

Control Systems

G V V Sharma*

CONTENTS

1	PID Controller	1
1.1	Introduction	1
2	Polar Plot	3
2.1	Introduction	3
2.2	Example	3
2.3	Example	3
2.4	Example	4
2.5	Example	4

Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

1 PID CONTROLLER

1.1 Introduction

1.1.1. Tabulate the transfer functions of a PID controller and its variants.

Solution: See Table 1.1.1.

Controller	Gain
PID	$K_p \left(1 + T_d s + \frac{1}{T_i s} \right)$
PD	$K_p (1 + T_d s)$
PI	$K_p \left(1 + \frac{1}{T_i s} \right)$

TABLE 1.1.1

1.1.2. For a unity Feedback system

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \quad (1.1.2.1)$$

Design a PD Controller with $K_v = 2$ and Phase Margin 30°

Solution: The gain after cascading the PD Controller with $G(s)$ is

$$G_c(s) = \frac{K_p(1 + T_d s)K}{s(s+2)(s+4)(s+6)} \quad (1.1.2.2)$$

Choosing $K_p = 1$ in ,

$$K_v = \lim_{s \rightarrow 0} s G_c(s) = 2 \quad (1.1.2.3)$$

$$\Rightarrow K = 96 \quad (1.1.2.4)$$

For Phase Margin 30° , at Gain Crossover Frequency ω ,

$$\tan^{-1}(T_d \omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = -60^\circ \quad (1.1.2.5)$$

$$|G_1(j\omega)| = \frac{96 \sqrt{T_d^2 \omega^2 + 1}}{\omega \sqrt{(\omega^2 + 4)(\omega^2 + 16)(\omega^2 + 36)}} = 1 \quad (1.1.2.6)$$

By Hit and Trial, one of the best combinations is

$$\omega = 4 \quad (1.1.2.7)$$

$$T_d = 1.884 \quad (1.1.2.8)$$

We get a Phase Margin of 30.31°

1.1.3. Verify using a Python Plot

Solution: The following code plots Fig. 1.1.3

```
codes/ee18btech11021/EE18BTECH11021_3.py
```

1.1.4. Design a PI Controller with $K_v = \infty$ and Phase Margin 30°

Solution: From Table 1.1.1, the open loop gain in this case is

$$G_1(s) = \frac{K_p \left(1 + \frac{1}{T_i s} \right) K}{s(s+2)(s+4)(s+6)} \quad (1.1.4.1)$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

