# Quadrature Oscillator

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Consider the quadrature-oscillator circuit given Fig. 0 without the limiter. Let the resistance $R_f$ be equal to $\frac{2R}{1+\Delta}$ where $\Delta << 1$. Show that the poles of the characteristic equation are in the right-half s plane and given by s $\approx \frac{1}{CR}(\frac{\Delta}{4} \pm j)$ ¡¡¡¡¡¡¡ .mine

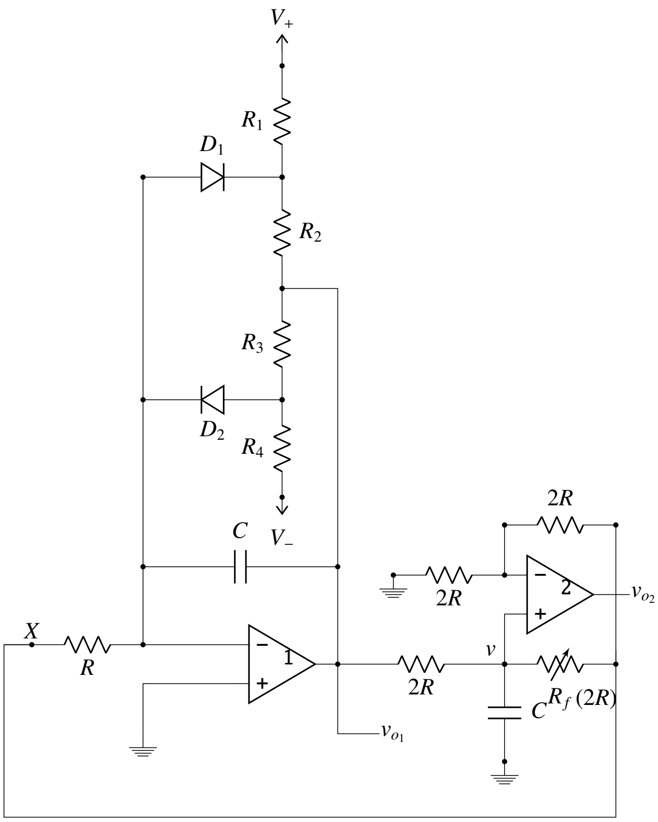


Fig. 0

1. Identify the the open loop gain and feedback components of the circuit.
   **Solution:** See Fig. 1.
2. Draw the block diagram and equivalent circuit for $H$.
   **Solution:** See Figs. 2.1 and 2.2.

$$H = \frac{v_f}{v_o} \tag{2.1}$$

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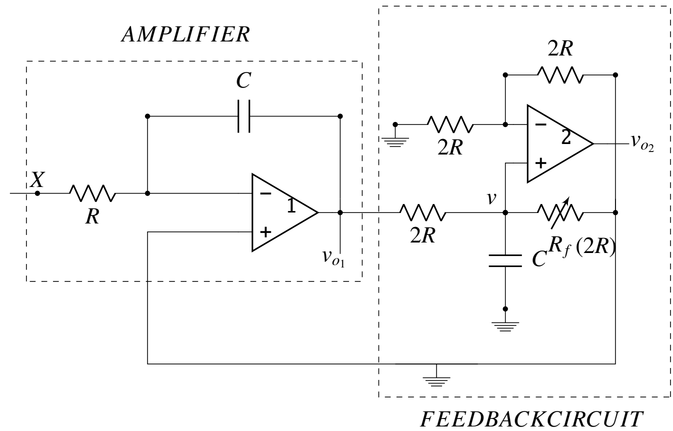


