# Hartley Oscillator

Harsh kabra *

Find the frequency of oscillation for the Hartley circuit in Fig. 0. Also find the condition on $g_m$.

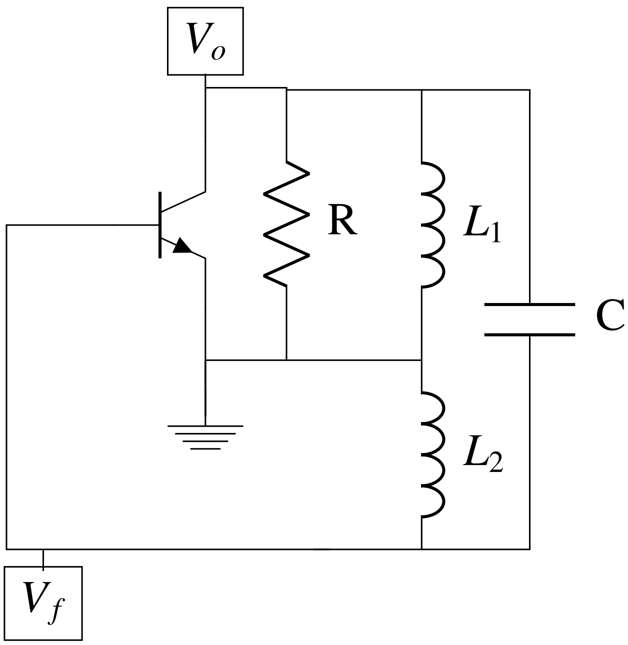


Fig. 0: Hartley oscillator

1. Draw the small signal model for Fig. 0 and the block diagram for the equivalent control system.
   **Solution:** See Figs. 1.1 and 1.2.
2. Draw the circuit for $H$ and find it.
   **Solution:** From Fig. 2,

$$H(s) = \frac{V_f}{V_o} \tag{2.1}$$

$$= \left(\frac{s^2 CL_2}{s^2 CL_2 + 1}\right) \tag{2.2}$$

3. Draw the circuit for $G$ and find it.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India. 

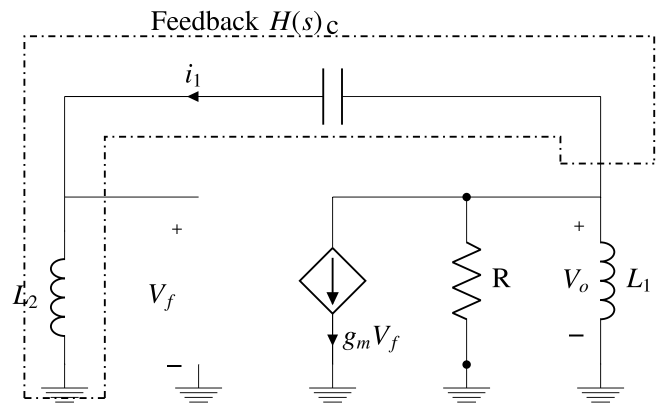


Fig. 1.1: Small signal model

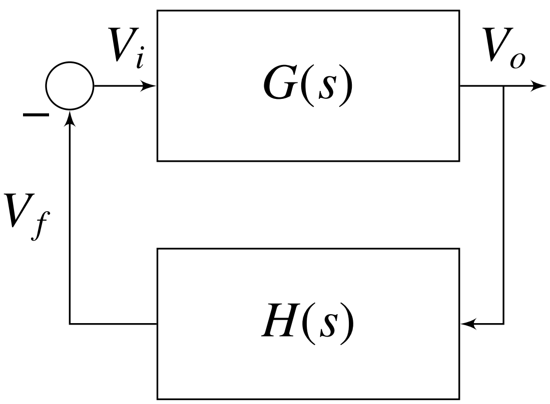


Fig. 1.2: Block diagram

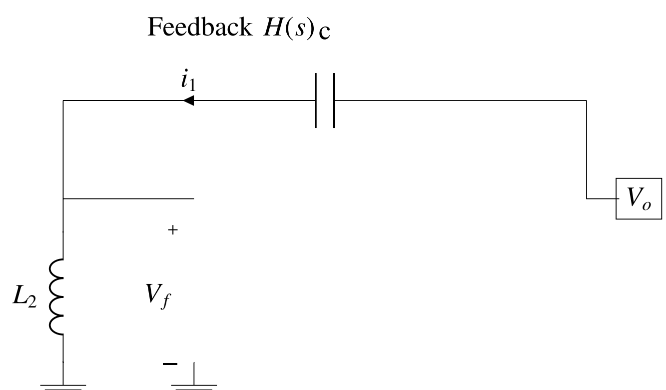


Fig. 2: Small signal H(s)

**Solution:** In Fig. 3

$$i_1 = \frac{V_i}{sL_2} \tag{3.1}$$

$$V_o = I(sL_1 \parallel R) \tag{3.2}$$

$$I = i_1 + g_m V_i \tag{3.3}$$

yielding

$$\frac{V_o}{V_i} = G(s) = \left(g_m + \frac{1}{sL_2}\right)\left(\frac{RsL_1}{R + sL_1}\right) \quad (3.4)$$

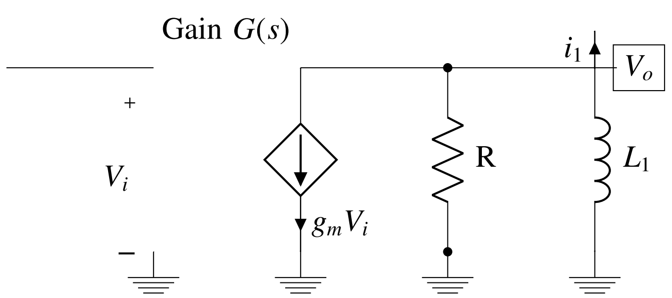


Fig. 3: Small signal G(s)

