Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

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svn co https://github.com/gadepall/school/trunk/control/codes
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1 Feedback Circuits

1.1. The CE BJT amplifier in Fig. 1.1 employs shunt–shunt feedback: Feedback resistor R_F senses the output Voltage V_o and provides a feedback current to the base node. $\left(R_f = 56k\Omega, R_C = 5.6k\Omega, R_S = 10k\Omega\right)$

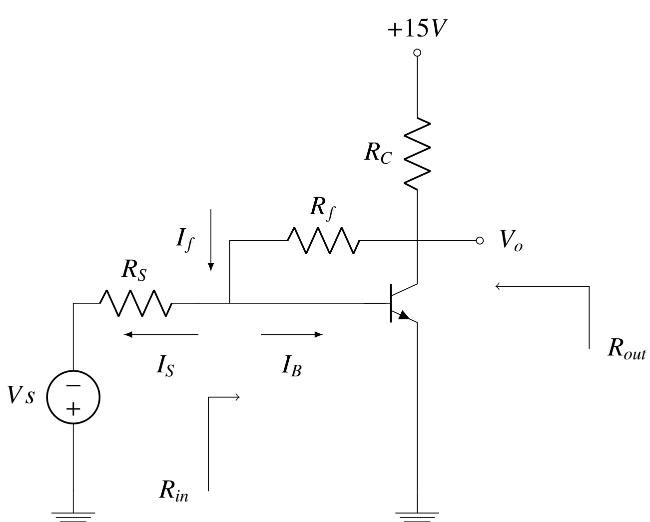

Fig. 1.1

1.2. If V_s has a zero dc component, find the dc collector current of the BJT. Assume the transistor H = 100.

Solution: Since, $V_E = 0$ and $V_S = V_{BE}$

$$I_S = \frac{V_{BE}}{R_S} = \frac{0.7}{10*10^3} \tag{1.2.1}$$

$$\implies I_S = 0.07mA \tag{1.2.2}$$

Applying KCL at feedback resistor output

$$-V_o + V_{BE} + I_f R_f = 0$$
$$\left(Since, I_f = I_B + I_S\right)$$
$$V_o = V_{BE} + (I_B + I_S) R_f$$
$$= 0.7 + \left(I_B + 0.07*10^{-3}\right)\left(56*10^3\right)$$
$$\implies V_o = \left(56*10^3\right) I_B + 4.62 \tag{1.2.3}$$

Applying KCL at collector node

$$\frac{V_o - 15}{5.6*10^3} + I_C + I_f = 0$$
$$(Since, I_C = HI_B)$$
$$\frac{V_o - 15}{5.6*10^3} + HI_B + (I_B + I_S) = 0$$
$$\frac{V_o - 15}{5.6*10^3} + (100+1) I_B + \left(0.07*10^{-3}\right) = 0$$
$$\implies V_o = 14.608 - \left(565.5*10^3\right) I_B \tag{1.2.4}$$

Subtracting 1.2.3 from 1.2.4, we get,

$$I_B = 16.06\mu A \tag{1.2.5}$$
$$I_C = I_E = HI_B \tag{1.2.6}$$
$$\text{Dc collector Current}, I_C = 1.606mA \tag{1.2.7}$$

1.3. Find the small-signal equivalent circuit of the amplifier with the signal source represented by its Norton equivalent (as we usually do when the feedback connection at the input is shunt).
Solution: In fig 1.3

1.4. Find the G circuit and determine the value of G, R_i , and R_o.
Solution: G circuit in fig. 1.4

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Parameter	Description
R_{in}	Total Input Resistance
R_{out}	Total Output Resistance
r_o	Output resistance of NPN
R_f	Feedback resistance
R_l	Input resistance of G circuit
R_o	Output resistance of G circuit
R_{if}	Input resistance of Feedback
R_{of}	Output resistance of Feedback
R_s	Resistance of Current Source
R_L	Output Load Resistance
g_m	Trans conductance
I_C	Collector current
I_E	Emitter Current
I_B	Base Current

TABLE 1.2

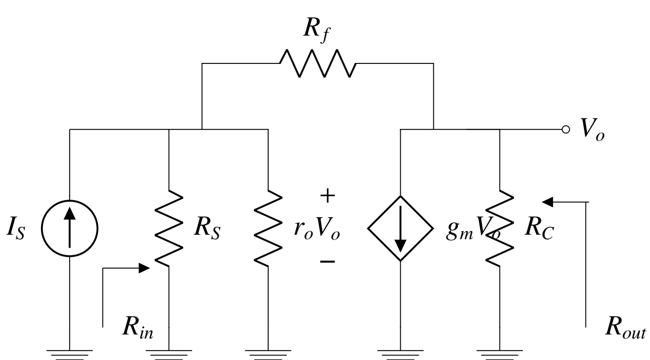

Fig. 1.3

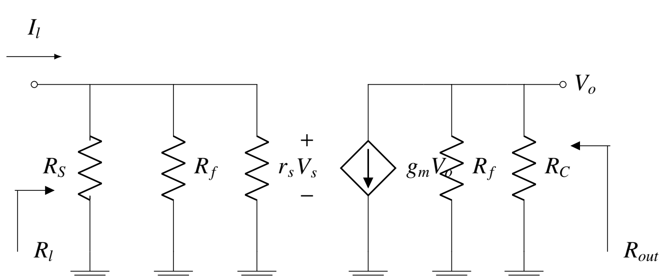

Fig. 1.4

$$g_m = \frac{I_C}{V_s} = \frac{1.606 * 10^{-3}}{25 * 10^{-3}} = 64mA/V \tag{1.4.1}$$