Fig. 1: Circuit without the limiter

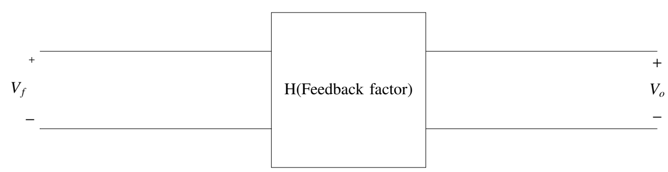


Fig. 2.1: Feedback Block diagram

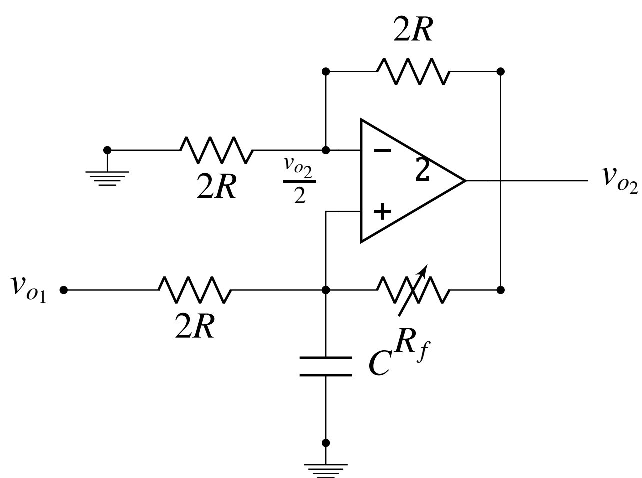


Fig. 2.2: Equivalent Feedback Circuit

3. Find $H$.
   **Solution:** In 2.2,

$$v_+ = v_- = \left(\frac{v_{o_2}}{2R + 2R}\right)(2R) = \frac{v_{o_2}}{2} \tag{3.1}$$

Using node analysis at the non- inverting terminal, and substituting

$$R_f = \frac{2R}{1+\Delta}, \tag{3.2}$$

$$\frac{\frac{v_{o_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{o_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{o_2}}{2} - v_{o_2}}{R_f} = 0 \tag{3.3}$$

$$\implies \frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{2R}(1+\Delta) = 0 \tag{3.4}$$

$$\text{or, } H = \frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \tag{3.5}$$

after some algebra.

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1. Identify the the open loop gain and feedback components of the circuit.
**Solution:** See Fig. 1.

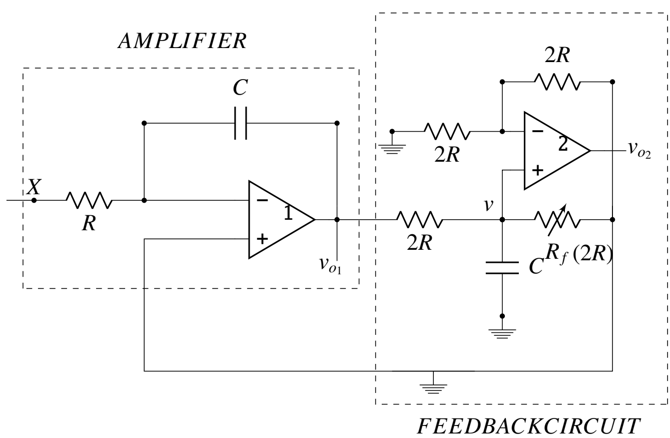


Fig. 3: Circuit without the limiter

2. Draw the block diagram and equivalent circuit for $H$.

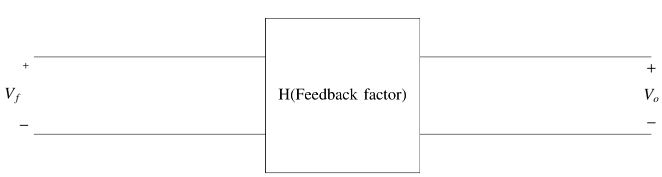


Fig. 3.2: Feedback Block diagram

**Solution:** See Figs. 2.1 and 2.2.

$$H = \frac{v_f}{v_o} \tag{3.6}$$

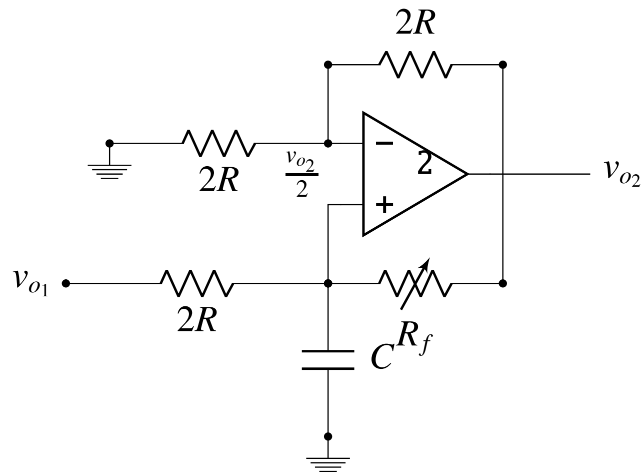


Fig. 3.3: Equivalent Feedback Circuit

3. Find $H$.
**Solution:** In 2.2,

$$v_+ = v_- = \left(\frac{v_{o_2}}{2R+2R}\right)(2R) = \frac{v_{o_2}}{2} \tag{3.7}$$

