Wein-bridge oscillator

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For the Wein-bridge oscillator of Fig 0, use the expression for loop gain to find the poles of the closed-loop system. Give the expression for the pole, Q and use it to show that to locate the poles in the right half of s plane, $\frac{R_2}{R_1}$ must be selected to be greater than 2.
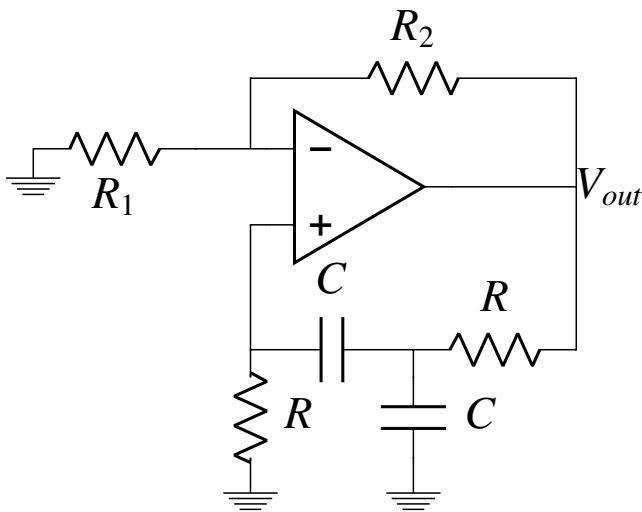

Fig. 0

1. Draw the equivalent control system block diagram for Fig. 0 and compute G and H.
   Solution:

2. Compare the basic structure for a sinusoidal oscillator with Wein-bridge oscillator and give expressions for G and H.
   Solution: See Fig 2.

3. Comparring Fig 0 and , we get

$$G = 1 + \frac{R_2}{R_1} \qquad (3.1)$$

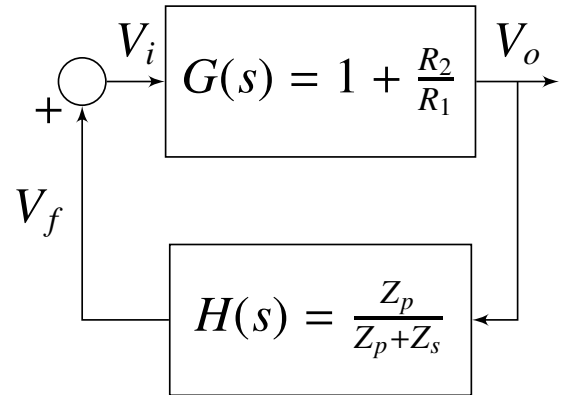$$H(s) = \frac{V_f}{V_o} = \frac{Z_p}{Z_p + Z_s} \qquad (3.2)$$

Fig. 2

where,

$$Z_p = \frac{R}{RSC + 1} \qquad (3.3)$$

$$Z_s = \frac{RSC + 1}{SC} \qquad (3.4)$$

4. Find the loop gain $L(s)$ and obtain the characteristic equation. for the Wein-bridge oscillator.
   Solution: From (3.1) and (3.2), the loop gain

$$L(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} = 0 \qquad (4.1)$$

$$(4.2)$$

and the characteristic equation is

$$1 - L(s) = 0 \qquad (4.3)$$

$$\implies s^2 + \frac{s}{RC}\left(2 - \frac{R_2}{R_1}\right) + \frac{1}{R^2C^2} = 0 \qquad (4.4)$$

after some algebra.

5. State the **Barkhausen criterion** for sustained oscillations with frequency ω_0.
   Solution: This is nothing but the condition for marginal stability in a control system.

$$L(j\omega_0) = G(j\omega_0)H(j\omega_0) = 1 \qquad (5.1)$$

- That is, at ω_0 the phase of the loop gain should be zero and the magnitude of loop

gain should be 1.

- Only for a ∞ gain,system will produce a finite output for zero input.

6. Give the definition of **Quality factor**(Q) and explain its significance.
   Solution:
   - It is a parameter of an oscillatory system expressing the relationship between stored energy and energy dissipation.
   - The "purity" of output sine waves will be a function of the selectivity feedback network.
   - That is, higher the value of Q for frequency selective network, the less the harmonic content of sine wave produced.

