Control Systems

Varum SM*

Consider the positive-feedback circuit shown in Fig. 0.

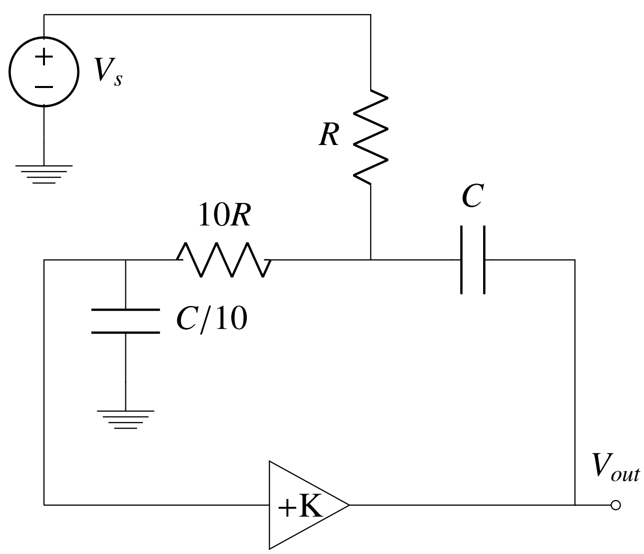

Fig. 0: Positive Feedback Circuit

(a) Find the loop transmission L(s) and the characteristic equation ,find the expressions for resulting pole frequency ω_o and Q factor?
(b) Sketch a Pole-Zero plot for varying K.
(c) For what value of K do the poles coincide? For what value of K does the response becomes maximally flat? For what value of K does the circuit oscillate?

Assume the amplifier has frequency-independent gain, infinite input impedance, and zero output impedance.

1. Compute G(s)
   Solution: :
   - For Gain G(s) greater than 1,the gain block in Fig:0 is built using LM741 op-amp.

$$G(s) = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} \tag{1.1}$$

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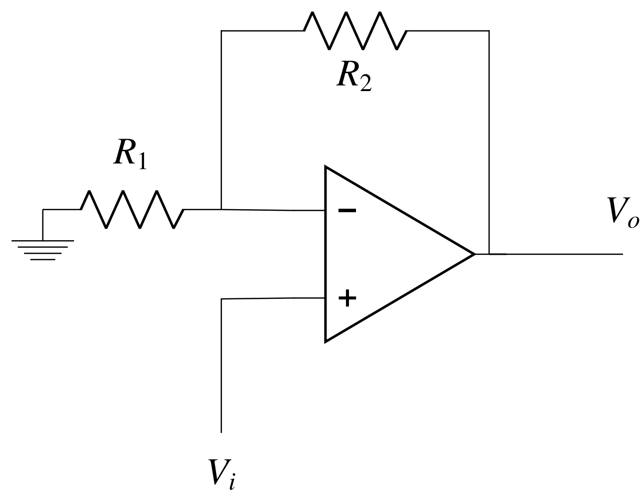

Fig. 1: LM741 op-amp

   - For Gain G(s) less than 1,the gain block(K) in Fig: 0 is built using the voltage divider.

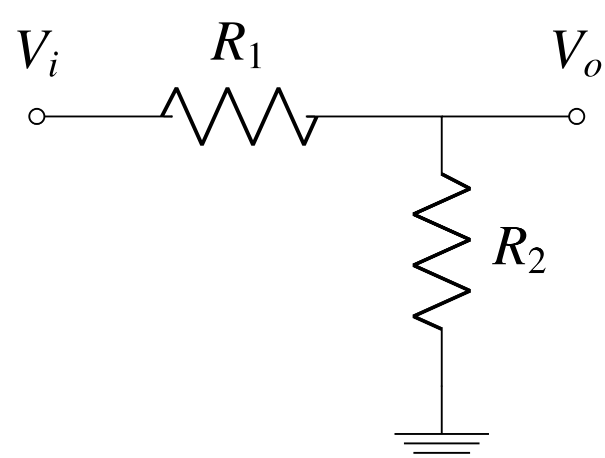

Fig. 1: Voltage Divider

$$G(s) = \frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2} \tag{1.2}$$

2. Compute H(s)
   Solution: :
   - Consider the voltage at the end of resistor R

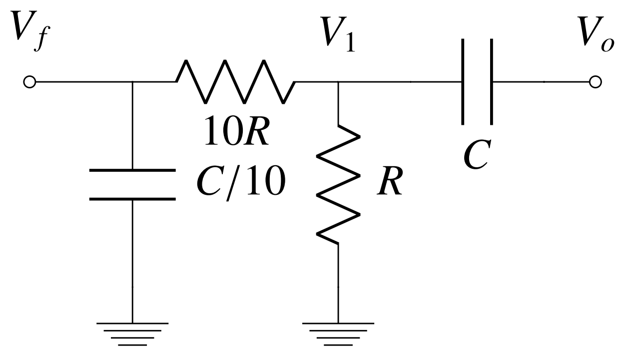

Fig. 2

to be V_1.Apply KCL at nodes.

