Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.1. The CE BJT amplifier in Fig. **??** employs shunt–shunt feedback: Feedback resistor R_F senses the output Voltage V_o and provides a feedback current to the base node. $\left(R_f = 56k\Omega, R_C = 5.6k\Omega, R_S = 10k\Omega\right)$

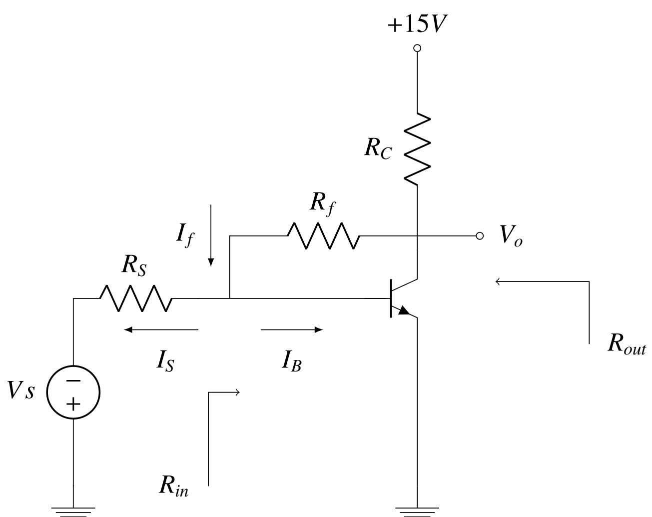

Fig. 1.1

1.2. If V_s has a zero dc component, find the dc collector current of the BJT. Assume the transistor H = 100.

Solution: Since, $V_E = 0$ and $V_S = V_{BE}$

$$I_S = \frac{V_{BE}}{R_S} = \frac{0.7}{10 * 10^3} \tag{1.2.1}$$

$$\implies I_S = 0.07mA \tag{1.2.2}$$

Applying KCL at feedback resistor output

$$-V_o + V_{BE} + I_f R_f = 0$$
$$\left(Since, I_f = I_B + I_S\right)$$
$$V_o = V_{BE} + (I_B + I_S) R_f$$
$$= 0.7 + \left(I_B + 0.07 * 10^{-3}\right)\left(56 * 10^3\right)$$
$$\implies V_o = \left(56 * 10^3\right) I_B + 4.62 \tag{1.2.3}$$

Applying KCL at collector node

$$\frac{V_o - 15}{5.6 * 10^3} + I_C + I_f = 0$$
$$(Since, I_C = HI_B)$$
$$\frac{V_o - 15}{5.6 * 10^3} + HI_B + (I_B + I_S) = 0$$
$$\frac{V_o - 15}{5.6 * 10^3} + (100 + 1) I_B + \left(0.07 * 10^{-3}\right) = 0$$
$$\implies V_o = 14.608 - \left(565.5 * 10^3\right) I_B \tag{1.2.4}$$

Subtracting **??** from **??**, we get,

$$I_B = 16.06\mu A \tag{1.2.5}$$
$$I_C = I_E = HI_B \tag{1.2.6}$$
$$\text{Dc collector Current}, I_C = 1.606mA \tag{1.2.7}$$

1.3. Find the small-signal equivalent circuit of the amplifier with the signal source represented by its Norton equivalent (as we usually do when the feedback connection at the input is shunt).
Solution: In fig **??**

1.4. Find the G circuit and determine the value of G, R_i , and R_o.
Solution: G circuit in fig. **??**

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Parameter	Description
R_{in}	Total Input Resistance
R_{out}	Total Ouput Resistance
r_o	Output resistance of NPN
R_f	Feedback resistance
R_l	Input resistance of G circuit
R_o	Output resistance of G circuit
R_{if}	Input resistance of Feedback
R_{of}	Output resistance of Feedback
R_s	Resistance of Current Source
R_L	Output Load Resistance
g_m	Trans conductance
I_C	Collector current
I_E	Emitter Current
I_B	Base Current

