2022-June Session-06-29-2022-shift-1

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1) The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to:

a) $\frac{133}{10^4}$
b) $\frac{18}{10^3}$
c) $\frac{19}{10^3}$
d) $\frac{271}{10^4}$

2) Let the solution of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to :

a) 15
b) 11
c) 13
d) 17

3) If the mirror image of the point $(2, 4, 7)$ in the plane $3x - y + 4z = 2$ is (a, b, c), then $2a + b + 2c$ is equal to:

a) 54
b) 50
c) -6
d) -42

4) Let f: $R \to R$ be a function defined by:

$$f(x) = \begin{cases} \max\{t^3 - 3t\} & ; t \le x,\ x \le 2 \\ x^2 + 2x - 6 & ; 2 < x < 3 \\ |x - 3| + 9 & ; 3 \le x \le 5 \\ 2x + 1 & ; x > 5 \end{cases}$$

Where $[t]$ is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)\, dx$. Then the ordered pair (m, I) is equal to:

a) $\left(3, \frac{27}{4}\right)$
b) $\left(3, \frac{23}{4}\right)$
c) $\left(4, \frac{27}{4}\right)$
d) $\left(4, \frac{23}{4}\right)$

5) Let $\mathbf{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\mathbf{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\mathbf{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in R$, be three vectors. If the projection of $\mathbf{a}$ on $\mathbf{c}$ is $\frac{10}{3}$ and $\mathbf{b} \times \mathbf{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to:

a) 3
b) 4
c) 5
d) 6

6) The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$, is equal to :

a) $\frac{16\sqrt{2}}{6}$
b) $\frac{11\sqrt{2}}{6}$
c) $\frac{13\sqrt{2}}{6}$
d) $\frac{5\sqrt{2}}{6}$

7) If the system of linear equations

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$$x + 4y + \delta z = k$$

where $\delta, k \in R$, has infinitely many solutions, then $\delta + k$ is equal to:

a) -3
b) 3
c) 6
d) 9

8) Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to:

a) 50
b) 250
c) 1250
d) 1500

9) Let $\triangle \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \triangle ((p \vee q) \Rightarrow q)$ is a tautology. Then $\triangle$ is equal to:

a) $\wedge$
b) $\vee$
c) $\Rightarrow$
d) $\Leftrightarrow$

10) Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all $i, j = 1, 2, 3$. Then, the matrix $A^2 + A^3 + \cdots + A^{10}$ is equal to:

a) $\left(\frac{3^{10}-3}{2}\right)$
b) $\left(\frac{3^{10}-1}{2}\right)$
c) $\left(\frac{3^{10}+1}{2}\right)$
d) $\left(\frac{3^{10}+3}{2}\right)$

11) Let a set $A = A_1 \cup A_2 \cup \cdots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j, 1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i,\ 1 \leq i \leq k\}$. Then, R is:

a) reflexive, symmetric but not transitive
b) reflexive, transitive but not symmetric
c) reflexive but not symmetric and transitive
d) an equivalence relation

12) Let $\{a_n\}_{n=0}^{\infty}$ be a sequence that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$.Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to:

a) $\frac{6}{343}$
b) $\frac{7}{216}$
c) $\frac{8}{343}$
d) $\frac{49}{216}$

13) The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and $A'B$ (where B is the point $(2, 3)$)subtend angle $\frac{\pi}{4}$ at the origin, is equal to :

a) 10
b) $\frac{48}{5}$
c) $\frac{52}{5}$
d) 3

14) A wire of length $22m$ is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is:

a) $\frac{22}{9+4\sqrt{3}}$
b) $\frac{66}{9+\sqrt{3}}$
c) $\frac{22}{4+9\sqrt{3}}$
d) $\frac{66}{4+9\sqrt{3}}$

15) The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is :

a) $R - \{-\frac{1}{2}, \frac{1}{2}\}$
b) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$
c) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$
d) $(-\infty, \frac{-1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, \infty) \cup \{0\}$