1-1.11-5

EE24BTECH11064 - Harshil Rathan

Question:
The scalar product of the vector $\hat{\mathbf{a}} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\hat{\mathbf{b}} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{\mathbf{c}} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\hat{\mathbf{b}} + \hat{\mathbf{c}}$.
Solution:
Find Sum,

Vertices	Given
$\hat{\mathbf{a}}$	$\left(1, 1, 1\right)$
$\hat{\mathbf{b}}$	$\left(2, 4, -5\right)$
$\hat{\mathbf{c}}$	$\left(\lambda, 2, 3\right)$
$\hat{\mathbf{u}}$	$\left(x, y, z\right)$

$$\hat{\mathbf{b}} + \hat{\mathbf{c}} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k}) \tag{0.1}$$

$$\hat{\mathbf{b}} + \hat{\mathbf{c}} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k} \tag{0.2}$$

Find Magnitude,

$$\|\hat{\mathbf{b}} + \hat{\mathbf{c}}\| = \sqrt{(2 + \lambda)^2 + 6^2 + (-2)^2} \tag{0.3}$$

$$\|\hat{\mathbf{b}} + \hat{\mathbf{c}}\| = \sqrt{(2 + \lambda)^2 + 40} \tag{0.4}$$

Unit vector $\hat{\mathbf{u}}$ in direction of $\hat{\mathbf{b}} + \hat{\mathbf{c}}$ is,

$$\hat{\mathbf{u}} = \frac{\hat{\mathbf{b}} + \hat{\mathbf{c}}}{\|\hat{\mathbf{b}} + \hat{\mathbf{c}}\|} \tag{0.5}$$

Scalar Product with $\hat{\mathbf{a}}$

$$\hat{\mathbf{a}} \cdot \hat{\mathbf{u}} = 1 \tag{0.6}$$

$$\hat{\mathbf{a}} \cdot \frac{\hat{\mathbf{b}} + \hat{\mathbf{c}}}{\|\hat{\mathbf{b}} + \hat{\mathbf{c}}\|} = 1 \tag{0.7}$$

$$\hat{\mathbf{a}} \cdot (\mathbf{b} \hat{+} \mathbf{c}) = \|\hat{\mathbf{b}} + \hat{\mathbf{c}}\| \tag{0.8}$$

$$(\hat{i} + \hat{j} + \hat{k}) \cdot (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k} = \|\hat{\mathbf{b}} + \hat{\mathbf{c}}\| \tag{0.9}$$

$$\lambda + 6 = \|\hat{\mathbf{b}} + \hat{\mathbf{c}}\| \tag{0.10}$$

$$\lambda + 6 = \sqrt{(2+\lambda)^2 + 40} \tag{0.11}$$

On solving,

$$\lambda = 1 \tag{0.12}$$

Unit Vector $\hat{\mathbf{u}}$ is,

$$\hat{\mathbf{u}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7} \tag{0.13}$$

$$\hat{\mathbf{u}} = \frac{3\hat{i}}{7} + \frac{6\hat{j}}{7} - \frac{2\hat{k}}{7} \tag{0.14}$$

Therefore

$$x = \frac{3}{7}, y = \frac{6}{7}, z = \frac{-2}{7} \tag{0.15}$$

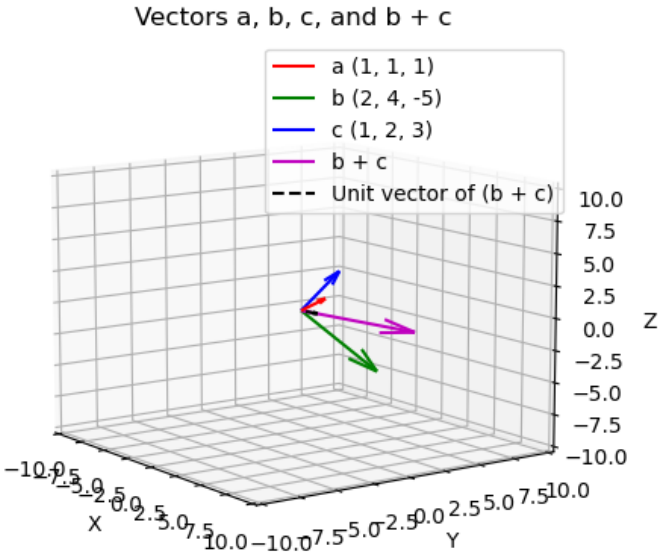

Fig. 0.1: Plot of Parallelogram ABCD