

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

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3.2 Example

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4.3 Stability

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8 PHASE MARGIN

8.1. The open loop transfer function of a system is

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (8.1.1)$$

Find its magnitude and phase response.

Solution: Substituting $s = j\omega$ in (8.1.1),

$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} \quad (8.1.2)$$

$$\Rightarrow |G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})} \quad (8.1.3)$$

$$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (8.1.4)$$

8.2. Find ω for which the gain of (8.1.1) first becomes 1.

Solution: From (8.1.3)

$$|G(j\omega)| = 1 \quad (8.2.1)$$

$$\Rightarrow \frac{2}{(\sqrt{\omega^2 + 1})(\sqrt{\omega^2 + 4})} = 1 \quad (8.2.2)$$

$$\Rightarrow \omega_{gc} = 0 \quad (8.2.3)$$

which is the desired frequency.

8.3. Find $\angle G(j\omega_{gc}) + 180^\circ$. This is known as the *phase margin*(PM)

Solution: From (8.1.4),

$$\angle G(j\omega) = 0^\circ \Rightarrow PM = 180^\circ \quad (8.3.1)$$

8.4. Verify your result by plotting the gain and phase plots of $G(j\omega)$.

Solution: The following code plots Fig. 8.4

```
codes/ee18btech11017.py
```

The Phase plot is as shown,

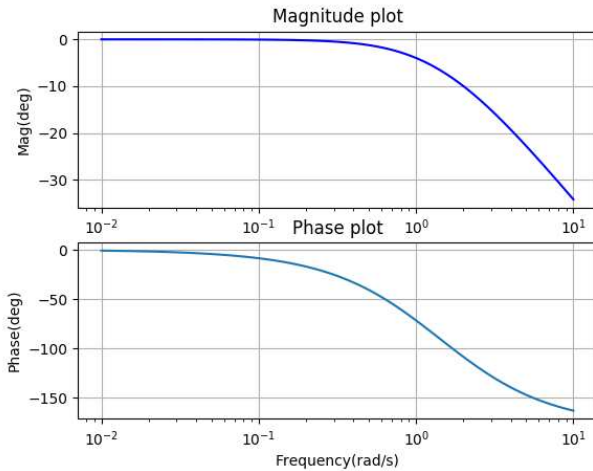


Fig. 8.4

8.5. A positive phase margin for the open loop system indicates a stable closed loop system. (8.3.1) indicates that $G(s)$ with unity feedback is stable. Show that the roots of $1 + G(s)$ lie in the left half plane proving closed loop stability.

Solution: Let the closed loop transfer function

$$T(s) = \frac{G(s)}{1 + G(s)} \quad (8.5.1)$$

Then

$$1 + G(s) = 0 \quad (8.5.2)$$

$$\Rightarrow s^2 + 3s + 4 = 0 \quad (8.5.3)$$

$$\text{or } s = -1.5 + 1.3j, -1.5 - 1.3j \quad (8.5.4)$$

Since the roots are in the left half plane, the system is stable.

8.6. Instead of unity feedback, consider a system with

$$H(s) = \frac{50}{s + 1} \quad (8.6.1)$$

Compute the open loop phase margin for this system.

Solution:

$$\therefore G(s)H(s) = \frac{100}{(s + 1)^2(s + 2)}, \quad (8.6.2)$$

the magnitude and phase are

$$|G(j\omega)H(j\omega)| = \frac{10^2}{\sqrt{(\omega^2 + 1)^2} \sqrt{\omega^2 + 4}} \quad (8.6.3)$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1} \frac{\omega}{2} - 2 \tan^{-1}(\omega) \quad (8.6.4)$$

The gain crossover frequency is given by

$$\frac{10^2}{\sqrt{\omega_{gc}^2 + 4} \sqrt{(\omega_{gc}^2 + 1)^2}} = 1 \quad (8.6.5)$$

$$(8.6.6)$$

$$\omega_{gc}^6 + 6\omega_{gc}^4 + 9\omega_{gc}^2 - 9996 = 0 \quad (8.6.7)$$

$$\Rightarrow \omega_{gc} = 4.42 \quad (8.6.8)$$

From (8.6.4) and (8.6.8), the phase margin is

$$PM = 180^\circ - 2 \tan^{-1}(\omega_{gc}) - \tan^{-1} \left(\frac{\omega_{gc}}{2} \right) \quad (8.6.9)$$

$$\Rightarrow P.M = -40.15^\circ \quad (8.6.10)$$

8.7. Verify your result through the magnitude and phase plot.

Solution: The following code plots Fig. 8.7

```
codes/ee18btech11017_2.py
```

8.8. Since the PM in (8.6.10) is negative, the closed loop system is unstable. Verify this using the Routh-Hurwitz criterion.

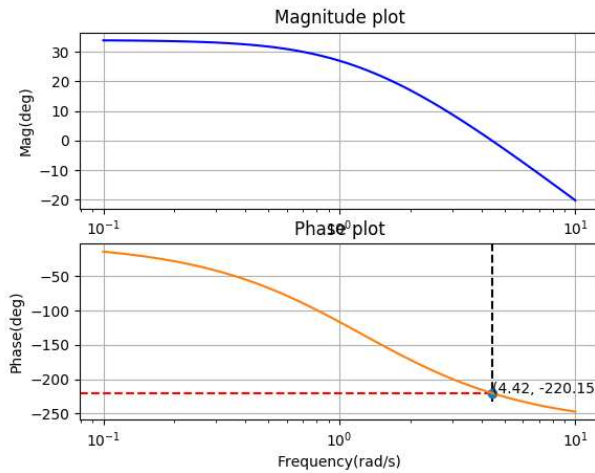


Fig. 8.7

Solution: The characteristic equation is

$$1 + G(s)H(s) = 0 \quad (8.8.1)$$

$$\Rightarrow s^3 + 4s^2 + 5s + 102 = 0 \quad (8.8.2)$$

Constructing the routh array for (8.8.2),

$$\begin{array}{c|ccc} s^3 & 1 & 5 & 0 \\ s^2 & 4 & 102 & 0 \\ s & -20.5 & 0 & 0 \end{array} \quad (8.8.3)$$

$$\begin{array}{c|ccc} s^3 & 1 & 5 & 0 \\ s^2 & 4 & 102 & 0 \\ s & -20.5 & 0 & 0 \\ s^0 & 102 & 0 & 0 \end{array} \quad (8.8.4)$$

\therefore there are two sign changes in the first column of the routh array, two poles lie on right half of s-plane. Therefore, the system is unstable.

9 GAIN MARGIN