7. The characteristic equation for a second order system is given by

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0 \qquad (7.1)$$

Obtain Q and ω_0 for (4.4)
Solution:

$$\omega_0 = \frac{1}{RC} \text{ and} \qquad (7.2)$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}\left(2 - \frac{R_2}{R_1}\right) \qquad (7.3)$$

$$\implies Q = \frac{1}{\left(2 - \frac{R_2}{R_1}\right)} \qquad (7.4)$$

8. Using (7.4) calculate the value of $\frac{R_2}{R_1}$ for which poles lie on right hand of s-plane.
   Solution:
   Poles lie on imaginary axis for $Q = \infty$

$$2 - \frac{R_2}{R_1} = 0 \qquad (8.1)$$

$$\frac{R_2}{R_1} = 2 \qquad (8.2)$$

$\therefore$ For poles to lie on right hand side of s-plane

$$\frac{R_2}{R_1} > 2 \qquad (8.3)$$

9. Verify the above theory using a Python code.
   Solution:

   ```
   codes/ee18btech11044/ee18btech11044_3_1.
      py
   ```

   - This figure shows how the location of poles vary if $\frac{R_2}{R_1}$ is varied for a fixed ω_0.
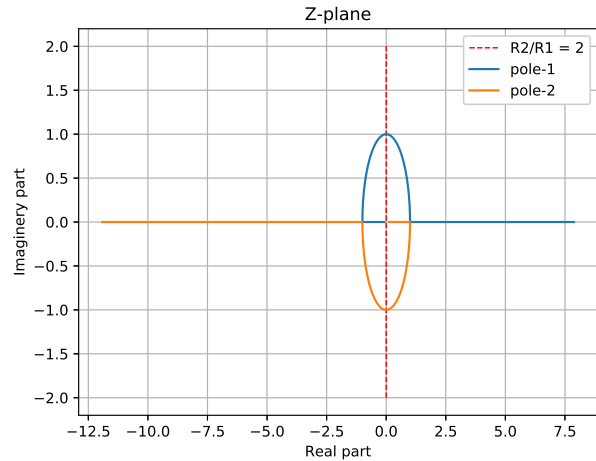   - I have varied $\frac{R_2}{R_1}$ from -10 to 10.

Fig. 9

10. Choose appropriate values of Resistances and Capacitors to simulate the circuit.
    Solution:

Parameter	Value
R_1	$10k\Omega$
R_2	$20.3k\Omega$
R_p	$10k\Omega$
R_s	$10k\Omega$
C_s	$16nF$
R_p	$10k\Omega$
C_p	$16nF$

TABLE 10

Where, according to Fig 0

$$R_p = R_s = R \qquad (10.1)$$
$$C_p = C_s = C \qquad (10.2)$$

11. Write the closed loop Transfer function T(s) for wein-bridge oscillator.
    Solution:

$$T(s) = \frac{(1 + \frac{R_2}{R_1})R^2C^2s^2 + 3RCs + 1}{R^2C^2s^2 + (2 - \frac{R_2}{R_1})RCs + 1} \qquad (11.1)$$

12. Calculate the frequency of sinusoid generated for the values given in table 10.
    Solution:
    - Calculating poles of transfer function for $\frac{R_2}{R_1} = 2.03$ using a python script.

    ```
    codes/ee18btech11044/
        ee18btech11044_3_1.py
    ```

marked in the Fig 15.

$$T_0 = 0.0856452 - 0.0846361 \qquad (15.1)$$
$$f_0 = 1/T_0 \qquad (15.2)$$
$$f_0 = 990.98Hz. \qquad (15.3)$$

- We get the frequencies calculated from the formulae and the plot to be approximately same.
- Use this script to generate Fig 15

  spice/ee18btech11044/
      ee18btech11044_spice_3_2.py

Fig. 17.8

Fig. 15

16. Calculate the poles of transfer function when the ratio $\frac{R_2}{R_1}$ is taken to be 2.
    Solution:
    - Calculating poles of transfer function for $\frac{R_2}{R_1} = 2$ using a python script.

      codes/ee18btech11044/
          ee18btech11044_3_4.py

    - We get the poles to be purely imaginery +6250j and -6250j, which correspond to a sinusoid of frequency 995.22Hz.

