MA - 2015

EE24BTECH11064 - Harshil Rathan

1) Let τ_1 be the usual topology on \mathbb{R} . Let τ_2 be the topology on \mathbb{R} generated by

$$\mathcal{B} = \{ (a, b) \subset \mathbb{R} : -\infty < a < b < \infty \}.$$

Then the set

$$\{x \in \mathbb{R} : 4\sin^2 x \le 1\} \cup \left\{\frac{n\pi}{2}\right\}_{n \in \mathbb{Z}}$$

is

- a) closed in (\mathbb{R}, τ_1) but NOT in (\mathbb{R}, τ_2)
- b) closed in (\mathbb{R}, τ_2) but NOT in (\mathbb{R}, τ_1)
- c) closed in both (\mathbb{R}, τ_1) and (\mathbb{R}, τ_2)
- d) neither closed in (\mathbb{R}, τ_1) nor closed in (\mathbb{R}, τ_2)
- 2) Let X be a connected topological space such that there exists a non-constant continuous function $f: X \to \mathbb{R}$, where \mathbb{R} is equipped with the usual topology. Let $f(X) = \{f(x) : x \in X\}$. Then
 - a) X is countable but f(X) is uncountable
 - b) f(X) is countable but X is uncountable
 - c) both f(X) and X are countable
 - d) both f(X) and X are uncountable
- 3) Let d_1 and d_2 denote the usual metric and the discrete metric on \mathbb{R} , respectively. Let $f:(\mathbb{R},d_1)\to(\mathbb{R},d_2)$ be defined by $f(x)=x,x\in\mathbb{R}$. Then
 - a) f is continuous but f^{-1} is NOT continuous
 - b) f^{-1} is continuous but f is NOT continuous
 - c) both f and f^{-1} are continuous
 - d) neither f nor f^{-1} is continuous
- 4) If the trapezoidal rule iwth single interval [0, 1] is exact for approximating the integral $\int_0^1 (x^3 cx^2) dx$, then the value of c is equal to _____.
- 5) Suppose that the Newton-Raphson method is applied to the equation $2x^2 + 1 e^{x^2} = 0$ with an initial approximation x_0 sufficiently close to zero. Then, for the root x=0, the order of convergence of the method is equal to ______.
- 6) The maximum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having $x^2 \sin x$ as a solution is equal to ______.

7) The Lagrangian of a system in terms of polar coordinates (r, θ) is given by

$$L = \frac{1}{2}mr^2 + \frac{1}{2}m(r^2 + r^2\theta^2) - mgr(1 - \cos\theta)$$

where m is the mass, g is the acceleration dur to gravity and s' denotes the derivative of s with respect to time. Then the equation of motion are

- a) $2r' = r\theta'^2 g(1 \cos\theta), \frac{d}{dt}(r^2\theta') = -gr\sin\theta$
- b) $2r' = r\theta^2 + g(1 \cos\theta), \frac{d}{dt}(r^2\theta) = -gr\sin\theta$
- c) $2r' = r\theta^2 g(1 \cos\theta)$, $\frac{d}{dt}(r^2\theta) = gr\sin\theta$
- d) $2r' = r\theta^2 + g(1 \cos\theta), \frac{d}{dt}(r^2\theta) = gr\sin\theta$
- 8) If y(x) satisfies the initial value problem

$$(x^2 + y)dx - xdy, y(1) = 2$$

then y(2) is equal to

9) It is known that Bessel functions $J_n(x)$, for $n \ge 0$, satisfy the identity

$$e^{\frac{x}{2}(t-\frac{1}{t})} = J_0(x) + \sum_{n=1}^{\infty} J_n(x) \left(t^n + \frac{(-1)^n}{t^n}\right)$$

for all t > 0 and $x \in \mathbb{R}$. The value of $J_0\left(\frac{\pi}{3}\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{3}\right)$ is equal to

10) Let X and Y be two random variables having the joint probability density function

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability $P(X \le \frac{2}{3}|Y - \frac{3}{4})$ is equal to

(A) $\frac{5}{9}$ (B) $\frac{2}{3}$

(C) $\frac{7}{9}$ (D) $\frac{8}{9}$

11) Let $\Omega = (0, 1]$ be the sample space and let P() be a probability function defined by

$$P((0, x]) = \begin{cases} \frac{x}{2} & \text{if } 0 \le x < \frac{1}{2} \\ x & \text{if } \frac{1}{2} \le x \le 1 \end{cases}$$

Then $P(\frac{1}{2})$ is equal to _____.

12) Let X_1, X_2 and X_3 be independent and identically distributed random variables with $E(X_1) = 0$ and $E(X_1^2 = \frac{15}{4})$. If $\phi: (0, \infty \to (0, \infty))$ si deifned through the conditional expectation

$$\phi t = E(X_1^2 | X_1^2 + X_2^2 + X_3^2 = t), t > 0$$

then $E(\phi((X_1 + X_2)^2))$ is equal to _____.

13) Let $X \sim \operatorname{Poisson}(\lambda)$, where $\lambda > 0$ is unknown. If $\delta(X)$ is the unbiased estimator of $g(\lambda) = e^{-\lambda} \left(3\lambda^2 + 2\lambda + 1 \right)$, then $\sum_{k=0}^{\infty} \delta(k)$ is equal to ______.