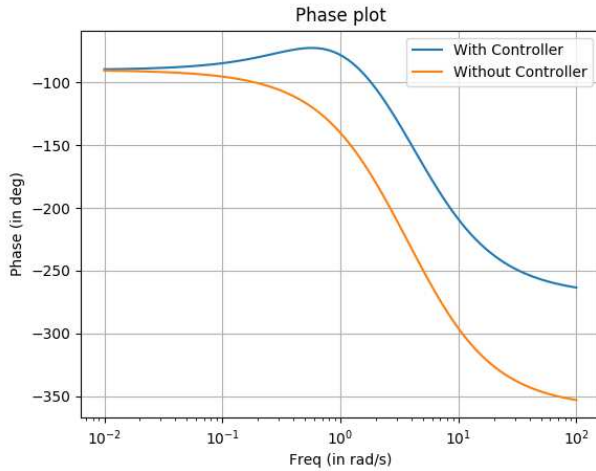


Fig. 1.1.3

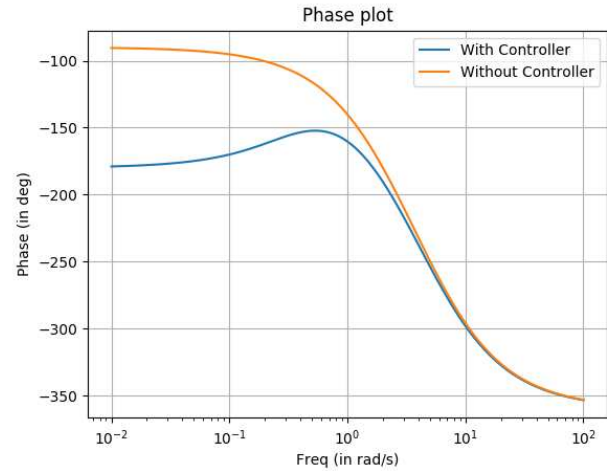


Fig. 1.1.5

Choose $K_p K = 96$. Then

$$G_1(s) = \frac{96(T_i s + 1)}{T_i s^2(s+2)(s+4)(s+6)} \quad (1.1.4.2)$$

For Phase Margin 30° , at Gain Crossover Frequency ω

$$\tan^{-1}(T_i \omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \quad (1.1.4.3)$$

and

$$|G_1(j\omega)| = \frac{96 \sqrt{T_i^2 \omega^2 + 1}}{T_i^2 \omega^2 \sqrt{(\omega^2 + 4)(\omega^2 + 16)(\omega^2 + 36)}} = 1 \quad (1.1.4.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 0.75 \quad (1.1.4.5)$$

$$T_i = 2.713 \quad (1.1.4.6)$$

We get a Phase Margin of 25.53°

1.1.5. Verify using a Python Plot

Solution: The following code plots Fig. 1.1.5.

```
codes/ee18btech11021/EE18BTECH11021_4.py
```

1.1.6. Design a PID Controller with $K_v = \infty$ and Phase Margin 30°

Solution:

$$G_1(s) = \frac{K_p \left(1 + T_d s + \frac{1}{T_i s}\right) K}{s(s+2)(s+4)(s+6)} \quad (1.1.6.1)$$

Choose $K_p K = 96$. The open loop gain is

$$G_1(s) = \frac{96(T_i T_d s^2 + T_i s + 1)}{T_i s^2(s+2)(s+4)(s+6)} \quad (1.1.6.2)$$

For Phase Margin 30° , at Gain Crossover Frequency ω ,

$$\tan^{-1}\left(\frac{T_i \omega}{1 - T_i T_d \omega^2}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) = 30 \quad (1.1.6.3)$$

$$|G_1(j\omega)| = \frac{96 \sqrt{(1 - T_i T_d \omega^2)^2 + T_i^2}}{T_i^2 \omega^2 \sqrt{(\omega^2 + 4)(\omega^2 + 16)(\omega^2 + 36)}} = 1 \quad (1.1.6.4)$$

By Hit and Trial, one of the best combinations is

$$\omega = 1 \quad (1.1.6.5)$$

$$T_i = 1.738 \quad (1.1.6.6)$$

$$T_d = 0.4 \quad (1.1.6.7)$$

We get a Phase Margin of 30°

1.1.7. Verify using a Python Plot

Solution: The following code plots Fig. 1.1.7

```
codes/ee18btech11021/EE18BTECH11021_5.py
```

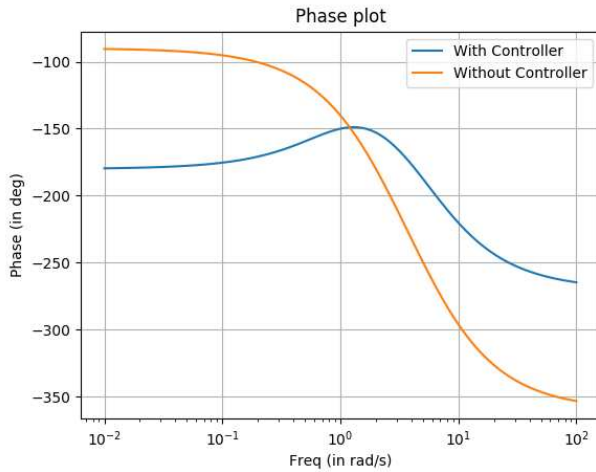


Fig. 1.1.7

2 POLAR PLOT

2.1 Introduction

2.1.1. Sketch the Polar Plot of

$$G(s) = \frac{\left(1 + \frac{s}{29}\right)(1 + 0.0025s)}{(s^3)(1 + 0.005s)(1 + 0.001s)} \quad (2.1.1.1)$$

