MA - 2019

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1) Let L denote the value of the line integral $\oint_C (3x-4x^2)y\,dx + (4xy^2+2y)\,dy$, where C, a circle of radius 2 with center at origin of the xy-plane, is traversed once in the anti-clockwise direction. Then $\frac{L}{\pi}$ is equal to ______.

2) The temperature $T : \mathbb{R}^3\backslash\{(0,0,0)\} \to \mathbb{R}$ at any point $P(x,y,z)$ is inversely proportional to the square of the distance of P from the origin. If the value of the temperature T at the point $R(0,0,1)$ is $\sqrt{3}$, then the rate of change of T at the point $Q(1,1,2)$ in the direction of $\overrightarrow{QR}$ is equal to ______ (round off to 2 places of decimal).

3) Let f be a continuous function defined on $[0,2]$ such that $f(x) \geq 0$ for all $x \in [0,2]$. If the area bounded by $y = f(x)$, $x = 0$, $y = 0$ and $x = b$ is $\sqrt{3+b^2} - \sqrt{3}$, where $b \in (0,2]$, then $f(1)$ is equal to ______ (round off to 1 place of decimal).

4) If the characteristic polynomial and minimal polynomial of a square matrix A are $(x-1)(x+1)^4(x-2)^5$ and $(x-1)(x+1)(x-2)$, respectively, then the rank of the matrix $A+I$ is ______, where I is the identity matrix of appropriate order.

5) Let ω be a primitive complex cube root of unity and $i = \sqrt{-1}$. Then the degree of the field extension $\mathbb{Q}\left(i, \sqrt{3}, \omega\right)$ over $\mathbb{Q}$ (the field of rational numbers) is ______.

6) Let
$$\alpha = \int_C \frac{e^{i\pi z}dz}{2z^2-5z+2}, C : \cos t + i\sin t, 0 \leq t \leq 2\pi, i = \sqrt{-1}$$
Then the gratest inetegr less tahn or equal to $|\alpha|$ is ______.

7) Consider the system:
$$3x_1 + x_2 + 2x_3 - x_4 = a$$
$$x_1 + x_2 + x_3 - 2x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$
If $x_1 = 1$, $x_2 = b$, $x_3 = 0$, $x_4 = c$ is a basic feasible solution of the above system (where a,b and c are real constants), then $a+b+c$ is equal to ______.

8) Let $f : \mathbb{C} \to \mathbb{C}$ be a function defined by $f(z) = z^6 - 5z^4 + 10$. Then the number of zeros of f in $\{z \in \mathbb{C} : |z| < 2\}$ is ______.
($\mathbb{C}$ is the set of all complex numbers)

9) Let

$$l^2 = \{x = (x_1, x_2, \cdots) : x_i \in \mathbb{C}, \sum_{i=1}^{\infty} |x_i|^2 < \infty$$

be a normed linear space with the norm

$$\|x\|_2 = (\sum_{i=1}^{\infty} |x_1|^2)^{\frac{1}{2}}$$

Let $g : l^2 \to \mathbb{C}$ be a bounded linear functional defined by

$$g(x) = \sum_{n=1}^{\infty} \frac{x_n}{3^n}, \text{ for all } x = (x_1, x_2, \cdots) \in l^2$$

Then $(\sup\{—g(x)| : \|x\|_2 \leq 1\})^2$ is equal to ______.(round off to 3 places of decimal).
($\mathbb{C}$ is the set of all complex numbers)

10) For the linear programming problem (LPP):

$$\text{Maximize } Z = 2x_1 + 4x_2,$$

subject to

$$-x_1 + 2x_2 \leq 4,$$

$$3x_1 + \beta x_2 \leq 6,$$

$$x_1, x_2 \geq 0, \quad \beta \in \mathbb{R},$$

($\mathbb{R}$ is the set of all real numbers).
Consider the following statements:

I. The LPP always has a finite optimal value for any $\beta \geq 0$.
II. The dual of the LPP may be infeasible for some $\beta \geq 0$.
III. If for some β, the point $(1, 2)$ is feasible to the dual of the LPP, then $Z \leq 16$, for any feasible solution (x_1, x_2) of the LPP.
IV. If for some β, x_1 and x_2 are the basic variables in the optimal table of the LPP with $x_1 = \frac{1}{2}$, then the optimal value of dual of the LPP is 10.

Then which of the above statements are **TRUE**?

a) I and III only
b) I, III and IV only
c) III and IV only
d) II and IV only

11) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} (x^2 + y^2) \sin\left(\frac{1}{x^2+y^2}\right), & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Consider the following statements:

I. The partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at $(0,0)$ but are unbounded in any neighbourhood of $(0,0)$.

II. f is continuous but not differentiable at $(0,0)$.

III. f is not continuous at $(0,0)$.

IV. f is differentiable at $(0,0)$.

Which of the above statements is/are **TRUE**?

a) I and II only
b) I and IV only
c) II only
d) III only

12) et $K = [k_{i,j}]_{i,j=1}^{\infty}$ be an infinite matrix over $\mathbb{C}$ (the set of all complex numbers) such that

(i) for each $i \in \mathbb{N}$ (the set of all natural numbers), the i^{th} row $(k_{i,1}, k_{i,2}, \dots)$ of K is in ℓ^1 and

(ii) for every $x = (x_1, x_2, \dots) \in \ell^1$, $\sum_{j=1}^{\infty} k_{i,j}x_j$ is summable for all $i \in \mathbb{N}$, and $(y_1, y_2, \dots) \in \ell^1$, where $y_i = \sum_{j=1}^{\infty} k_{i,j}x_j$.

Let the set of all rows of K be denoted by E. Consider the following statements:

P: E is a bounded set in ℓ^1.

Q: E is a dense set in ℓ^∞.

$$\ell^1 = \left\{(x_1, x_2, \dots) : x_i \in \mathbb{C}, \sum_{i=1}^{\infty} |x_i| < \infty\right\}$$

$$\ell^\infty = \left\{(x_1, x_2, \dots) : x_i \in \mathbb{C}, \sup_{i \in \mathbb{N}} |x_i| < \infty\right\}$$

Which of the above statements is/are TRUE?

a) Both P and Q
b) P only
c) Q only
d) Neither P nor Q

13) Consider the following heat conduction problem for a finite rod

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - xe^{-t} - 2t, \quad t > 0, \quad 0 < x < \pi,$$

with the boundary conditions $u(0,t) = -t^2$, $u(\pi, t) = -\pi e^{-t} - t^2$ and the initial condition $u(x, 0) = \sin x - \sin^3 x - x, \quad 0 \le x \le \pi$. If $v(x,t) = u(x,t) + xe^{-t} + t^2$, then which one of the following is CORRECT?

(A) $v(x,t) = \frac{1}{4}\left(e^t \sin x + e^{-9t} \sin 3x\right)$

(B) $v(x,t) = \frac{1}{4}\left(7e^t \sin x - e^{-9t} \sin 3x\right)$

(C) $v(x,t) = \frac{1}{4}\left(e^{t}\sin x + e^{-3t}\sin 3x\right)$

(D) $v(x,t) = \frac{1}{4}\left(3e^{t}\sin x - e^{-3t}\sin 3x\right)$