11.16.3.3.1

EE24BTECH11064 - Harshil Rathan

Question:
A die is thrown, find the probability of following events :

i) A prime number will appear

Theoretical Solution:
The sample space S of a fair six-sided die is

$$S = 1, 2, 3, 4, 5, 6 \tag{0.1}$$

The prime numbers in the Sample space are

$$A = 2, 3, 5 \tag{0.2}$$

Thus, the number of favorable outcomes = 3

$$|S| = 6 \tag{0.3}$$

$$|A| = 3 \tag{0.4}$$

The probability of getting a prime number when a fair die is rolled

$$P(A) = \frac{|A|}{|S|} \tag{0.5}$$

on substituing 0.3, 0.4

$$P(A) = \frac{1}{2} \tag{0.6}$$

Computational solution:

Computation of Probabilities for Rolling a Die

To compute the probability of obtaining specific outcomes when rolling a six-sided die, we rely on two key concepts: the Probability Mass Function (PMF) and the Cumulative Distribution Function (CDF).

Definitions

Probability Mass Function (PMF): The PMF represents the probability of each individual outcome in the sample space S. For a six-sided die:

$$S = \{1, 2, 3, 4, 5, 6\},$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{3}, & x \in \{2, 3, 5\}, \\ 0, & x \notin \{2, 3, 5\}. \end{cases}$$

Cumulative Distribution Function (CDF): The CDF represents the cumulative probability of outcomes up to a given value x, defined as:

$$F(x) = P(X \leq x) = \sum_{k=1}^{x} P(X = k).$$

For the event "A prime number will appear" on a six-sided die:

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{3}, & 2 \leq x < 3, \\ \frac{2}{3}, & 3 \leq x < 5, \\ 1, & x \geq 5. \end{cases}$$

Simulation Process

We simulate the rolling of a die to compute the probability of a prime number appearing using the following steps:

1) A six-sided die produces outcomes in the set:
$$S = \{1, 2, 3, 4, 5, 6\}.$$
2) Identify the subset of outcomes corresponding to prime numbers:
$$S_{\text{prime}} = \{2, 3, 5\}.$$
3) For each simulated roll, a random integer X is generated such that $X \in S$, using a random number generator function:
$$X = (\text{rand}() \mod 6) + 1.$$
4) Check if the outcome $X \in S_{\text{prime}}$. Track the number of occurrences of prime outcomes over N trials, where N is the total number of simulations.
5) Compute both the PMF and CDF for the event "A prime number appears":
   - PMF: The frequency of each prime outcome ($\{2, 3, 5\}$) is divided by the total trials to compute the probability of each prime face. Non-prime outcomes ($\{1, 4, 6\}$) have probabilities of zero.
   - CDF: The cumulative probabilities are calculated by summing the PMF values of the prime outcomes up to a given x. For $x \notin S_{\text{prime}}$, the CDF remains constant.

Calculation of Probabilities

Probability of Each Outcome (PMF): The probability of rolling each face i ($i \in \{2, 3, 5\}$) is computed as:

$$P(i) = \frac{\text{Number of rolls resulting in } i}{N},$$

where i is restricted to the set of prime numbers $\{2, 3, 5\}$. For non-prime outcomes ($i \in \{1, 4, 6\}$), $P(i) = 0$.

Cumulative Probability (CDF): The cumulative probability up to face i is:

$$F(i) = \begin{cases} 0, & i < 2, \\ \frac{1}{3}, & 2 \leq i < 3, \\ \frac{2}{3}, & 3 \leq i < 5, \\ 1, & i \geq 5. \end{cases}$$

Here, $F(i)$ accumulates probabilities only for the prime outcomes. For non-prime values of i, $F(i)$ remains constant.

Output Representation

The computed probabilities are represented in two forms:

- **PMF**: The probabilities of rolling each face from the set of prime numbers $\{2, 3, 5\}$. Non-prime outcomes $\{1, 4, 6\}$ have probabilities of zero.
- **CDF**: The cumulative probabilities up to each face $\{2, 3, 5\}$, showing the cumulative likelihood of rolling a prime number. For non-prime outcomes, the cumulative probability remains constant.

