

9-9.3-5

EE24BTECH11064 - Harshil Rathan

Question:

Find the area of the region enclosed by the line $y = \sqrt{3}x$, semi-circle $y = \sqrt{4 - x^2}$ and X axis in first quadrant.

Solution:

The parameters of the conic are

Equations
$y = \sqrt{3}x$
$y = \sqrt{4 - x^2}$

TABLE 0: Given Equations

Conic	Parameters
V	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
u	0
f	-4
h	0
m	$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

$$L : x_i = h + \kappa_i m \quad (0.1)$$

Where,

$$\kappa_i = \frac{1}{m^\top V m} (-m^\top (Vh + u)) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top V m)} \quad (0.2)$$

For the line $y = \sqrt{3}x$, the parameters are

$$h = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, m = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{3}} \end{pmatrix} \quad (0.3)$$

Substituting the above parameters in (0.2)

$$\kappa_i = \sqrt{3}, -\sqrt{3} \quad (0.4)$$

We consider only +ve value of κ_i , yielding the points of intersection

$$x_i = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \quad (0.5)$$

Angle between given line and X axis is,

$$\theta = 30^\circ \quad (0.6)$$

$$\frac{\theta}{360} \pi r^2 = \frac{\pi}{3} \quad (0.7)$$

Area of sector is $\frac{\pi}{3}$.

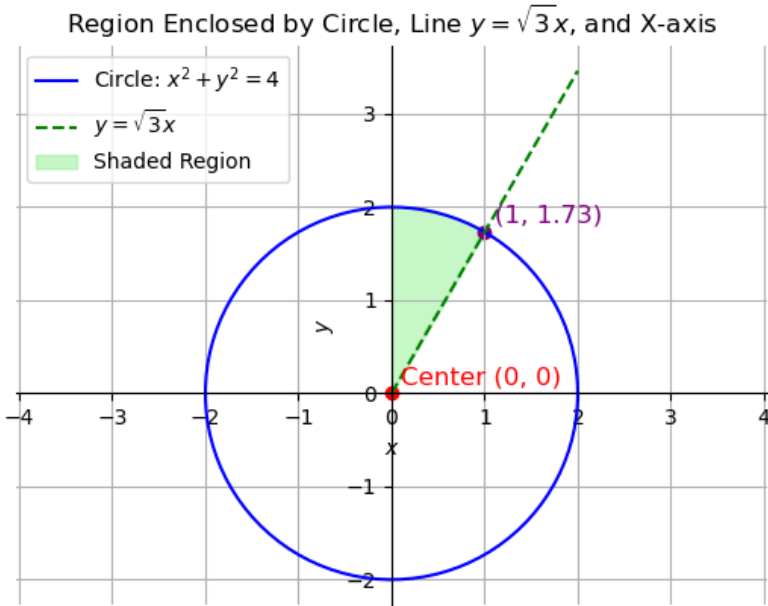


Fig. 0.1