10.3.2.2.1

EE24BTECH11064 - Harshil Rathan

Question:
On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident

$$5x - 4y + 8 = 0$$
$$7x + 6y - 9 = 0$$

Solution:
The general form of a pair of linear equations is given as:

$$a_1x + b_1y + c_1 = 0,$$
$$a_2x + b_2y + c_2 = 0.$$

Here, the coefficients are:

$$a_1 = 5, \quad b_1 = -4, \quad c_1 = 8,$$
$$a_2 = 7, \quad b_2 = 6, \quad c_2 = -9.$$

We calculate the ratios:

$$\frac{a_1}{a_2} = \frac{5}{7}, \quad \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{-9} = -\frac{8}{9}.$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the given lines intersect at a unique point.
Solution using LU Decomposition
Simplifying and using matrix notation,

$$5x - 4y = -8 \tag{0.1}$$
$$7x + 6y = 9 \tag{0.2}$$
$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.3}$$

Any non-sigular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \tag{0.4}$$

The upper triangular matrix U is found by row reducing A,

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \xrightarrow{R_2 -> R_2 - \frac{7}{5}R_1} \begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \tag{0.5}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{0.6}$$

l_{21} is the multiplier used to zero a_{21}, so $l_{21} = \frac{7}{5}$.

Now,

$$A = \begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \tag{0.7}$$

Now we can get the solution to our problem by the two step process,

$$L\mathbf{y} = \mathbf{b} \tag{0.8}$$

$$U\mathbf{x} = \mathbf{y} \tag{0.9}$$

Using forward substitution to solve the first equation,

$$\begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.10}$$

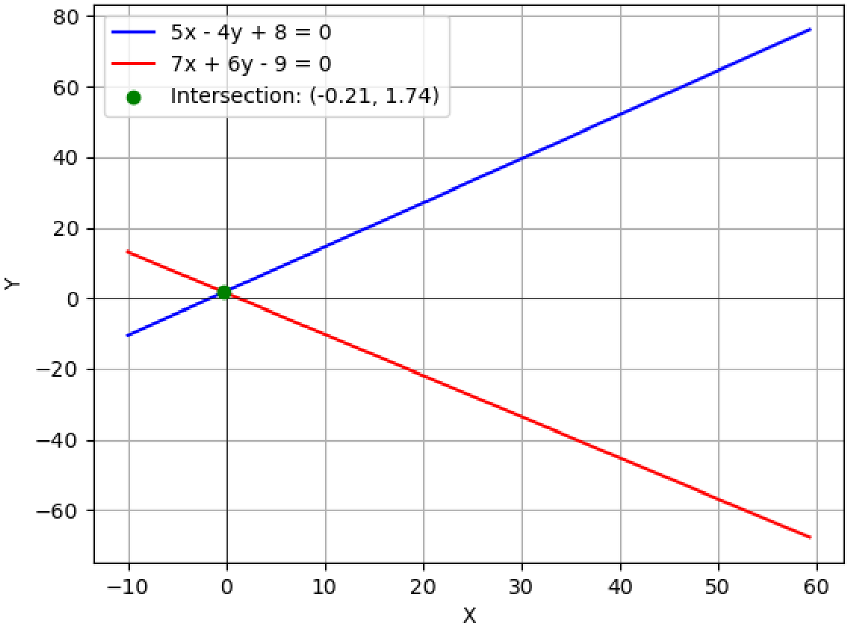
$$\rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{0.11}$$

Now using back-substitution for the second equation,

$$\begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{0.12}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix} \tag{0.13}$$

The given pair of lines intersect at $\begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix}$ and hence they are not parallel


5x - 4y + 8 = 0
7x + 6y - 9 = 0
Intersection: (-0.21, 1.74)
80
60
40
20
0
−20
−40
−60
Y
−10
0
10
20
30
40
50
60
X