Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Op-Amp RC Oscillator Circuits

1.1. For the circuit shown in Fig. 1.1.1, find $L(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

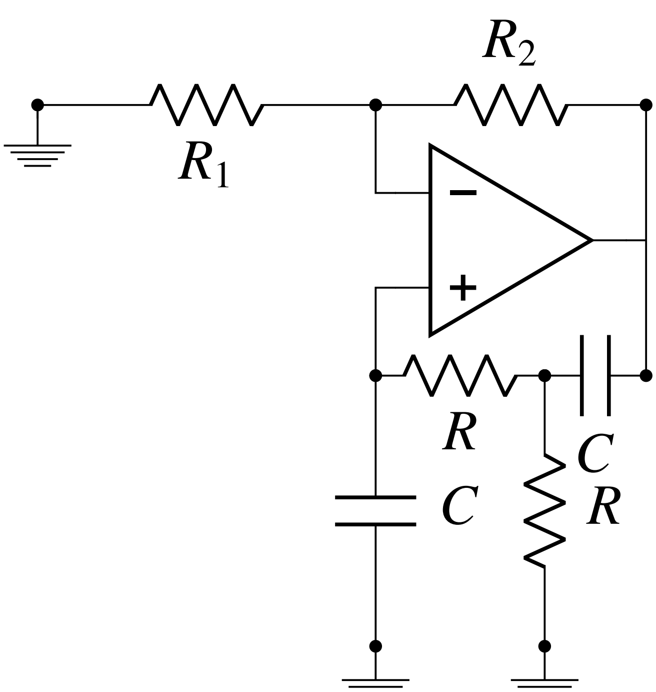

Fig. 1.1.1

Solution: The equivalent control system representation is shown in Fig. 1.1.2. Oscillators do not include input signal.

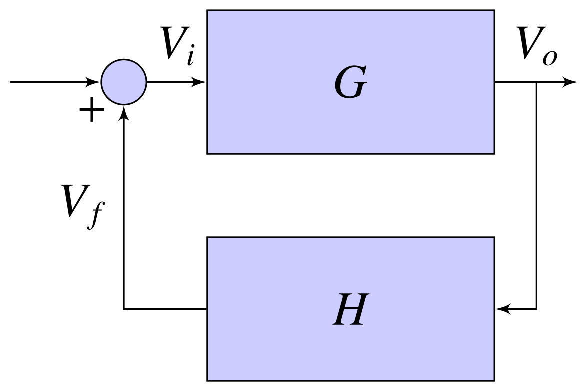

Fig. 1.1.2

1.2. Find the open loop gain G.
Solution: Let the closed loop gain, open-loop gain of op-amp connected in non-inverting configuration be T_1 and G_1 respectively. From Table **??**

$$T_1 = \frac{G_1(R_1+R_2)}{(R_1+R_2)+G_1R_1} \tag{1.2.1}$$

$$T_1 = \frac{(R_1+R_2)}{(R_1+R_2)/G_1+R_1} \tag{1.2.2}$$

Assuming $G_1 \to \infty$

$$T_1 = 1+\frac{R_2}{R_1} \tag{1.2.3}$$

The open loop gain of the circuit shown in Fig. 1.1.1 is equal to the closed loop gain of an op-amp connected in non-inverting configuration.