4. Find the frequency of oscillation.
   **Solution:** From 3.4 and 3.4,

$$1 + G(s)H(s) = 0 \quad (4.1)$$

$$\implies s^3(g_m CL_1L_2 + CL_1L_2) + s^2(RCL_1 + RCL_2) + sL_1 + R = 0 \quad (4.2)$$

   For oscillations, substituting $s = \jmath\omega$,

$$(R - \omega^2(RC(L_1 + L_2)) + j(\omega L_1 - \omega^3(g_m R + 1)CL_1L_2) = 0 \quad (4.3)$$

   Equating the real part of the above to 0,

$$\omega^2(RC(L_1 + L_2) = R \quad (4.4)$$

$$\implies \omega = \frac{1}{\sqrt{C(L_1 + L_2)}} \quad (4.5)$$

   Similarly, from the imaginary part,

$$g_m R + 1 = \frac{C(L_1 + L_2)}{CL_2} \quad (4.6)$$

$$\implies g_m R = \frac{L_1}{L_2} \quad (4.7)$$

   For stable oscillations,

$$g_m R >= \frac{L_1}{L_2} \quad (4.8)$$

5. Simulate the oscillator using Fig. 5.1 and Table 5.
   **Solution:** The closed loop impulse response is plotted in Fig. 5.2 using the following code.

```
codes/ee18btech11019_1.py
```

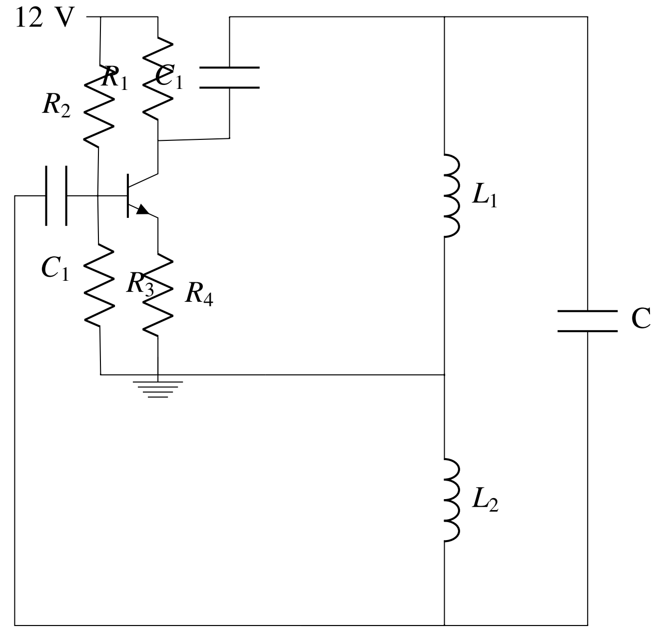


Fig. 5.1: Simulation circuit

| Parameter | Value |
|---|---|
| $R_1$ | $1.2k\Omega$ |
| $R_2$ | $50.5k\Omega$ |
| $R_3$ | $10.5k\Omega$ |
| $R_3$ | $298\Omega$ |
| $C_1$ | $22\mu F$ |
| $C_2$ | $22\mu F$ |
| $C$ | $1.1\mu F$ |
| $L_1$ | $1mH$ |
| $L_2$ | $1mH$ |

TABLE 5

The spice simulations are done using the following netlist

```
spice/Draft3.net
```

and plotted in Fig. 5.3 using the following code.

```
spice/ee18btech11019_2.py
```

From Fig. 5.3, the frequency obtained is 3.384 kHz. From 4.5, the expected frequency is

$$f = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}} \quad (5.1)$$

$$= 3.394\,kHz \quad (5.2)$$

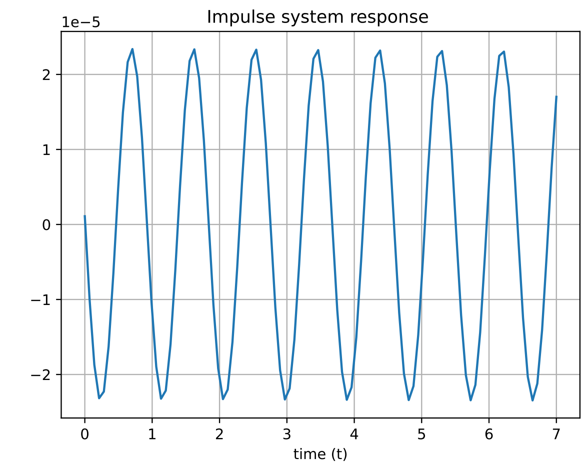


Fig. 5.2: Output when taken from transfer function

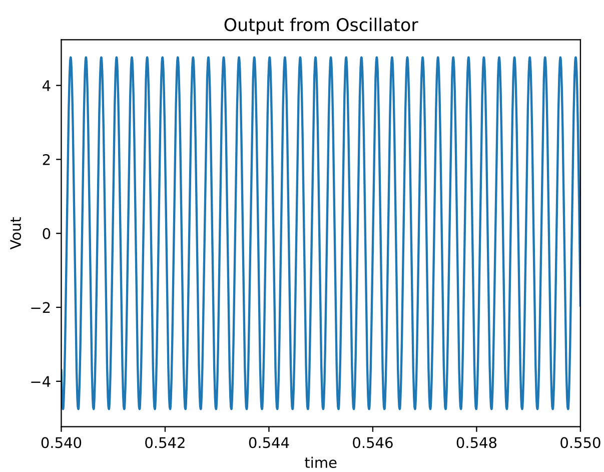


Fig. 5.3: Simulation result