Pair Instability Supernovae

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February 21, 2021

Documentation of iSURP 2020

1 Preliminaries

1.1 Celestial Sphere

A few terms and definitions - The **meridian** is a great circle; it is defined as passing through the observer's zenith and intersecting the horizon due north and south.
The Sun moves through those constellations along a path referred to as the ecliptic.
The **celestial equator**, which is defined by passing a plane through Earth at its equator and extending that plane out to the celestial sphere.
The Sun crosses the celestial equator, once moving northward along the ecliptic and later moving to the south. In the first case, the point of intersection is called the **vernal equinox** and the southern crossing occurs at the **autumnal equinox**

The Altitude–Azimuth Coordinate System - Based on the measurement of the azimuth angle along the horizon together with the altitude angle above the horizon. The altitude h is defined as that angle measured from the horizon to the object along a great circle that passes through that object and the point on the celestial sphere directly above the observer, a point known as the zenith. Equivalently, the zenith distance z is the angle measured from the zenith to the object, so $z + h = 90°$. The azimuth A is simply the angle measured along the horizon eastward from north to the great circle used for the measure of altitude.
It is Problematic since it is highly dependant on latitude and longitude, coordinates of the stars constantly change, and change in rise times of stars (due to motion around the sun, leading to a greater tilt to reach the meridian) results in a daily shift of these coordinates.

The **Equatorial Coordinate System** is based on the latitude–longitude system of Earth but does not participate in the planet's rotation. Declination is the equivalent of latitude and is measured in degrees north or south of the celestial equator. Right ascension is analogous to longitude and is measured eastward along the celestial equator from the vernal equinox to its intersection with the object's hour circle . Right ascension is traditionally measured in hours, minutes, and seconds.
Since the changes in the latitude and longitude of the observer do not affect the values of right ascension and declination. Values of α and δ are similarly unaffected by the annual motion of Earth around the Sun. Precession (the wobble of Earths rotation axis), along with the proper motion of stars (velocity perpendicular to the radial direction) also causes these coordinates to change.

Measurements of Time The time that is universally used as a reference is noon on January 1, 4713 b.c., as specified by the Julian calendar. This time is designated as JD 0.0, where JD indicates Julian Date.

Spherical Geometry - Some of the results used to finding the angular distance are based on spherical trigonometry, the basic laws of which are summarized below -

$$\frac{sin(a)}{sin(A)} = \frac{sin(b)}{sin(B)}$$
$$cos(a) = cos(b)cos(c) + sin(b)sin(c)cos(A)$$
$$cos(A) = cos(B)cos(C) + sin(B)sin(C)cos(a)$$

Using these we can arrive at the expression for the angular distance traveled in terms of the changes in right ascension and declination: $(\Delta\theta)^2 = (\Delta\alpha cos\delta)^2 + (\Delta\delta)^2$

1.2 Celestial Mechanics

The Kepler laws on planetary motion, formulated by Kepler based on observational data of elliptic motion of planets around the sun are as follows -
Kepler's First Law- A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse. According to Kepler's first law, a planet orbits the Sun in an ellipse, with the Sun located at one focus of the ellipse, the principal focus.
Kepler's Second Law A line connecting a planet to the Sun sweeps out equal areas in equal time intervals.
Kepler's Third Law The Harmonic Law $P^2 = a^3$ where P is the orbital period of the planet, measured in years, and a is the average distance of the planet from the Sun, in astronomical units, or AU.

The point on the ellipse that is closest to the principal focus (located on the major axis) is called perihelion; the point on the opposite end of the major axis and farthest from the principal focus is known as aphelion.

Using the definition and pre-defined terms for an ellipse, we find that the magnitude of r as a function of the angle wrt the foci is given by $r = \frac{a-ae^2}{1+ecos\theta}$, and each type of conic section is related to a specific form of celestial motion, with bounded motion referring to $e = 0$ (circle) and $e < 1$ (elliptical orbits), along with unbound motion $e = 1$ (parabola) and $e > 1$ (hyperbola).

The Newtons Laws are as follows -
Newton's First Law The Law of Inertia - An object at rest will remain at rest and an object in motion will remain in motion in a straight line at a constant speed unless acted upon by an external force.
Newton's Second Law The net force (the sum of all forces) acting on an object is proportional to the object's mass and its resultant acceleration.
Newton's Third Law For every action there is an equal and opposite reaction - acting on different objects. Another major law, the **the Law of Universal Gravitation** $F = \frac{GMm}{r^2}$, where G is the Universal Gravitational Constant. The amount of energy (the work) necessary to raise an object of mass m a height h against a gravitational force is equal to the change in the potential energy of the system, against a reference. **Work Energy Theorem** Thus work done on the particle results in an equivalent change in the particle's kinetic energy. This statement is simply one example of the conservation of energy.

Finally, using Newtons Laws, we can finally re-derive Keplers Laws, in the Centre of Mass frame. In general, the two-body problem may be treated as an equivalent one body problem with the reduced mass $\mu = mM/m + M$ moving about a fixed mass M at a distance r, giving the total Energy as - $E = \frac{\mu v^2}{2} - G\frac{m\mu}{r}$, and $\mathbf{L} = \mu \mathbf{r} x \mathbf{v}$. Evaluating the time derivative, we see that the angular momentum for such a system is conserved.
A mathematical derivation of the first law assuming Both objects in a binary orbit move about the center of mass in ellipses, with the center of mass occupying one focus of each ellipse gives $r = \frac{L^2/\mu^2}{GM(1+ecos\theta)}$ and $L = \mu\sqrt{GMa(1-e^2)}$.
The second law can be derived by computing $\frac{dA}{dt} = \frac{L}{2\mu}$, and by seeing that L is a constant, the area swept per unit time is also a constant.
The general form of Kepler's third law is - $P^2 = \frac{4\pi^2 a^3}{G(m_1+m_2)}$, not only demonstrating that the relationship

between the semi major axis of an elliptical orbit and the orbital period, but also is dependant on the total mass of the system.

For gravitationally bound systems in equilibrium, it can be shown that the total energy is always one-half of the time-averaged potential energy; this is known as the **Virial theorem**, and is applicable to a wide range of systems.

1.3 Spectrum of Light

A measurement of the parallax angle p (one-half of the maximum change in angular position) allows the calculation of the distance d to a star, when measured 6 months apart, that is $d = \frac{1AU}{tanp}$. Defining a new unit of distance, the parsec (parallax-second, abbreviated pc), leads to $d = \frac{1pc}{p''}$. Thus 1 parsec is the distance from which the radius of Earth's orbit, 1AU, subtends an angle of 1. This method is only useful for for surveying the local neighborhood of the Sun, since the angle subtended by farther stars are much lesser.

A numerical scale to describe how bright each star appeared in the sky, known as the apparent magnitude of an object, with a smaller apparent magnitude means a brighter-appearing star. This is a logarithmic scale, and a difference of 1 in the apparent magnitude implies a 2.512 factor increase in brightness. The "brightness" of a star is actually measured in terms of the radiant flux F(total amount of light energy of all wavelengths that crosses a unit area oriented perpendicular to the direction of the light's travel per unit time) received from the star, and $F = \frac{L}{4\pi r^2}$, L is the intrinsic luminosity and r is the distance.
The absolute magnitude, M, is defined to be the apparent magnitude a star would have if it were located at a distance of 10 pc. The relation between m (resp. M) and L is - $m_1 - m_2 = -2.5log_{10}(\frac{F_1}{F_2})$, and that of d, m and M is $m - M = 5log_{10}(\frac{d}{10pc})$.

Light can be determined as a propagating transverse wave of the Electric and Magnetic fields (perpendicular to each other), which are concisely defined by Maxwell in his 4 equations. The total spectrum of light consists of electromagnetic waves of all wavelengths, ranging from very short-wavelength gamma rays to very long-wavelength radio waves.
Like all waves, electromagnetic waves carry both energy and momentum in the direction of propagation. The rate at which energy is carried by a light wave is described by the Poynting vector, $\mathbf{S} = \frac{\mathbf{E}x\mathbf{B}}{\mu_0}$, pointing in the direction of the electromagnetic wave's propagation.
Because an electromagnetic wave carries momentum, it can exert a force on a surface hit by the light. The resulting radiation pressure depends on whether the light is reflected from or absorbed by the surface. ($\frac{<S>Acos\theta}{c}$ for absorption and $\frac{2<S>Acos^2\theta}{c}$ for reflection).
A blackbody is an ideal emitter, and absorbs all of light energy incident on it, and emits the characteristic blackbody radiation. Weins Displacement law gives the relation between maximum intensity wavelength and temperature of the blackbody, as $\lambda T = b = 0.0028$ The Stefan–Boltzmann equation, derives the relation $L = \sigma AT^4$, where σ is the SB constant, A is $4\pi R^2$, and T is the effective temperature of the star (modelling them as blackbodies upto first order).
To fit the blackbody curve, Planck made the assumption that a wave can have only specific allowed values of energy, in integral multiples of a minimum wave energy. This minimum energy, a quantum of energy, is given by h or hc/, leading to the function of form -

$$B_\mu(T) = \frac{2h\mu^3/c^2}{e^{\frac{h\mu}{kT}} - 1}$$

which when integrated over a model star emitting blackbody radiation isotropically (equally in all directions) over the outward hemisphere, the energy per second having wavelengths between λ and $\lambda + d\lambda$ emitted by the star is -

$$L_\lambda d\lambda = \frac{8\pi^2 R^2 hc^2/\lambda^5}{e^{\frac{hc}{k}} - 1} d\lambda$$

The apparent and absolute magnitudes measured over all wavelengths of light emitted by a star, are known as bolometric magnitudes and are denoted by m_{bol} and M_{bol}, respectively.
A star's U - B color index is the difference between its ultraviolet and blue magnitudes, and a star's B - V color index is the difference between its blue and visual magnitudes.