$$\frac{V_f - V_1}{10R} + \frac{V_f - 0}{\frac{10}{sC}} = 0 \implies V_1 = V_f(sCR + 1) \tag{2.1}$$

$$\frac{V_1 - V_f}{10R} + \frac{V_1 - 0}{R} + \frac{V_1 - V_o}{\frac{1}{sC}} = 0 \tag{2.2}$$

- Substitute V_1 from Eq: 2.1 in Eq: 2.2

$$H(s) = \frac{V_f}{V_o} = \frac{1}{sRC + \frac{1}{sRC} + 2.1} \tag{2.3}$$

3. Find $L(s)$.
Solution: To obtain the loop transmission L(s),

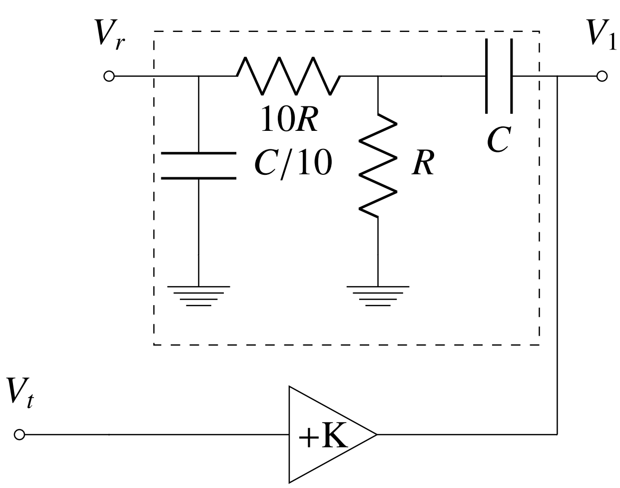

Fig. 3

The loop transmission is given by

$$L(s) = -\frac{V_r(s)}{V_t(s)} = -KH(s) \tag{3.1}$$

where H(s) is the transfer function of the two-port RC network shown inside the broken-line box in Figure: 3.

$$L(s) = \frac{-s(\frac{K}{CR})}{s^2 + s(\frac{2.1}{CR}) + (\frac{1}{CR})^2} \tag{3.2}$$

The characteristic equation is

$$1 + L(s) = 0 \tag{3.3}$$

$$s^2 + s(\frac{2.1 - K}{CR}) + (\frac{1}{CR})^2 = 0 \tag{3.4}$$

The standard characteristic equation of a second order network can be written as

$$s^2 + \frac{\omega_o}{Q}s + \omega_o^2 = 0 \tag{3.5}$$

ω_o is called pole frequency , Q is called pole Qfactor. By comparing the Eq:3.4 with the standard characteristic equation Eq:3.5

$$\omega_o = \frac{1}{RC}; Q = \frac{1}{2.1 - K} \tag{3.6}$$

4. Equivalent control system model of Fig. 3
Solution:

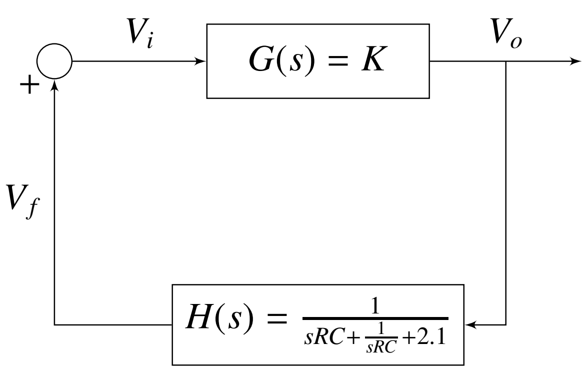

Fig. 4: Positive Feedback Circuit

Closed Loop gain

$$T = \frac{k(s^2 + s(\frac{2.1}{RC}) + (\frac{1}{RC})^2)}{s^2 + s(\frac{2.1-K}{CR}) + (\frac{1}{CR})^2} \tag{4.1}$$

