Phase Margin

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An amplifier has a dc gain of 10^5 and poles at 10^5 Hz , 3.16×10^5 Hz and 10^6 Hz . Find the value of H, and the corresponding closed-loop gain, for which a phase margin of 45° is obtained.

1. Find $G(s)$.
Solution: For a 3-pole amplifier open loop transfer function is

$$G(s) = \frac{G_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)} \tag{1.1}$$

where the Gain and Poles are listed in Table 1. Thus,

Parameters	Value
p_1	$2\pi\times10^5$ rad/sec
p_2	$2\pi(3.16\times10^5)$ rad/sec
p_3	$2\pi\times10^6$ rad/sec
G_0	10^5

TABLE 1

$$G(f) = \frac{G_0}{\left(1+\jmath\frac{f}{f_1}\right)\left(1+\jmath\frac{f}{f_2}\right)\left(1+\jmath\frac{f}{f_3}\right)} \tag{1.2}$$

$$= \frac{10^5}{\left(1+\jmath\frac{f}{10^5}\right)\left(1+\jmath\frac{f}{3.16\times10^5}\right)\left(1+\jmath\frac{f}{10^6}\right)} \tag{1.3}$$

2. Given that $PM = 45°$, find the crossover frequency f_c.
Let $L(s) = G(s)H$ be the loop gain. Then

$$PM = 180° - \underline{/L(f_c)} \tag{2.1}$$

where

$$|L(f_c)| = 1 \tag{2.2}$$

From (2.1),

$$\underline{/L(f_c)} = -135° \tag{2.3}$$

$$\implies -135° = -\tan^{-1}\left(\frac{f_c}{10^5}\right) - \tan^{-1}\left(\frac{f_c}{3.16\times10^5}\right) - \tan^{-1}\left(\frac{f_c}{10^6}\right) \tag{2.4}$$

$$\text{or, } f_c = 315\,kHz. \tag{2.5}$$

3. Verify your result using a Bode plot.
Solution: The following code id used to verify the value of f_c Fig. 3

```
codes/ee18btech11016/ee18btech11016_1.py
```

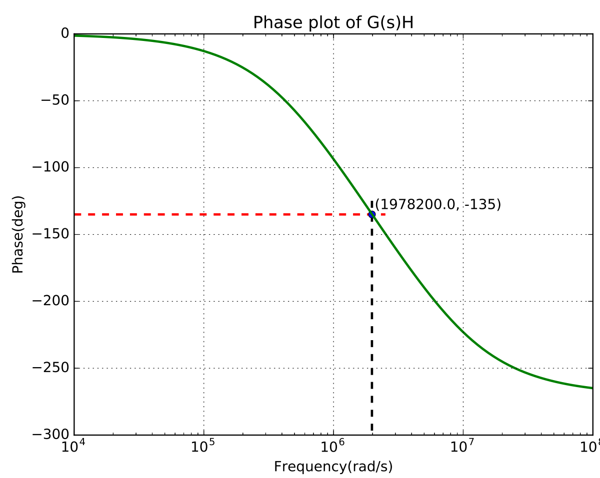

Fig. 3

4. Find the value of H.
Solution:

$$\because |L(f_c)| = |G(f_c)H| = 1, \tag{4.1}$$

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$$H\left(\frac{10^5}{\sqrt{1+\left(\frac{315\times10^3}{10^5}\right)^2}\sqrt{1+\left(\frac{315\times10^3}{3.16\times10^6}\right)^2}\sqrt{1+\left(\frac{315\times10^3}{10^6}\right)^2}}\right) = 1 \quad (4.2)$$

upon substituting from (1.3) and (2.5)

$$\implies H = 34.651\times10^{-6} \quad (4.3)$$

The following code provides the method to calculate the unit step response and the values of H,G(fc).

```
codes/ee18btech11016/
    ee18btech11016_verifyingvalues.py
```

5. Sketch the block diagram for the given closed loop system with $PM = 45°$.
Solution: See Fig. 5.

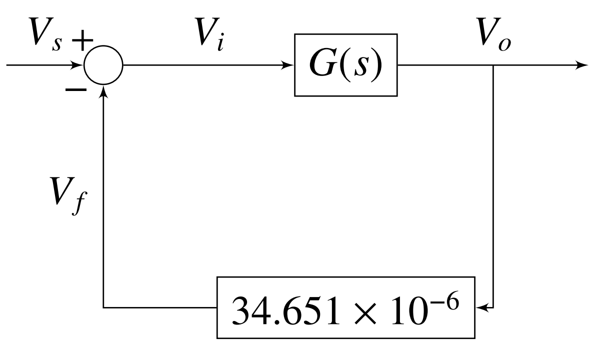

Fig. 5

6. Design the circuit for H
Solution: In Fig. 6,

$$H = \frac{V_f}{V_o} = \frac{R_{f_1}}{R_{f_1}+R_{f_2}} \approx 34.651\times10^{-6} \quad (6.1)$$

$$\implies \begin{aligned} R_{f_1} &= 100\Omega \\ R_{f_2} &= 4.057M\Omega \end{aligned} \quad (6.2)$$

