Quadrature Oscillator

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Consider the quadrature-oscillator circuit given Fig. 0 without the limiter. Let the resistance R_f be equal to $\frac{2R}{1+\Delta}$ where $\Delta << 1$. Show that the poles of the characteristic equation are in the right-half s plane and given by s $\approx \frac{1}{CR}(\frac{\Delta}{4} \pm j)$

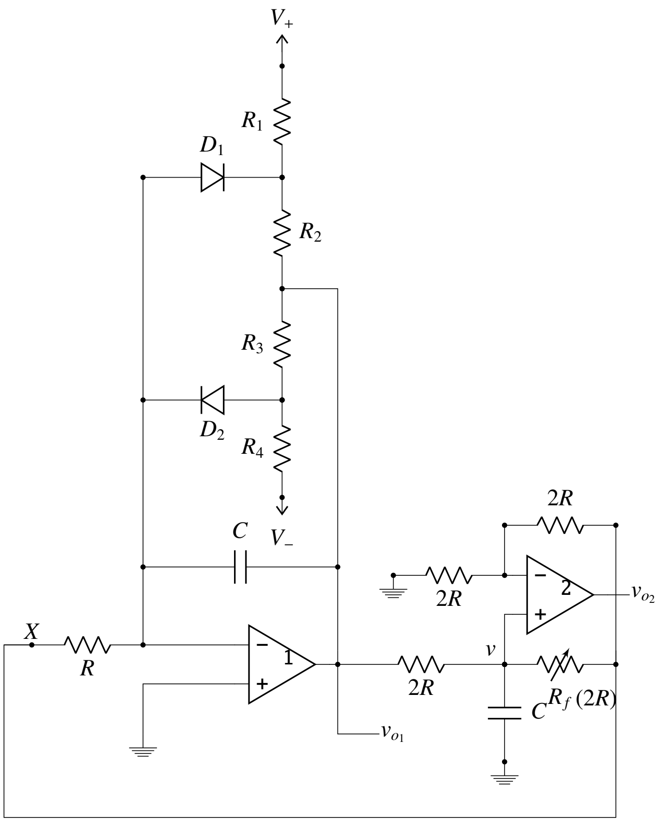

Fig. 0

1. Identify the the open loop gain and feedback components of the circuit.
   Solution: See Figs. 1.1 and 1.2
2. Draw the block diagram and equivalent circuit for H.
   Solution: See Figs. 2.1 and 2.2.

$$H = \frac{v_f}{v_o} \tag{2.1}$$

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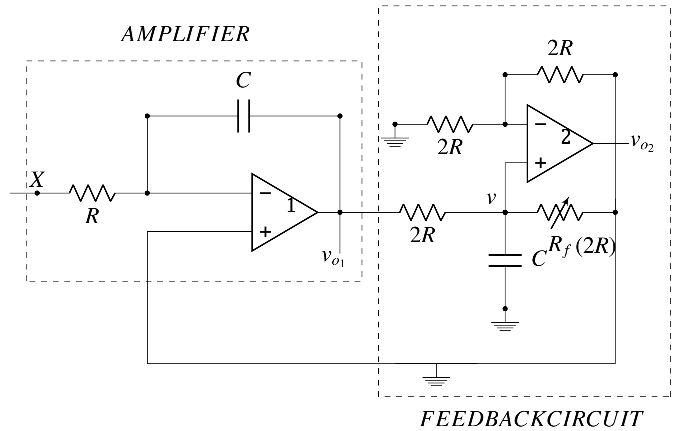