5. Sketch the Normalised closed loop gain of T for various Q values
Solution:
The following is code for the plot

```
codes/ee18btech11030/ee18btech11030.py
```

From Figure:5 ,

- It is observed that maximally flat response is obtained when Q = 0.71

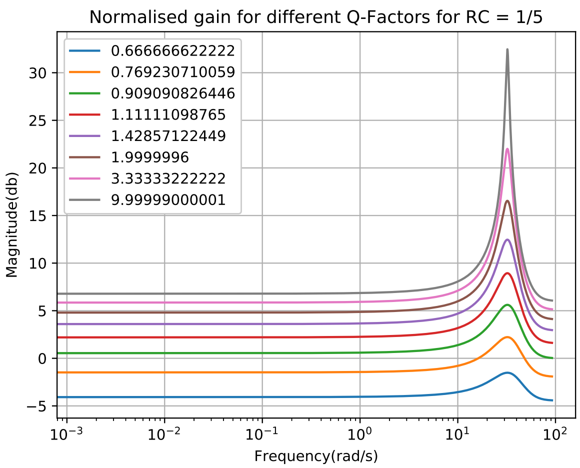

Fig. 5

- It will be seen that response of the feedback amplifier under consideration shows almost no peaking for Q$\leq$ 0.71

6. Sketch a Pole-Zero Plot to Eq:4.1 for a varying K
Solution: :

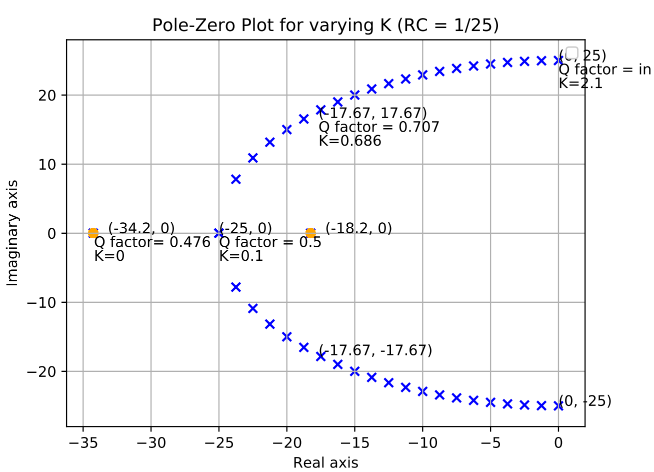

Fig. 6

The following is code for the plot

codes/ee18btech11030/ee18btech11030_1.py

From Figure : 6

- For K = 0,the poles have Q = 0.476 and therefore located on negative real axis.
- As K increases poles are brought closer together and eventually coincide at K = 0.1 and Q = 0.5
- Further increase in K results in poles becoming complex conjugate
- Maximally flat response is obtained when Q = 0.707,which results when K = 0.686.In this case poles are at 45°.
- Oscillating response is obtained when poles are completely imaginary when Q = inf which results when K = 2.1

Q-Factor	K	Requirement
0.5	0.1	Poles are coincident
0.707	0.686	Maximally flat response
∞	2.1	Oscillatory response

TABLE 6

7. Verify the response in time domain using parameters in Table : 8 for K = 2.13
Solution: :

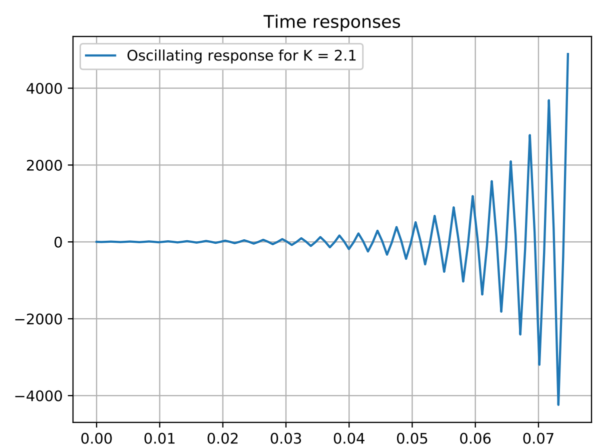

Fig. 7

The following is code for the plot

codes/ee18btech11030/ee18btech11030_2.py

8. Choose the appropriate values of R and C to simulate the circuit for K = 2.1
Solution: :
9. Verify the response using the spice model
Solution: :
   - Figure 9 is the spice simulated output for K = 2.1 using Table 8 parameters.
   - The following is the netlist for simulated circuit.