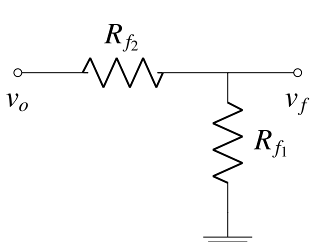

Fig. 6

7. Design the feedback circuit using an OPAMP.
Solution: See Fig. 7

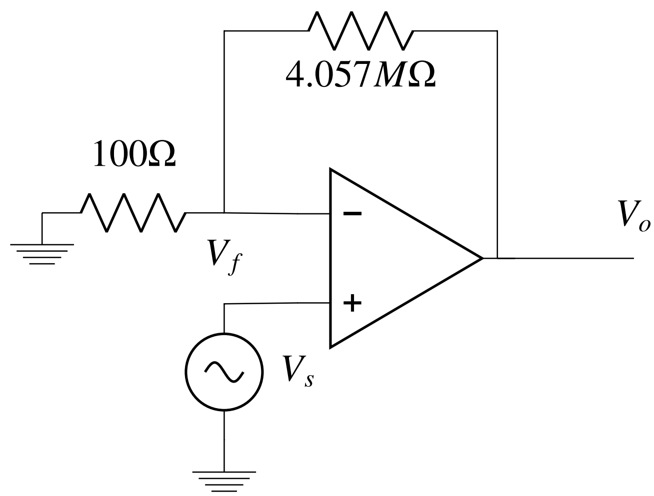

Fig. 7

8. Simulate the circuit in ngspice.
Solution: For $H = 34.651\times10^{-6}$, the closed loop response is

$$|T| \approx \frac{1}{H} = 28.8588\times10^3 \quad (8.1)$$

The following code provides instructions about the simulation.

```
codes/ee18btech11016/spice/README.md
```

The following netlist simulates the unity feedback system for a DC input

```
codes/ee18btech11016/spice/
    ee18btech1016_sim.net
```

which is plotted using the following code in Fig. 8.1. Note that the DC gain in Fig. 8.1 is the same as (8.1)

```
codes/ee18btech11016/spice/
    ee18btech11016_simulation.py
```

Fig. 8.2 shows how the OPAMP circuit is actually implemented in spice using the parameters in Table 8

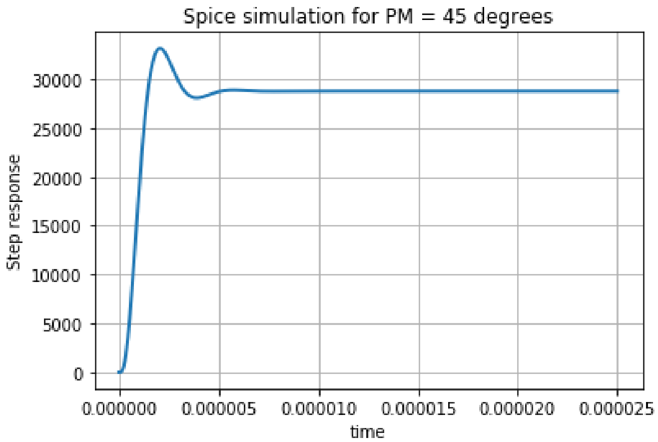

Fig. 8.1

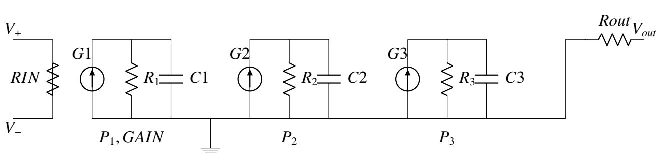

Fig. 8.2: Circuit resembling G(s)

Elements	**Value**
G_1	$10^{-1}(V_+ - V_-)A/V$
G_2	$10^{-6}A/V$
G_3	$10^{-6}A/V$
R_1	$1M\Omega$
R_2	$1M\Omega$
R_3	$1M\Omega$
C_1	$1.59pF$
C_2	$0.503pF$
C_3	$0.159pF$
R_{IN}	$1000M\Omega$
R_{OUT}	100Ω
R_{f_1}	100Ω
R_{f_2}	$4.057M\Omega$
R_s	$1M\Omega$

TABLE 8