000
dominant pole	1000Hz
insignificant pole	$-p_2$
dc closed loop gain	10

TABLE 1

$$G_0 = 1000 \tag{1.7}$$

$$\text{Therefore.,} G(s) = \frac{1000}{(1 + \frac{s}{p_1})(1 + \frac{s}{p_2})} \tag{1.8}$$

2. Find p_2.
Solution: From (1.3) and (1.1),

$$T(s) = \frac{p_1 p_2 G_0}{s^2 + (p_1 + p_2)s + (HG_0 + 1)p_1 p_2} \tag{2.1}$$

$$= \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2.2}$$

$$\omega_n = \sqrt{(HG_0 + 1)p_1 p_2}$$

$$\implies \zeta = \frac{p_1 + p_2}{2\sqrt{(HG_0 + 1)p_1 p_2}} \tag{2.3}$$

using the standard formulation for a second order system. Also, for maximally flat response, the quality factor

$$Q = \frac{1}{2\zeta} = \frac{1}{\sqrt{2}} \tag{2.4}$$

$$\implies \zeta = \frac{1}{\sqrt{2}} \tag{2.5}$$

$$\implies \frac{p_1 + p_2}{2\sqrt{(HG_0 + 1)p_1 p_2}} = \frac{1}{\sqrt{2}} \tag{2.6}$$

$$\implies \sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}} = \sqrt{2(HG_0 + 1)} \tag{2.7}$$

The above equation is of the form

$$x + \frac{1}{x} = a \tag{2.8}$$

$$\implies x = \frac{a \pm \sqrt{a^2 - 4}}{2} \tag{2.9}$$

where

$$x = \sqrt{\frac{p_2}{p_1}} \tag{2.10}$$

$$a = \sqrt{2(HG_0 + 1)}, \tag{2.11}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India.

Thus, from (2.10), (2.11) and (2.9),

$$p_2 = p_1 \left[\frac{\sqrt{2\left(HG_0 + 1\right)} \pm \sqrt{2\left(HG_0 + 1\right) - 4}}{2} \right]^2 \tag{2.12}$$

From the following code,

codes/ee18btech11005/ee18btech11005_1.py

$$\begin{aligned} p_2 &= 1244038.9567529503 \\ &\text{and } 31.734068607786863 \end{aligned} \tag{2.13}$$

3. Draw the equivalent circuit system diagram.
Solution: The equivalent circuit system is shown in the Fig. 3

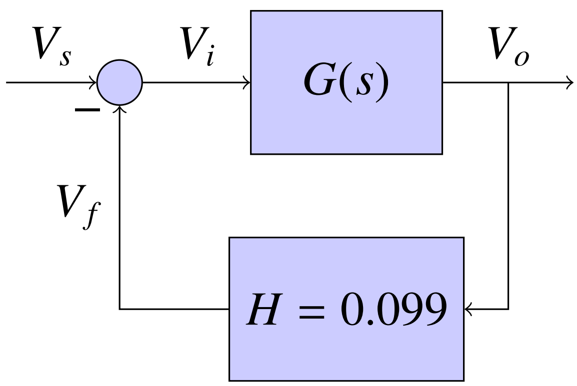

Fig. 3

4. Obtain $G(s)$ and $T(s)$
Solution: Substituting the value of p_2 in (1.1) and (2.1),

$$G(s) = \frac{1000}{\left(1 + \frac{s}{2\pi 10^3}\right)\left(1 + \frac{s}{1.244 \times 10^6}\right)} \tag{4.1}$$

$$T(s) = \frac{10}{0.128 \times 10^{-11} s^2 + 1.599 \times 10^{-6} s + 1} \tag{4.2}$$

5. Verify from the Bode plot of above closed loop transfer function that it has maximally flat response.
Solution: The following code generates the bode plot of the transfer function in Fig. 5.

