Assignment - 3

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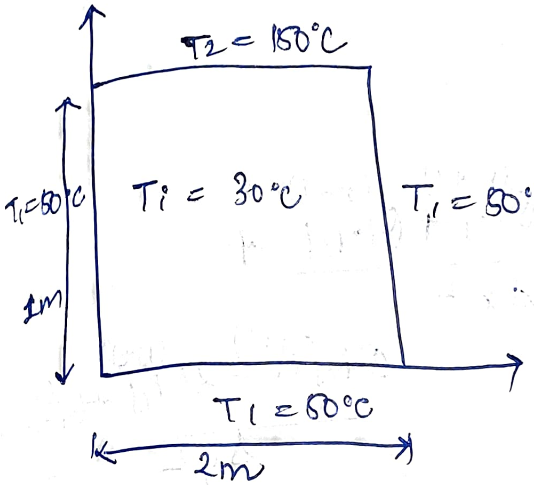

$\alpha = 9.7 \times 10^{-5}\ m^3/s$

Governing eqn is given by,

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

* Explicit method

$\cancel{\Theta_{i+1,j}}$

$$\frac{\Theta_{i,j}^{(n+1)} - \Theta_{i,j}^{(n)}}{\Delta t} = \alpha \left[\frac{\Theta_{i+1,j}^{(n)} - 2\Theta_{i,j}^{(n)} + \Theta_{i-1,j}^{(n)}}{\Delta x^2} + \frac{\Theta_{i,j+1}^{(n)} - 2\Theta_{i,j}^{(n)} + \Theta_{i,j-1}^{(n)}}{\Delta y^2} \right]$$

considering, $r_x = \frac{\alpha \Delta t}{\Delta x^2}$ and $r_y = \frac{\alpha \Delta t}{\Delta y^2}$

$$\theta_{i,j}^{(n+1)} = (1 - 2r_x - 2r_y)\,\theta_{i,j}^{(n)} + r_x\left(\theta_{i-1,j}^{(n)} + \theta_{i+1,j}^{(n)}\right) + r_y\left(\theta_{i,j+1}^{(n)} + \theta_{i,j+1}^{(n)}\right)$$

Implicit scheme

$$\frac{\theta_{i,j}^{(n+1)} - \theta_{i,j}^{(n)}}{\Delta t} = \alpha\left[\frac{\theta_{i+1,j}^{(n+1)} - 2\theta_{i,j}^{(n+1)} + \theta_{i-1,j}^{(n+1)}}{\Delta x^2} + \frac{\theta_{i,j+1}^{(n+1)} - 2\theta_{i,j}^{(n+1)} + \theta_{i-1,j}^{(n+1)}}{\Delta y^2}\right]$$

$$\Rightarrow (1 + 2r_x + 2r_y)\,\theta_{i,j}^{(n+1)} = \theta_{i,j}^{(n)} + r_x\left(\theta_{i+1,j}^{(n+1)} + \theta_{i-1,j}^{(n+1)}\right) + r_y\left(\theta_{i,j+1}^{(n+1)} + \theta_{i,j-1}^{(n+1)}\right)$$

Cranck Nickolson Method

$$\frac{\partial \theta}{\partial t} = \frac{1}{2}\left[\left.\frac{\partial \theta}{\partial \tau}\right|^{n} + \left.\frac{\partial \theta}{\partial \tau}\right|^{n+1}\right]$$

$$\frac{\theta^{n+1} - \theta^{n}}{\Delta t} = \frac{\alpha}{2}\left[\frac{\theta_{i+1,j}^{(n)} - 2\theta_{i,j}^{(n)} + \theta_{i-1,j}^{(n)}}{\Delta x^2} + \frac{\theta_{i,j+1}^{(n)} - 2\theta_{i,j}^{(n)} + \theta_{i,j-1}^{(n)}}{\Delta y^2}\right] + \frac{\alpha}{2}\left[\frac{\theta_{i+1,j}^{(n+1)} - 2\theta_{i,j}^{(n+1)} + \theta_{i+1,j}^{(n+1)}}{\Delta x^2} + \frac{\theta_{i,j+1}^{(n+1)} - 2\theta_{i,j}^{(n+1)} + \theta_{i,j}^{(n+1)}}{\Delta y^2}\right]$$

