OPAMP Stability

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An op amp having a low-frequency gain of 10^3 and a single-pole rolloff at 10^4 rad/s is connected in a negative feedback loop via a feedback network having a transmission k and a two-pole rolloff at 10^4 rad/s. Find the value of k above which the closed-loop amplifier becomes unstable.

1. Find the OPAMP gain $G(s)$
   Solution: The given oscillator has a low frequency gain 10^3 and a single-pole rolloff at 10^4 rad/s. So we have a open loop amplifier gain
   $$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \tag{1.1}$$
2. Find the feedback $H(s)$
   Solution:
   $$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \tag{2.1}$$
3. Find the loop-gain.
   Solution: The loop gain is given by
   $$L(s) = G(s)H(s) = \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3} \tag{3.1}$$
   and the various gains summarised in Table 3

Parameters	Definition	For given question
Open loop gain	G	$\frac{10^3}{1+\frac{s}{10^4}}$
Feedback factor	H	$\frac{k}{\left(1+\frac{s}{10^4}\right)^2}$
Loop gain	GH	$k\left(\frac{10}{1+\frac{s}{10^4}}\right)^3$

TABLE 3

4. Find the PM and the condition for stability.
   Solution: For stability, $PM > 0$ For the given system :
   $$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} = -3tan^{-1}\left(\frac{\omega}{10^4}\right) \tag{4.1}$$
   So,
   $$180^\circ = -3tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) \tag{4.2}$$
   $$\implies \omega_{180} = -\sqrt{3} \times 10^4 \tag{4.3}$$
   The Loop gain at ω_{180} is $G(j\omega_{180})H(j\omega_{180})$. The system becomes unstable if
   $$G(j\omega_{180})H(j\omega_{180}) \geq 1 \tag{4.4}$$
   $$\implies \left|\frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3}\right| \geq 1 \tag{4.5}$$
   $$\left|\frac{10^3 k}{\left(1 - \sqrt{3}j\right)^3}\right| \geq 1 \tag{4.6}$$
   $$\frac{10^3 k}{\left|\sqrt{1 + \sqrt{3}^2}\right|} \geq 1 \tag{4.7}$$
   $$\frac{10^3 k}{8} \geq 1 \tag{4.8}$$
   $$\implies k \geq 0.008 \tag{4.9}$$
   Hence, the value of k above which the system becomes unstable is 0.008.
5. Design the feedback circuit H.
6. Design the closed loop circuit. You may choose a suitable vale of k such that the system is stable.
7. Sketch the Bode plot of the closed loop system.
8. Find the output of the circuit for an appropriate input using spice.

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