Fig. 1.1: Circuit without the limiter

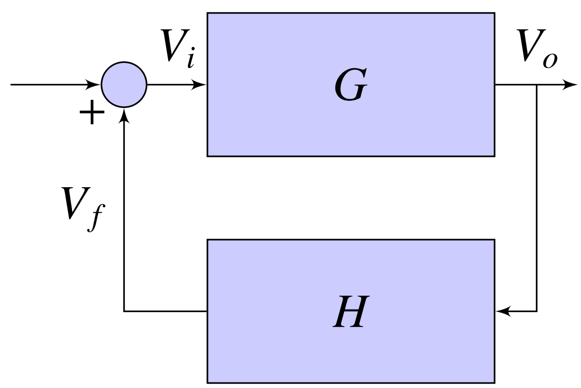

Fig. 1.2: Simplified equivalent block diagram

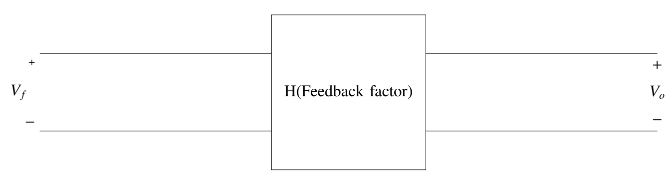

Fig. 2.1: Feedback Block diagram

3. Find H.
   Solution: In Fig. 2.2,

$$v_+ = v_- = \left(\frac{v_{o_2}}{2R + 2R}\right)(2R) = \frac{v_{o_2}}{2} \tag{3.1}$$

Using node analysis at the non- inverting ter-

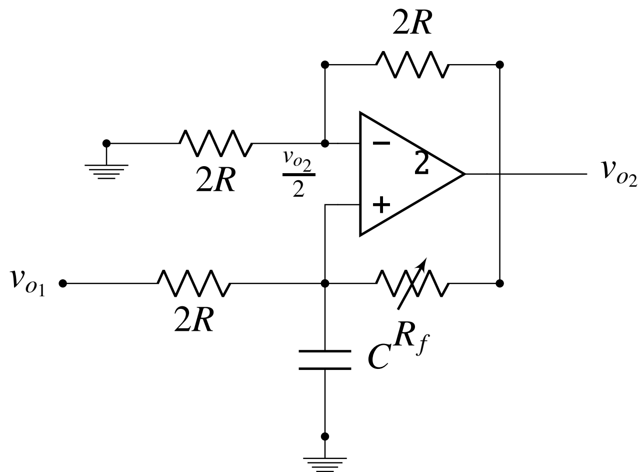

Fig. 2.2: Equivalent Feedback Circuit

minal, and substituting

$$R_f = \frac{2R}{1+\Delta}, \tag{3.2}$$

$$\frac{\frac{v_{o_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{o_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{o_2}}{2} - v_{o_2}}{R_f} = 0 \tag{3.3}$$

$$\implies \frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{2R}(1+\Delta) = 0 \tag{3.4}$$

$$\text{or, } H = \frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \tag{3.5}$$

after some algebra.

4. Find R_{11} and R_{22} from Fig 4
Solution:

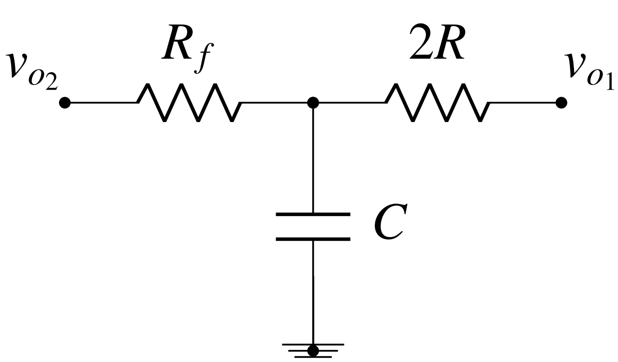

Fig. 4: From Feedback Circuit

Shorting v_{o_1} to ground,

$$R_{11} = R_f + \left(2R \parallel \frac{1}{sC}\right) \tag{4.1}$$

From (3.2),

$$R_{11} = \left(\frac{2R}{1+\Delta}\right) + \left(2R \parallel \frac{1}{sC}\right) \tag{4.2}$$

Shorting v_{o_2} to ground,

$$R_{22} = 2R + \left(R_f \parallel \frac{1}{sC}\right) \tag{4.3}$$

From (3.2),

$$R_{22} = 2R + \left(\left(\frac{2R}{1+\Delta}\right) \parallel \frac{1}{sC}\right) \tag{4.4}$$