$$r_s = \frac{H}{g_m} = \frac{100}{64 * 10^{-3}} = 1.56k\Omega \tag{1.4.2}$$

$$\text{Gain}, G = \frac{V_o}{I_l} \tag{1.4.3}$$

$$G = \frac{V_S}{I_l}\left(\frac{V_o}{V_o}\right) \tag{1.4.4}$$

$$V_o = -g_m V_s \left(R_f \| R_C\right) \tag{1.4.5}$$

$$V_s = I_l \left(R_S \| R_f \| r_s\right) \tag{1.4.6}$$

Substituting V_o V_s in 1.4.4,

$$G = -g_m \left(R_f \| R_c\right)\left(R_s \| R_f \| r_s\right) \tag{1.4.7}$$

$$G = -429k\Omega \tag{1.4.8}$$

Input Resistance

$$R_I = \left(R_s \| R_f \| r_s\right) = 1.31k\Omega \tag{1.4.9}$$

$$R_o = \left(R_f \| R_C\right) \tag{1.4.10}$$

$$\text{Output Resistance}, R_o = 5.09k\Omega \tag{1.4.11}$$

1.5. Find H and hence AH and 1+AH.
Solution:

$$H = \frac{I_f}{V_o} = -\frac{1}{R_f} \tag{1.5.1}$$

$$\implies H = -17.85 * 10^{-4} \tag{1.5.2}$$

$$GH = 7.662 \tag{1.5.3}$$

$$1 + GH = 8.66 \tag{1.5.4}$$

1.6. Find T , R_{if} and R_{of} and hence R_{in} and R_{out} .
Solution:

$$T = \frac{G}{1 + GH} \tag{1.6.1}$$

$$= -49.54k\Omega \tag{1.6.2}$$

$$R_{if} = \frac{R_l}{1 + GH} \tag{1.6.3}$$

$$= \frac{1.31 * 10^3}{8.66} \tag{1.6.4}$$

$$= 151.27\Omega \tag{1.6.5}$$

$$R_{of} = \frac{R_o}{1 + GH} \tag{1.6.6}$$

$$= \frac{5.09 * 10^3}{8.66} \tag{1.6.7}$$

$$= 587.7\Omega \tag{1.6.8}$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \tag{1.6.9}$$

$$= 153.2\Omega \tag{1.6.10}$$

1.7. What voltage gain V_o/V_s is realized? How does this value compare to the ideal value obtained

if the loop gain is very large and thus the signal voltage at the base becomes almost zero (like what happens in an inverting op-amp circuit).
Solution:

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} \tag{1.7.1}$$

$$= \frac{T}{R_s} \tag{1.7.2}$$

$$\text{Since, } T = \frac{V_o}{I_S} \tag{1.7.3}$$

$$\frac{V_o}{V_s} = \frac{-49.54 * 10^3}{10 * 10^3} \tag{1.7.4}$$

$$= -4.95V/V \tag{1.7.5}$$

If the loop gain is very large, then the gain with feedback T is:

$$T = \frac{1}{H} \tag{1.7.6}$$

$$= \frac{1}{(-17.85 * 10^{-6})} \tag{1.7.7}$$

$$= -56k\Omega \tag{1.7.8}$$

∴ the closed loop gain, T = - R_f

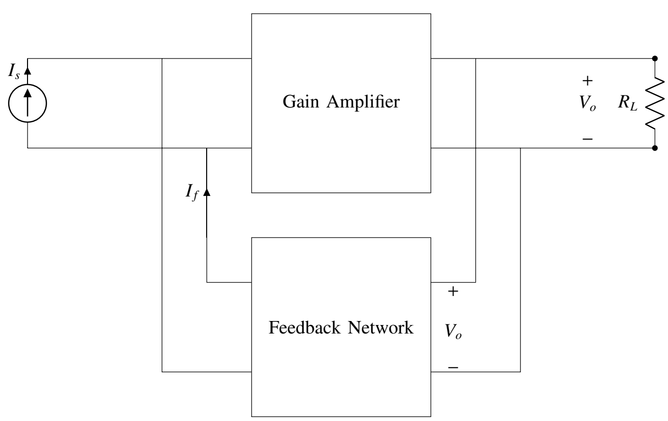

Fig. 1.7: Shunt-Shunt Amplifier Block Diagram

1.8. Verify your solution using spice
Solution: Doing operational Analysis on the Circuit 1.1
Table 1.2 is close to the numerical Calculation done above.

Parameter	**Value**
R_f	$56k\Omega$
R_S	$10k\Omega$
R_C	$5.6k\Omega$
I_S	$0.07mA$
I_B	$16.06\mu A$
I_C	$1.606mA$
I_E	$1.606mA$
g_m	$64mA/V$
r_s	$1.56k\Omega$
G	$-429k\Omega$
R_I	$1.31k\Omega$
R_o	$5.09k\Omega$
H	$-17.85 * 10^{-4}$
GH	7.662
$1 + GH$	8.66
T	$-49.54k\Omega$
R_{if}	$151.27k\Omega$
R_{of}	$587.7k\Omega$
R_{in}	153.2Ω

TABLE 1.7

Parameter	**Value**
I_C	$2.1mA$
I_E	$2.1mA$
I_B	$2.1\mu A$
I_S	$0.07mA$

TABLE 1.8