Using node analysis at the non- inverting terminal, and substituting

$$R_f = \frac{2R}{1+\Delta}, \tag{3.8}$$

$$\frac{\frac{v_{o_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{o_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{o_2}}{2} - v_{o_2}}{R_f} = 0 \tag{3.9}$$

$$\implies \frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{2R}(1+\Delta) = 0 \tag{3.10}$$

$$\text{or, } H = \frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \tag{3.11}$$

after some algebra.
¿¿¿¿¿¿¿ .r28

4. And the Equivalent circuit at the input of op-amp 2 is given in Fig 4.1
5. **Solution:** The voltage $v_{o_1}$ and 2R are replaced with the Norton equivalent composed of a current source $\frac{v_{o_1}}{2R}$ and parallel resistance 2R. The direction of current in $R_f$ would be from output to input. Thus $R_f$ gives rise to a negative input resistance $-R_f$ as indicated in equivalent circuit given in Fig 4.1
When we consider the circuit without the limiter and break the loop at X, The circuit

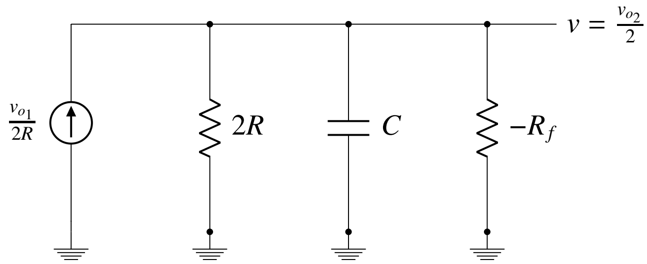


Fig. 3.4

looks as shown in Fig 1

6. Find the open loop gain.
   Consider the general open loop block diagram as shown in Fig 6.1

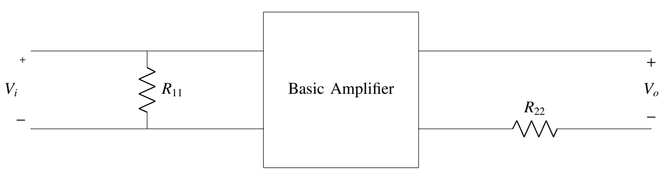


Fig. 3.5: Open Loop Block diagram

$$G = \frac{v_o}{v_i} \tag{3.12}$$

7. Find $R_{11}$ and $R_{22}$ from Fig **??**

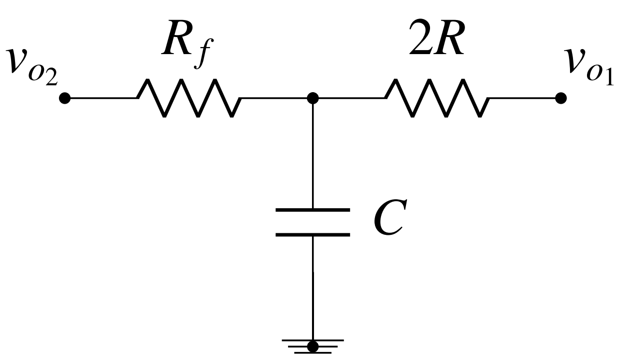


Fig. 3.6: From Feedback Circuit

Shorting $v_{o_1}$ to ground,

$$R_{11} = R_f + \left(2R \parallel \frac{1}{sC}\right) \tag{3.13}$$

From (**??**),

$$R_{11} = \left(\frac{2R}{1+\Delta}\right) + \left(2R \parallel \frac{1}{sC}\right) \tag{3.14}$$

Shorting $v_{o_2}$ to ground,

$$R_{22} = 2R + \left(R_f \parallel \frac{1}{sC}\right) \tag{3.15}$$

From (**??**),

$$R_{22} = 2R + \left(\left(\frac{2R}{1+\Delta}\right) \parallel \frac{1}{sC}\right) \tag{3.16}$$

8. ¡¡¡¡¡¡¡ .mine ———————— .r26 Consider the general block diagram for Feedback network in Fig 2.1