1.4 Special Theory of Relativity

The postulates, or general statements which provide for a basis of the theory of STR are as follows

- *The Principle of Relativity* - The laws of physics are the same in all inertial frames
- *The Principle of constancy of c* - The speed of light in free space has the same value c in all inertial reference frames

These 2 postulates lead to another consequence - it is impossible to accelerate a particle to a speed greater than c, no matter how much Kinetic energy we give it.
A coordinate system in STR is called inertial if the distance between points is independent of time, the clocks that sit at every point ticking off the time coordinate t are synchronized and all run at the same rate and that the geometry of space at any constant time t is Euclidean.

There are multiple ways to derive the time dilation and length contraction equations in STR, however we will focus on the methods of Lorentz Transformations. The transformation equations in general have - an observer S in an inertial frame, another observer S' in another inertial frame moving relative to S, and a common event observed. Without loss of generality, assuming the relative motion of frames is in the xx' direction, with velocity $\vec{u}$. Galilean transformations gives the relationships as -

$$\begin{aligned} x' &= x - ut \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

The relativistic corrections to these, which can derived from the postulates given above by assuming space-time homogeneity and symmetry are -

$$\begin{aligned} x' &= \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma(x - ut) \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \frac{ux}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma(t - \frac{ux}{c^2}) \end{aligned}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ is known as the Lorentz factor. These equations also reduce to the Galilean equations in the limit $u << c$ or $c \to \infty$. From the above equations, we can see the effects of time dilation (the clock of an observer in relative motion seems to run slowly when timed by the clocks in the frame of reference which is "at rest") and length contraction (the distance between 2 points measured simultaneously, by an observer in relative motion is observed to be lesser than the one "at rest", but only along the axis of relative motion). We can also derive the relation between velocities as -

$$\begin{aligned} {v_x}' &= \frac{\Delta x'}{\Delta t'} = \frac{\gamma(\Delta x - u\Delta t)}{\gamma(\Delta t - \frac{u\Delta x}{c^2})} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \\ {v_y}' &= \frac{v_y}{\gamma(1 - \frac{uv_x}{c^2})} \\ {v_z}' &= \frac{v_z}{\gamma(1 - \frac{uv_x}{c^2})} \end{aligned}$$

We can also put $c = v_x$ and check that $v_x{}' = c$, always, irrespective of u.

Applying the classical definition of momentum to frames in relative motion, we see that momentum is not conserved in such inertial frames. Einstein gave a new definition of momentum, so that it would be conserved in all inertial frames, and would reduce to the classical definition at lower speeds. The relativistic formula for the momentum of a particle moving with velocity $\vec{v}$ is -

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

Since energy of a particle is defined from Work-Energy theorem, we get the total KE of a particle as -

$$KE = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 \tag{2}$$

which, once again reduces to the classical formula $KE = \frac{mv^2}{2}$ at low speeds. We call the mc^2 term as rest energy, or the total energy of a particle as measured from a frame in which it is at rest. Only the total energy, the combination of rest energy and kinetic energy is conserved in all inertial frames. A little manipulation of the above equation yield the total energy (E) as -

$$E^2 = p^2c^2 + m^2c^4 \tag{3}$$

Similarly, deriving the equation for the relativistic Doppler shift for the source moving at u, at an angle θ, so that the radial velocity is $v_r = ucos\theta$ as -

$$v_{recieved} = \frac{v_{source}\sqrt{1 - u^2/c^2}}{1 + v_r/c}$$

If the source of light is moving away from the observer ($v_r > 0$), then this shift to a longer wavelength is called a redshift. Similarly, if the source is moving toward the observer ($vr < 0$), then there is a shift to a shorter wavelength, a blueshift. Unlike the classical case, There is also a transverse Doppler shift for motion perpendicular to the observer's line of sight ($\theta = 90\circ, v_r = 0$). This transverse shift is entirely due to the effect of time dilation, and is a redshift of the light from the source.
A redshift parameter z is used to describe the change in wavelength; it is defined as $z = \frac{\Delta\lambda}{\lambda_{source}}$, which for radial motion becomes $z + 1 = \sqrt{\frac{1+v_r/c}{1-v_r/c}}$

1.5 Light and Matter Interaction

The 3 laws for spectroscopy, found by Kirchoff are -

- A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines (the continuous spectrum of blackbody radiation emitted at any temperature above absolute zero)
- A hot, diffuse gas produces bright spectral lines (produced when an electron makes a downward transition from a higher orbit to a lower orbit, carried away by a photon).
- A cool, diffuse gas in front of a source of a continuous spectrum produces dark spectral lines (absorption lines) in the continuous spectrum (when an electron makes a transition from a lower orbit to a higher orbit).

Spectrographs are used to measure spectra of stars and galaxies - After passing through a narrow slit, the starlight is collimated by a mirror and directed onto a diffraction grating, a piece of glass onto which narrow, closely spaced lines have been evenly ruled, which acts like a long series of neighboring double slits. Using $dsin\theta = n\lambda$, we can get the wavelengths for specific angles. The spectrum is then focused onto a photographic plate or electronic detector for recording. The smallest difference in wavelength that the grating can resolve is $\Delta\lambda = \frac{\lambda}{nN}$, where n is the order of the spectrum, , and the total number of lines of the grating that are illuminated is N.

The photoelectric effect, leading to the consequences of number of emitted electrons depending on intensity of light, and maximum KE of emitted electron only on frequency, can be summed up in - $KE_{max} = \frac{hc}{\lambda} - \phi$, where ϕ is the work function of the metal. The Compton effect, describing the scattering of a photon by a free electron, as the change in wavelength after scattering at an angle θ by - $\Delta\lambda = \frac{h(1-cos\theta)}{m_e c}$, providing verification that though massless, photons do carry momentum.

The physical reason behind the spectral lines came i 1913, when Niels Bohr proposed, that like the energy, value of the angular momentum of the hydrogen atom could assume only integral multiples of Planck's constant divided by $2\pi : L = nh/2$. Bohr hypothesized that in orbits with precisely these allowed values of the angular momentum, the electron would be stable and would not radiate in spite of its centripetal acceleration. Using this, we only get allowed radii for the electron around the nuclues, in n^2 multiples of the Bohr radius. The integer n, known as the principal quantum number, completely determines the characteristics of each orbit of the Bohr atom. These orbits, then correspond to the spectral lines, and was in excellent agreement with the experimental findings of Balmer, et al.

Extending the discussion to all particles with the de Broglie wavelength ($\lambda = \frac{h}{p}$), we get the **Probability** wave of a particle, the square of amplitude of which, gives us information about the probabilty of that particle being found in that location. Added to this description, Heisenbergs Uncertainity Principle states the fundamental inability of a particle to simultaneously have a well-defined position and a well-defined momentum as a direct result of the wave–particle duality of nature ($\Delta x \Delta p \geq \frac{h}{4\pi}$). The wave–particle duality of nature implies that particles can also tunnel through a region of space (a barrier) in which they cannot exist classically.

Applying these new concepts to the Bohr atom, the electron orbitals are clouds of probability, with them being more dense in certain regions where it is likely to be found. Schrodinger equations can be solved for the probability waves that describe the allowed values of a particle's energy, momentum, and so on, as well as the particle's propagation through space. Additional to the n quantum number, the l quantum number (between 0 to n-1) describing the angular momentum, m_l describing the z component of L (between -l to l), and the spin quantum number (-1/2 or 1/2) describe an electron completely.
The splitting of spectral lines under magnetic fields along a certain direction, called the Zeeman Effect, leads to the splitting of m_l degenerate orbitals, leading to frequencies of the form $v_0, v_0 + \frac{eB}{4\pi\mu}, v_0 - \frac{eB}{4\pi\mu}$.

Certain selection rules restrict the allowed transitions to only those having $\Delta l = 1$. For Zeeman Effect, another set of selection rules require that $m_l = 0, 1 or - 1$ and forbidding transitions between orbitals if both orbitals have $m_l = 0$. Although forbidden transitions may occur, they require much longer times if they are to occur with any significant probability.

I am skipping the discussion on Telescopes, but the book does provide a very beautiful description on how we actually view the universe.

2 Binary Systems

Binary Star Systems are classified as (according to observational characteristics) -

- **Optical Double-** Seperated by large distances but have same ra, dec, not gravitatioanlly bound.
- **Visual binary** Both stars can be resolved independantly

- **Astrometric binary-** One member of binary is brighter than other, existence of unseen by oscillatory motion of the first
- **Eclipsing binary-** Orbital planes along LOS, eclipse each other, observations of these light curves provide lot of information
- **Spectrum binary-** System with 2 superimposed, discernible spectra, can be seen by periodic shifts in wavelength, and simultaneous blue and red shift of the 2 companions (similar to spectroscopic binary)

Visual binaries can yield the ratio of the mass information, as the ratio of angle subtended by the respective semi major axis ($\frac{m_1}{m_2} = \frac{\alpha}{\alpha}$), and the third law of Kepler gives the sum of the masses (assuming either the distance or the radial velocities are known). The sum of masses needs to be corrected if the plane of rotation is not perpendicular to the line of sight.

Eclipsing systems usually have sinusoidal radial velocity curves, with almost circular orbits, since they tend to circularize due to tidal interactions over short timescales. Hence, this becomes simplified, and we can determine ratio of masses by ratio of radial velocities ($\frac{m_1}{m_2} = \frac{v_{2r}}{v_{1r}}$), and the sum of masses as as a function of the tilt of orbit $m_1 + m_2 = \frac{P}{2\pi G}\frac{(v_{1r}+v_{2r})^3}{sin^3 i}$.
If the spectrum of only one star is visible, then the mass of the other star can be derived as $\frac{m_2^3}{(m_1+m_2)^3} sin^3 i = \frac{P}{2\pi G} v_{1r}^3$, where the RHS is the mass function.
The value of i can be determined if its an eclipsing binary, by the amount one star is eclipsed by the other. The duration of the eclipse gives the radii of the stars, where $r_s = (v_s + v_l)\frac{\Delta t}{2}$, and similarly for r_l.
Comparing the amount of light received during eclipse, versus when both are visible, we can find the ratio of effective temperatures of the 2 stars. Let $B_0 = k(\pi r_l^2 F_{lr} + \pi r_s^2 F_{rs})$ when both are visible, $B_p = k\pi r_l^2 F_{lr}$ when the smaller is eclipsed, and $B_s = k((\pi r_l^2 - \pi r_s^2)F_{lr} + \pi r_s^2 F_{rs})$, we get the relation $\frac{B_o - B_p}{B_o - B_s} = (\frac{T_s}{T_l})^4$.

