Tutorial - II

Time Domain Analysis of Discrete-time LTI Systems

1. Plotting discrete time signals:

Plot the discrete-time signal 0.5^{n} for $-10 \leq n \leq 10$

```
clc;
clf;
clear
n=-10:10;
x1= 0.5.^n;
stem(n,x1)
```

2. Generation of signals:

A small set of elementary signals is needed in this exercise. To begin, create the following signals of length 16 ($0 \leq n \leq 15$)

- b[n], a 16-point block sequence with unit amplitude.
- r[n), the first 16 points of the ramp function, defined as nu[n].
- t[n], 16 points of a periodic triangular wave with period 8, a maximum value of one, a minimum value of 0, and starting point n = 0.
- e[n], the first 16 points of the one-sided exponential, (5/6)nu[n).

Write the MATLAB code to plot the above signals. Using the elementary signals just created, write the code to plot the following new signals:

i. $v[n] = r[n-6]u[n]$
ii. $z[n] = t[n](u[n] - u[n-10])$

```
%% Workspace Initialization.
 clc; clear; close all;

%% Generate the basic signals of common length 16.
N = 16;
n = 0:N-1;

b = ones(1,N);  % Block of ones.
r = n;              % Ramp function.

P = 8;  % Triangular wave period.
n1 = 0:P/2-1;
n2 = P/2:P-1;
P1 = P*ones(1,length(n2));
A = 1;

tri_block = [2*A*n1/P 2*A*(P1-n2)/P]  ;
t = [tri_block tri_block]; % % Periodic Triangular wave
```

```
e = (5/6).^n; % One sided exponential.

figure(1)
stem(n,b);
figure(2)
stem(n,r);
figure(3)
stem(n,t);
figure(4)
stem(n,e);

%% i. Create and display  r[n-6]*u[n].
figure('Name', 'Tutorial-2. Elementary Signals');
stem(n,r);
grid;
hold on;
stem(n+6,r,'r*');
title('r[n] (blue) and v[n]=r[n-6]*u[n] (red)');

%% ii. Create and display z[n]=t[n]*(u[n]-u[n-10]).
z = [t(1:10) zeros(1,6);];
figure('Name',' Tutorial-2. Elementary Signals');
stem(n,t);
grid;
hold on;
stem(n,z,'r*');
title('t[n] (blue) and z[n]=t[n]*(u[n]-u[n-10]) (red)');
```

3. Check the output of the following code:

i)
```
t=-10:0.01:10;
f=heaviside(t);
plot(t,f)
```

ii)
```
t=-10:0.01:10;
g=heaviside(t-3);
figure(1)
plot(t,g)
axis([-15 15 -1 2])
```

iii)
```
t=-10:0.01:10;
l=heaviside(-2*t+2);
figure(4)
plot(t,l)
axis([-15 15 -1 2])
```

Addition & Subtraction of signals:

4. Perform the operations $x_1 + x_2$and $x_1 - x_2$ where $x_1 = 0.5^{\mathrm{n}}$ and $x_2 = 0.8^{\mathrm{n}}$ for $-10 \leq n \leq 10$

```
clf;
clear
n=-10:10;
x1= 0.5.^n;
x2= 0.8.^n;
x=x1+x2;
 y=x1-x2;
 subplot(221)
stem(n,x1)
 subplot(222)
stem(n,x2)
subplot(223)
stem(n,x)
 subplot(224)
 stem(n,y)
```

5. Compute and plot the output response of the LTI system whose impulse response

h[n] = 1 ; 0≤ n ≤ 5
0 ; otherwise for the input, x[n] = {1 4 2 6}
↑

```
clf;
clear
x=input('enter input sequence')  %x = [1 4 2 6];
h=input('enter h(n)')   %h = [1 1 1 1 1 1];
x1=length(x);
h1=length(h);
s1=x1+h1-1; n=0:s1-1;
y = conv(x,h)
stem(n,y)
```

6. For the LTI systems described by the following difference equations, generate its impulse response, and unit step response.

i. *y[n] = x[n] + 2x[n-1]*
ii. *y[n] = 0.9 y[n-1] + x[n]* Also find the analytical expression.
iii. *y[n]-0.3695y[n-1]+0.1958y[n-2]=0.2066x[n]+0.4131x[n-1] +0.2066x[n-2]*

