Indian Institute of Technology Bombay

PH 821 **Gravitational Waves Physics and Astronomy**

Problem set-1 **Date 8-08-2022** **Deadline: 17-08-2022**

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1. Consider the following metric that describes a weak gravitational field:

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) dt^2 - dx^2 - dy^2 - dz^2 \tag{1}$$

where $\phi = \phi(x, y, z)/c^2 \ll 1$. Calculate Riemann curvature tensor, Ricci tensor and Ricci tensor up to the lowest non-trivial order in ϕ/c^2.

2. **Kretschmann scalar**

Scalar quantities are useful as they are invariant under coordinate transformation. In the case of Schwarzschild, Ricci scalar is zero. To understand the properties of the space-time, it is important to construct other scalar invariants. For the 4-D Schwarzschild coordinate, calculate the Kretschmann scalar (K):

$$K = R_{\lambda\mu\nu\kappa} R^{\lambda\mu\nu\kappa} \tag{2}$$

3. **Friedmann Robertson-Walker space-time**

Consider the following 4-dimensional line-element

$$ds^2 = dt^2 - a^2(t) \left[dx^2 - dy^2 - dz^2\right] \tag{3}$$

(a) Calculate all the non-zero components of the Riemann tensor $R_{\mu\nu\rho\sigma}$ for the above line-element.

(b) Calculate Ricci tensor and Ricci scalar

(c) Calculate Weyl tensor

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}\left[g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}\right] + \frac{1}{6}\left[g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}\right] R$$

(d) Show that the stress-tensor corresponding to the above line-element (3) must be of the form:

$$T^{\mu}_{nu} = dia[\rho(t), -P(t), -P(t), -P(t)]$$

(e) Substituing the stress tensor in the Einstein field equation, show that $a(t), \rho(t)$ and $P(t)$ are related by the equation:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho(t) \qquad \frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G P(t) \tag{4}$$

These are called Friedmann equations.

(f) Show that the Friedmann equations can be written as:

$$\frac{d}{dt}\left(\rho(t) a^3\right) = -P(t)\frac{da^3}{dt} \tag{5}$$