codes/ee18btech11005/ee18btech11005_2.py

6. Find the step response of $T(s)$
Solution: The following code generates the desired response of in Fig. 6.

codes/ee18btech11005/ee18btech11005_3.py

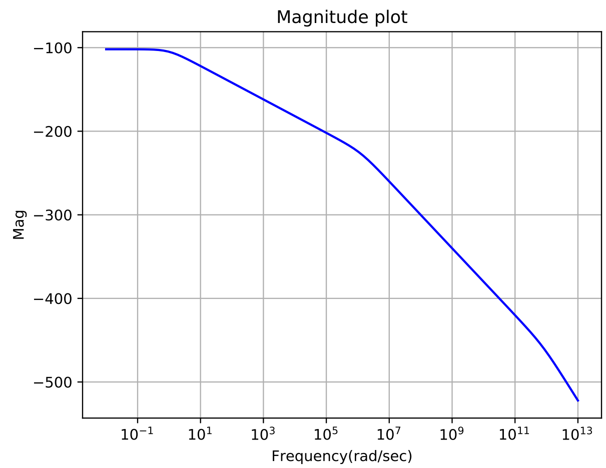

Fig. 5

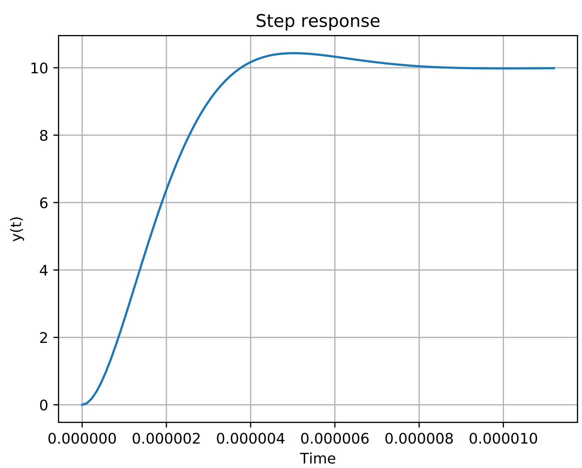

Fig. 6

7. Design a circuit for $T(s)$.
Solution: See Fig. 7.1. Assume the gain of all the amplifiers are large. For the first amplifier,

$$\frac{V_{in} - V_1^1}{R_1} = \frac{V_1^1 - V_1}{\frac{1}{sC_1}} \tag{7.1}$$

$$\frac{V_{in}}{R_1} = \frac{V_1^1}{R_1} + sC_1 V_1^1 - sC_1 V_1 \tag{7.2}$$

$$\frac{V_{in}}{R_1} = V_1^1 \left[sC_1 + \frac{1}{R_1} \right] - sC_1 V_1 \tag{7.3}$$

$$V_{in} = V_1^1 (sC_1 R_1 + 1) - sC_1 R_1 V_1 \tag{7.4}$$

For the second amplifier,

$$\frac{V_1 - V_b}{R_2} = (V_b - V_{out})sC_2 \tag{7.5}$$

$$\implies V_1 = -sC_2R_2V_{out} \tag{7.6}$$

Using Voltage division at node C,

$$\frac{V_1^1}{V_{out}} = \frac{1 + \frac{1}{sC}}{\frac{1}{sC}} \tag{7.7}$$

$$\implies V_1^1 = (sCR + 1)V_{out} \tag{7.8}$$

From (7.4), (7.6) and (7.8)

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2(CRC_1R_1 + C_1R_1C_2R_2) + s(CR + C_1R_1) + 1} \tag{7.9}$$

Comparing (4.2) and (7.9),

$$C_1R_1(CR + C_2R_2) = 0.128 \times 10^{-11} \tag{7.10}$$

$$CR + C_1R_1 = 1.599 \times 10^{-6} \tag{7.11}$$

$$\tag{7.12}$$

Letting

$$CR = 10^{-6} \tag{7.13}$$

$$\implies C_1R_1 = 0.599 \times 10^{-6} \text{ and} \tag{7.14}$$

$$C_2R_2 = 0.681 \times 10^{-6} \tag{7.15}$$

The parameters can be chosen as shown in the Table 7 The final circuit is shown in Fig. 7.2.

