Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

1.1 Introduction

1.1.1. Fig. 1.1.1.1 shows a non-inverting op-amp configuration with parameters described in Table 1.1.1. Draw the equivalent control system.

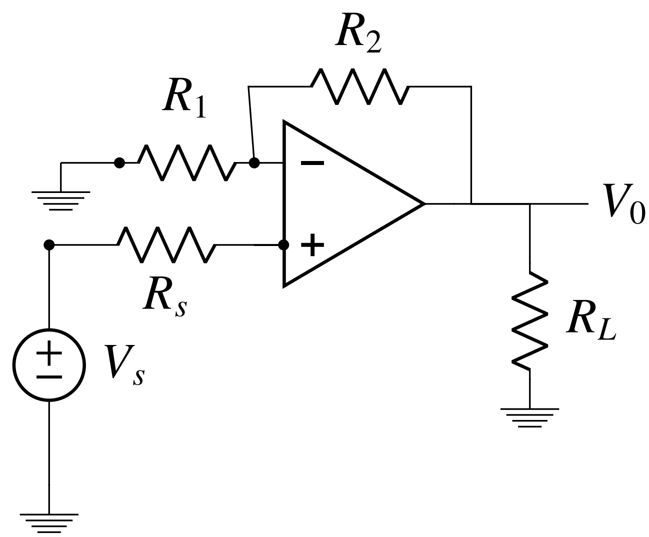

Fig. 1.1.1.1

Parameter	Value
input resistance	∞
output resistance	**0**
Input voltage	V_s
Output Voltage	V_o
Feedback resistance	R_2
load resistance	R_L

TABLE 1.1.1

Solution: See Fig. 1.1.1.2

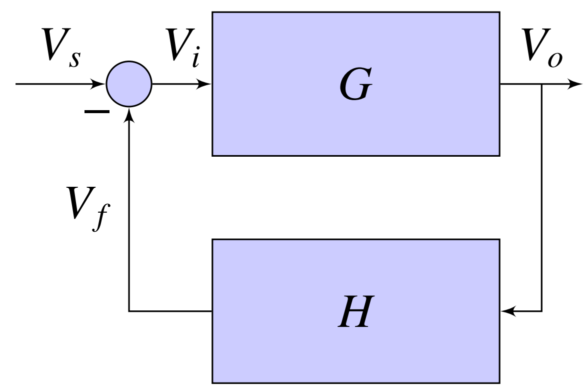

Fig. 1.1.1.2

1.1.2. Draw the small signal model for Fig. 1.1.1.1. **Solution:** The equivalent circuit of the amplifier is in Fig. 1.1.2

1.1.3. Assuming that the operational amplifier has infinite input resistance and zero output resistance, find the *feedback factor H*. **Solution:** Let the gain of the operational amplifier be G. From the equivalent circuit, Applying Ohms law,

$$V_0 = G(V_i) \tag{1.1.3.1}$$

$$\text{and,} V_i = V_+ - V_- \tag{1.1.3.2}$$

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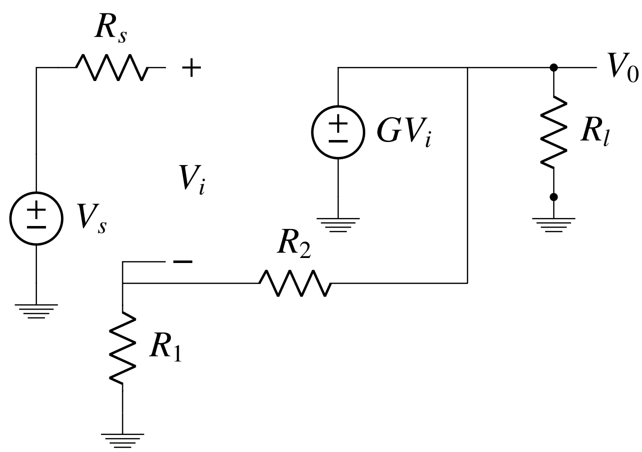

