10.3.2.3.3

EE24BTECH11064 - Harshil Rathan

Question:
A die is thrown, find the probability of following events :
i) A prime number will appear

Theoretical Solution:
The sample space S of a fair six-sided die is

$$S = 1, 2, 3, 4, 5, 6 \tag{0.1}$$

The prime numbers in the Sample space are

$$A = 2, 3, 5 \tag{0.2}$$

Thus, the number of favorable outcomes = 3

$$|S| = 6 \tag{0.3}$$

$$|A| = 3 \tag{0.4}$$

The probability of getting a prime number when a fair die is rolled

$$P(A) = \frac{|A|}{|S|} \tag{0.5}$$

on substituing 0.3, 0.4

$$P(A) = \frac{1}{2} \tag{0.6}$$

Computational Solution:
The goal of this task was to compute the probability distribution of outcomes when rolling a six-sided die. The outcomes $1, 2, 3, 4, 5, 6$ represent the faces of the die, and each face is expected to have an approximately equal probability if the die is fair. The computed probabilities (PMF) were plotted as a stem plot to visualize the distribution.

Process Overview

The process involved two main steps: 1. Computation of the probabilities using a C program. 2. Visualization of the results using Python.

Step 1: Probability Computation in C

- A simulation was performed by rolling a virtual six-sided die N times, where $N = 1,000,000$, to ensure accurate probabilities. - Each roll was simulated using a random number generator that produced values between 1 and 6. - A count was maintained for how many times each outcome occurred during the simulation. - The probability of each

outcome (PMF) was calculated by dividing the count of each outcome by the total number of rolls.

Step 2: Data Export via Shared Library

- The C program was compiled into a shared library (`.so` file) that could be accessed from Python. - This ensured that the computationally heavy task of rolling the die and calculating probabilities was handled efficiently in C.

Step 3: Visualization in Python

- The computed probabilities were imported from the shared library into Python. - A stem plot was used to visualize the probability distribution. Each outcome $1, 2, 3, 4, 5, 6$ was plotted on the x-axis, and its corresponding probability was plotted on the y-axis. - The stem plot highlighted the uniform distribution of probabilities for a fair die, with each outcome having a probability close to $1/6$.

Results and Insights

Probability Mass Function (PMF)

The PMF represents the probability of each individual outcome $x \in \{1, 2, 3, 4, 5, 6\}$. The table below shows the PMF for a six-sided die based on simulation:

Outcome (x)	$P_X(x)$
1	0.1667
2	0.1665
3	0.1666
4	0.1668
5	0.1669
6	0.1665

As expected, the probabilities are close to $1/6 \approx 0.1667$, with minor variations due to random sampling.

Cumulative Distribution Function (CDF)

The CDF represents the cumulative probability up to each outcome $x \in \{1, 2, 3, 4, 5, 6\}$. The table below shows the CDF for a six-sided die:

Outcome (x)	$F_X(x)$
1	0.1667
2	0.3332
3	0.4998
4	0.6666
5	0.8335
6	1.0000

The CDF starts with the PMF of $x = 1$ and accumulates to 1.0 at $x = 6$, confirming the correctness of the cumulative probabilities.

Conclusion

This process effectively demonstrated the computation and visualization of the probability distribution for a six-sided die. The use of C for efficient computation and Python for visualization provided a robust framework for handling such probabilistic simulations.