Stemplot Distribution

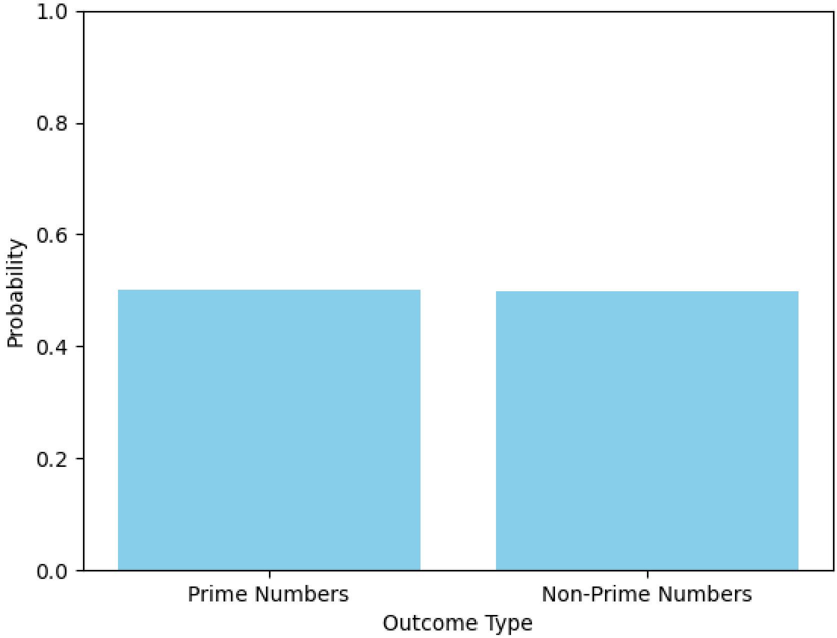
When a die is rolled, the prime outcomes are $2, 3, 5$. Each face of a fair die has an equal probability of occurring, which is $\frac{1}{6}$. This means that each of the outcomes 2,3,5 have probability $\frac{1}{6}$

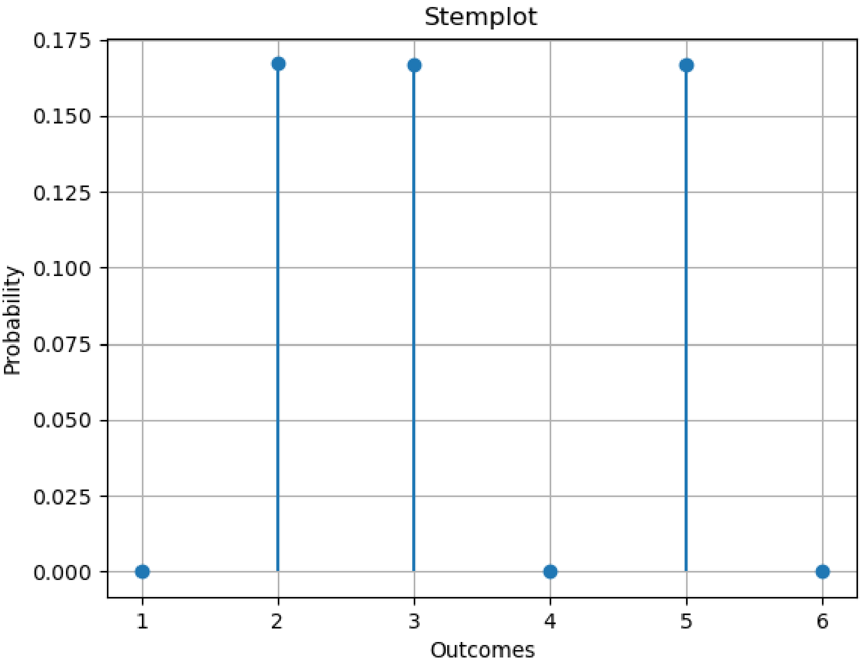
- The stem plot shows vertical lines (stems) at the positions 2, 3, 5 on the x-axis
- The height of each stem corresponds to the probability of that particular prime number outcome $\frac{1}{6}$

$$P(A) = P(2) + P(3) + P(5) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2} \tag{5.1}$$

Conclusion

This task demonstrates the integration of C and Python for simulating and visualizing a probabilistic experiment. By combining the computational efficiency of C with the graphical capabilities of Python, we achieve an effective solution for analyzing and representing data. The code clearly shows that the probability of the given event is equal to **half**


1.0
0.8
0.6
0.4
0.2
0.0
Probability
Prime Numbers
Non-Prime Numbers
Outcome Type



Stemplot
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
Probability
1
2
3
4
5
6
Outcomes