12.6.5.3.2

EE24BTECH11064 - Harshil Rathan

Question:
Find the local maxima and local minima, if any of the following functions. Find also the local maximum and local minimum values, as the case may be

$$g(x) = x^3 - 3x$$

Solution:
Theory
The first derivative, $g'(x)$ gives us the critical points (where the slope is zero or undefined)

$$g'(x) = 3x^2 - 3 \tag{0.1}$$

Set derivative equal to 0

$$g'(x) = 3x^2 - 3 = 0 \tag{0.2}$$

$$x = \pm 1 \tag{0.3}$$

The critical points are 1 and -1
The second derivative $g''(x)$ helps us determine the nature of critical points

$$g''(x) = 6x \tag{0.4}$$

At $x = 1$

$$g''(1) = 6 > 0 \tag{0.5}$$

indicating a local minimum
For $x = -1$

$$g''(-1) = -6 < 0 \tag{0.6}$$

indicating a local maximum
Calculating local maxima and minimum values
At $x = 1$

$$g(1) = 1 - 3 = -2 \tag{0.7}$$

Local minimum value is -2
At $x = -1$

$$g(-1) = (-1)^3 - 3(-1) = 2 \tag{0.8}$$

Local maximum value is 2
Local maximum : At $x = -1$, the maximum value is $g(-1) = 2$.
Local minimum : At $x = 1$, the minimum value is $g(1) = -2$.

Computational Solution Using Gradient Descent and Ascent

To verify the analytical results, we use gradient descent and gradient ascent to find the local minimum and maximum, respectively.

Gradient Descent for local minimum :
- Start with $x_0 = 0.5$ (close to 1.0)
- Update x iteratively using

$$x_1 = x_0 - \eta \cdot g'(x_0) \tag{0.9}$$

where :

$$\eta = 0.1 \tag{0.10}$$

$$g'(x) = 3x^2 - 3 \tag{0.11}$$

$$x_{n+1} = x_n - \eta \cdot (3x_n^2 - 3) \tag{0.12}$$

Stop the iteration when the change in x is smaller than a specified tolerance (10^{-6})
Gradient Ascent for local maxima
- Start with $x_0 = -1.5$ (close to -1)
- Update x iteratively

$$x_1 = x_0 + \eta \cdot g'(x_0) \tag{0.13}$$

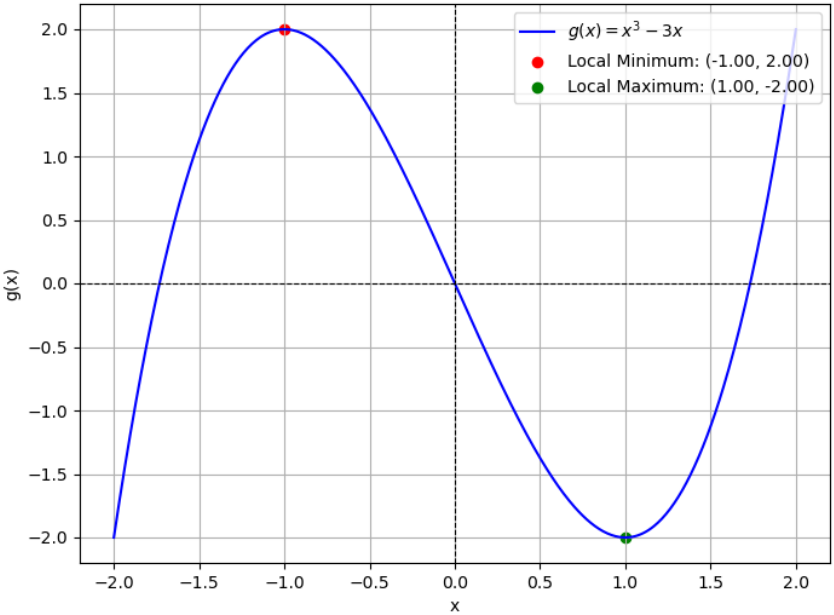
$$x_{n+1} = x_n + \eta \cdot (3x_n^2 - 3) \tag{0.14}$$

Computational Results
- Local minimum

$$x \approx 1.000, \ \ g(x) \approx -2.000 \tag{0.15}$$

- Local maximum

$$x \approx -1.000, \ \ g(x) \approx 2.000 \tag{0.16}$$


$g(x) = x^3 - 3x$
Local Minimum: (-1.00, 2.00)
Local Maximum: (1.00, -2.00)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
g(x)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x