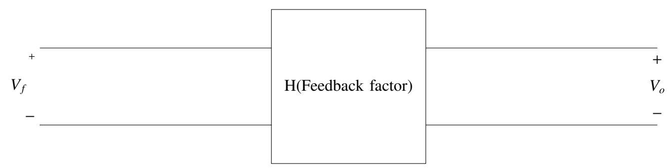


Fig. 3.7: Feedback Block diagram

$$H = \frac{v_f}{v_o} \tag{3.17}$$

9. The equivalent Circuit is shown in Fig 2.2

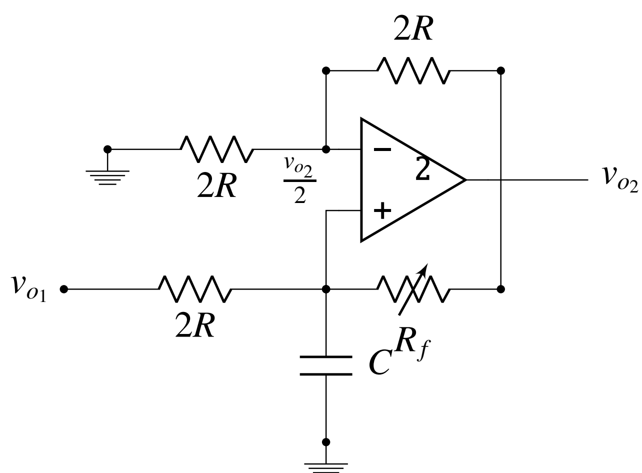


Fig. 3.8: Equivalent Feedback Circuit

$$H = \frac{v_{o_2}}{v_{o_1}} \tag{3.18}$$

10. ======= Equivalent circuit diagram for Fig 6.1 is shown in 7.1
    where $G_0$ is the gain of the op-amp.
    The expression for open loop gain is

$$G = \frac{v_{o_1}}{v_X} = -\frac{1}{sCR} \tag{3.19}$$

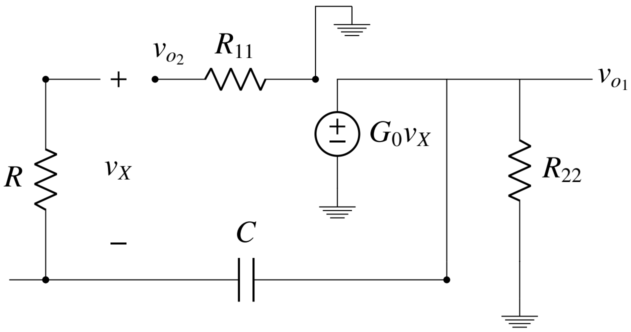


Fig. 3.9: Equivalent Circuit for open loop block diagram

11. ¿¿¿¿¿¿¿ .r28 Draw the equivalent control system representation for the circuit in Fig. 1 which is given in 8.1

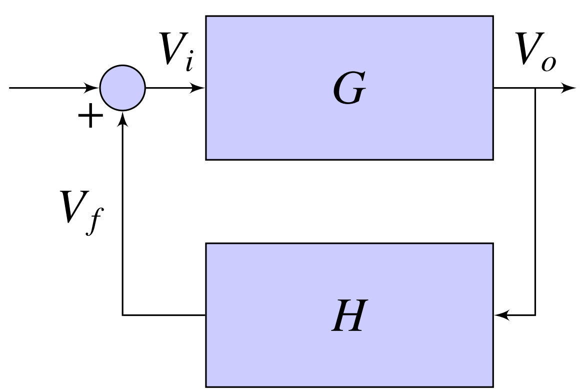


Fig. 3.10: Simplified equivalent block diagram

12. The Quadrature oscillator is based on the second integrator.
As an active filter, the loop is damped to locate the poles in the left half of the s-plane. In the quadrature oscillator, the op amp 1 is connected as an inverting miller integrator with a limiter in the feedback for controlling the amplitude. The op amp 2 is connected as a non-inverting integrator.
13.
14. The transfer function of the equivalent positive feedback circuit in Fig. 2.2 is

$$T = \frac{G}{1 - GH} \tag{3.20}$$

Therefore, loop gain is given by

$$L = GH \tag{3.21}$$

From (7.1) and (3.5)

$$L(s) = \frac{-1}{sCR} \frac{1}{sCR - \frac{\Delta}{2}} \tag{3.22}$$

$$L(s) = \frac{1}{-s^2C^2R^2 + \frac{sCR\Delta}{2}} \tag{3.23}$$

Consider the characteristic equation of the transfer function (11.1),

$$1 - L(s) = 0 \tag{3.24}$$

$$L(s) = 1 \tag{3.25}$$

$$-s^2C^2R^2 + \frac{sCR\Delta}{2} = 1 \tag{3.26}$$

$$\left(C^2R^2\right)s^2 + \left(-\frac{CR\Delta}{2}\right)s + 1 = 0 \tag{3.27}$$

15. Write the expression for roots of a general quadratic equation

$$s_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{3.28}$$

Substitute $a = C^2R^2$, $b = -\frac{CR\Delta}{2}$, $c = 1$ in (12.1),

$$s_p = \frac{-\left(-\frac{CR\Delta}{2}\right) \pm \sqrt{\left(-\frac{CR\Delta}{2}\right)^2 - 4\left(C^2R^2\right)(1)}}{2C^2R^2} \tag{3.29}$$

$$= \frac{RC\left(\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}\right)}{2C^2R^2} \tag{3.30}$$

$$= \frac{\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}}{2RC} \tag{3.31}$$

$$= \frac{\frac{\Delta}{2} \pm 2j\sqrt{1 - \left(\frac{\Delta}{4}\right)^2}}{2RC} \tag{3.32}$$