$$G = T_1 \tag{1.2.4}$$

$$\implies G = 1+\frac{R_2}{R_1} \tag{1.2.5}$$

1.3. Find the feedback factor H.
Solution: The small signal model is shown in Fig. 1.3 Applying KCL at node V_f

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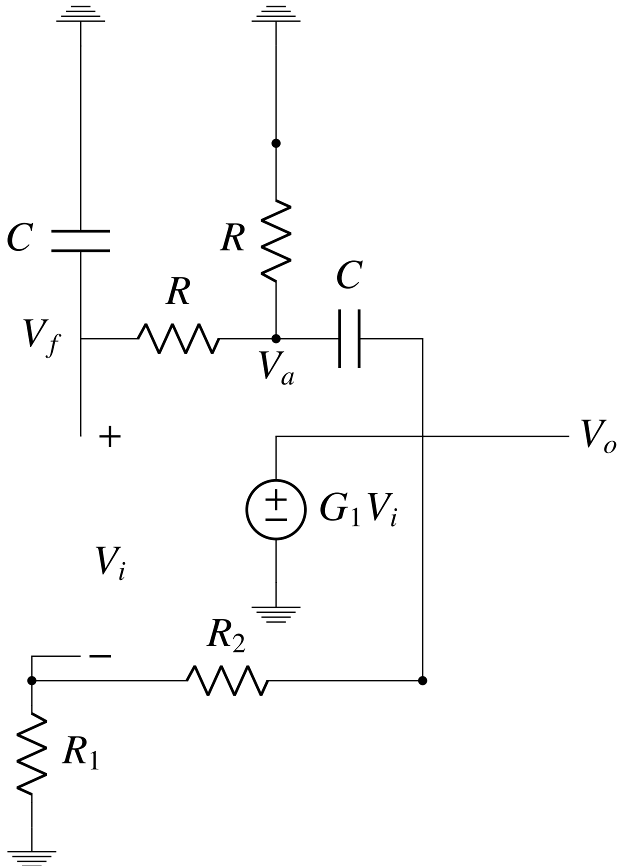

Fig. 1.3

$$\frac{V_f - 0}{\frac{1}{sC}} + \frac{V_f - V_a}{R} = 0 \quad (1.3.1)$$

$$V_f\left(sC + \frac{1}{R}\right) = \frac{V_a}{R} \quad (1.3.2)$$

$$V_a = V_f\,(sRC + 1) \quad (1.3.3)$$

Applying KCL at node V_a

$$\frac{V_a - V_f}{R} + \frac{V_a - 0}{R} + \frac{V_a - V_o}{\frac{1}{sC}} = 0 \quad (1.3.4)$$

$$V_a\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \quad (1.3.5)$$

Substitute V_a value from equation(1.3.3)

$$V_f(sRC + 1)\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \quad (1.3.6)$$

$$V_f\left(3 + sRC + \frac{1}{sRC}\right) = V_o \quad (1.3.7)$$

The feedback factor H is given by

$$H = \frac{V_f}{V_o} \quad (1.3.8)$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \quad (1.3.9)$$

1.4. Find the loop gain L(s).
Solution: The transfer function of the equivalent positive feedback circuit in Fig. 1.1.2 is

$$T = \frac{G}{1 - GH} \quad (1.4.1)$$

Therefore, loop gain is given by

$$L = GH \quad (1.4.2)$$

From equations (1.2.5) and (1.3.9)

$$L(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3 + sRC + \frac{1}{sRC}}\right) \quad (1.4.3)$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}}\right) \quad (1.4.4)$$

1.5. Find the loop gain in terms of $j\omega$.
Solution: Substitute $s = j\omega$ in equation (1.4.4)

$$L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\omega RC + \frac{1}{j\omega RC}}\right) \quad (1.5.1)$$

$$\implies L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}\right) \quad (1.5.2)$$

1.6. Find the frequency for zero loop phase.
Solution: The frequency at which loop phase will be zero (i.e. loop gain will be a real number).To obtain the required frequency, equate the imaginary part of the loop gain $L(j\omega)$ to zero.

$$j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \quad (1.6.1)$$

$$\omega^2 = \frac{1}{(RC)^2} \quad (1.6.2)$$

$$\implies \omega = \frac{1}{RC} \quad (1.6.3)$$

1.7. Find R_2/R_1 for oscillation.

Solution: For oscillations to start,

- the imaginary part of the loop gain should become zero.
- the loop gain must be at least equal to unity.

