MA - 2018

EE24BTECH11064 - Harshil Rathan

1) Which of the following statements is true?
   a) Every group of order 12 has non-trivial proper normal subgroup
   b) Some group of order 12 does not have a non-trivial proper normal subgroup
   c) Every grooup of order 12 has a subgroup of order 6
   d) Every group of order 12 has an element of order 12

2) For an odd prime p, consider the ring $\mathbb{Z}[\sqrt{-p}] = \{a + b\sqrt{-p} : a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Then the element 2 in $\mathbb{Z}[\sqrt{-p}]$ is

   a) a unit
   b) a square
   c) a prime
   d) irreducible

3) Consider the following two statements:
   P: The matrix $\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$ has infinitely many LU factorizations, where L is lower triangular with each diagonal entry 1, and U is upper triangular.
   Q: The matrix $\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$ has no LU factorization, where L is lower triangular with each diagonal entry 1, and U is upper triangular.
   Then which one of the following options is correct?
   a) P is TRUE and Q is FALSE
   b) Both P and Q are TRUE
   c) P is FALSE and Q is TRUE
   d) Both P and Q are FALSE

4) If the characteristic curves of the partial differential eqauation $xu_{xx} + 2x^2u_{xy} = u_x - 1$ are $\mu(x, y) = c_1$ and $\nu(x, y) = c_2$, where c_1 and c_2 are constants, then
   a) $\mu(x, y) = x^2 - y$, $\nu(x, y) = y$
   b) $\mu(x, y) = x^2 + y$, $\nu(x, y) = y$
   c) $\mu(x, y) = x^2 + y$, $\nu(x, y) = x^2$
   d) $\mu(x, y) = x^2 - y$, $\nu(x, y) = x^2$

5) Let $f : X \to Y$ be a continuous map from a Hausdorff topological space X to a metric space Y. Consider the following two statements:
   P: f is a closed map and the inverse image $f^{-1}(y) = \{x \in X : f(x) = y\}$ is compact for each $y \in Y$.

Q: For every compact subset $K \subset Y$, the inverse image $f^{-1}(K)$ is a compact subset of X.

Which one of the following is true?

a) Q implies P but P does NOT imply Q
b) P implies Q but Q does NOT imply P
c) P and Q are equivalent
d) neither P implies Q nor Q implies P

6) Let X denote $\mathbb{R}^2$ endowed with the usual topology. Let Y denote $\mathbb{R}$ endowed with the co-finite topology. If Z is the product topological space $Y \times Y$, then

a) the topology of X is the same as the topology of Z
b) the topology of X is strictly coarser (weaker) than that of Z
c) the topology of Z is strictly coarser (weaker) than that of X
d) the topology of X cannot be compared with that of Z

7) Consider $\mathbb{R}^n$ with the usual topology for $n = 1, 2, 3$. Each of the following options gives topological spaces X and Y with respective induced topologies. In which option is X home-omorphic to Y?

a) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}, \quad Y = \{(x, y, z) \in \mathbb{R}^3 : z = 0,\ x^2 + y^2 \neq 0\}$
b) $X = \{(x, y) \in \mathbb{R}^2 : y = \sin(1/x),\ 0 < x \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0,\ -1 \leq y \leq 1\}, \quad Y = [0, 1] \subseteq \mathbb{R}$
c) $X = \{(x, y) \in \mathbb{R}^2 : y = x \sin(1/x),\ 0 < x \leq 1\}, \quad Y = [0, 1] \subseteq \mathbb{R}$
d) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}, \quad Y = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \neq 0\}$

8) Let $\{X_i\}$ be a sequence of independent Poisson(λ) variables and let $W_n = \frac{1}{n} \sum_{i=1}^{n} X_i$. Then the limiting distribution of $\sqrt{n}(W_n - \lambda)$ is the normal distribution with zero mean and variance given by

(A) 1
(B) $\sqrt{\lambda}$
(C) λ
(D) λ^2

9) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with probability density function given by

$$f_X(x; \theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Also, let $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$. Then the maximum likelihood estimator of θ is

a) $\frac{1}{\overline{X}}$
b) $\frac{1}{\overline{X}} - 1$
c) $\frac{1}{\overline{X}-1}$
d) $\overline{X}$

10) Consider the Linear Programming Problem (LPP):

$$\text{Maximize } \alpha x_1 + x_2$$

Subject to

$$2x_1 + x_2 \leq 6, \quad -x_1 + x_2 \leq 1, \quad x_1 + x_2 \leq 4, \quad x_1 \geq 0, \quad x_2 \geq 0,$$

where α is a constant. If $(3,0)$ is the only optimal solution, then

a) $\alpha < -2$
b) $-2 < \alpha < 1$
c) $1 < \alpha < 2$
d) $\alpha > 2$

11) Let $M_2(\mathbb{R})$ be the vector space of all 2×2 real matrices over the field $\mathbb{R}$. Define the linear transformation $S : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by $S(X) = 2X + X^T$, where X^T denotes the transpose of the matrix X. Then the trace of S equals ____________.

12) Consider $\mathbb{R}^3$ with the usual inner product. If d is the distance from $(1,1,1)$ to the subspace span$(\{1,1,0\},\{0,1,1\})$ of $\mathbb{R}^3$, then $3d^2 =$ ____________.

13) Consider the matrix $A = I_9 - 2uu^T$ with $u = \frac{1}{3}[1,1,1,1,1,1,1,1,1]^T$, where I_9 is the 9×9 identity matrix and u^T is the transpose of u. If λ and μ are two distinct eigenvalues of A, then $|\lambda - \mu| =$ ____________.