Fig. 1.1.2

Now,Applying voltage dividing rule

$$V_- = \left[\frac{R_1}{R_1 + R_2}\right] V_0 \tag{1.1.3.3}$$

Substituting in equ.1.1.3.1

$$V_0 = G(V_+ - \left[\frac{R_1}{R_1 + R_2}\right] V_0) \tag{1.1.3.4}$$

$$\implies V_0 = GV_+ - G\left[\frac{R_1}{R_1 + R_2}\right] V_0 \tag{1.1.3.5}$$

$$G(V_+) = V_0 + G\left[\frac{R_1}{R_1 + R_2}\right] V_0 \tag{1.1.3.6}$$

But,

$$V_s = V_+ \tag{1.1.3.7}$$

because, no current flows through resistor.

$$V_0 = G\left[\frac{1}{1 + \frac{GR_1}{R_1+R_2}}\right] V_s \tag{1.1.3.8}$$

$$\text{Gain} = \frac{V_0}{V_s} = \left[\frac{G}{1 + \frac{GR_1}{R_1+R_2}}\right] \tag{1.1.3.9}$$

For a negative feedback system,

$$\frac{V_0}{V_i} = \frac{G}{1 + GH} \tag{1.1.3.10}$$

$$\text{where.,}H = \frac{R_1}{R_1 + R_2} \tag{1.1.3.11}$$

The equation.1.1.3.1 looks exactly similar to the Gain of a negative feedback system with

- Open loop gain = G
- Loop gain = P
- Amount of feedback = F
- Feedback factor = f
- closed loop gain = T

Parameters	**Definition**	**For given circuit**
Open loop gain	G	G
Feedback factor	H	$\frac{R_1}{R_1+R_2}$
Loop gain	GH	$G\frac{R_1}{R_1+R_2}$
Amount of feedback	1+GH	$1 + \frac{GR_1}{R_1+R_2}$
Closed loop gain	$\frac{G}{1+GH}$	$\frac{G(R_1+R_2)}{R_1+R_2+GR_1}$

TABLE 1.1.3

Therefore, So, the feedback factor ..,

$$f = H = \frac{R_1}{R_1 + R_2} \tag{1.1.3.12}$$

1.1.4. Find the condition under which closed loop gain T is almost entirely determined by the feedback network. **Solution:** For T to entirely dependent on feedback network, it should be independent on G(open loop gain) T is given by..,

$$T = \frac{G}{1 + \frac{GR_1}{R_1+R_2}} \tag{1.1.4.1}$$

$$\tag{1.1.4.2}$$

For T to be independent on G..,

$$GH >> 1 \tag{1.1.4.3}$$

$$G\frac{R_1}{R_1 + R_2} >> 1 \tag{1.1.4.4}$$

$$G >> 1 + \frac{R_2}{R_1} \tag{1.1.4.5}$$

Under such condition..,

$$T = \frac{1}{H} \tag{1.1.4.6}$$

$$T = \frac{R_1 + R_2}{R_1} \tag{1.1.4.7}$$

$$T = 1 + \frac{R_2}{R_1} \tag{1.1.4.8}$$

so, the necessary condition for T depend only on feedback network is

$$G >> T \tag{1.1.4.9}$$

1.1.5. If the open loop voltage gain

$$G = 10^4 \tag{1.1.5.1}$$

Find the ratio of R2 and R1 to obtain a closed loop gain of 10. **Solution:** The closed loop gain gain T is given by