TABLE 1.2

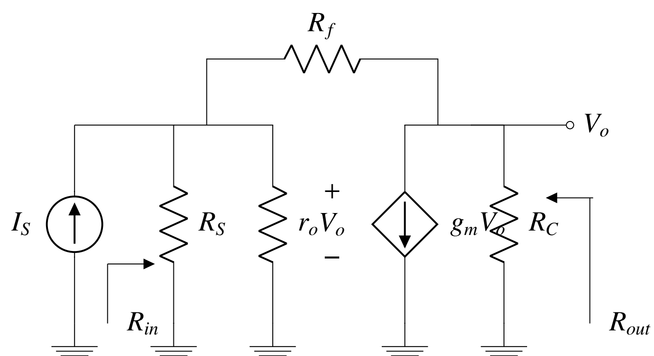

Fig. 1.3

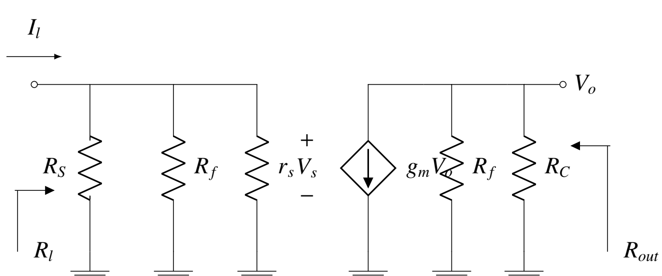

Fig. 1.4

$$g_m = \frac{I_C}{V_s} = \frac{1.606 * 10^{-3}}{25 * 10^{-3}} = 64mA/V \quad (1.4.1)$$

$$r_s = \frac{H}{g_m} = \frac{100}{64 * 10^{-3}} = 1.56k\Omega \quad (1.4.2)$$

$$\text{Gain}, G = \frac{V_o}{I_l} \quad (1.4.3)$$

$$G = \frac{V_S}{I_l}\left(\frac{V_o}{V_o}\right) \quad (1.4.4)$$

$$V_o = -g_m V_s \left(R_f \| R_C\right) \quad (1.4.5)$$

$$V_s = I_l \left(R_S \| R_f \| r_s\right) \quad (1.4.6)$$

Substituting V_o V_s in **??**,

$$G = -g_m \left(R_f \| R_c\right)\left(R_s \| R_f \| r_s\right) \quad (1.4.7)$$

$$G = -429k\Omega \quad (1.4.8)$$

Input Resistance

$$R_I = \left(R_s \| R_f \| r_s\right) = 1.31k\Omega \quad (1.4.9)$$

$$R_o = \left(R_f \| R_C\right) \quad (1.4.10)$$

$$\text{Output Resistance}, R_o = 5.09k\Omega \quad (1.4.11)$$

1.5. Find H and hence AH and 1+AH.
Solution:

$$H = \frac{I_f}{V_o} = -\frac{1}{R_f} \quad (1.5.1)$$

$$\implies H = -17.85 * 10^{-4} \quad (1.5.2)$$

$$GH = 7.662 \quad (1.5.3)$$

$$1 + GH = 8.66 \quad (1.5.4)$$

1.6. Find T , R_{if} and R_{of} and hence R_{in} and R_{out} .
Solution:

$$T = \frac{G}{1+GH} \quad (1.6.1)$$

$$= -49.54k\Omega \quad (1.6.2)$$

$$R_{if} = \frac{R_l}{1+GH} \quad (1.6.3)$$

$$= \frac{1.31 * 10^3}{8.66} \quad (1.6.4)$$

$$= 151.27\Omega \quad (1.6.5)$$

$$R_{of} = \frac{R_o}{1+GH} \quad (1.6.6)$$

$$= \frac{5.09 * 10^3}{8.66} \quad (1.6.7)$$

$$= 587.7\Omega \quad (1.6.8)$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \quad (1.6.9)$$

$$= 153.2\Omega \quad (1.6.10)$$

1.7. What voltage gain V_o/V_s is realized? How does this value compare to the ideal value obtained