17. Substituting the values shown in table 10 in the (11.1), Consider the ratio of $\frac{R_2}{R_1} = 2$ and plot the impulse response and step response using a python code.
    Solution:
    - Refer Fig 17.8 for impulse response of transfer function.
    - Code for generating impulse response

Fig. 17.9

  codes/ee18btech11044/
      ee18btech11044_3_5.py

- Refer D=Fig 17.9 for step response of transfer function.
- Code for generating step response

  codes/ee18btech11044/
      ee18btech11044_3_6.py

- As expected from the poles we are getting pure sinusoid as both impulse response and step response.

18. Simulate the circuit shown in Fig 0 using spice simulators. Consider the ratio of $\frac{R_2}{R_1} = 2$ and plot the output using a python script.
    Solution:
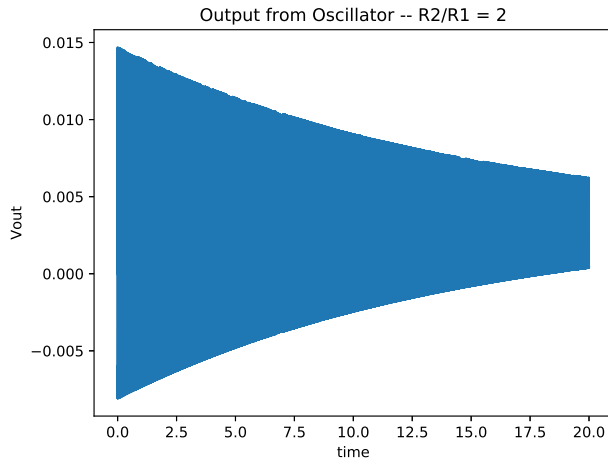
Fig. 18
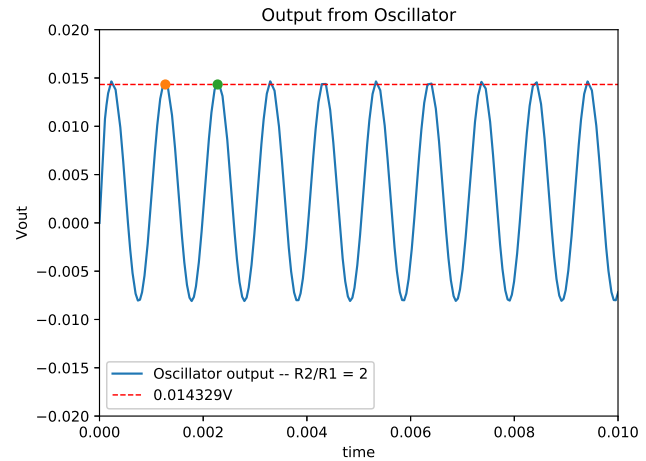
Fig. 19

- Refer Fig 18 for the spice simulation output.
- You can find the netlist for the simulated circuit here:
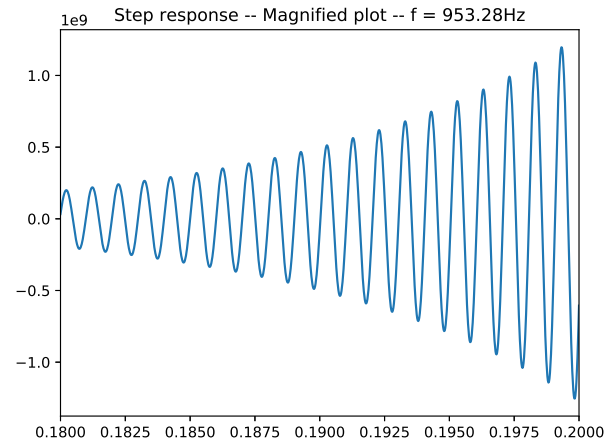
spice/ee18btech11044/
    ee18btech11044_3_2.net

