10.3.2.3.3

EE24BTECH11064 - Harshil Rathan

Question:
A die is thrown, find the probability of following events :
i) A prime number will appear

Theoretical Solution:
The sample space S of a fair six-sided die is

$$S = 1, 2, 3, 4, 5, 6 \tag{0.1}$$

The prime numbers in the Sample space are

$$A = 2, 3, 5 \tag{0.2}$$

Thus, the number of favorable outcomes = 3

$$|S| = 6 \tag{0.3}$$

$$|A| = 3 \tag{0.4}$$

The probability of getting a prime number when a fair die is rolled

$$P(A) = \frac{|A|}{|S|} \tag{0.5}$$

on substituing 0.3, 0.4

$$P(A) = \frac{1}{2} \tag{0.6}$$

Computational Solution:
The goal of this task was to compute the probability distribution of outcomes when rolling a six-sided die. The outcomes $1, 2, 3, 4, 5, 6$ represent the faces of the die, and each face is expected to have an approximately equal probability if the die is fair. The computed probabilities (PMF) were plotted as a stem plot to visualize the distribution.

Process Overview

The process involved two main steps: 1. Computation of the probabilities using a C program. 2. Visualization of the results using Python.

Step 1: Probability Computation in C

- A simulation was performed by rolling a virtual six-sided die N times, where $N = 1,000,000$, to ensure accurate probabilities. - Each roll was simulated using a random number generator that produced values between 1 and 6. - A count was maintained for how many times each outcome occurred during the simulation. - The probability of each

outcome (PMF) was calculated by dividing the count of each outcome by the total number of rolls.

Step 2: Data Export via Shared Library

- The C program was compiled into a shared library (`.so` file) that could be accessed from Python. - This ensured that the computationally heavy task of rolling the die and calculating probabilities was handled efficiently in C.

Step 3: Visualization in Python

- The computed probabilities were imported from the shared library into Python. - A stem plot was used to visualize the probability distribution. Each outcome $1, 2, 3, 4, 5, 6$ was plotted on the x-axis, and its corresponding probability was plotted on the y-axis. - The stem plot highlighted the uniform distribution of probabilities for a fair die, with each outcome having a probability close to $1/6$.

Results and Insights

The results confirmed the uniform distribution of probabilities for a fair die:

$$P(X = x) \approx \frac{1}{6}, \quad \text{for } x \in \{1, 2, 3, 4, 5, 6\}.$$

Minor variations in probabilities were observed due to random sampling, but they were negligible given the large number of rolls.

Conclusion

This process effectively demonstrated the computation and visualization of the probability distribution for a six-sided die. The use of C for efficient computation and Python for visualization provided a robust framework for handling such probabilistic simulations.

Definitions of PMF and CDF

Probability Mass Function (PMF)

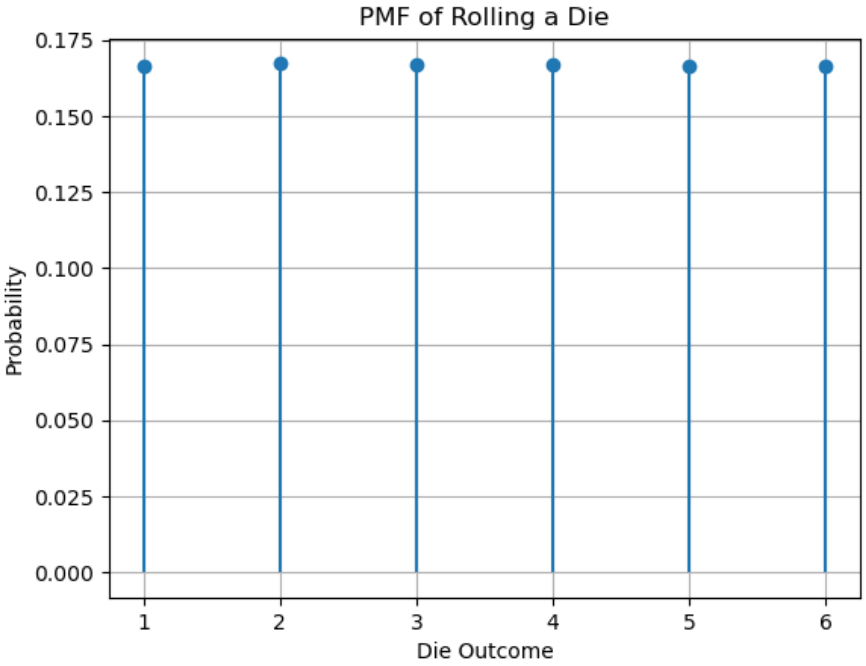
The PMF represents the probability of each individual outcome x of a discrete random variable X:

$$P(X = x) = \frac{\text{Number of times } X = x}{\text{Total number of trials}}.$$

Cumulative Distribution Function (CDF)

The CDF represents the cumulative probability of the random variable X being less than or equal to a specific value x:

$$F(X = x) = P(X \leq x) = \sum_{i=1}^{x} P(X = i).$$


PMF of Rolling a Die
Probability
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
1
2
3
4
5
6
Die Outcome