Extra solar planets are discovered by radial velocity measurements (variations due to gravitational pull), astrometric wobbles and eclipses.

3 Stellar Spectra

The Harvard classification scheme of "OBAFGKM" becomes a temperature sequence, running from the hottest blue O stars to the coolest red M stars. Stars nearer the beginning of this sequence are referred to as early-type stars, and those closer to the end are called late-type stars. These labels also distinguish the stars within the spectral subdivisions, so astronomers may speak of a K0 star as an "early K star" or refer to a B9 star as a "late B star." The distinctions between the spectra of stars with different temperatures are due to electrons occupying different atomic orbitals in the atmospheres of these stars. A I refers to the ground state of an element A.

Spectral Type	Characteristics
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).
G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.
K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.
M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.
L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.
T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.

Figure 1: Harvard Classification

To study this, we understand the **Maxwell-Boltzmann velocity distribution function**, given as -

$$n_\nu d\nu = n(\frac{m}{2\pi kT})^{3/2} \exp\{-mv^2/2kT\} 4\pi\nu^2 d\nu \tag{4}$$

where n_ν is the number of gas particles per unit volume having speeds between $\nu and d\nu + d\nu$, where the most probable speed is given as $v_{mp} = \sqrt{\frac{2kT}{m}}$ and the root mean square speed as $v_{rms} = \sqrt{\frac{3kT}{m}}$. This gives the energies of the atoms as a whole.
The distribution of electrons is governed by a fundamental result of statistical mechanics: Orbitals of higher energy are less likely to be occupied by electrons. Lets_a stand for the specific set of quantum numbers that identifies a state of energy E_a for a system of particles. Similarly, let s_b stand for the set of quantum numbers that identifies a state of energy E_b. Then the ratio of the probability $P(s_b)$ that the system is in state s_b to the probability $P(s_a)$ that the system is in state s_a is given by $\frac{P(s_b)}{P(s_a)} = \frac{\exp\{-E_b/kT\}}{\exp\{-E_a/kT\}}$. where $\exp\{-E/kT\}$ is the Boltzmann factor. Similarly, accounting degeneracy states -

$$\frac{P(E_b)}{P(E_a)} = \frac{N_b}{N_a} = \frac{g_b \exp\{-E_b/kT\}}{g_a \exp\{-E_a/kT\}} \tag{5}$$

Considering the relative number of atoms in different stages of ionization we get the **Saha Equation**.

We first define the partition function, basically the weighted sum of the number of ways the atom can arrange its electrons with the same energy. It is defined as $Z = \sum_{j=1}^{\infty} g_j \exp\{-(E_j - E_1)/kT\}$. This leads us to the number of atoms in stage (i+1) to stage i as -

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{Z_i n_e} (\frac{2\pi m_e kT}{h^2})^{1/2} e^{-\chi_i/kT} \tag{6}$$

where χ_i is the the ionization energy needed to remove an electron from an atom.
The narrow region inside a star where hydrogen is partially ionized is called a hydrogen partial ionization zone and has a characteristic temperature of approximately 10,000 K for a wide range of stellar parameters. The diminishing strength of the Balmer lines at higher temperatures is due to the rapid ionization of hydrogen above 10,000 K. Doing some calculations, it becomes apparent that the strength of the H and K lines is not due to a greater abundance of calcium in the Sun. Rather, the strength of these Ca II lines reflects the sensitive temperature dependence of the atomic states of excitation and ionization.

The **Hertzsprung–Russell** (H–R) diagram, has either the luminosity and effective temperature are plotted for each star, or the observationally determined quantities of absolute magnitude vs color index or spectral type. A band, called the main sequence, contains between 80% and 90% of all stars in the H–R diagram. The giant stars occupy the region above the lower main sequence, with the supergiants, in the extreme upper right-hand corner. The white dwarfs lie well below the main sequence. The radius of a star can be easily determined from its position on the H–R diagram.On a logarithmically plotted H–R diagram, the locations of stars having the same radii fall along diagonal lines that run roughly parallel to the main sequence. This implies that the position of a star on the HR diagram is governed by its mass.

The MKK Atlas established the two-dimensional Morgan–Keenan (M–K) system of spectral classification, which display the effect of temperature and luminosity on stellar spectra. A luminosity class, designated by a Roman numeral, is appended to a star's Harvard spectral type. The numeral "I" (subdivided into classes Ia and Ib) is reserved for the supergiant stars, and "V" denotes a main-sequence star. In general, for stars of the same spectral type, narrower lines are usually produced by more luminous stars. Once the star's absolute magnitude, M, has been read from the vertical axis of the H–R diagram, the distance to the star can be calculated from its apparent magnitude, m as $d = 10^{(m-M+5)/5}$, in parsecs.

4 Stellar Atmosphere

The specific intensity of rays, I_λ is defined in relation with the amount of electromagnetic radiation energy having a wavelength between λ and $\lambda + d\lambda$ that passes in time dt through the area dA into a solid angle $d\omega = sin\theta d\theta d\phi$, and the mean intensity as -

$$E_\lambda = I_\lambda d\lambda dt dA cos(\theta) sin(\theta) d\theta d\phi$$
$$< I_\lambda > = \frac{1}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda sin\theta d\theta d\phi$$

Specific Energy density, energy per unit volume between frequencies $\mu and \mu + d\mu$ defined in a similar manner is $u_\lambda d\lambda = \frac{4\pi}{c} < I_\lambda > d\lambda$ and for blackbody radiation as -

$$u_\mu d\mu = \frac{8\pi h \mu^3/c^3}{e^{h\mu/kT} - 1} d\mu \tag{7}$$

The specific radiative flux F_λ is the net energy having a wavelength between $\lambda and \lambda + d\lambda$ that passes each second through a unit area in the direction of the z axis and is given by -

$$F_\lambda d\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda cos\theta sin\theta d\theta d\phi \tag{8}$$

(0 for isotropic) For resolved sources, energy is measured by specific energy density, and unresolved sources it is measured by specific radiative flux, since it ends up integrating over all directions in the Airys disc. Radiation pressure exerted by photons, on account of their energy, is found by calculating the change of momentum per unit time per unit area in a radiation field. For a reflecting surface the change is twice, whereas for transmission, the total radiation pressure is given by -

$$P_{rad,\lambda} = \frac{2}{c}\int_{sphere} I_\lambda d\lambda cos^2(\theta) sin(\theta) d\theta d\phi \quad \text{reflection}$$

$$P_{rad,\lambda} = \frac{1}{c}\int_{sphere} I_\lambda d\lambda cos^2(\theta) sin(\theta) d\theta d\phi \quad \text{transmission}$$

$$P_{rad,\lambda} = \frac{4\pi}{3c} I_\lambda d\lambda \quad \text{isotropic radiation field}$$

The total radiation pressure produced by photons of all wavelengths is found by integrating $P_{rad,\lambda}$, and for blackbody radiation, the total pressure in $P_{rad} = \frac{u}{3}$, one third of the energy density.

4.1 Opacity

The "surface" of a star is defined as the region where the emergent visual continuum forms, namely the photosphere. Solar absorption lines remove light from the Sun's continuous spectrum at certain wavelengths. The decrease in intensity produced by the dense series of metallic absorption lines in the solar spectrum is especially effective; this effect is called **line blanketing**, and in other regions emission lines augment the spectrum. The types of temperature definitions are -

- The effective temperature, which is obtained from the Stefan–Boltzmann law is uniquely defined for a specific level within a star and is an important global descriptor of that star.
- The excitation temperature is defined by the Boltzmann equation.
- The ionization temperature is defined by the Saha equation
- The kinetic temperature is contained in the Maxwell–Boltzmann distribution
- The color temperature is obtained by fitting the shape of a star's continuous spectrum to the Planck function

If every process (e.g., the absorption of a photon) occurs at the same rate as its inverse process (e.g., the emission of a photon), it is called thermodynamic equilibrium. We can define a local thermodynamic equilibrium (LTE), if the distance over which the temperature changes significantly is large compared with the distances traveled by the particles and photons between collisions (their mean free paths). Classically, the mean free path is given by $l = \frac{1}{n\sigma}$, where n is no of atoms per m^3 and $\sigma = \pi 4 a_0^2$, where a_0 is the Bohr radius.

Change in the intensity,of a ray of wavelength λ as it travels through a gas,

$$dI_\lambda = \kappa_\lambda \rho I_\lambda ds \tag{9}$$

κ_λ is called the absorption coefficient, or opacity, and is wavelength dependent. Both absorption and scattering can remove photons from a beam of light, and so contribute to the opacity, of the stellar material. Bound-Bound transitions (for discrete wavelenghts), photoionization (continuum, $\sigma = 1.31x10^{-19}\frac{1}{n^5}(\frac{\lambda}{500nm})^3 m^2$, H^- plays a major role in this), free-free absorption (continuum), and electron scattering or Thompson scattering (continuum, $\sigma = 6.65x10^{-29}m^2$) are the sources of opacity. Compton scattering if photon wavelength is less than atom, or Rayleigh if larger (prop to $1/\lambda^4$). The total opacity is the sum of the opacities due to all of the preceding sources.
We define an optical depth, τ, back along a light ray by $d\tau = \kappa\rho ds$. The outermost layers of a star may be taken to be at $\tau = 0$ for all wavelengths, and hence -

$$\tau_{\lambda,0} = \int_0^s \kappa_\lambda \rho ds \tag{10}$$

The optical depth may be thought of as the number of mean free paths from the original position to the surface, as measured along the ray's path.
The Rosseland mean opacity is defined as -

$$\frac{1}{\bar{\kappa}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu} \tag{11}$$

and approximation formulae have been developed for both the average bound–free and free–free opacities. Any opacity having $\bar{\kappa} \propto \frac{\rho}{T^{3.5}}$ is referred to as a Kramers opacity law. Opacity increases with increasing density for a given temperature. For a constant-density plot as opacity rises steeply with increasing temperature. This reflects the increase in the number of free electrons produced by the ionization of hydrogen and helium. After this peak, it follows Kramers law, and declines with increase in temperature, and is due primarily to the bound–free and free–free absorption of photons. Electron scattering dominates at the highest temperatures, when nearly all of the stellar material is ionized.