$$
\begin{aligned}
\theta_{i,j}^{(n+1)} = \theta_{i,j}^{(n)} &+ \frac{r_x}{2}\left[\theta_{i-1,j}^{(n)} - 2\theta_{i,j}^{(n)} + \theta_{i+1,j}^{(n)}\right] \\
&+ \frac{r_y}{2}\left[\theta_{i,j-1}^{(n)} - 2\theta_{i,j}^{(n)} + \theta_{i,j+1}^{(n)}\right] \\
&+ \frac{r_x}{2}\left[\theta_{i-1,j}^{(n+1)} - 2\theta_{i,j}^{(n+1)} + \theta_{i+1,j}^{(n+1)}\right] \\
&+ \frac{r_y}{2}\left[\theta_{i,j+1}^{(n+1)} - 2\theta_{i,j}^{(n+1)} + \theta_{i,j-1}^{(n+1)}\right]
\end{aligned}
$$

Alternating Direct Implicit (ADI).

$$\frac{\partial T}{\partial t}\Big|^{n} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\Big|^{*} + \frac{\partial^2 T}{\partial y^2}\Big|^{n}\right)$$

$$\frac{T_{i,j}^{*} - T_{i,j}^{(n)}}{\Delta t/2} = \alpha\left\{\frac{T_{i+1,j}^{*} - 2T_{i,j}^{*} + T_{i-1,j}^{*}}{\Delta x^2} + \frac{T_{i,j-1}^{n} - 2T_{i,j}^{n} + T_{i,j+1}^{n}}{\Delta y^2}\right\}$$

(First time step)

$$r_x = \frac{\alpha\,\Delta t}{\Delta x^2}$$

$$r_y = \alpha\frac{\Delta t}{\Delta y^2}$$

$$(1 + r_x + \cancel{r_y})T_{i,j}^{*} - \frac{r_x}{2}T_{i+1,j}^{*} - \frac{r_x}{2}T_{i-1,j}^{*} = \frac{r_y}{2}T_{i,j+1}^{(n)} + (1 - r_y)T_{i,j}^{(n)} + \frac{r_y}{2}T_{i,j-1}^{(n)}$$

$$\Rightarrow (1+r_x)T_{i,j}^{*} - \frac{r_x}{2}T_{i+1,j}^{*} - \frac{r_x}{2}T_{i-1,j}^{*} = \frac{r_y}{2}T_{i,j+1}^{(n)} + \frac{r_y}{2}T_{i,j-1}^{(n)} + (1-r_y)T_{i,j}^{(n)}$$

(second time step)

$$\frac{T_{i,j}^{(n+1)} - T_{i,j}^{*}}{\Delta t/2} = \alpha\left\{\frac{T_{i+1,j}^{*} - 2T_{i,j}^{*} + T_{i-1,j}^{*}}{\Delta x^2} + \frac{T_{i,j+1}^{(n+1)} - 2T_{i,j}^{(n+1)} + T_{i,j-1}^{(n+1)}}{\Delta y^2}\right\}$$

$$\Rightarrow (1+r_y)T_{i,j}^{(n+1)} - \frac{r_y}{2}T_{i,j+1}^{(n+1)} - \frac{r_y}{2}T_{i,j-1}^{(n+1)}$$

$$= \cancel{\frac{r_x}{2}T_{i+1,j}^{(n)}}\ \frac{r_x}{2}T_{i+1,j}^{*} + \frac{r_x}{2}T_{i-1,j}^{*} + (1 - r_x)T_{i,j}^{(n)}$$