Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

1.1. Fig. 1.1.1 shows a non-inverting op-amp configuration with parameters described in Table 1.1. Draw the equivalent control system.

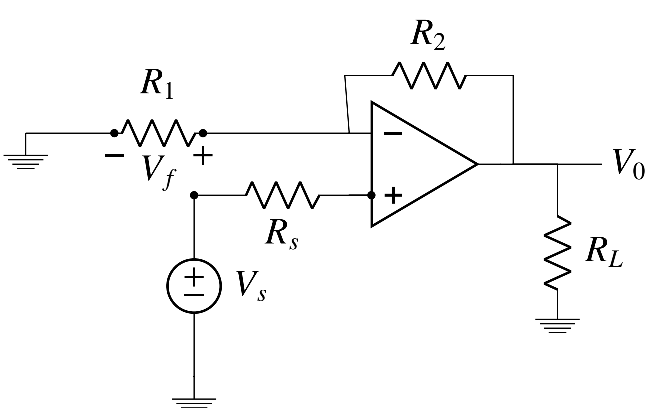

Fig. 1.1.1

Solution: See Fig. 1.1.2

1.2. Draw the small signal model for Fig. 1.1.1.
Solution: The equivalent circuit of the amplifier is in Fig. 1.2

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Parameter	**Value**
input resistance	∞
output resistance	**0**
Input voltage	V_s
Output Voltage	V_o
Feeding resistance	R_1
Feedback resistance	R_2
Source resistance	R_s
load resistance	R_L

TABLE 1.1

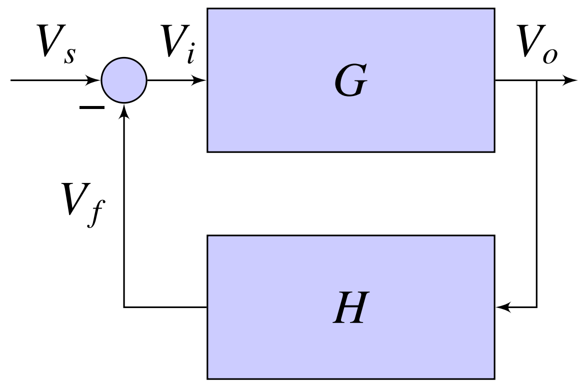

Fig. 1.1.2

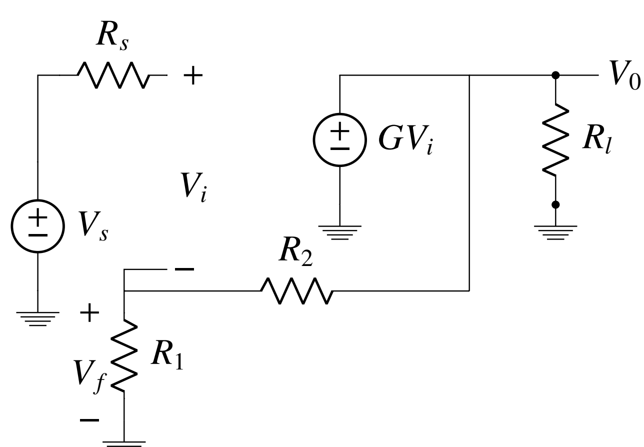

Fig. 1.2

1.3. Assuming that the operational amplifier has

infinite input resistance and zero output resistance, find the *feedback factor* H.
Solution: From Fig. 1.2,

$$V_0 = GV_i \tag{1.3.1}$$
$$V_i = V_s - V_f \tag{1.3.2}$$
$$V_f = \frac{R_1}{R_1 + R_2} V_o \tag{1.3.3}$$

assuming that the current through R_s is very small. Thus,

$$H = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \tag{1.3.4}$$

1.4. Obtain the closed loop gain T and summarize your results through a Table.
Solution: Table 1.4 provides a summary.

$$T = \frac{V_0}{V_i} = \frac{G}{1 + GH} \tag{1.4.1}$$
$$= \frac{G\left(R_1 + R_2\right)}{\left(R_1 + R_2\right) + GR_1} \tag{1.4.2}$$

Parameters	Definition	For given circuit
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1+R_2}$
Loop gain	GH	$G\frac{R_1}{R_1+R_2}$
Amount of feedback	1+GH	$1 + \frac{GR_1}{R_1+R_2}$
Closed loop gain	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$

