

1-1.2-17

EE24BTECH11064 - Harshil Rathan

Question:

Three vertices of a parallelogram $ABCD$ are $\mathbf{A} = (3, -1, 2)$, $\mathbf{B} = (1, -2, 4)$ and $\mathbf{C} = (-1, 1, 2)$. Find the coordinates of the fourth vertex.

Solution:

Vertices	Given
A	$(3, -1, 2)$
B	$(1, -2, 4)$
C	$(-1, 1, 2)$
D	(x, y, z)

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (0.1)$$

$$\mathbf{P} = \frac{\mathbf{B} + \mathbf{D}}{2} \quad (0.2)$$

$$\mathbf{M} = \mathbf{P} \quad (0.3)$$

$$\mathbf{M} = \frac{\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}}{2} \quad (0.4)$$

$$\mathbf{M} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad (0.5)$$

$$\mathbf{P} = \frac{\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}}{2} \quad (0.6)$$

$$\mathbf{P} = \begin{pmatrix} \frac{1+x}{2} \\ \frac{y-2}{2} \\ \frac{z+4}{2} \end{pmatrix} \quad (0.7)$$

Since,

$$\mathbf{M} = \mathbf{P} \quad (0.8)$$

On comparing both sides,

$$x = 1, y = 2, z = 0 \quad (0.9)$$

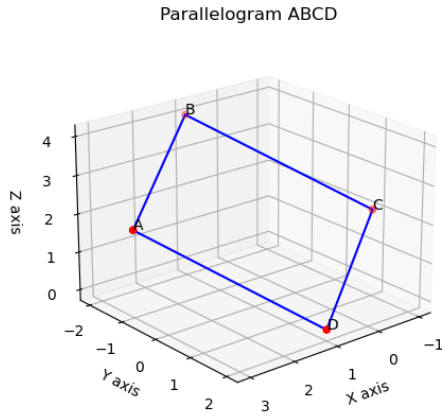


Fig. 0.1: Plot of Parallelogram ABCD