$$T = \frac{G}{1+GH} = \frac{G}{1+\frac{GR_1}{R_1+R_2}} = 10 \tag{1.1.5.2}$$

$$\text{where..,} G = 10^4 \tag{1.1.5.3}$$

$$10 = \frac{10^4}{1+\frac{10^4}{1+\frac{R_2}{R_1}}} \tag{1.1.5.4}$$

$$\implies 1+\frac{R_2}{R_1} = \frac{10^4}{\frac{10^4}{10}-1} \tag{1.1.5.5}$$

$$1+\frac{R_2}{R_1} = 10.010 \tag{1.1.5.6}$$

$$\frac{R_2}{R_1} = 9.010 \tag{1.1.5.7}$$

1.1.6. What is the amount of feedback in decibels? **Solution:** The value of F in decibals is given by

$$F(dB) = 20\log(F) \tag{1.1.6.1}$$

$$\text{where..,} F = 1+GH \tag{1.1.6.2}$$

$$F = \frac{G}{T} \tag{1.1.6.3}$$

$$\text{where..,} G = 10^4 \tag{1.1.6.4}$$

$$T = 10 \tag{1.1.6.5}$$

$$F(dB) = 20\log(\frac{10^4}{10}) = 20\log(1000) \tag{1.1.6.6}$$

$$F(dB) = 60dB \tag{1.1.6.7}$$

1.1.7. If G decreases by 20%,what is the corresponding decrease in T? **Solution:** Given

$$G = 10^4 \tag{1.1.7.1}$$

If G decrease by 20% then, the value of G is..,

$$G = (1-0.2)10^4 \tag{1.1.7.2}$$

$$= 8000 \tag{1.1.7.3}$$

For this value of G and ,

$$\frac{R_2}{R_1} = 9.010 \tag{1.1.7.4}$$

The value of T can be solved as follows,

$$T = \frac{G}{1+\frac{G}{1+\frac{R_2}{R_1}}} \tag{1.1.7.5}$$

$$T = \frac{8000}{1+\frac{8000}{1+0.9010}} \tag{1.1.7.6}$$

$$T = 9.99749 \tag{1.1.7.7}$$

The percentage change in T is..,

$$fractionalchange = \frac{10-9.99749}{10} \tag{1.1.7.8}$$

$$= 2.51x10^{-4} \tag{1.1.7.9}$$

$$\%changeinT = 0.00251 \tag{1.1.7.10}$$

Therefore T decreases by 0.0025% when G decreases by 20%

1.1.8. Write a python code that can compute closed loop gain,loop gain,amount of feedback given all input parameters. **Solution:** Code to compute different gains.,

```
codes/ee18btech11005/ee18btech11005.py
```

2 Feedback Current Amplifier: Shunt-Series

2.1 Introduction

2.1.1. Draw the equivalent control system for the feedback current amplifier shown in 2.1.1.4
Solution: See Fig. 2.1.1.5.

2.1.2. For the feedback current amplifier shown in 2.1.1.4, draw the Small-Signal Model. Neglect the Early effect in Q_1 and Q_2.
Solution: See Fig. 2.1.2.
While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