TABLE 1.4

1.5. Find the condition under which closed loop gain T is almost entirely determined by the feedback network.
Solution: If

$$GH \gg 1, \tag{1.5.1}$$
$$T \approx \frac{1}{H} = 1 + \frac{R_2}{R_1} \tag{1.5.2}$$

1.6. If

$$G = 10^4 \tag{1.6.1}$$
$$T = 10, \tag{1.6.2}$$

find H.
Solution: From Table 1.4

$$T = \frac{G}{1 + GH} = 10 \tag{1.6.3}$$
$$\implies H = 0.0999 \tag{1.6.4}$$

1.7. *Gain Desensitivity:* If G decreases by 20%,what is the corresponding decrease in T? Comment.
Solution: From From Table 1.4, Given

$$T = \frac{G}{1 + GH} \tag{1.7.1}$$
$$\implies dT = \frac{dG}{(1 + GH)^2} \tag{1.7.2}$$
$$\implies \frac{dT}{T} = \frac{1}{1 + GH}\frac{dG}{G} \tag{1.7.3}$$

From the information available so far,

$$dG = 20\%, G = 10^4, H = 0.0999 \implies \frac{dT}{T} = 0.025\% \tag{1.7.4}$$

using the following code.

```
codes/ee18btech11005/ee18btech11005.py
```

Thus the closed loop gain is almost invariant to a relatively large (20%) variation in the open loop gain G. This is known as gain desensitivity.

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.1.1. Draw the equivalent control system for the feedback current amplifier shown in 2.1.1.4
Solution: See Fig. 2.1.1.5.

2.1.2. For the feedback current amplifier shown in 2.1.1.4, draw the Small-Signal Model. Neglect the Early effect in Q_1 and Q_2.
Solution: See Fig. 2.1.2.
While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as