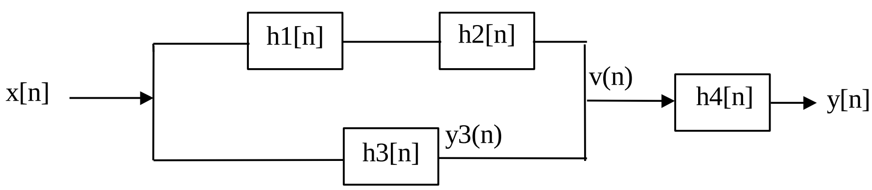
Sample Solution (iii)

```
% Time domain response of difference equations
% y[n] - 0.3695 y[n-1]+0.1958y[n-2]=0.2066x[n]+0.4131x[n-1]+0.2066x[n-2]
```

```
        b1 = [0.2066   0.4131   0.2066];
        a1 = [1   –0.3695    0.1958];
    %   Impulse response of the system
             impz(b1,a1,20);
             title('Impulse response');

    %   Step response of the system
        step_n = [ones(1,20)];
        y = filter(b1,a1,step_n);
        stem(y);
        title('Step response');
```

7. Compute the overall impulse response of the system shown in figure.



$h1[n] = (½)^n \quad 0 \le n \le 5$
$\qquad\quad 0 \quad$ otherwise

$h2[n] = 1 \quad 0 \le n \le 5$
$\qquad\quad 0 \quad$ otherwise

$y3[n] = 0.25\, x[n] + 0.5x[n-1] + 0.25\, x[n-2]$

$y[n] = 0.9\, y[n-1] – 0.81\, y[n-2] + v[n] + v[n-1]$

Code to find the overall impulse response of the system given:

```
n=0:5;
h1=(0.5).^n; h2=ones(1,6);
h5=conv(h1,h2);
% y3(n) = 0.25 x(n)+0.5 x(n-1)+0.25 x(n-2);
h3 = impz([0.25 0.5 0.25],1,max(size(h5)));
h6 = h5 + h3';
% y(n) = 0.9 y(n-1) - 0.81 y(n-2)+v(n)+v(n-1);
n4=[1 1 0]; d4=[1 -0.9 0.81];
 h4 = impz(n4,d4,20);
h = conv (h6, h4); x=0:29;
stem(x,h); grid on; title('Overall Impulse Response')
```

Transform domain representation

$$\boldsymbol{H(z) = \frac{1 - 5z^{-1} + 6z^{-2}}{1 - 2.5z^{-1} + z^{-2}}}$$

- *To enter a transfer function*

```
%   Z - domain
%   H(z) = (1-5z^(-1)+6z^(-2))/(1-2.5z^(-1)+z^(-2))
      b2 = [1 -5 6]; a2 = [1 -2.5 1];
      h  = [b2,a2];
      sys=tf(b2,a2,1e-3)
%   To display H(z) as a rational function of ‘z’
      printsys(b2,a2,’z’);
```

- *Transfer function to zero-pole conversion (tf2zp)*

```
[z,p,k] = tf2zp(b2,a2)
```

- *To obtain the pole-zero map*

```
zplane(b2,a2);
title(‘ Pole- zero plot of H(z) ‘)
```

- *To find the Partial Fractions of the Transfer function*

```
[r,p,k] = residuez(b2,a2)
```

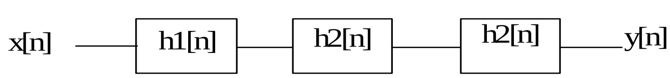
Questions for practice:

1) A small set of elementary signals is needed in this exercise. To begin, create the following signals of length 16 ($0 \leq n \leq 15$)
   - b[n], a 16-point block sequence with unit amplitude.
   - e[n], the first 16 points of the one-sided exponential, $\left(\frac{5}{6}\right)^n u[n]$.

Write the MATLAB code to plot the above signals. Using the elementary signals just created, write the code to plot the following new signals:

i. $y[n] = e[n+10]b[n]$
ii. $e_e[n] = even\{e[n]\}(u[n+5] - u[n-5])$

2) A cascade of three LTI systems is shown below



If h2[n] = U[n] – U[n-2] and overall impulse response is {1 5 10 11 8 4 1} starting at n=0, find h1[n] and verify the result analytically. (use MATLAB function deconv) Also find the response of the overall system to the input x[n]= δ[n] - δ[n-1].

3) Transform the system described by y[n]-0.3695y[n-1]+0.1958y[n-2]=0.2066x[n] +0.4131x[n-1] +0.2066x[n-2] to zero-pole form and residue form. Plot pole-zero map and comment on stability.

4) Compute the causal inverse of $H(z) = \dfrac{z^{-1} + 0.5z^{-2}}{1 - 0.6z^{-1} + .08z^{-2}}$