- You can find the python script used to generate the output here:
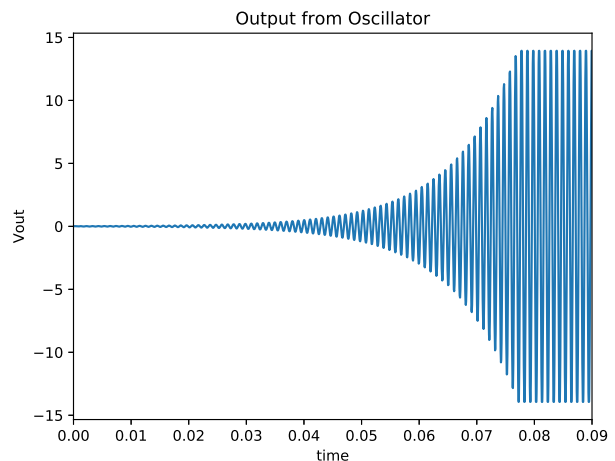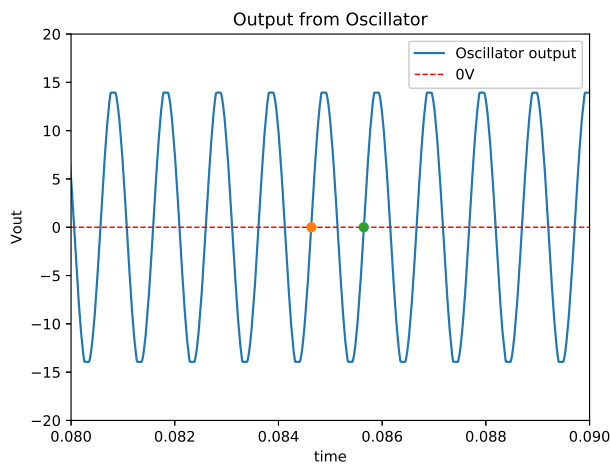
spice/ee18btech11044/
    ee18btech11044_spice_3_3.py

19. Calculate the frequency of sinusoidal wave using plot generated from simulationfor the case where $\frac{R_2}{R_1} = 2$.
Solution:
  - Consider a part of plot generated from simulation shown in the Fig 19.
  - Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 19.

$$T_0 = 0.00227823 - 0.00127016 \quad (19.1)$$
$$f_0 = 1/T_0 \quad (19.2)$$
$$f_0 = 991.9944 Hz. \quad (19.3)$$

  - We get the frequencies calculated from the formulae and the plot to be approximately same.
  - Use this script to generate Fig 19

spice/ee18btech11044/
    ee18btech11044_spice_3_4.py

20. Calculate the poles of transfer function when the ratio $\frac{R_2}{R_1}$ is taken to be 9.
Solution:
  - Calculating poles of transfer function for $\frac{R_2}{R_1} = 9$ using a python script.

codes/ee18btech11044/
    ee18btech11044_3_7.py

  - We get the poles two real and distinct poles at 42838.13728906053 and 911.8627109394729, which correspond to an exponentially increasing function.

21. Substituting the values shown in table 10 in the (11.1). Consider the ratio of $\frac{R_2}{R_1} = 9$ and plot the impulse response and step response using a python code.
Solution:
  - Refer Fig 21 for impulse response of transfer function.
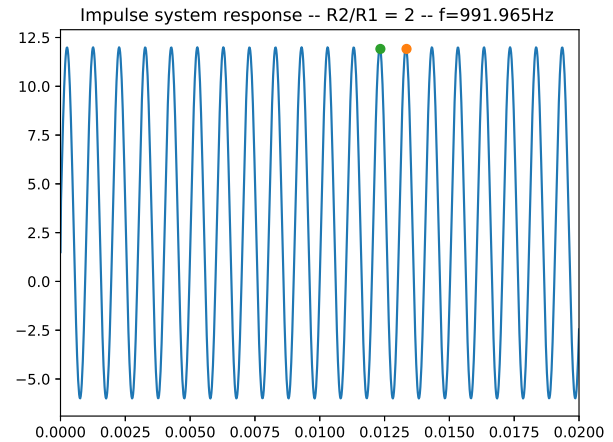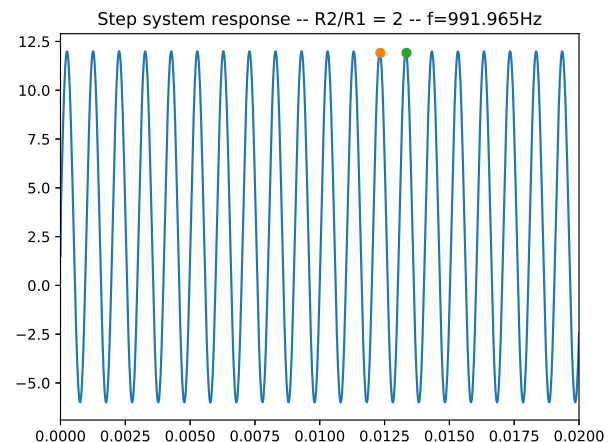 - Code for generating impulse response