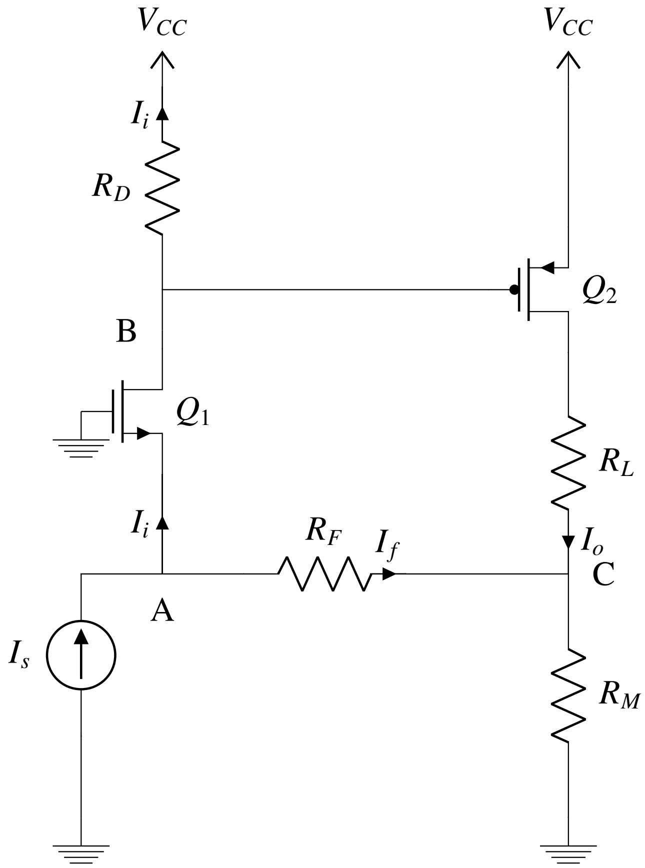

Fig. 2.1.1.1

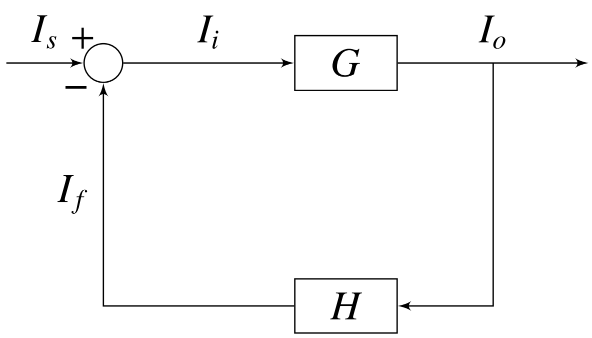

Fig. 2.1.1.2

a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

2.1.3. Write all the node/loop equations using KCL/KVL.

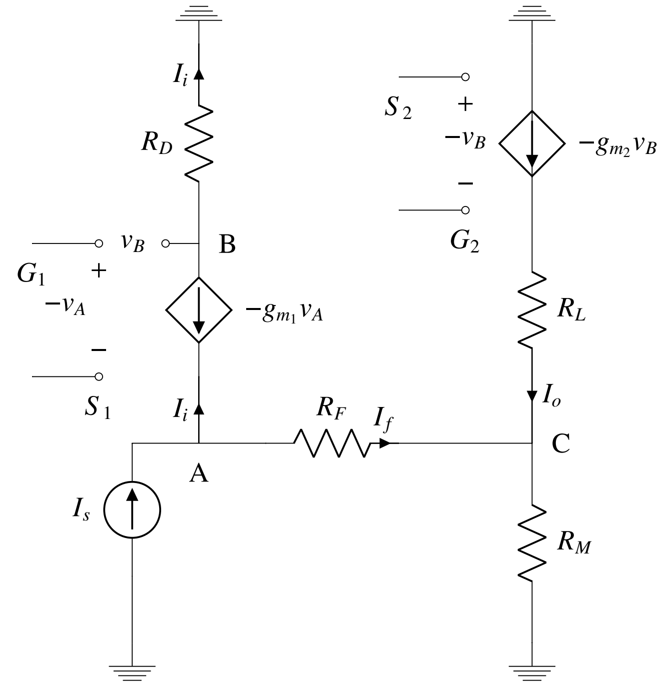

Fig. 2.1.2: Small Signal Model

Solution: From Figs. 2.1.1.4 and 2.1.2,

$$I_i = \frac{v_B}{R_D} \tag{2.1.3.1}$$

$$I_o = -g_{m_2} v_B \tag{2.1.3.2}$$

$$v_C - v_A = -I_f R_F \tag{2.1.3.3}$$

$$v_C = \left(I_o + I_f\right) R_M \tag{2.1.3.4}$$

$$I_i = g_{m_1} v_A \tag{2.1.3.5}$$

2.1.4. Find the Expression for the Open-Loop Gain G.
Solution: From (2.1.3.1) and (2.1.3.2),

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.1}$$

2.1.5. Find the Expression of the Feedback Factor H.
Solution:

$$H = \frac{I_f}{I_o}, \tag{2.1.5.1}$$

From (2.1.3.3) and (2.1.3.4),

$$\left(I_o + I_f\right) R_M - v_A = -I_f R_F \tag{2.1.5.2}$$

$$\implies \left(I_o + I_f\right) R_M + \frac{I_i}{g_{m_1}} = -I_f R_F \tag{2.1.5.3}$$

from (2.1.3.5). Dividing by I_o,

$$\implies (1+H)R_M + \frac{1}{g_{m_1}G} = -HR_F \quad (2.1.5.4)$$

upon substituting from and . Simplifying further, we obtain

$$\implies H = \frac{\frac{1}{g_{m_1}g_{m_2}R_D} - R_M}{R_F + R_M} \quad (2.1.5.5)$$

$$\approx -\frac{R_M}{R_F + R_M} \quad (2.1.5.6)$$

for $R_M \gg \frac{1}{g_{m_1}g_{m_2}R_D}$.

