10.4.2.5

EE24BTECH11064 - Harshil Rathan

Question:
The altitude of a right triangle is 7cm less than its base. If the hypotenuse is 13cm, find the other two sides
Solution:
We get the equation

$$x^2 + (x-7)^2 = 13^2 \tag{0.1}$$

$$x^2 - 7x - 60 = 0 \tag{0.2}$$

We can solve the above equation using fixed point iterations. First we separate x, from the above equation and make an update equation of the below sort.

$$x = g(x) = \frac{x^2 - 60}{7} \tag{0.3}$$

Applying the above update equation on our equation, we get

$$x_{n+1} = \frac{x_n^2 - 60}{7} \tag{0.4}$$

Now we start with an initial guess $x_0 = 10$
But we realize that the updated values always approach infinity for any initial value.
Thus we will alternatively use Newton's Method for solving equations.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.5}$$

Where we define $f(x)$ as,

$$f(x) = x^2 - 7x - 60 \tag{0.6}$$

$$f'(x) = 2x - 7 \tag{0.7}$$

Thus, the new update equation is,

$$x_{n+1} = x_n - \frac{x_n^2 - 7x_n - 60}{2x_n - 7} \tag{0.8}$$

Taking the initial guess as $x_0 = 10$, we can see that x_n converges with x as,

$$x \approx 12.002 \tag{0.9}$$

$$x = 12cm \tag{0.10}$$

Alternatively, we can use the Secant method for solving equations.

$$x_{n+1} = x_n + f(x_n)\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{0.11}$$

The altitude is

$$12 - 7 = 5cm \tag{0.12}$$

The base is 12cm and the altitude is 5cm