5. Draw the block diagram and equivalent circuit for the open loop gain G.
Solution: See Figs. 5.1 and 5.2

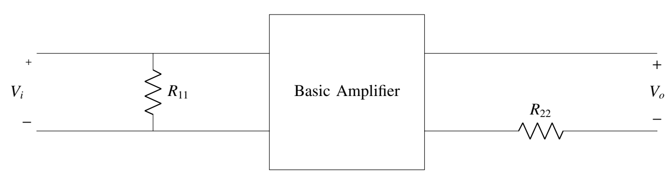

Fig. 5.1: Open Loop Block diagram

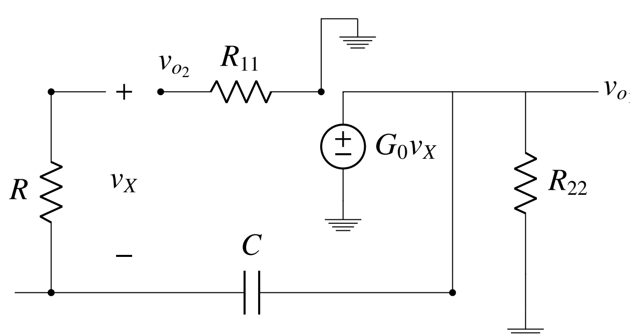

Fig. 5.2: Equivalent Circuit for open loop block diagram

6. Find G.
Solution: From Fig. 5.2,

$$G = \frac{v_{o_1}}{v_X} = -\frac{1}{sCR} \tag{6.1}$$

7. Find the loop gain L and the frequency of oscillation.

Solution: From (6.1) and (3.5),

$$L(s) = G(s)H(s) = \frac{-1}{sCR}\frac{1}{sCR - \frac{\Delta}{2}} \tag{7.1}$$

$$= \frac{1}{-s^2C^2R^2 + \frac{sCR\Delta}{2}} \tag{7.2}$$

Oscillations occur for

$$L(s) = 1 \tag{7.3}$$

$$\implies -s^2C^2R^2 + \frac{sCR\Delta}{2} = 1 \tag{7.4}$$

$$\text{or, } s = \frac{\frac{\Delta}{2} \pm 2j\sqrt{1 - \left(\frac{\Delta}{4}\right)^2}}{2RC} \tag{7.5}$$

$$\implies \omega_0 \approx \frac{1}{RC}, \quad \Delta \ll 1 \tag{7.6}$$

which is the desired frequency of oscillation.

8. What is the significance of Δ?
9. Find the step and impulse response of $T(s)$ for the parameters given in Table 9.
Solution:

$$T(s) = \frac{G(s)}{1 - G(s)H(s)} \tag{9.1}$$

$$= \frac{-SCR + \frac{\Delta}{2}}{s^2C^2R^2 - \frac{sCR\Delta}{2} + 1} \tag{9.2}$$

From Table 9,

$$T(s) = \frac{-0.05s + 0.05}{0.0025s^2 - 0.0025s + 1} \tag{9.3}$$

The following code plots the step response of the system in Fig. 9.1

```
codes/es17btech11009/es17btech11009_1_1.
    py
```

The following code plots the impulse response of the system in Fig. 9.2

```
codes/es17btech11009/es17btech11009_imp.
    py
```