codes/ee18btech11044/
    ee18btech11044_3_8.py

  - Refer D=Fig 21 for step response of transfer function.
  - Code for generating step response

codes/ee18btech11044/
    ee18btech11044_3_9.py

  - As expected from the poles we are getting an exponentially increasing function as both impulse response and step response.
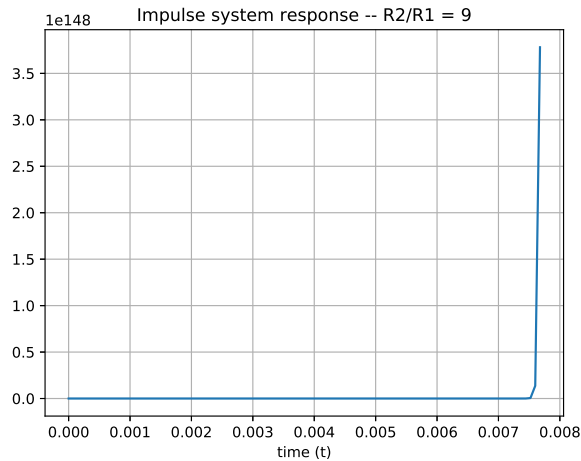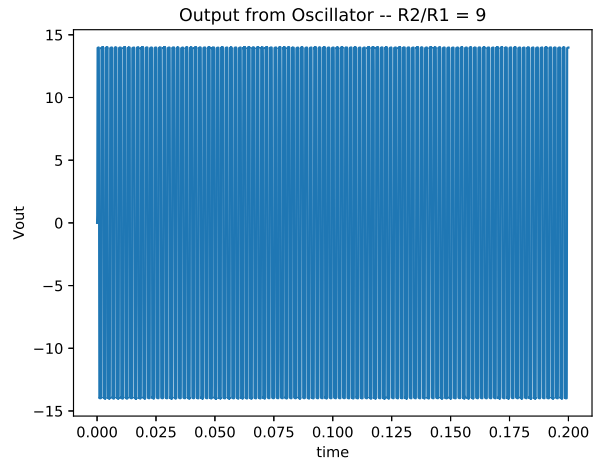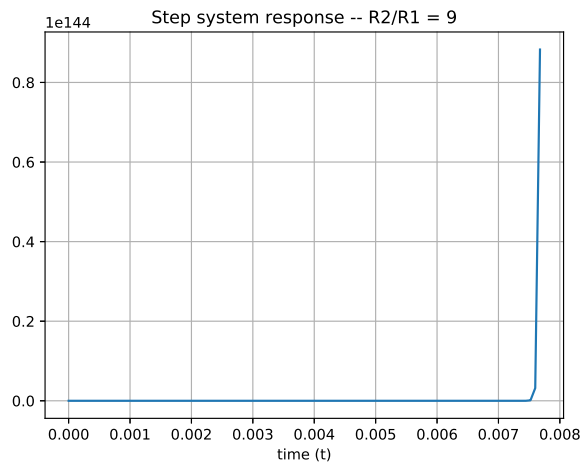
Fig. 21

Fig. 22

Fig. 21

22. Simulate the circuit shown in Fig 0 using spice simulators. Consider the ratio of $\frac{R_2}{R_1} = 9$ and plot the output using a python script.
Solution:
- Refer to Fig. 22 for the spice simulation output.
- You can find the netlist for the simulated circuit here:

  spice/ee18btech11044/
  ee18btech11044_3_3.net

- You can find the python script used to generate the output here:

  spice/ee18btech11044/
  ee18btech11044_spice_3_5.py

23. Calculate the frequency of sinusoidal wave using plot generated from simulationfor the case where $\frac{R_2}{R_1} = 9$.
Solution:
- Consider a part of plot generated from simulation shown in the Fig 23.
- Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 23.

$$T_0 = 0.00453629 - 0.00270161 \quad (23.1)$$
$$f_0 = 1/T_0 \quad (23.2)$$
$$f_0 = 545.054 Hz. \quad (23.3)$$

- We get the frequencies calculated from the formulae and the plot to be approximately same.
- Use this script to generate Fig 23

  spice/ee18btech11044/
  ee18btech11044_spice_3_6.py

- In this case because of the saturation voltage in op-amp we are getting a curve similar to square wave.

