Assignment - 2

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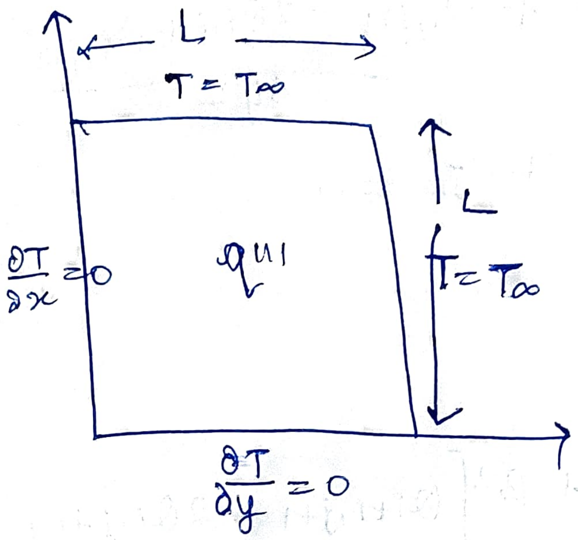

Governing eqn is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q'''}{k} = 0$$

Non dimensionalisation,

$$\theta = \frac{T - T_\infty}{q''' L^2 / K}$$

@ $x = L$ $\quad \theta = 0$

@ $y = L$ $\quad \theta = 0$

@ $x = 0$ $\quad \frac{\partial \theta}{\partial x} = 0$

@ $y = 0$ $\quad \frac{\partial \theta}{\partial y} = 0$

After normalisation equation becomes,

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + 1 = 0$$

Discretising using central difference,

$$\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta x^2} + \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{\Delta y^2} + 1 = 0$$

$$\beta = \frac{\Delta x^2}{\Delta y^2}$$

$$\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j} + \beta^2\left[\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}\right] + \Delta x^2 = 0$$

$$\Rightarrow \theta_{i,j} = \frac{1}{2(1+\beta^2)}\left\{\theta_{i+1,j} + \theta_{i-1,j} + \beta^2\theta_{i,j+1} + \beta^2\theta_{i,j-1} + \Delta x^2\right\}$$

Iterative method

* (Gauss Siedel)

$$\theta_{i,j}^{(k+1)} = \frac{1}{2(1+\beta^2)}\left\{\theta_{i+1,j}^{(k)} + \theta_{i-1,j}^{(k+1)} + \beta^2\left(\theta_{i,j+1}^{(k)} + \theta_{i,j-1}^{(k+1)}\right) + \Delta x^2\right\}$$

* Gauss Siedel with over and Under Relaxation

General form

$$x_i^{(k+1)} = (1-\alpha)\, x_i^{(k)} + \alpha\left(x_i^{(k+1)}\right)_{G.S.}$$

↓

(computed using gauss siedel)

α, is the relaxation parameter

$$\theta_{i,j}^{(k+1)} = (1-\alpha)\,\theta_{i,j}^{(k)} + \alpha \cdot \frac{1}{2(1+\beta^2)}\left\{\theta_{i+1,j}^{(k)} + \theta_{i-1,j}^{(k+1)} + \beta^2\left(\theta_{i,j+1}^{(k)} + \theta_{i,j-1}^{(k+1)}\right) + \Delta x^2\right\}$$

$0 < \alpha < 1$ → under relaxation

$1 < \alpha < 2$ → over relaxation

Line by line gauss siedel

$$2(1+\beta^2)\,\theta_{i,j} - \theta_{i-1,j} + \theta_{i+1,j} = \Delta x^2 + \beta^2\theta_{i,j+1} + \beta^2\theta_{i,j-1}$$

Sweep in x direction assuming known in y direction,

Generates a tridiagonal system.

⇒ TDMA is used to solve the generated equations then.