Similar to what we did in absorption, scattering of photons into a beam, as well as downward transitions both will count as emission, and eavh of the absorption processes above have an inverse emission process. Photons perform random walk to approach the surface of the star, and for a large number of scattering, the average number of steps needed for a photon to travel the distance d before leaving the surface is then $N = \tau_\lambda^2$. Looking into a star at any angle, we always look back to an optical depth of about $\tau_\lambda = 2/3$, as measured straight back along the line of sight, the photosphere.
The line of sight of an observer on Earth viewing a star is vertically downward at the center of the star's disk but makes an increasingly larger angle with the vertical near the edge, or limb, of the star. Looking near the limb, the observer will not see as deeply into the solar atmosphere and will therefore see a lower temperature at optical depth of 2/3, and a lower intensity, leading to the star appearing darker at the limb, or **Limb Darkening**.
A decrease in temperature outwards leads to a decrease in radiation pressure, which produces the slight net movement of photons toward the surface that carries the radiative flux and is described by $\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c} F_{rad}$.

4.2 Transfer Equation

For pure emission (no absorption of the radiation), $dI = j_\lambda \rho ds$, where j_λ is the emission coefficient of the gas. The ratio of the emission coefficient to the absorption coefficient is called the source function, S_λ. The equation of radiative transfer (usually referred to as the transfer equation) is written as -

$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda = \frac{dI_\lambda}{d\tau_\lambda} \tag{12}$$

For the case of thermodynamic equilibrium, the source function is equal to the Planck function, $S_{lambda} = B_\lambda$ for blackbody radiation. Assuming a plane parallel atmosphere, and $\tau_{\lambda,\nu}$ as the optical depth perpendicular to the surface, and the transfer equation becomes $I_\lambda - S_\lambda = cos\theta \frac{dI_\lambda}{d\tau_{\lambda,\nu}}$, we also assume that the opacity is independant of wavelength (equal to $\bar{\kappa}$), called a gray atmosphere. Integrating the transfer equation over all wavelengths, we finally get for a gray atmosphere -

$$cos\theta \frac{dI}{d\tau_\nu} = I - S \tag{13}$$

Integrating over all solid angles, taking the source function to be independent of distance -

$$\frac{dF_{rad}}{d\tau_\nu} = 4\pi(<I> - S) \tag{14}$$

Multiplying by $cos\theta$ and integrating, we find -

$$\frac{dP_{rad}}{d\tau_\nu} = \frac{F_{rad}}{c} \tag{15}$$

which leads to the equation described above. Since F_{rad} is a constant (in eqb), the change is 0, hence $< I >= S$, and the equation above can be simply integrated. The Eddington approximation gives us -

$$
\begin{aligned}
< I > &= \frac{1}{2}(I_{out} + I_{in}) \\
F_{rad} &= \pi(I_{out} - I_{in}) \\
P_{rad} &= \frac{4\pi}{3c} < I >
\end{aligned}
$$

Using this, we get $< I >= \frac{3\sigma}{4\pi}T_e^4(\tau_\nu + 2/3)$ Finally, assuming LTE, $< I >= \frac{\sigma T^4}{\pi}$, after which we finally get -

$$T^4 = \frac{3}{4}T_e^4(\tau_\nu + 2/3) \tag{16}$$

and that $T = T_e$ at $\tau_\nu = 2/3$, thus defining the surface of the star. we see down to a vertical optical depth of 2/3, averaged over the disk of the star.
Reconsidering limb darkening, assuming plane-parallel atmosphere and the vertical optical depth,and taking the origination of the ray to have a large optical depth, we get the emergent intensity as - $I(0) = \int_0^\infty Ssec\theta e^{-\tau_\nu sec\theta}d\tau_\nu$. (I and τ_ν are still λ dependent). A good approximation is to consider $S = a + b\tau_\nu$.

The strength of a spectral line is measured in terms of its equivalent width. The equivalent width W of a spectral line is defined as the width of a box reaching up to the continuum that has the same area as the spectral line, or - $W = \int \frac{F_c - F_\lambda}{F_c}d\lambda$. Full width at half maximum is the denoted by $(\Delta\lambda)_{1/2}$and is the wavelength till which the difference in continuum is half od the maximum. If the radiant flux is never blocked, the spectral line is called optically thin. If the F_λ/F_c dips below, implying it is formed in cooler regions of the stellar atmosphere and away from it occurs at more interior layers until the continuum layer. There are 3 processes responsible for broadening of spectral lines -

- Due to Heisenbergs uncertainty principle, spectral lines cannot be infinitely sharp, resulting in an uncertainty in the photon's wavelength has a magnitude of roughly $\Delta\lambda = \frac{\lambda^2}{2\pi c}(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f})$, $T_i and t_f$ are at initial and final stages, and using an average waiting time for a specific transition, $(\Delta\lambda)_{1/2} = \frac{\lambda^2}{\pi c\Delta t_0}$
- A more substantial effect is due to the doppler shift, producing $\Delta\lambda = \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}}$. Doppler shifts caused by the large-scale turbulent motion of large masses of gas can also be accommodated, and are important to describe spectra of giant and supergiant stars, leading to $(\Delta)_{1/2} = \frac{2\lambda}{c}\sqrt{(\frac{2kT}{m} + V_{turb}^2)ln2}$.
- The results of individual collisions of elements, and the statistical effects of the electric fields of large numbers of closely passing ions is termed pressure broadening. An estimate for atoms of single element, using the mean time as mean free path per most probable speed gives us $\Delta\lambda = \frac{\lambda^2 n\sigma}{\pi c}\sqrt{\frac{2kT}{m}}$, narrower lines are observed in supergiant stars due to lower number densities.

The line profile shared by natural and pressure broadening is called the damping profile. Total line profiles (Voigt profiles) tend to have Doppler cores and damping wings. The oscillator strength is the probability of a transition happening, and thus the oscillator strength is the effective number of electrons per atom participating in a transition, and so multiplying the number of absorbing atoms per unit area by the f -value gives the number of atoms lying above each square meter of the photosphere that are actively involved in producing a given spectral line. A curve of growth, is a logarithmic graph of the equivalent width, W, as a function of the number of absorbing atoms, N_a. The curve of growth is initially linear with lnN_a. As a rule, the flux at any wavelength cannot fall below the value of the source function at an optical depth of 2/3. Increasing the number of absorbing atoms still, it log W starts going as $\sqrt{N_a}$.

5 Interior of stars

The gravitational force is always attractive, implying that an opposing force must exist if a star is to avoid collapse. This force is provided by pressure. Assume there are no shear stresses, on a cylinder of dm mass.

Assuming these are the only forces on the cylinder, assuming spherical symmetry and that the star is static, we get the condition of hydrostatic equilibrium as -

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2} = -\rho g \tag{17}$$

in order for a star to be static, a pressure gradient $\frac{dP}{dr}$ must exist to counteract the force of gravity. It is not the pressure that supports a star, but the change in pressure with radius. There is also the mass conservation equation -

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \tag{18}$$

The pressure integral computes the pressure given a distribution function of momenta, which can be converted to one in velocity(for massive particles, non relativistic in an ideal gas)is given as -

$$P = \frac{1}{3}\int_0^\infty n_p p v dp \tag{19}$$

$$n_v dv = n(\frac{m}{2\pi kT})^{3/2} e^{-\frac{mv^2}{2kT}} 4\pi v^2 dv \tag{20}$$

where n is the particle number density.

For a vareity of masses, we get $P_g = \frac{\rho kT}{\mu m_H}$, where $\mu = \frac{\rho}{n m_H}$, the mean molecular weight. The mean molecular weight depends on the composition of the gas as well as on the state of ionization of each species. The level of ionization enters because free electrons must be included in the average mass per particle m. For a neutral gas $\frac{1}{\mu_n} = X + Y/4 + < \frac{1}{A_n} > Z$. $< \frac{1}{A_n} >$ is the weighted average of all elements in the gas heavier than helium, where X, Y and Z are the mass fractions of Hydrogen, Helium and metals respectively. Similarly, for a completely ionized gas, we use $\frac{1}{\mu_i} = \sum_i \frac{1+z_i}{A_i} X_i$, where z is the number of free electrons. From these, we also get the average kinetic energy per particle as $1/2 m\bar{v}^2 = 3/2 kT$, each 1/2kT per degree of freedom. This is the Maxwell-Boltzmann statistics, however, Fermi-Dirac distribution function incorporates quantum mechanics and leads to different equations, for fermions and similarly bosons follow the Bose-Einstein statistics.
Finally, we combine the radiation pressure, which using the Planck distribution function, comes as - $P_{rad} = \frac{aT^4}{3}$, a is the radiation constant.