2.1.3. Describe how the given circuit is a Negetive Feedback Current Amplifier.

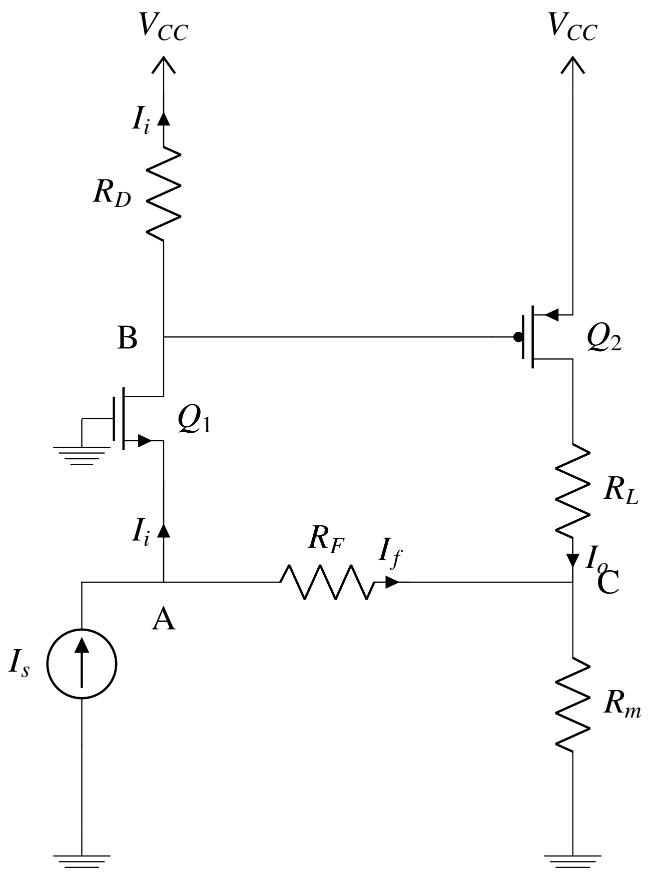

Fig. 2.1.1.4

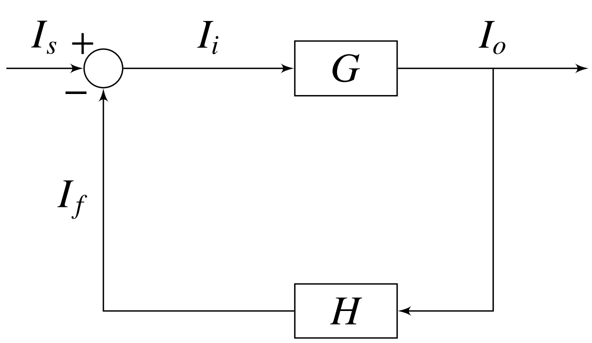

Fig. 2.1.1.5

Solution: For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the p-channel device Q_2 , its increase will cause I_o , the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed

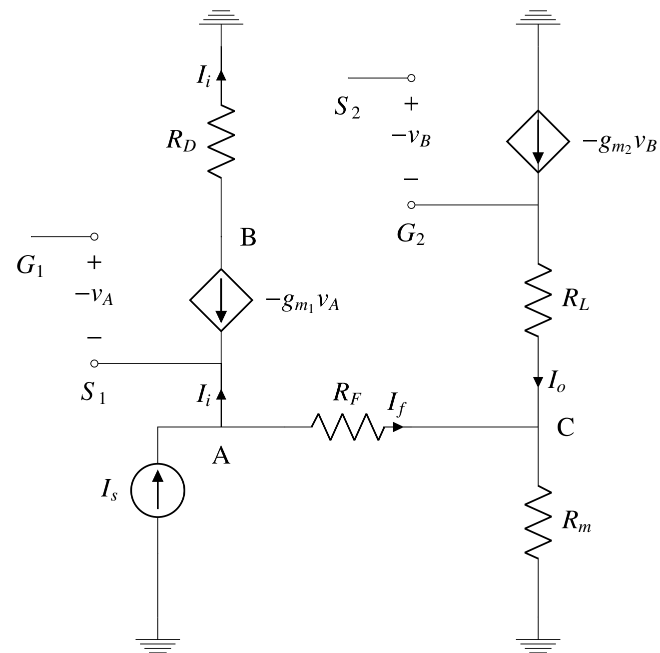

Fig. 2.1.2: Small Signal Model

for the initial change in I_s, verifying that the feedback is indeed negative.

2.1.4. Find the Expression for the Open-Loop Gain $G = \frac{I_o}{I_i}$, from the Small-Signal Model. in Fig. 2.1.2.
Solution: In Small-Signal Model,

$$v_B = I_i R_D \tag{2.1.4.1}$$

$$v_{gs_2} = v_B = I_i R_D \tag{2.1.4.2}$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.4.3}$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.4}$$

2.1.5. Find the Expression of the Feedback Factor $H = \frac{I_f}{I_o}$, from Small-Signal Model.
Solution: I_o is fed to a current divider formed by R_M and R_F. R_F is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it leaving a small current as input to Amplifier. Hence the voltage at point 'A' is very small and is

considered, $v_A \simeq 0$. So R_F and R_M are parallel and Voltage Drop across them is same.

$$(I_o + I_f)R_M \simeq -I_f R_F \tag{2.1.5.1}$$

$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.2}$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.3}$$

2.1.6. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$.

Solution: From (2.1.4.4) and (2.1.5.3),

$$T = \frac{I_o}{I_s} = \frac{G}{1 + GH} \tag{2.1.6.1}$$

$$= -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.2}$$

$$\implies T = -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.3}$$