From equation (1.5.2)

$$\left(\frac{1+\frac{R_2}{R_1}}{3+j(0)}\right) \geq 1 \tag{1.7.1}$$

$$1+\frac{R_2}{R_1} \geq 3 \tag{1.7.2}$$

$$\implies \frac{R_2}{R_1} \geq 2 \tag{1.7.3}$$

1.8. Draw the block diagram and circuit diagram for H.

Solution: See figs 1.8.4 and 1.8.5 .From Fig.

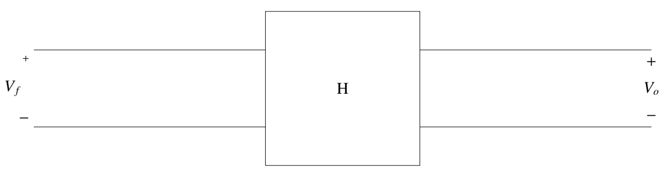

Fig. 1.8.4: Feedback block diagram

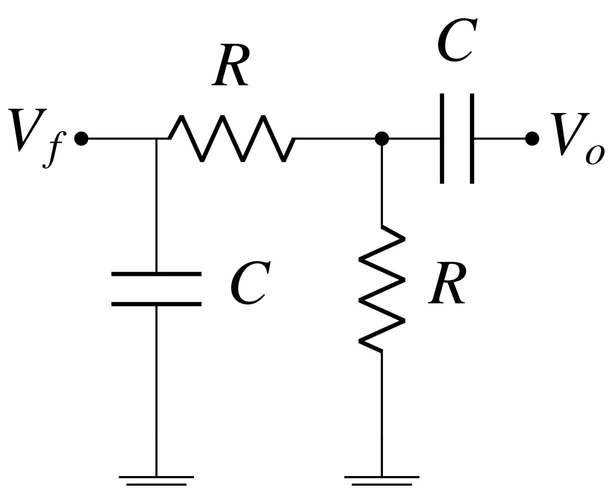

Fig. 1.8.5: Feedback circuit

1.8.5,the analysis is same as problem 1.3

$$\frac{V_f}{V_o} = \frac{1}{\left(3+sRC+\frac{1}{sRC}\right)} \tag{1.8.1}$$

$$\implies H = \frac{1}{\left(3+sRC+\frac{1}{sRC}\right)} \tag{1.8.2}$$

1.9. Find the input and output resistances of the feedback network.

Solution: To find the input resistance R_{11} short the output node V_o to ground.

$$R_{11} = Z||(R+(R||Z)) \tag{1.9.1}$$

where $Z = \frac{1}{sC}$ is the impedance of the capacitor.

$$\implies R_{11} = \left(\frac{1}{sC}||\left(R+R||\frac{1}{sC}\right)\right) \tag{1.9.2}$$

To find the output resistance R_{22} short the input node V_f to ground.

$$R_{22} = Z+(R||R) \tag{1.9.3}$$

$$\implies R_{22} = \frac{1}{sC}+\frac{R}{2} \tag{1.9.4}$$

1.10. Draw the block diagram and circuit diagram for G.

Solution: See figs 1.10.6 and 1.10.7.From

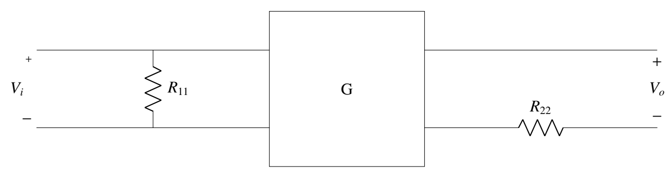

Fig. 1.10.6: Open loop block diagram

Fig. 1.10.7 using same analysis as problem 1.2

$$G = \frac{V_o}{V_i} \tag{1.10.1}$$

$$G = \frac{R_1+R_2}{R_1} \tag{1.10.2}$$

$$\implies G = 1+\frac{R_2}{R_1} \tag{1.10.3}$$

Hence verified with equation (1.2.5).

