Assignment - 1

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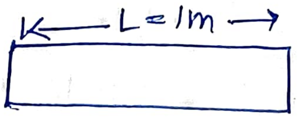
Q. 1

Soln :-



$T_a = 300K$ $\quad$ $T_L = 400K$

Assuming 1D heat transfer (given)

Governing eqn is, $\frac{\partial^2 T}{\partial x^2} = 0$

$$T(0) = 300K$$
$$T(L) = 400K$$

Analytical solution

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$\Rightarrow \int \frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow \frac{\partial T}{\partial x} = c_1$$

$$\Rightarrow \int \frac{\partial T}{\partial x} = \int c_1 \Rightarrow T(x) = c_1 x + c_2$$

@ $x = 0, T = 300 \Rightarrow c_2 = 300$

@ $x = L, T = 400 \Rightarrow c_1 = 100$

$$\therefore \boxed{T(x) = 100x + 300}$$

Using second order central difference approximations

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1} - 2T_i + T_{i+1}}{h^2} = 0$$

$$\Rightarrow T_i = \frac{1}{2}\left(T_{i-1} + T_{i+1}\right)$$

$$\Rightarrow \left(\frac{1}{h^2}\right)T_{i-1} + \left(\frac{-2}{h^2}\right)T_i + \left(\frac{1}{h^2}\right)T_{i+1} = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & & \\ 1/h^2 & -2/h^2 & 1/h^2 & - & - & \\ - & - & & - & & \\ \vdots & - & & - & & \\ - & - & & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ \vdots \\ \vdots \\ T_n \end{Bmatrix} = \begin{Bmatrix} 300 \\ 0 \\ \vdots \\ \vdots \\ 400 \end{Bmatrix}$$

The generated coefficient matrix is a tridiagonal system.

Iterative methods:

* **Jacobi method**

$$a_{ii} x_i^{(k+1)} = b_i - \sum_{j \neq i} a_{ij} x_j^{(k)}$$

Since it is tridiagonal system (most coefficient are zero) we get relation,

$$T_i^{(k+1)} = \frac{1}{2}\left[T_{i-1}^{(k)} + T_{i+1}^{(k)}\right]$$

* **Gauss Siedel**

$$T_i^{(k+1)} = \frac{1}{2}\left[T_{i-1}^{(k+1)} + T_{i+1}^{(k)}\right]$$

Direct method

TDMA (Tridiagonal matrix Algorithm)

$a\phi_i = b\phi_{i+1} + c\phi_{i-1} + d_i$. (general form) — ①

we have

$2T_i = T_{i+1} + T_{i-1}$

$a = 2, \quad b = 1, \quad c = 1$

for 1st point

$T_0 = 300K$ (comparing with ①) (also $T_N = 400$)

$P_1 = b_1/a_1, \quad Q_1 = d_1/a_1$

$P_1 = 0, \quad Q_1 = 300$

for 2nd to n points

~~$Q_i = P_i Q_{i+1}$~~

$P_i = \left(\frac{b_i}{a_i - c_i P_{i-1}}\right), \quad Q_i = \left(\frac{d_i + c_i Q_{i-1}}{a_i - c_i P_{i-1}}\right)$

end.

$T_N = Q_N$

for N-1 to 1 point,

$T_i = P_i T_{i+1} + Q_i$

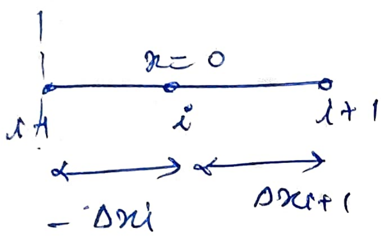
Non uniform grid

function used (as taught in class)

$$\frac{x_i}{L} = \left(\frac{i-1}{N-1}\right)^n$$

$n > 1$ finer mesh near $x = 0$

$n < 1$ finer mesh near $x = L$



$$R = \frac{\Delta x_{i+1}}{\Delta x_i} \quad , \quad R = \frac{x_{i+1} - x_i}{x_i - x_{i-1}}$$

$$\phi = Ax^2 + Bx + C$$

$$\phi_i = C$$

$$\phi_{i-1} = A(\Delta x_i)^2 - B\Delta x_i + \phi_i$$

$$\phi_{i+1} = AR^2 \Delta x_i^2 + BR\Delta x_i + \phi_i$$

$$\phi_{i+1} + R\phi_{i-1} = A(R+R^2)\Delta x_i^2 + (1+R)\phi_i$$

$$A = \frac{\phi_{i+1} + R\phi_{i-1} - (1+R)\phi_i}{\Delta x_i^2 (R+1) R} + \underline{\underline{O(\Delta x)}}$$

$$\frac{\partial \phi}{\partial x} = 2Ax + B$$

$$\frac{\partial^2 \phi}{\partial x^2} = 2A$$

$$\frac{\partial^2\phi}{\partial x^2} = 2A = 2\left\{\frac{\phi_{i+1} + R\phi_{i-1} - (1+R)\phi_i}{\Delta x_i^2 (R+1)R}\right\}$$