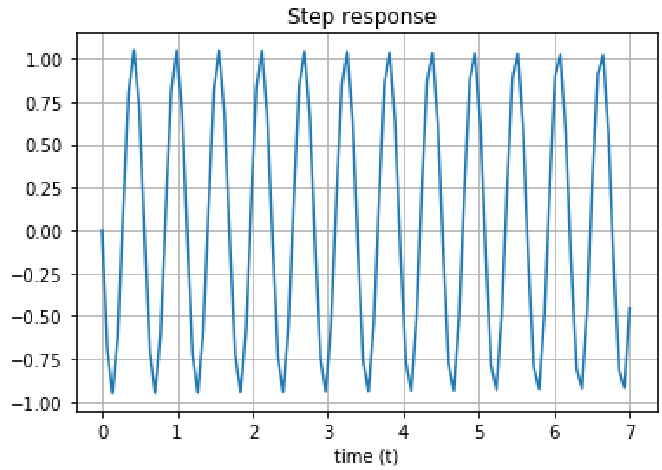

Fig. 9.1

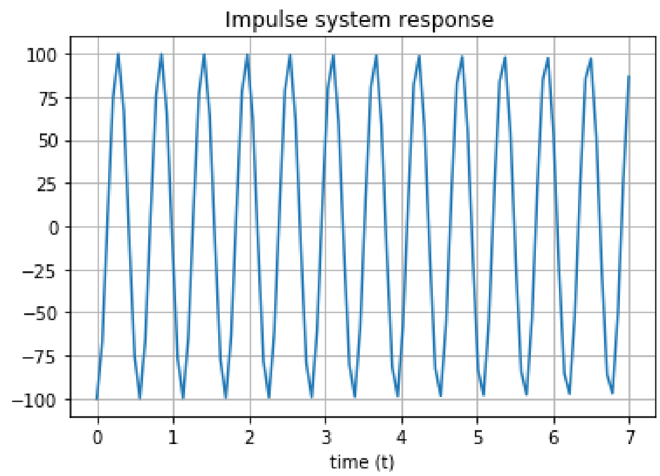

Fig. 9.2

Parameter	Value
R	$1k\Omega$
C	$10\mu F$
Δ	0.1
$R_f = \frac{2R}{1+\Delta}$	1818.18

TABLE 9

10. Verify your results using spice for the parameters given in Table 9. **Solution:** The loop will oscillate at frequency ω_o, given by

$$\omega_o = \frac{1}{RC} \tag{10.1}$$

$$= 20 rad/s \tag{10.2}$$

$$\text{or, } f = 3.184 Hz \tag{10.3}$$

The following spice netlist generates the output

```
spice/es17btech11009.net
```

which is plotted by the following python code in Fig. 10

```
codes/es17btech11009/es17btech11009_spice.
    py
```

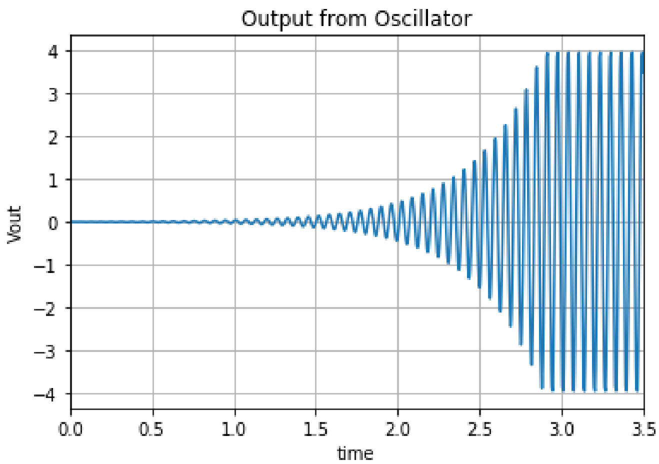

Fig. 10

A zoomed version is plotted in Fig. 10 by the following code.

```
codes/es17btech11009/
    es17btech11009_spice1.py
```

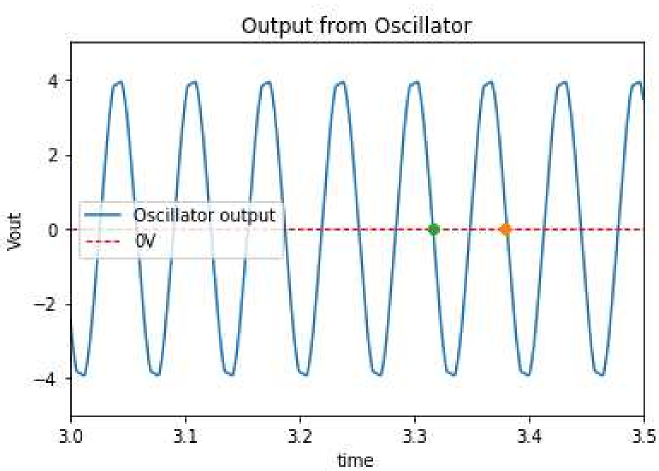

Fig. 10

From Fig 10, Time period of oscillation

$$T = 3.3801 - 3.317 \tag{10.4}$$

$$f = \frac{1}{T} = 15.847Hz \tag{10.5}$$

which is close to the theoretical value in (10.3)