5.1 Sources of Energy

If we assume that the star converts half of its gravitational energy into heat and radiate (half since thats the total energy it has), and assuming that the density is constant and equal to its average value, we get the total energy as $E = -\frac{3GM^2}{10R}$. The energy time scale till which it will emit energy at current luminosity is is known as the Kelvin–Helmholtz timescale as $t = \frac{E}{L}$. Chemical processes release very small amount of energy for H and He, hence they cannot account for stars luminosity. However, if we consider the binding energy of hydrogen atoms while converting to helium atoms, we find that the resulting energy change is enough to power the sun, indicating most of its energy comes from nuclear processes.

Far away from the nucleus, a coloumbic repulsion dominates, and the nuclei tend to fly apart. However, the portion inside the nucleus forms a potential well governed by the strong nuclear force that binds the nucleus together. The strong nuclear force is a very short-range force that acts between all nucleons within the atom. It is an attractive force that dominates the Coulomb repulsion between protons.If we assume that the energy required to overcome the Coulomb barrier is provided by the thermal energy of the gas, and that all nuclei are moving nonrelativistically, then the temperature $T_{classical}$ required to overcome the barrier can be estimated. The turn-around point is given by - $T_{classical} = \frac{Z_1 Z_2 e^2}{6\pi\epsilon_0 kr}$, which is very high, and classical physics is unable to explain how a sufficient number of particles can overcome the Coulomb barrier to produce the

Sun's observed luminosity. Quantum mechanical tunnelling provides a solution in the uncertainty of position of the nuclei, and assuming that a proton must be within approximately one de Broglie wavelength of its target in order to tunnel through the Coulomb barrier, we get $T_{quantum} = \frac{Z_1^2 Z_2^2 e^4 \mu}{12\pi^2 \epsilon_0^2 h^2 k}$, consistent with estimated central temperatures.
The reaction rate per energy interval must be described in terms of the number density of particles having energies within a specific range, combined with the probability that those particles can actually tunnel through the Coulomb barrier of the target nucleus. The Maxwell-Boltzmann statistics describes the no of particles in a range dE, and the cross-section $\sigma(E)$ describes the probability that the particles will actually interact, defined as the number of reactions per target nucleus per unit time, divided by the flux of incident particles. Although $\sigma(E)$ is strictly a measure of probability, it can be thought of as roughly the cross-sectional area of the target particle; any incoming particle that strikes within that area, centered on the target, will result in a nuclear reaction. Using this, we can determine the total number of reactions per unit volume per unit time, integrated over all possible energies as -

$$r_{ix} = \int_0^\infty n_x n_i \sigma(E) v(E) n_E / n dE \tag{21}$$

n_E is the number of total particles having energy between E and E+dE, n_x are the number of targets per unit volume, and n_i are the incident particles. Using the fact that cross-section is proportional to the area, and counting for the ratio of the barrier height to the KE of the particle we get -

$$\sigma(E) \propto \frac{1}{E} \tag{22}$$

$$\sigma(E) \propto e^{-2\pi^2 U_c/E} \tag{23}$$

$$\sigma(E) = \frac{S(E)}{E} e^{-bE^{-1/2}} \tag{24}$$

$$\tag{25}$$

where b is a constant, and can be computed by U_c/E, depends on the masses and electric charges of the two nuclei involved in the interaction, and S(E) is some slowly varying function of energy. Hence r_{ix} has 2 exponential factors, from penetration possibility, and velocity distribution. The product of these two factors produces a strongly peaked curve, known as the Gamow peak. The top of the curve, or the greatest contribution to reaction rate is at $E_0 = (\frac{bkT}{2})^{2/3}$. Electron screening and resonance have a major effect on these energies. Taking ϵ to be the total energy released per kg per second, we get -

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \tag{26}$$

During every nuclear reaction it is necessary to conserve electric charge, the number of nucleons, and the number of leptons.

5.1.1 Proton-Proton Chains

Applying the conservation laws, one chain of reactions that can convert hydrogen into helium is the first proton–proton chain (PPI). The ultimate reaction is -

$$4\,{}^1_1\mathrm{H} \longrightarrow {}^4_2\mathrm{He} + 2\,\mathrm{e}^+ + 2\nu_e + 2\gamma \tag{27}$$

With the intermediate steps being -

$$\begin{gathered} {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^2_1\mathrm{H} + \mathrm{e}^+ + \nu_e \\ {}^2_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^3_2\mathrm{He} + \gamma \\ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} \longrightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{H} \end{gathered}$$

The inital step is the slowest, requiring the decay of a proton to a neutron using the weak force. 31% of the time, in the second step the PPII chain occurs -

$$
\begin{aligned}
{}^3_2\mathrm{He} + {}^4_2\mathrm{He} &\longrightarrow {}^7_4\mathrm{Be} + \gamma \\
{}^7_4\mathrm{Be} + \mathrm{e}^- &\longrightarrow {}^7_3\mathrm{Li} + \nu_e \\
{}^7_3\mathrm{Li} + {}^1_1\mathrm{H} &\longrightarrow 2\,{}^4_2\mathrm{He}
\end{aligned}
$$

Only a 0.3% of the time, a PP III chain occurs -

$$
\begin{aligned}
{}^7_4\mathrm{Be} + {}^1_1\mathrm{H} &\longrightarrow {}^8_5\mathrm{B} + \gamma \\
{}^8_5\mathrm{B} &\longrightarrow {}^8_4\mathrm{Be} + \nu_e + e^+ \\
{}^8_4\mathrm{Be} &\longrightarrow 2\,{}^4_2\mathrm{He}
\end{aligned}
$$

5.1.2 CNO cycle

In the CNO cycle, carbon, nitrogen, and oxygen are used as catalysts, being consumed and then regenerated during the process. The first branch is -

$$
\begin{aligned}
{}^{12}_6\mathrm{C} + {}^1_1\mathrm{H} &\longrightarrow {}^{13}_7\mathrm{N} + \gamma \\
{}^{13}_7\mathrm{N} &\longrightarrow {}^{13}_6\mathrm{C} + e^+ + \nu_e \\
{}^{13}_6\mathrm{C} + {}^1_1\mathrm{H} &\longrightarrow {}^{14}_7\mathrm{N} + \gamma \\
{}^{14}_7\mathrm{N} + {}^1_1\mathrm{H} &\longrightarrow {}^{15}_8\mathrm{O} + \gamma \\
{}^{15}_8\mathrm{O} &\longrightarrow {}^{15}_7\mathrm{N} + e^+ + \nu_e \\
{}^{15}_7\mathrm{N} + {}^1_1\mathrm{H} &\longrightarrow {}^{12}_6\mathrm{C} + {}^4_2\mathrm{He}
\end{aligned}
$$

The second branch arises with 0.04% of the time, with the last step changed -

$$
\begin{aligned}
{}^{15}_7\mathrm{N} + {}^1_1\mathrm{H} &\longrightarrow {}^{16}_8\mathrm{O} + \gamma \\
{}^{16}_8\mathrm{O} + {}^1_1\mathrm{H} &\longrightarrow {}^{17}_9\mathrm{F} + \gamma \\
{}^{17}_9\mathrm{F} &\longrightarrow {}^{17}_8\mathrm{O} + e^+ + \nu_e \\
{}^{17}_8\mathrm{O} + {}^1_1\mathrm{H} &\longrightarrow {}^{14}_7\mathrm{N} + {}^4_2\mathrm{He}
\end{aligned}
$$

The CNO cycle is much more strongly temperature-dependent than the pp chain. This property implies that low-mass stars, which have smaller central temperatures, are dominated by the pp chains during their "hydrogen burning" evolution, whereas more massive stars, with their higher central temperatures, convert hydrogen to helium by the CNO cycle. When hydrogen is converted into helium by either the pp chain or the CNO cycle, the mean molecular weight of the gas increases. If neither the temperature nor the density of the gas changes, the ideal gas law predicts that the central pressure will necessarily decrease. As a result, the star would no longer be in hydrostatic equilibrium and would begin to collapse. This collapse has the effect of actually raising both the temperature and the density to compensate for the increase in μ.

5.1.3 Triple Alpha Process

The reaction sequence by which helium is converted into carbon is known as the triple alpha process. It is given by -

$$
\begin{aligned}
{}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} &\rightleftharpoons {}^{8}_{4}\mathrm{Be} \\
{}^{4}_{2}\mathrm{He} + {}^{8}_{4}\mathrm{Be} &\longrightarrow {}^{12}_{6}\mathrm{C} + \gamma
\end{aligned}
$$

This process exhibits a very strong temperature dependence, of the order of 41.

5.1.4 Carbon and Oxygen burning

After sufficient carbon has been generated by the triple alpha process, it becomes possible for carbon nuclei to capture alpha particles, producing oxygen. Some oxygen captures alpha particles to form Neon. If the star is very massive, higher temperatures result in Carbon burning (forming O, Na, Ne and Mg) and Oxygen burning reactions (forming Mg, Si, P and S).

Checking the Binding Energy per nucleon $E_b/A = [Zm_p + (A - Z)m_n - m_{nucleus}]c^2/A$, the most stable nuclei (called magic nuclei) are H, He, O, C, Ne, N, Mg, Si and Fe, those isotopes having same number of protons and neutrons.