For our eq^n, $\frac{\partial^2 T}{\partial x^2} = 0$

$$\Rightarrow \cancel{\theta}\ \frac{T_{i+1} + RT_{i-1} - (1+R)T_i}{\Delta x_i^2 (R+1)R} = 0$$

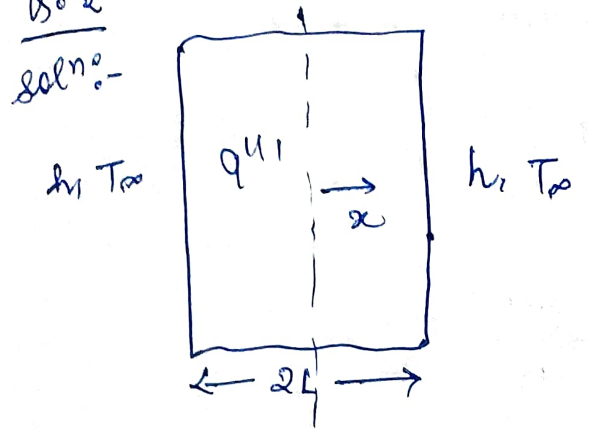
$$\frac{T_{i+1} + RT_{i-1}}{1+R} = T_i$$

$$\Rightarrow T_i \cancel{= T_{i+1} + RT_{i-1}} \over (1+R$$

$$\therefore T_i = \frac{T_{i+1} + RT_{i-1}}{1+R}$$

Q. 2

Soln:-



$T_\infty = 25°C$

$h = 22\ W/m^2K$

$K = 0.5\ W/mK$

$q''' = 5\times10^4\ W/m^2$

$2L = 50\ cm$

~~1D~~ Assume 1D heat transfer (given)

Governing eqn: $\frac{\partial^2 T}{\partial x^2} + \frac{q'''}{k} = 0$

Analytical soln

$$\int \frac{d^2T}{\partial x^2} = -\int \frac{q'''}{k}$$

$$\Rightarrow T = -\frac{q''' x^2}{2k} + c_1 x + c_2$$

~~at~~ Boundary condition

$$-K\frac{\partial T}{\partial x}\Big|_{x=L} = h(T_s - T_\infty)$$

$$q''' L = h(T_s - T_\infty)$$

$$\Rightarrow T_s = T_\infty + \frac{q''' L}{h}$$

$T_{s_1} = T_{s_2} = T_s$ @ $x = L$ & $x = -L$

(same surface temp. on both sides)

$$\Rightarrow T(x) = \frac{q''' L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_{s}$$

Using second order central difference

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{h^2}$$

$$\Rightarrow \frac{T_{i+1} - 2T_i + T_{i-1}}{h^2} + \frac{q'''}{k} = 0$$

$$\Rightarrow \left\{\frac{1}{h^2}\right\} T_{i+1} + \left\{\frac{-2}{h^2}\right\} T_i + \left\{\frac{1}{h^2}\right\} T_{i-1} = -\frac{q'''}{k}$$

Let $c = \frac{q''' h^2}{k}$

$$\therefore T_i = \frac{1}{2}\left[c + T_{i-1} + T_{i+1}\right]$$

<u>Iterative method</u>

* <u>Jacobi Method</u>

$$T_i^{(k+1)} = \frac{1}{2}\left[c + T_{i-1}^{(k)} + T_{i+1}^{(k)}\right]$$

* Gauss - Seidel

$$T_i^{(k+1)} = \frac{1}{2}\left[c + T_{i-1}^{(k+1)} + T_{i+1}^{(k)}\right]$$

<u>Direct method</u>

* <u>TDMA</u>

$$\frac{2T_i}{h^2} = \frac{1}{h^2}T_{i-1} + \frac{1}{h^2}T_{i+1} + \frac{q'''}{k}$$

$$a = \frac{2}{h^2}, \quad b = \frac{1}{h^2}, \quad c = \frac{1}{h^2}, \quad d = \frac{q'''}{k}$$

Then proceeded with TDMA algo.

~~Grid~~