2.1.6. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$.
Solution: From (2.1.5) and (2.1.5.6),

$$T = \frac{I_o}{I_s} = \frac{G}{1+GH} \quad (2.1.6.1)$$

$$= -\frac{g_{m_2}R_D}{1 + g_{m_2}R_D/\left(1+\frac{R_F}{R_M}\right)} \quad (2.1.6.2)$$

2.2 Practical Case

2.2.1. Draw the Block Diagram and Circuit Diagram for H.
Solution: The Block Diagram is available in Fig. 2.2.1.1

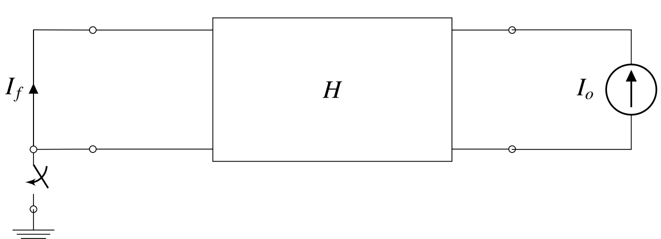

Fig. 2.2.1.1: Feedback Block Diagram

and the corresponding circuit diagram in Fig. 2.2.1.2

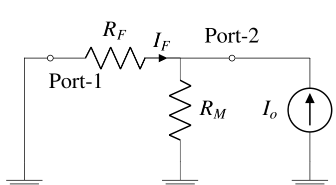

Fig. 2.2.1.2: Feedback Network

2.2.2. Find H from Fig. 2.2.1.2.
Solution: Using current division,

$$\frac{I_f}{I_o} = -\frac{R_M}{R_F + R_M} \quad (2.2.2.1)$$

$$\implies H = -\frac{R_M}{R_F + R_M} \quad (2.2.2.2)$$

2.2.3. Find R_{11} and R_{22} of Feedback Network where R_{11} is input resistance through Port-1 and R_{22} is Input Resistance through Port-2.
Solution: R_{11} is calculated by opening the current source at Port-2. Hence,

$$R_{11} = R_F + R_M \quad (2.2.3.1)$$

While calculating R_{22}, Port-1 should be shorted. Hence,

$$R_{22} = R_F \| R_M \quad (2.2.3.2)$$

$$= \frac{R_F R_M}{R_F + R_M} \quad (2.2.3.3)$$

2.2.4. Draw the block diagram and circuit diagram for calculating G.
Solution: See Figs. 2.2.4.1 and 2.2.4.2

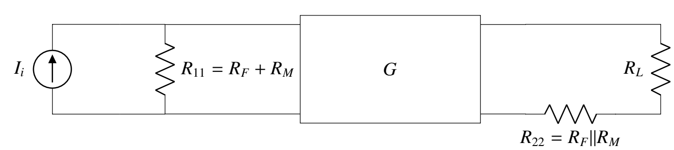

Fig. 2.2.4.1: Open-Loop Block Diagram

2.2.5. Find G.
Solution: The analysis is the same as Problem 2.1.4.

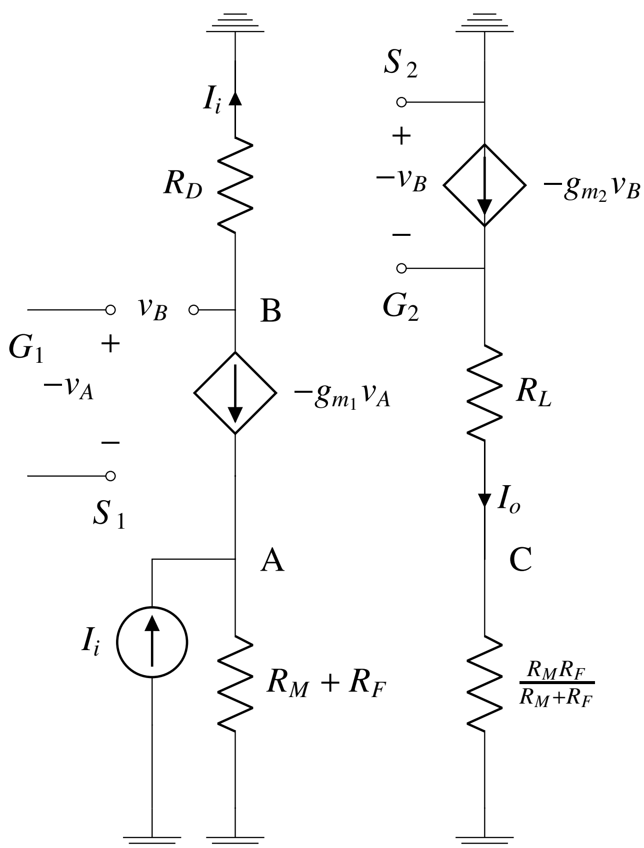

Fig. 2.2.4.2: Open-Loop Network