24. Calculate the poles of transfer function when the ratio $\frac{R_2}{R_1}$ is taken to be 1.5.
Solution:
- Calculating poles of transfer function for $\frac{R_2}{R_1} = 1.5$ using a python script.
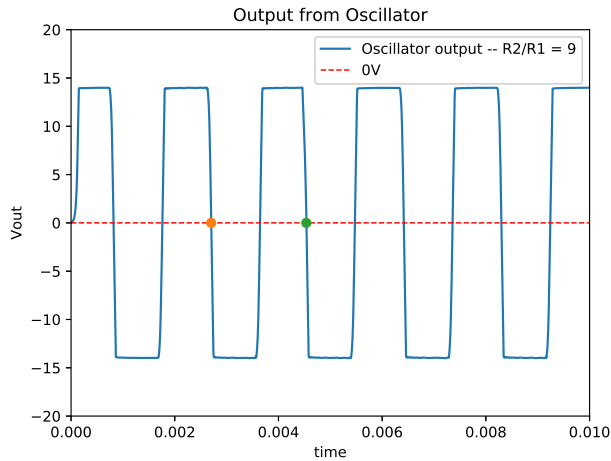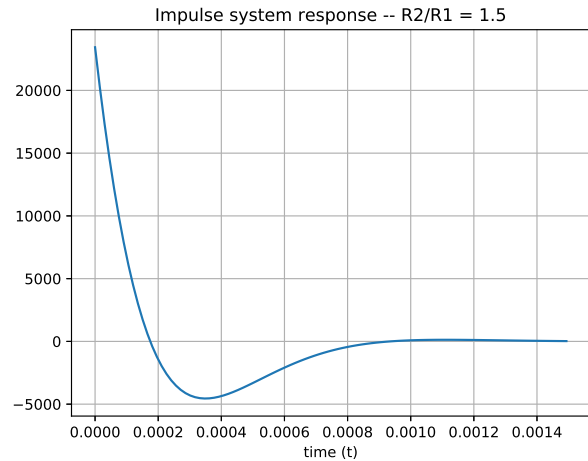
  codes/ee18btech11044/
  ee18btech11044_3_10.py

Fig. 23

Fig. 25

- We get the poles two complex conjugate poles in Left Hand Side of s-plane at -1563+6051.5364784490885j and -1563-6051.5364784490885j, which correspond to an exponentially decreasing sinusoid function, corresponding to a frequnecy of 963.27Hz.

25. Substituting the values shown in table 10 in the Eq 11.1. Consider the ratio of $\frac{R_2}{R_1} = 1.5$ and plot the impulse response and step response using a python code.
    Solution:
    - Refer Fig 25 for impulse response of transfer function.
    - Code for generating impulse response

    codes/ee18btech11044/
        ee18btech11044_3_11.py

    - Refer Fig 25 for step response of transfer function.
    - Code for generating step response

    codes/ee18btech11044/
        ee18btech11044_3_12.py

    - As expected from the poles we are getting an exponentially increasing function as both impulse response and step response.

26. Simulate the circuit shown in Fig 0 using spice simulators. Consider the ratio of $\frac{R_2}{R_1} = 1.5$ and plot the output using a python script.
    Solution:
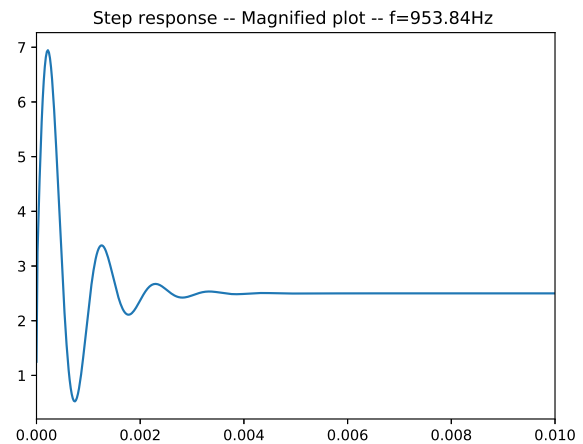   - Refer Fig 26 for the spice simulation output.