spice/ee18btech11030/ee18btech11030.net

Parameter	Value
R_1	$10k\Omega$
R_2	$11.3k\Omega$
R	$10k\Omega$
$10R$	$100k\Omega$
$C/10$	$1.6nF$
C	$16nF$

TABLE 8

- The following is code for generating output

```
spice/ee18btech11030/ee18btech11030.py
```

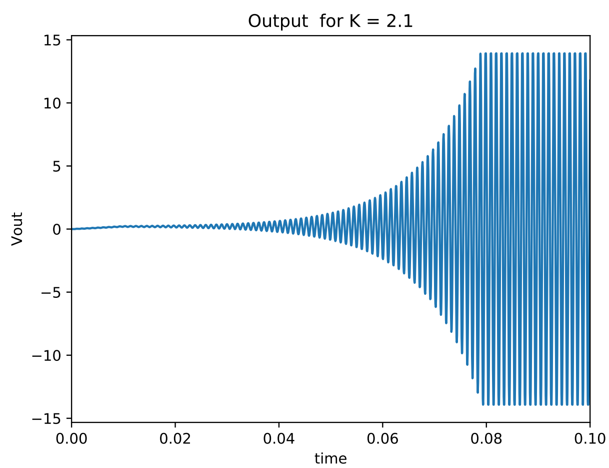

Fig. 9

10. Choose the appropriate values of R and C to simulate the circuit for K = 0.1 and K = 0.686
Solution: :

Parameter	K = 0.686	K = 0.1
R_1	$4.14k\Omega$	$1k\Omega$
R_2	$6.86k\Omega$	$9k\Omega$
R	$10k\Omega$	$10k\Omega$
$10R$	$100k\Omega$	$100k\Omega$
$C/10$	$1.6nF$	$1.6nF$
C	$16nF$	$16nF$

TABLE 10

11. Verify the response using the spice model
Solution:
    - Figure 11 is the spice simulated output for K = 0.686 using Table 10 parameters.

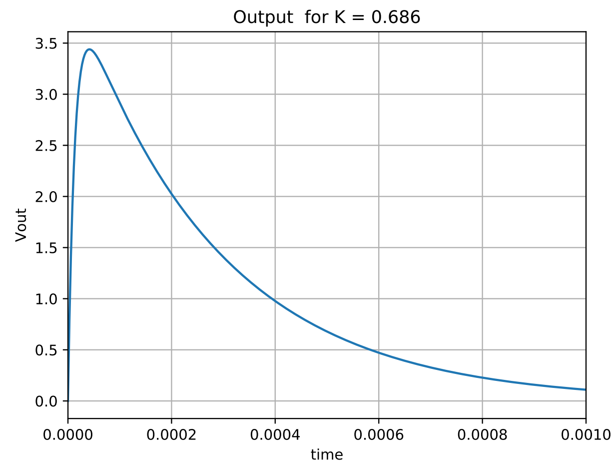

Fig. 11

- The following is the netlist for simulated circuit.

```
spice/ee18btech11030/ee18btech11030_1.
    net
```

- The following is code for generating output

```
spice/ee18btech11030/ee18btech11030_1.
    py
```

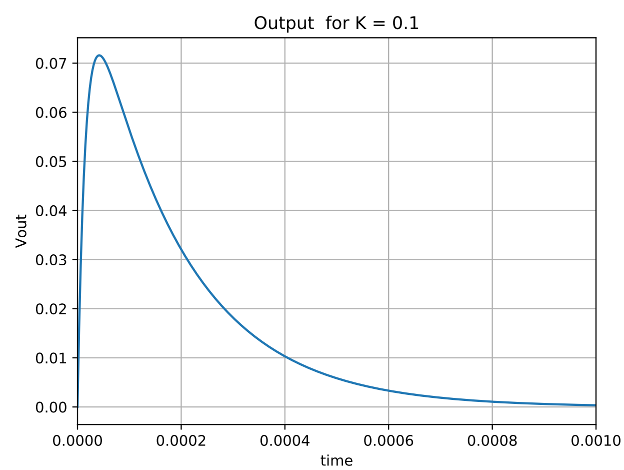

Fig. 11

- Figure 11 is the spice simulated output for K = 0.1 using Table 10 parameters.
- The following is the netlist for simulated circuit.

```
spice/ee18btech11030/ee18btech11030_2.
    net
```

- The following is code for generating output

```
spice/ee18btech11030/ee18btech11030_2.py
```