Solution: The following code generates the polar plot in Fig. 2.1.1

```
codes/ee18btech11029.py
```

- The polar plots use open loop transfer function to determine the stability and hence reference point is shifted to $(-1, 0)$
- If $(-1, 0)$ is left of the polar plot or $(-1, 0)$ is not enclosed, then it is stable
- If $(-1, 0)$ is on right side of the polar plot or $(-1, 0)$ is enclosed by polar plot then it is unstable.
- If $(-1, 0)$ is on the polar plot then it is marginally stable

In Fig. 2.1.1, $(-1, 0)$ is on the polar plot so the system is marginally stable.

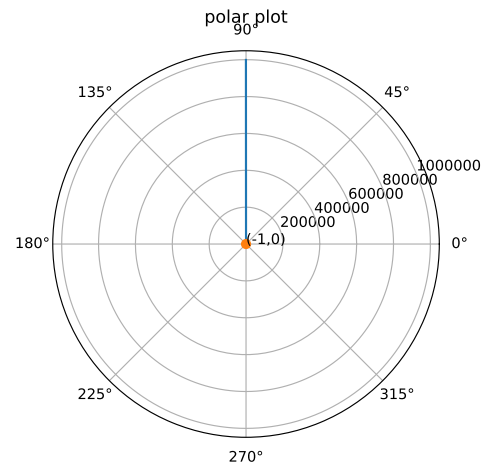


Fig. 2.1.1

2.2 Example

2.2.1. Sketch the Polar Plot of

$$G(s) = \frac{1}{s(1 + s^2)} \quad (2.2.1.1)$$

Solution: From (2.2.1.1),

$$G(j\omega) = \frac{1}{j\omega(1 - \omega^2)} \quad (2.2.1.2)$$

$$|G(j\omega)| = \frac{1}{|\omega(1 - \omega^2)|} \quad (2.2.1.3)$$

$$\angle G(j\omega) = \begin{cases} \frac{\pi}{2} & \omega > 1 \\ -\frac{\pi}{2} & 0 < \omega < 1 \end{cases} \quad (2.2.1.4)$$

The corresponding polar plot is generated in Fig. 2.2.1 using

```
codes/ee18btech11023.py
```

In Fig. 2.2.1, $(-1, 0)$ is exactly on the polar plot. Hence, the system is marginally stable.

2.3 Example

2.3.1. Sketch the Polar Plot for

$$G(s) = \frac{1}{(1 + s)(1 + 2s)} \quad (2.3.1.1)$$

Solution: The following code generates Fig. 2.3.1

```
codes/ee18btech11012.py
```