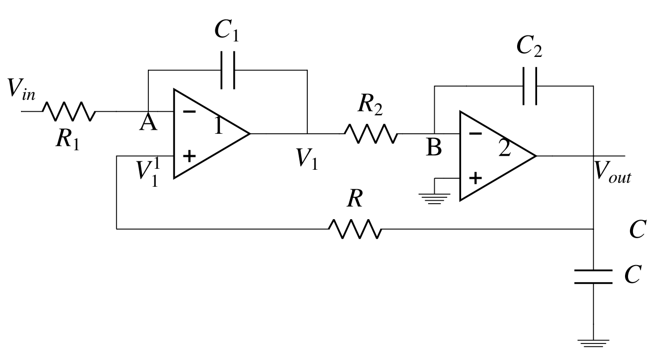

Fig. 7.1

8. Verify the closed loop DC gain using NGSPICE simulator.
Solution: The following README file gives the procedure to be followed.

```
codes/ee18btech11005/spice/README
```

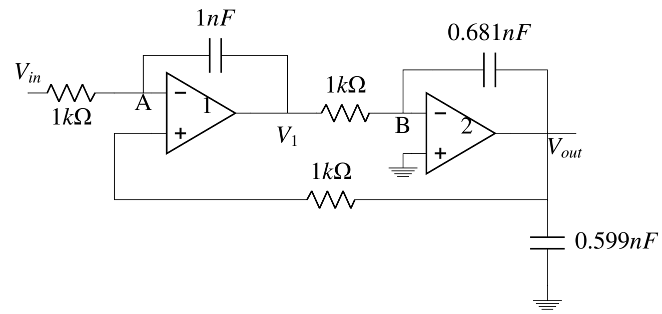

Fig. 7.2

Parameter	Value
R_1	1000 Ω
R_2	1000 Ω
R	1000 Ω
C_1	0.1 nF
C_2	0.681 nF
C	0.599 nF

TABLE 7

From equation.4.2. The DC closed loop gain is 10.
The following netlist file, gives the DC gain of the closed loop function.

```
codes/ee18btech11005/spice/gvv_ngspice.net
```

We can observe from simulation that the value of DC closed loop gain is 9.997.
Error analysis:-
ERROR in DC GAIN = 10-9.993 = 0.007
Thus, the predicted value in ngspice is almost accurate. Therefore, the value is verified using ngspice.

9. Verify the step response of the output from ngspice simulation.
Solution: The following netlist file does the transient anaylsis and store the Vout values with respect to time in a dat file.

```
codes/ee18btech11005/spice/gvv_ngspice.net
```

Following python code is to plot the step response.

```
codes/ee18btech11005/spice/
    ee18btech11005_spice.py
```

The step response obtained is shown in the Fig. 9.The graph has steady state value equal to 10.

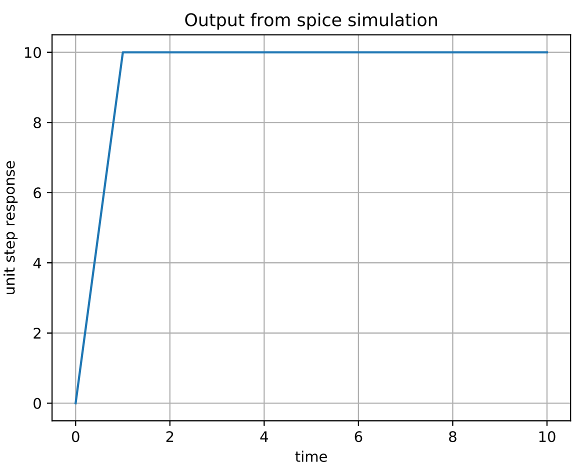
Output from spice simulation
10
8
6
4
2
0
unit step response
0
2
4
6
8
10
time

Fig. 9