As $\Delta << 1$,

$$\left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{\frac{1}{2}} = 1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2 \tag{3.33}$$

$$s_p = \frac{\frac{\Delta}{2} \pm 2j\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{3.34}$$

$$s_p = \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{3.35}$$

From (12.8),

$$Re\left(s_p\right) > 0 \tag{3.36}$$

Hence, the poles of the characteristic equation are in the right half of the s plane. As $\Delta << 1$, higher order terms are neglected.

$$s_p = \frac{\frac{\Delta}{2} \pm 2j}{2RC} \tag{3.37}$$

$$s_p = \frac{\frac{\Delta}{4} \pm j}{RC} \tag{3.38}$$

16. Find the frequency for arbitrary R,C values as given in Table 13

| **Parameter** | **Value** |
|---|---|
| $R$ | $5k\Omega$ |
| $C$ | $10\mu F$ |
| $\Delta$ | 0.1 |
| $R_f = \frac{2R}{1+\Delta}$ | 9090.9 |

TABLE 3

The loop will oscillate at frequency $\omega_o$, given by

$$\omega_o = \frac{1}{RC} \tag{3.39}$$

From Table 13,

$$\omega_o = 20rad/s \tag{3.40}$$

$$f = \frac{\omega_o}{2\pi} = 3.184Hz \tag{3.41}$$

From (11.1),

$$T = \frac{-SCR + \frac{\Delta}{2}}{s^2C^2R^2 - \frac{sCR\Delta}{2} + 1} \tag{3.42}$$

From Table 13,

$$T = \frac{-0.05s + 0.05}{0.0025s^2 - 0.0025s + 1} \tag{3.43}$$

The following code plots the oscillating response of the system as shown in 13.1

```
codes/es17btech11009/
    es17btech11009_1_1.py
```

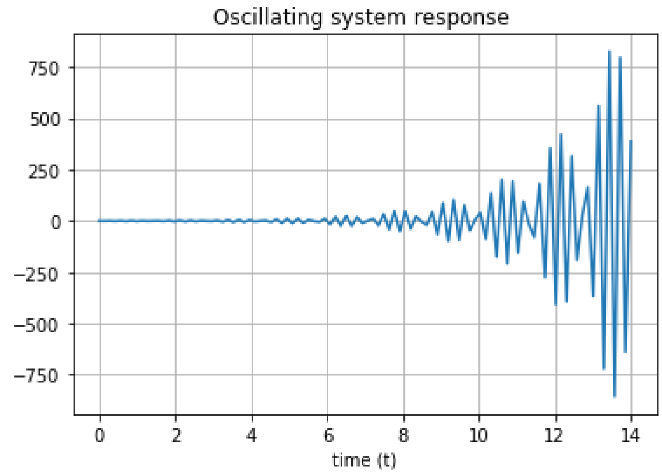


Fig. 3.11

17. Simulate the circuit Fig 2.2 using spice simulators and plot the generated output using python script.
Find the netlist for the simulated circuit here:

```
spice/es17btech11009.net
```

Python code used for generating the output:

```
codes/es17btech11009/
    es17btech11009_spice.py
```

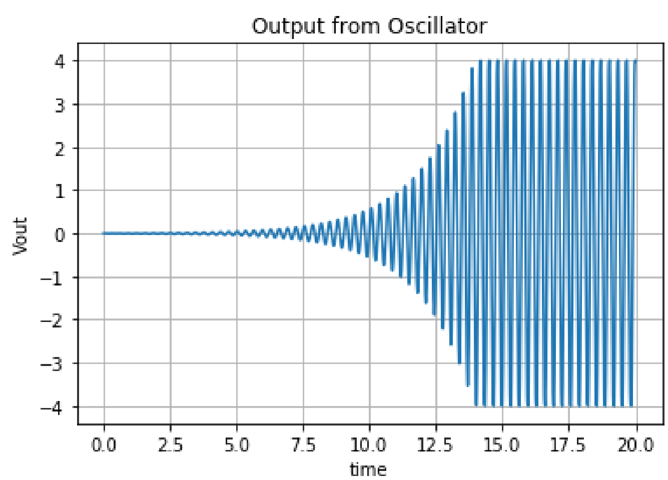


Fig. 3.12

18. Consider part of the spice simulation and the following code plots the part of the output as shown in Fig 15.1