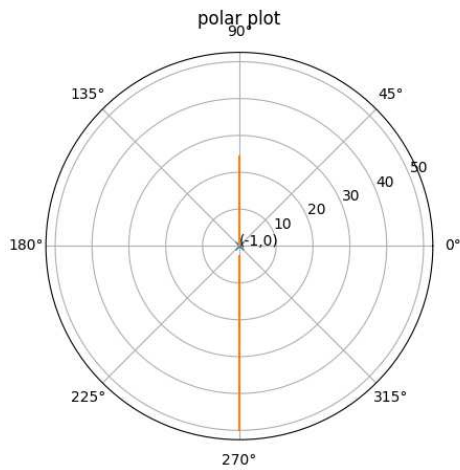


Fig. 2.2.1

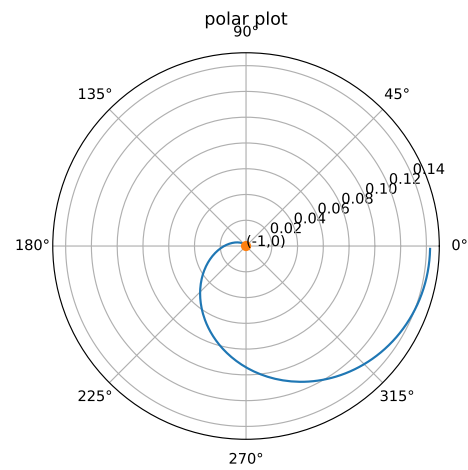


Fig. 2.4.1

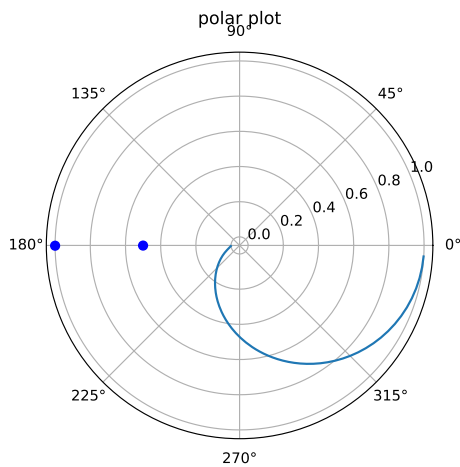


Fig. 2.3.1

The polar plot is to the right of $(-1, 0)$. Hence the closed loop system is stable.

2.4 Example

2.4.1. Plot the polar plot of

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}. \quad (2.4.1.1)$$

Solution:

The following python code generates the polar plot in Fig. 2.4.1

```
codes/ee18btech11033.py
```

$\therefore (-1, 0)$ is on the right side of the polar plot, the system is unstable.

2.5 Example

2.1. Sketch the direct polar plot for a unity feedback system with open loop transfer function

$$G(s) = \frac{1}{s(1+s)^2} \quad (2.1.1)$$

Solution: The polar plot is obtained by plotting (r, ϕ)

$$r = |H(j\omega)||G(j\omega)| \quad (2.1.2)$$

$$\phi = \angle H(j\omega)G(j\omega), 0 < \omega < \infty \quad (2.1.3)$$

The following code plots the polar plot in Fig. 2.1

```
codes/ee18btech11002/polarplot.py
```

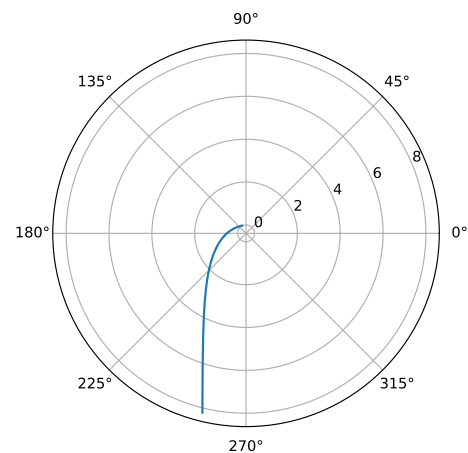


Fig. 2.1: Polar Plot

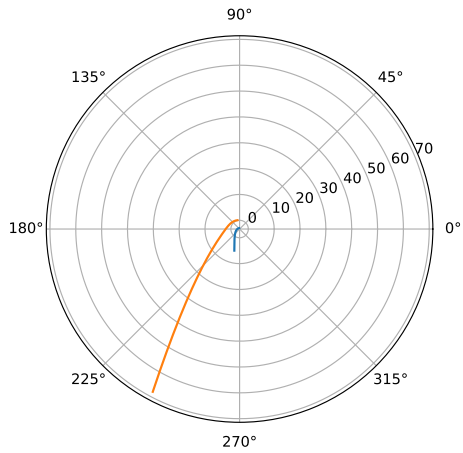


Fig. 2.2: Inverse Polar Plot

2.2. Sketch the inverse polar plot for (2.1.1)

Solution: The above code plots the polar plot in Fig. 2.2 by plotting $\left(\frac{1}{r}, -\phi\right)$