1.11. Find the amplitude and frequency for some arbitrary values given in Table 1.11.

Solution: From equation (1.2.5)

$$G = 1+\frac{R_2}{R_1} = 3 \tag{1.11.1}$$

From equation (1.3.9)

$$H = \frac{1}{3+0.25s+\frac{1}{0.25s}} \tag{1.11.2}$$

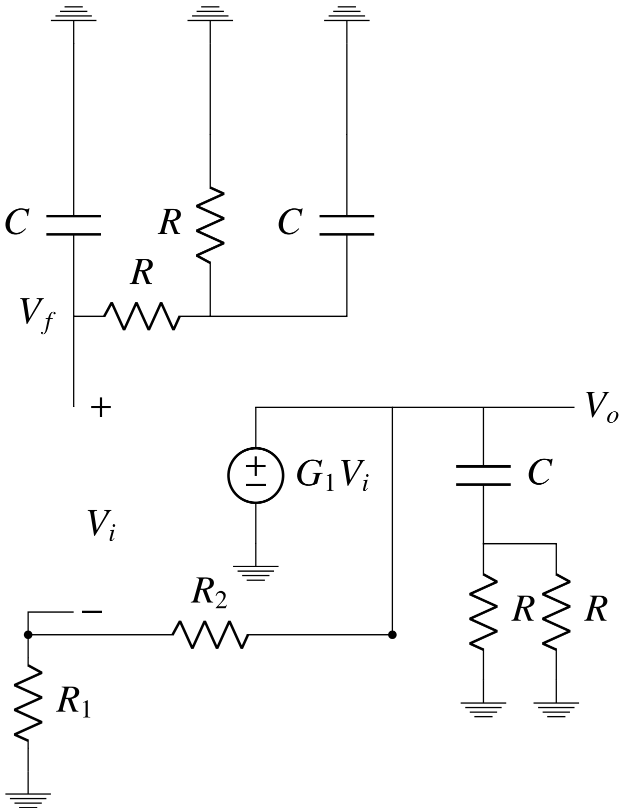

Fig. 1.10.7: Open loop circuit diagram

Parameter	Value
R	250Ω
C	$1mF$
R_2	$2k\Omega$
R_1	$1k\Omega$

TABLE 1.11

From equation (1.4.1)

$$T = \frac{3\left(0.0625s^2 + 0.75s + 1\right)}{0.0625s^2 + 1} \tag{1.11.3}$$

The following code plots the oscillating response of the system.

```
codes/ee18btech11047/ee18btech11047.py
```

Amplitude: From Fig. 1.11 V(peak-peak) is

$$V_{p-p} = 18.12 \tag{1.11.4}$$

$$V_{max} = \frac{V_{p-p}}{2} = 9.06 \tag{1.11.5}$$

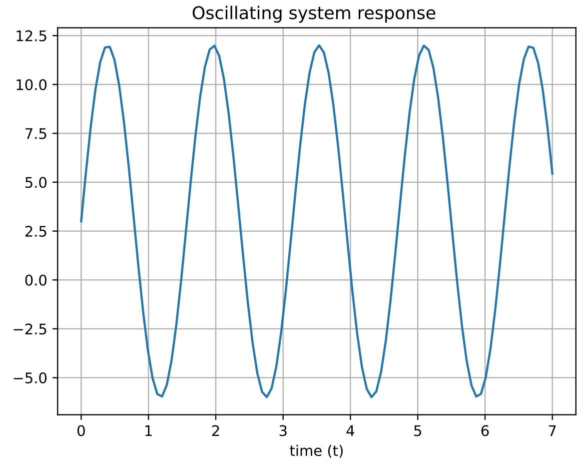

Fig. 1.11

Frequency: From equation (1.6.3)

$$\omega = \frac{1}{RC} = 4rad/sec \tag{1.11.6}$$

$$f = \frac{\omega}{2\pi} = 0.636Hz \tag{1.11.7}$$