5.2 Energy Transport mechanisms

Three different energy transport mechanisms operate in stellar interiors. Radiation allows the energy produced by nuclear reactions and gravitation to be carried to the surface via photons. Convection can be a very efficient transport mechanism in many regions of a star, with hot, buoyant mass elements carrying excess energy outward while cool elements fall inward. Finally, conduction transports heat via collisions between particles. Although conduction can play an important role in some stellar environments, it is generally insignificant in most stars throughout the majority of their lifetimes. the temperature gradient for radiative transport becomes –

$$\frac{dT}{dr} = -\frac{3\bar{\kappa}_r}{4acT^3 4\pi r^2} \tag{28}$$

If the temperature gradient becomes too steep, convection can begin to play an important role in the transport of energy. Physically, convection involves mass motions: hot parcels of matter move upward as cooler, denser parcels sink. Turbulent flows emerge, and require detailed viscosity and heat dissipation knowledge. Convection is strongly coupled to the stars behaviour.
To estimate the size of a convective region in a star, consider the pressure scale height, H_p defined as, and using the stars pressure equation,

$$\frac{1}{H_p} = -\frac{dP}{Pdr} \tag{29}$$

$$H_p = \frac{P}{\rho g} \tag{30}$$

The first law of thermodynamics gices us $dQ = dU + dW$, The internal energy of a system U is a state function, meanwhile dQ and dW are inexact diffrentials.$U = \frac{3}{2}nRT$. The specific heat is defined as the amount of heat required to raise the temperature of a unit mass of a material by a unit temperature interval, $C_p = \frac{\partial Q}{\partial T}|_p$ and $C_v = \frac{\partial Q}{\partial T}|_v$. $dW = Pdv$. We get for a monoatomic gas, $C_v = 3/2nR$ and always $C_p = C_v + nR$. $\gamma = \frac{C_p}{C_v}$ is defined. As both the specific heat increase, γ approaches unity. The adiabatic gas law is given as $PV^\gamma = constant$.
The sound speed is defined as $v_s = \sqrt{\frac{B}{\rho}}$, and $B = -V\frac{\partial P}{\partial V}$, and the adiabatic sound speed becomes $v_s = \sqrt{\frac{\gamma P}{\rho}}$.
In convection, we first consider the situation where a hot convective bubble of gas rises and expands adiabatically. After it has traveled some distance, it finally thermalizes, giving up any excess heat as it loses its

identity and dissolves into the surrounding gas. Using the equation for adiabatic expansion, and assuming that μ is a constant, we get -

$$\frac{dT}{dr}|_{ad} = -(1-\frac{1}{\gamma})\frac{\mu m_H G M_r}{k r^2} = -\frac{g}{C_p} \tag{31}$$

To see the convection, we consider the buoyant force as $f = (\rho^s - \rho^b)g > 0$. Assuming that the bubble and surrounding are in thermal eqb initially, we get the condition $|\frac{dT}{dr}|_{ad}| < |\frac{dT}{dr}|_{act}|$, or equivalently, $\frac{dlnP}{dlnT} < \frac{\gamma}{\gamma-1}$. radiation. In general, convection will occur when (1) the stellar opacity is large, implying that an unachievably steep temperature gradient would be necessary for radiative transport, (2) a region exists where ionization is occurring, causing a large specific heat and a low adiabatic temperature gradient, and (3) the temperature dependence of the nuclear energy generation rate is large, causing a steep radiative flux gradient and a large temperature gradient. In the atmospheres of many stars, the first two conditions can occur simultaneously, whereas the third condition would occur only deep in stellar interiors. In particular, the third condition can occur when the highly temperature-dependent CNO cycle or triple alpha processes are occurring. The temperature gradient must be only slightly superadiabatic in the deep interior in order for convection to carry most of the energy. The mixing length is the point at which it thermalizes with its surroundings, giving up its excess heat at constant pressure, and is given by $l = \alpha H_p$. Using $\delta T = \delta(\frac{dT}{dr})dr$, as the difference between surrounding and bubble temperature. Using the average value of the buoyant force, and the mixing length we get the expression for convective flux (the amount of energy per unit area per unit time carried by a bubble) as -

$$F_c = \rho C_p (\frac{k\alpha}{\mu m_H})^2 (\frac{T}{g})^{3/2} \beta^{1/2} [\delta T]^{3/2} \tag{32}$$

Using this and assuming all the flux is carried by convection, we can get the difference in temperature gradient. This excess comes out very small, so can be adequately approximated by adiabatic processes, in the interior, and being more significant outwards.

5.3 Building a stellar model

The static stellar equations are as summarized below -

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2} \tag{33}$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \tag{34}$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \tag{35}$$

$$\begin{aligned} \frac{dT}{dr} &= -\frac{3\bar{\kappa} L_r}{4acT^3 4\pi r^2} \quad \text{(radiation)} \\ &= -(1-\frac{1}{\gamma})\frac{\mu m_H G M_r}{k r^2} \quad \text{(adiabatic convection)} \end{aligned} \tag{36}$$

These require information concerning the physical properties of the matter from which the star is made. The required conditions are the equations of state of the material and are collectively referred to as constitutive relations. Specifically, we need relationships for the pressure, the opacity, and the energy generation rate, in terms of fundamental characteristics of the material: the density, temperature, and composition. The pressure equation of state, including ideal gas and radiation pressure, works well enough. The opacity is calculated for different regions at specific compositions, densities and temperatures. For energy generation we use the pp chain and the CNO cycle.

The boundary conditions imply $M_r, L_r = 0$ as r = 0, and $T, \rho, P = 0$ as r=R. The Vogt-Russell theorem states that : The mass and the composition structure throughout a star uniquely determine its radius, luminosity, and internal structure, as well as its subsequent evolution.

Hypothetical stellar models in which the pressure depends on density in the form $P = K\rho^\gamma$ are known as polytropes. Changing the above equations to obtain a poisson equation, and using the polytrope formula, we get the Lane-Emden equation as -

$$\frac{1}{\xi^2}\frac{d}{d\xi}[\xi^2\frac{dD_n}{d\xi}] = -D_n^n \tag{37}$$

where ξ is r scaled with a constant (units of distance squared), $D_n(r)$ is a scaled dimensionless function of density, and n is the polytropic index $\gamma = \frac{n+1}{n}$. This provides the density and pressure profile, and assuming ideal gas law and radiation pressure, we can calculate T(r). This imposes the boundary conditions that $D_n(\xi_1) = 0$ at the surface $\xi = \xi_1$, and $\frac{dD_n}{d\xi} = 0$ at $\xi = 0$. These can be directly integrated to find the total mass of the star, given the derivative at the surface. n is usually limited to between 0 and 5, and is 1.5 for ideal monoatomic gas, and 3 for a star in radiative equilbrium.

For extremely massive stars, the temperatures are high, and densities are low, it is possible for radiation pressure to dominate over gas pressure, in the outer layers of the star. We then get the maximum radiative luminosity that a star can have and still remain in hydrostatic equilibrium $L_{ED} = \frac{4\pi Gc}{\bar{\kappa}}M$, and this maximum is known as the Eddington limit. Main-sequence lifetimes (till when they burn hydrogen in thier core) decrease with increasing luminosity. In the upper portion of the main sequence, where energy generation is due to the strongly temperature-dependent CNO cycle, convection is dominant in the core. This occurs because the rate of energy generation changes quickly with radius, and radiation is not efficient enough to transport all of the energy being released in nuclear reactions. Outside of the hydrogen-burning core, radiation is again capable of handling the flux, and convection ceases. As the stellar mass decreases, so does the central temperature and the energy output of the CNO cycle until, near 1.2M, the pp chain begins to dominate and the core becomes radiative. On the surfaces, the effective temperature decreases with decreasing mass, the opacity increases, in part because of the location of the zone of hydrogen ionization. The increase in opacity makes convection more efficient than radiation near the surfaces of stars with masses less than approximately 1.3 M. practically the whole star becomes convective near 0.3M.

6 Main sequence and Post Main Sequence evolution

6.1 Evoltuion on the main sequence

The existence of the main sequence is due to the nuclear reactions that convert hydrogen into helium in the cores of stars. The timescale of nuclear reactions is the largest, which governs the life on the main sequence comapared to the Kelvin-Hemholtz timecale (to radiative the gravitational binding energy) or the free fall timescale.

6.1.1 Low mass main sequence evolution

Zero Age Min Sequence (ZAMS) stars with masses more than 1.2M have convective cores due to the highly temperature-dependent CNO cycle. On the other hand, ZAMS stars with masses less than 1.2 M are dominated by the less temperature dependent pp chain. This implies that ZAMS stars in the range 0.3 M to 1.2 M possess radiative cores. However, the lowest-mass ZAMS stars again have convective cores because their high surface opacities drive surface convection zones deep into the interior, making the entire star convective.

The evolution on the main sequence occurs because, as the pp chain converts hydrogen into helium, the mean molecular weight μ of the core increases. According to the ideal gas law, unless the density and/or temperature of the core also increases, there will be insufficient gas pressure to support the overlying layers of the star. As a result, the core must be compressed. While the density of the core increases, gravitational potential energy is released, and, as required by the virial theorem, half of the energy is radiated away and half of the energy goes into increasing the thermal energy and hence the temperature of the gas. One consequence of this temperature increase is that the region of the star that is hot enough to undergo nuclear reactions increases slightly during the main-sequence phase of evolution. This also leads to increase in T and ρ of the core, increasing the luminosity of the star (by increase of both T and R). With the depletion of

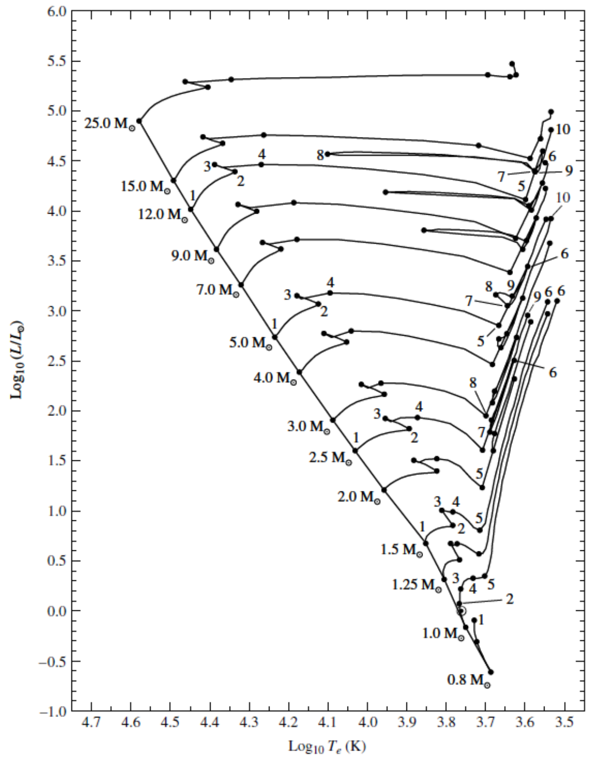

Figure 2: Main-sequence and post-main-sequence evolutionary tracks of stars with a predetermined initial composition. The model calculations include mass loss and convective overshooting. The diagonal line connecting the locus of points 1 is the zero-age main sequence.

hydrogen in the core, the generation of energy via the pp chain must stop. However, by now the core temperature has increased to the point that nuclear fusion continues to generate energy in a thick hydrogen-burning shell around a small, predominantly helium core. For an isothermal core to support the material above it in hydrostatic equilibrium, the required pressure gradient must be the result of a continuous increase in density as the center of the star is approached. At this point, the luminosity being generated in the thick shell actually exceeds what was produced by the core during the phase of core hydrogen burning.As a result, the evolutionary track continues to rise. As the envelope of the star expands, the effective temperature begins to decrease slightly and the evolutionary track bends to the right. As the hydrogen-burning shell continues to consume its nuclear fuel, the ash from nuclear burning causes the isothermal helium core to grow in mass while the star moves farther to the red in the H–R diagram.