Fig. 25

- You can find the netlist for the simulated circuit here:

    spice/ee18btech11044/
        ee18btech11044_3_4.net

- You can find the python script used to generate the output here:

    spice/ee18btech11044/
        ee18btech11044_spice_3_7.py

27. Calculate the frequency of sinusoidal wave using plot generated from simulationfor the case where $\frac{R_2}{R_1} = 1.5$.
    Solution:
    - Consider a part of plot generated from simulation shown in the Fig 27.
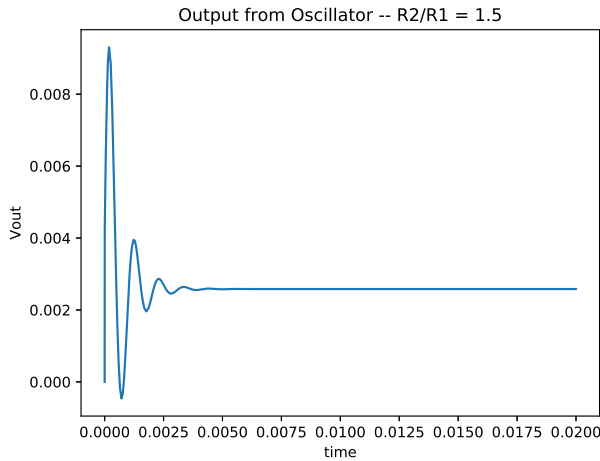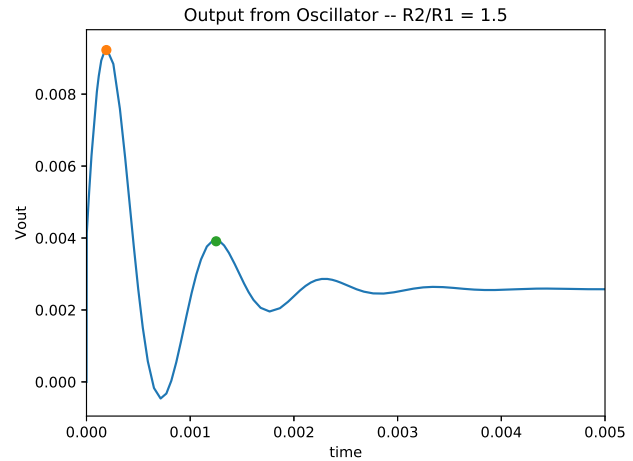
Fig. 26

Fig. 27

- Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 27.

$$T_0 = 0.00125 - 0.000191532 \quad (27.1)$$
$$f_0 = 1/T_0 \quad (27.2)$$
$$f_0 = 944Hz. \quad (27.3)$$

- We get the frequencies calculated from the formulae and the plot to be very different.
- THe output predicted by python and spice simulation are matching in this case.
- Use this script to generate Fig 23

spice/ee18btech11044/
    ee18btech11044_spice_3_8.py

- In this case because of the saturation voltage in op-amp we are getting a curve similar to square wave.

28. Describe the output depending upon the nature of poles.
   Solution:
   - For purely imaginery poles, we expect a sinusoidal output (As in the case $\frac{R_2}{R_1} = 2$).
   - For complex conjugate poles in right hand side of s-plane, we expect an exponentially increasing sinusoid as the output.(As in the case $2 < \frac{R_2}{R_1} < 4$).
   - For real and distinct poles in right hand side of s-plane, we expect an exponential output.(As in the case $\frac{R_2}{R_1} > 4$).
   - For complex conjugate poles on the left hand side of splane, we expect an exponentially

decreasing sinusoid.(As in the case $0 < \frac{R_2}{R_1} < 2$.
- It is not practical to simulate for real and distinct poles in left hand side of s plane as R_2 should be negative for that case.
- As you can observe behaviour of output generated by python scripts can be explained from the location of poles.
- In the case of spice simulation, the output is always sinusoidal when $\frac{R_2}{R_1} > 2$ which is not the expected shape.
- For $\frac{R_2}{R_1} <= 2$ spice simulation is producing output of expected shape and frequency.
- Do not consider the frequency of impulse response from python plot in case-1 and case-4, as python is not able to calculate the Inverse laplace completely i have plotted using function signal.impulse(system) which generates a python plot of correct shape but frequency cannot be calculated from this plot.
- Please find the code to calculate impulse response in Case-1.

codes/ee18btech11044/
    ee18btech11044_3_13.py

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- Please find the code to calculate impulse response in Case-2.

codes/ee18btech11044/
    ee18btech11044_3_14.py