8-8.3-3

EE24BTECH11064 - Harshil Rathan

Question:
Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.
Solution:
THe equation of the curve is

$$y = 4x^2 \tag{0.1}$$

$$x = \frac{\sqrt{y}}{2} \tag{0.2}$$

w.k.t that 0.1 is an upward parabola symmetric about y-axis
Theory

$$\int_1^4 x\,dx = \int_1^4 \left|\frac{\sqrt{y}}{2}\right| dy \tag{0.3}$$

$$\frac{1}{2}\int_1^4 y^{\frac{1}{2}}\,dy = \frac{1}{2}\left|\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right|_1^4 \tag{0.4}$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] \tag{0.5}$$

$$= \frac{1}{3}\left(4\sqrt{4} - 1\right) = \frac{1}{3}(8-1) = \frac{7}{3} \quad = 2.3333 \text{ sq. units.} \tag{0.6}$$

Solution using Trapezoidal Rule
Divide the interval [1,4], into n = 1000 sub intervals
The step size is:

$$h = \frac{4-1}{1000} = 0.003 \tag{0.7}$$

The y-values at sub interval points are

$$y_i = 1 + i \cdot h \text{ for i = 1,2,3} \cdots \text{,1000} \tag{0.8}$$

$$y_0 = 1,\ y_1 = 1.003,\ y_2 = 1.006,\ \cdots, y_{1000} = 4 \tag{0.9}$$

Compute the corresponding x_i values

$$x_i = \frac{\sqrt{y_i}}{2} \tag{0.10}$$

$$x_0 = \sqrt{\frac{1}{4}} = 0.5,\ x_1 = \sqrt{\frac{1.003}{4}},\ x_2 = \sqrt{\frac{1.006}{4}} \cdots, x_{1000} = 1 \tag{0.11}$$

Apply Trapezoidal rule

$$Area \approx \frac{h}{2}\left[x_0 + 2(x_1 + x_2 + \cdots + x_{n-1}) + x_n\right]. \tag{0.12}$$

General difference equation
Substitute $y_i = 1 + ih$ into x_i

$$x_i = \sqrt{\frac{1+ih}{4}} \tag{0.13}$$

$$x_i = \sqrt{\frac{1+\frac{3i}{n}}{4}}, \quad i = 0, 1, 2, \ldots, n. \tag{0.14}$$

$$A \approx \frac{h}{2}\left[\sqrt{\frac{1}{4}} + 2\sum_{i=1}^{n-1}\sqrt{\frac{1+\frac{3i}{n}}{4}} + \sqrt{\frac{4}{4}}\right] \tag{0.15}$$

$$A \approx \frac{3}{2n}\left[\frac{1}{2} + 2\sum_{i=1}^{n-1}\sqrt{\frac{1+\frac{3i}{n}}{4}} + 1\right] \tag{0.16}$$

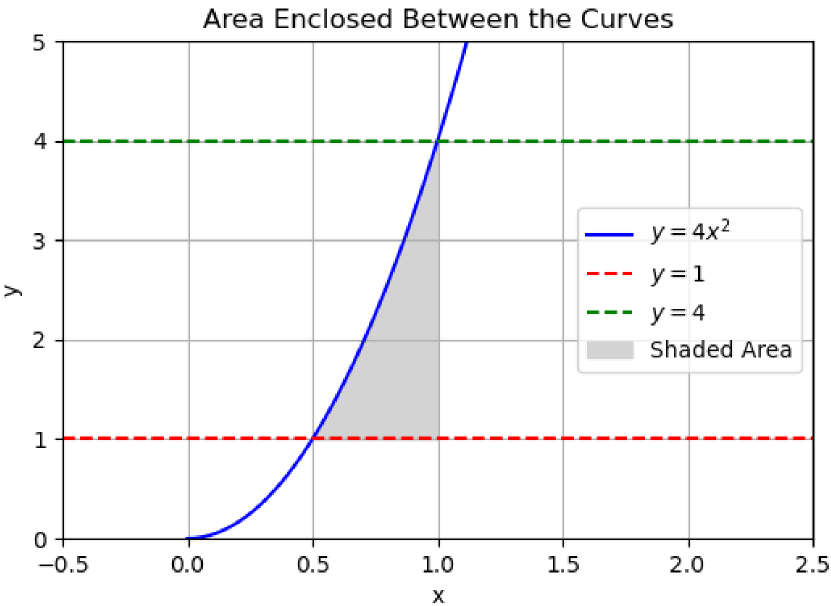
Substituting n = 1000 in 0.16

$$A \approx \frac{0.003}{2}\left[\frac{1}{2} + 2\sum_{i=1}^{999}\sqrt{\frac{1+0.003i}{4}} + 1\right] \tag{0.17}$$

On simplying 0.17 we get

$$\text{Area} = 2.3333$$

The solution is hence verified with the theoretical solution


Area Enclosed Between the Curves
$y = 4x^2$
$y = 1$
$y = 4$
Shaded Area
x
y
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
0
1
2
3
4
5