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1) The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits $0, 1, 3, 5, 7$ and 9 without repetition, is equal to

a) 120
b) 132
c) 72
d) 96

2) Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to

a) 729
b) 72
c) 81
d) 9

3) Let $< a_n >$ be a sequence such that
$$a_1 + a_2 + \cdots + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}.$$
If, $28 \sum_{k=1}^{10} \frac{1}{a_k} = p_1 \cdot p_2 \cdot p_3 \cdots p_m$, where $p_1, p_2, \cdots, p_m$ are the first m prime numbers, then m is equal to:

a) 7
b) 6
c) 5
d) 8

4) Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to

a) 12
b) 14
c) 10
d) 8

5) Let $P\left(\frac{2\sqrt{3}}{7}, \frac{6}{\sqrt{7}}\right)$, Q, R and S be four points on th ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are co-prime, then $p + q$ is equal to

a) 143
b) 137
c) 157
d) 147

6) Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

a) 1
b) -1
c) -2
d) 2

7) If the local maximum value of the function $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to

a) $e^5 + e^6 + e^{11}$
b) $e^3 + e^5 + e^{11}$
c) $e^3 + e^6 + e^{11}$
d) $e^3 + e^6 + e^{10}$

8) Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to R$ defined by $g(x) = x - [x]$, ($[x]$ is the greatest integer function), is (α, β), then $\alpha + \frac{5}{\beta}$ is equal to

a) 46
b) 135
c) 136
d) 45

9) Let $y = y(x), y > 0$, be a solution curve of the differential equation $\left(1 + x^2\right)dy = y(x - y)dx$. If $y(0) = 1$ and $y\left(2\sqrt{2}\right) = \beta$, then

a) $e^{3\beta^{-1}} = e\left(3 + 2\sqrt{2}\right)$
b) $e^{\beta^{-1}} = e^{-2}\left(5 + \sqrt{2}\right)$
c) $e^{\beta^{-1}} = e^{-2}\left(3 + 2\sqrt{2}\right)$
d) $e^{3\beta^{-1}} = e\left(5 + \sqrt{2}\right)$

10) Among the two statements

$(S1) : (p \Rightarrow q) \wedge (q \wedge (\neg q))$ is a contradiction

$(S2) : (p \wedge q) \vee ((\neg p) \wedge q) \vee (p \wedge (\neg q) \vee ((\neg p)) \wedge (\neg q))$

is a tautology

a) only$(S2)$ is true
b) only $(S1)$ is true
c) both are false
d) both are true

11) Let $\lambda \in Z$, $\mathbf{a} = \lambda\hat{i}+\hat{j}-\hat{k}$ and $\mathbf{b} = 3\hat{i}-\hat{j}+2\hat{k}$. Let $\mathbf{c}$ be a vector such that $(\mathbf{a}+\mathbf{b}+\mathbf{c})\times\mathbf{c} = \mathbf{0}$, $\mathbf{a}\cdot\mathbf{c} = -17$ and $\mathbf{b}\cdot\mathbf{c} = -20$. Then $|\mathbf{c}\times\left(\lambda\hat{i}+\hat{j}+\hat{k}\right)|^2$ is equal to

a) 46
b) 135
c) 136
d) 45

12) The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to

a) $-{}^{101}C_{50}$
b) ${}^{99}C_{49}$
c) $-{}^{99}C_{49}$
d) ${}^{101}C_{50}$

13) The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1,-1)$ is

a) $\frac{27}{4}$
b) $\frac{19}{4}$
c) $\frac{23}{4}$
d) $\frac{31}{4}$

14) Let $A = \begin{pmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{pmatrix}$. If $B = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix} A \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

a) 100
b) 50
c) 75
d) 125

15) Let the plane $P: 4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2,3,-4)$ from the new position of the plane P, then 35α is

a) 90
b) 85
c) 105
d) 126