As the core mass increases, the maximum pressure at the surface of the core decreases. At some point, it may no longer be possible for the core to support the overlying layers of the star's envelope. This phase of evolution ends when the mass of the isothermal core has become too great and the core is no longer capable of supporting the material above it. The maximum fraction of a star's mass that can exist in an isothermal core and still support the overlying layers is given by -

$$\left(\frac{M_{ic}}{M}\right) = 0.37\left(\frac{\mu_{env}}{\mu ic}\right)^2 \tag{38}$$

where env and ic stand for the envelope and isothermal core respectively, and this is known as the Schönberg–Chandrasekhar limit. When the mass of the isothermal helium core exceeds this limit, the core collapses on a Kelvin–Helmholtz timescale, and the star evolves very rapidly relative to the nuclear timescale of main-sequence evolution. For stars below about 1.2 M, this defines the end of the main-sequence phase.

The mass of an isothermal core may exceed the Schönberg–Chandrasekhar limit if an additional source of pressure can be found to supplement the ideal gas pressure. This can occur if the electrons in the gas start to become degenerate. With increasing density, the electrons in the gas are forced to occupy the lowest available energy levels. Obeying the Pauli Exclusion Principle, electrons are stacked into progressively higher states, and in complete degeneracy, the pressure becomes independent of temperature, and is given by $P = K\rho^{5/3}$, in the non relativistic case.

6.1.2 Main sequence evolution of massive stars

The evolution of more massive stars on the main sequence is similar to that of their lower-mass cousins with one important difference: the existence of a convective core, mixing the material and keeping it homogeneous, since the convection time scale is much lesser than the nuclear timescale. Moving up the main sequence, as the star evolves the convection zone in the core retreats more rapidly with increasing stellar mass, disappearing entirely before the hydrogen is exhausted for those stars with masses greater than about 10 M. When the mass fraction of hydrogen reaches about X = 0.05 in the core of a 5 M star, the entire star begins to contract. With the release of some gravitational potential energy, the luminosity increases slightly. Since the radius decreases, the effective temperature must also increase. For stars with masses greater than 1.2 M, this stage of overall contraction is defined to be the end of the main-sequence phase of evolution.

6.2 Evolution off the main sequence

6.2.1 The Subgiant Branch

For both low- and intermediate-mass stars, as the shell continues to consume the hydrogen that is available at the base of the star's envelope, increasing the mass of the helium core till it reaches the above limit. The core begins to contract rapidly, causing the evolution to proceed on the much faster Kelvin–Helmholtz timescale. The gravitational energy released by the rapidly contracting core again causes the envelope of the star to expand and the effective temperature cools, resulting in redward evolution on the H–R diagram. This phase of evolution is known as the subgiant branch (SGB). As the core contracts, a nonzero temperature the temperature and density of the hydrogen-burning shell increase, and, although the shell begins to narrow significantly, the rate at which energy is generated by the shell increases rapidly.

6.2.2 Red Giant Branch

With the expansion of the stellar envelope and the decrease in effective temperature, the photospheric opacity increases due to the additional contribution of the H^- ion. The result is that a convection zone develops near the surface for both low- and intermediate-mass stars. With the nearly adiabatic temperature gradient associated with convection throughout much of the stellar interior, and the efficiency with which the energy is transported to the surface, the star begins to rise rapidly upward along the red giant branch (RGB) of the H-R diagram. When these convection zones reach deep enough into the chemically modified regions, observable changes occur in the photosphere, decrease in Li and increase in 3-He. This transport of materials from the deep interior to the surface is referred to as the first dredge-up phase. At the tip of the RGB, tempreatures and densities of the core have become high enough to initiate triple alpha burning, also converting to O. With the onset of a new and strongly temperature-dependent source of energy, the core expands, cooling it, abruptly decreasing the luminosity. The envelope contracts and the T starts increasing.

6.2.3 Helium Core Flash

The evolution changes for stars lesser than 1.8M here. Significant neutrino losses from the core of the star prior to reaching the tip of the RGB result in a negative temperature gradient near the center (i.e., a temperature inversion develops); the core is actually refrigerated somewhat because of the energy that is carried away by the neutrinos. When the temperature and density become high enough to initiate the triple alpha process, the ensuing energy release is almost explosive. The ignition of helium burning occurs initially in a shell around the center of the star, but the entire core quickly becomes involved and the temperature inversion is lifted, resulting in an very short energy burst, and is absorbed by the overlying layers, known as the helium core flash. The origin of the explosive energy release is in the very weak temperature dependence of electron degeneracy pressure and the strong temperature dependence of the triple alpha process.

6.2.4 Horizontal Branch

For both low- and intermediate-mass stars, as the envelope of the model contracts following the red giant tip, the increasing compression of the hydrogen-burning shell eventually causes the energy output of the shell, and the overall energy output of the stars, to begin to rise again. With the associated increase in effective temperature, the deep convection zone in the envelope rises toward the surface, while at the same time, a convective core develops. The appearance of a convective core is due to the high temperature sensitivity of the triple alpha process. This generally horizontal evolution is the blueward portion of the horizontal branch (HB) loop. The redward portion starts after the end, due to the increse in μ of the core, causing it to contract.

With the increase in core temperature associated with its contraction, a thick helium burning shell develops outside the CO core. As the core continues to contract, the helium burning shell narrows and strengthens, forcing the material above the shell to expand and cool. This results in a temporary turn-off of the hydrogen-burning shell.

6.2.5 Asymptotic Giant Branch

When the redward evolution reaches the Hayashi track, the evolutionary track bends upward along a path referred to as the asymptotic giant branch (AGB). The AGB may be thought of as the helium-burning-shell analog to the hydrogen-burning-shell RGB. The expanding envelope initially absorbs much of the energy produced by the helium burning shell. As the effective temperature continues to decrease, the convective envelope deepens again, this time extending downward to the chemical discontinuity between the hydrogen-rich outer layer and the helium-rich region above the helium-burning shell. The mixing that results during this second dredge-up phase increases the helium and nitrogen content of the envelope. This is the early phase.

Near the upper portion of the AGB, the dormant hydrogen-burning shell eventually reignites and again dominates the energy output of the star. However, during this phase of evolution, the narrowing helium burning shell begins to turn on and off quasi-periodically, due to the dumping of helium ash. This leads to the helium shell flash, turning off hydrogen shell for a while. This leads to thermal pulses.

In the third dredge up phase, the carbon-rich material is brought to the surface, decreasing the ratio of oxygen to carbon, after a helium flash due to convetive zones in intermediate mass stars.

6.2.6 Mass Loss and AGB evolution

AGb stars lose mass at a rapid rate, hence dust grains tend to form. The composition of the ISM may be related to the relative numbers of carbon- and oxygen-rich stars. For stars with masses less than 8M, the helium burning shell converts more and more of the helium into carbon and then into oxygen,increasing the mass of the carbon–oxygen core. At the same time, the core continues to contract slowly, causing its central density to increase. hence, electron degeneracy pressure starts dominating. For stars with ZAMS masses less than about 4 M, the carbon–oxygen core will never become large enough and hot enough to ignite nuclear burning. On the other hand, if the important contribution of mass loss is ignored for stars between 4 M and 8 M suggests that the C–O core would reach a sufficiently large mass that it could no longer remain in hydrostatic equilibrium, even with the assistance of pressure from the degenerate electron gas. The outcome of this situation is catastrophic core collapse. The maximum value of 1.4M for a completely degenerate core is known as the Chandrasekhar limit. Including mass loss, they transition to ONeMg cores, with core masses below the Chandrasekhar limit. The rate of mass loss accelerates with time because the luminosity and radius are increasing while the mass is decreasing during continued evolution up the AGB.

6.2.7 Post AGB

reactions. With only a very thin layer of material remaining above them, the hydrogen- and helium-burning shells are extinguished, and the luminosity of the star drops rapidly. The hot central object, now revealed, will cool to become a white dwarf star, which is essentially the old red giant's degenerate C–O core (or ONeMg core in the case of the more massive stars), surrounded by a thin layer of residual hydrogen and helium. These then expel the envelopes into the ISM, resulting in planetary nebulae. The (thus far hypothetical) original stars that formed immediately after the Big Bang are referred to as Population III stars, metal-poor stars are referred to as Population II, and metal-rich stars are called Population I. In the formation of stellar clusters, according to the Vogt-Russell theorem, the difference in evolutionary states is due to their masses. Bigger clusters are globular clusters, and the smaller and younger ones are are called open clusters.

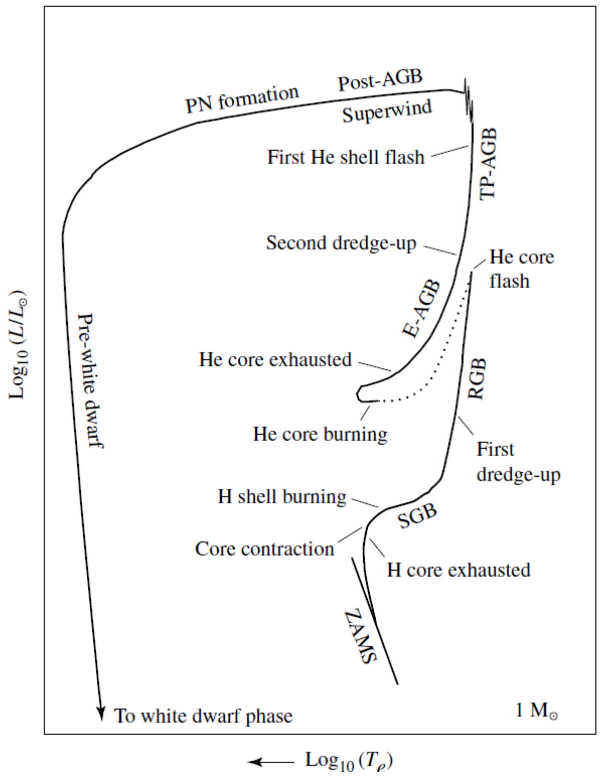

Figure 3: The evolution of a low-mass star of 1 M from the zero-age main sequence to the formation of a white dwarf star. The dotted phase of evolution represents rapid evolution following the helium core flash. The various phases of evolution are labeled as follows: Zero-Age-Main-Sequence (ZAMS), Sub-Giant Branch (SGB), Red Giant Branch (RGB), Early Asymptotic Giant Branch (E-AGB), Thermal Pulse Asymptotic Giant Branch (TP-AGB), Post- Asymptotic Giant Branch (Post-AGB), Planetary Nebula formation (PN formation), and Pre-white dwarf phase leading to white dwarf phase.

7 Stellar pulsation

Pulsating stars are stars that dims and brightens as its surfaces expands and contracts. After linear fitting, a period-luminosity relation was found as $M_v = -2.81log(P_d) - 1.43$. M_v is the average absolute V magnitude, and P is the pulsation period in days. For the infrared band, adding the color index, $H = -3.428logP_d + 1.54(J - K) + 15.637$. In δ Cephei, the main change is due to change in temperature, changing the spectral type from F5 to G2. Pulsating stars are a transient phenomenon, and occupy a vertical instability strip on the HR Diagram.

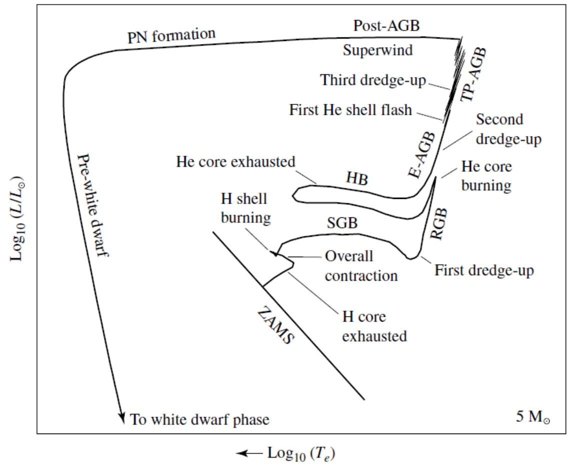

Figure 4: A schematic diagram of the evolution of an intermediate-mass star of 5 M from the zero-age main sequence to the formation of a white dwarf star. It is labelled similarly to the above, with an additional Horizontal Branch.

7.1 Physics of stellar pulsation

The radial oscillations of a pulsating star are the result of sound waves resonating in the star's interior. A rough estimate of the pulsation period may be obtained by considering how long it would take a sound wave to cross the diameter of a model star of radius R and constant density ρ. Using the boundary condition, adiabatic sound speed, we finally get the time period as -

$$\Pi = \sqrt{\frac{3\pi}{2\lambda G\rho}} \tag{39}$$

This also shows the period-mean density relation. The vast majority of stars oscillate in their fundamental mode or at the most their first overtone. The motion of the stellar material occurs in the surface region but also oscillates deep in the star (node at centre and antinode at surface). Layers do positive work or negative work on each other, and in equilibrium the total work is 0. The driving layers (when they do positive work) absorb heat at the time of maximum compression, and hence maximum pressure will occur after.
If a layer of a star become more opaque upon compression, it could "dam up" the energy flowing towards the surface and push the surface layers upward. The, as this expanding layer became more opaque, the trapped heat could escape and the layer could fall back down to begin the cycle anew. For this to work, the opacity must increase with compression. However, in most cases opacity decreases with compression. This is because the Kramer's law opacity is usually $\frac{\rho}{T^{3.5}}$ which means it is more sensitive to temperature than density. As a layer is compressed, the density and temperature increase, resulting in an overall decrease in opacity.

The answer to this problem lies in the partial-ionization zones in stars. In these layers of the stars, part of the work done on the gases as they are compressed produces further ionization rather than raising the temperature of the gas. With a smaller temperature rise, the increase in compression causes a corresponding increase in Kramer's opacity. Similarly during expansion, the temperature does not decrease much as the ions recombine with the electrons, releasing energy. The location of these ionization zones within the star determine its pulsational properties.
At the minimum radius, the luminosity incident at the bottom of the H ionization zone is the maximum, but this just propels the zone outward most rapidly at that instant. The emergent L is greatest after minimum radius, when the zone is nearest the surface. This delay of the partial ionization zone of H produces the phase lag observed in classical Cepheids.

7.2 Modelling Stellar Pulsation

The state equations derived before will not work for pulsatory solutions, for example, Newtons 2nd lay must be used here, $\rho\frac{d^2r}{dt^2} = -G\frac{M_r\rho}{r^2} - \frac{dP}{dr}$, resulting in non linear calculations. We then proceed to linearlize the DE by considering small amplitude oscillations. However this lead to sinusoidal solutions, and hence limiting values cannot be determined.

We now consider an unrealistic, yet informative model of stellar pulsation called the one-zone model. According to this model, the star consists of a central point mass equal to the entire mass of the star M, surrounded by a single thin spherical shell or radius R and mass m representing the surface of the star. The interior of the shell is filled with a massless gas of pressure P whose sole function is to support the shell against the gravitational pull of the central mass M. In the newtons law above, we linearize $R = R_0 + \delta R$ and $P = P_0 + \delta P$, where R_0, P_0 are equilibrium values. Substiutuing this in the Newtons law, and assuming our model to be adiabatic, thats is $PR^{3\gamma} = constant$ we finally get -

$$\frac{d^2\delta R}{dt^2} = -(3\gamma - 4)\frac{GM}{R_0^3}\delta R \tag{40}$$

Here, if $\gamma > 4/3$, then this is the equation for a SHM, with -

$$\Pi = \frac{2\pi}{\sqrt{\frac{4}{3}\pi G\rho_0(3\gamma - 4)}} \tag{41}$$

If $\gamma < 4/3$ we have a dynamically unstable model where $\delta R = Ae^{-kt}$ which means that the increase in gas pressure is not enough to overcome the pull of gravity, resulting in a collapse.

8 Fate of Massive stars

Type I are those that do not have any H lines in thier spectra (stripped of H envelopes). They are further subdivided into Type Ia showing strong Si II lines, Ib based on presence of helium lines and Ic based on absence of He lines. Type II supernovae contain strong H lines. They are further classified as Type II-P (plateau) or Type II-L (linear). This is summarized below -

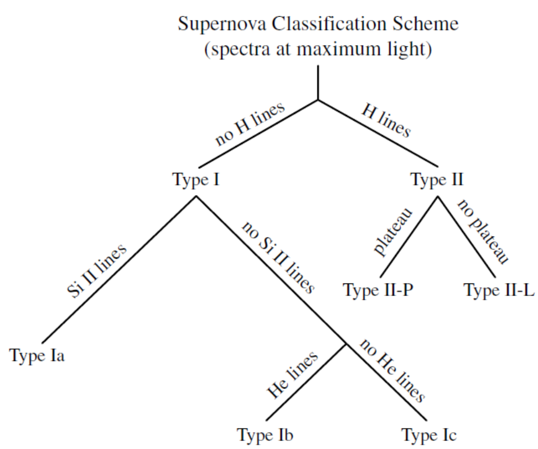

Figure 5: Classification of supernovae based on spectra at maximum light.

8.1 Core Collapse Supernovae

Type Ib, Ic and II are known as core collapse supernovae. The helium burning shell adds ash to the Co core, which generates a lot of by-products. Assuming they reach equilibrium, following C burning, O will ignite, producing Si. This will then commence at higher temperatures, producing a 54 and 56 isotopes of Fe and Ni. This produces an iron core. Since these make nuclei near the iron peak, the timescales for these reactions are shorter. Photodisintegration occurs, from Fe to He and then to protons and neutrons, and is a highly endothermic process, leading to thermal energy being removed from the core, with lesser pressure to sustain the mass.

This then leads to electron capture by the protons, and energy escapes in the form of neutrinos, leading to a very high percentage of energy lost. Due to this loss of pressure, the core begins to collapse rapidly, with velocity proportional to the distance from the centre. This can lead to supersonic collapse of the outer core. At roughly 3 times the density of the atomic nuclei, the inner core faces the repulsive strong force, a result of Pauli exclusion principle on neutrons. The inner core then rebounds, sending pressure waves to the outer core, which then becomes shockwaves at sound speed. The material then starts accreting in it, making it an accretion shock. Type II and Ib and Ic depend on the composition and mass of the envelope at ejection time. Ib and Ic lose envelope prior to collapse. For ZAMs mass less than 25 M, the core remnant will become a neutron star, supported by neutron degenerate pressure. For higher mass, this will result in a blackhole.

The pleateau in type II-P occurs because of the energy deposited by shock into envelope triggering recombination, at constant temperature. We now move on to the basics of some Modern Particle Physics