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1) Let

$$P = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{pmatrix}$$

where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = \frac{-k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to

2) Let $B_i\ (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$. All the probabilities are assumed to lie in the interval $(0, 1)$. Then $\left(\frac{P(B_1)}{P(B_3)}\right)$ is equal to

3) The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is

4) If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose centre is at $(2, 1)$, then its radius is

5) $\lim_{n \to \infty} \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right)$ is equal to

6) If

$$\int_{-a}^{a} (|x| + |x - 2|)\, dx = 22$$

and $[x]$ denotes the greatest integer $\leq x$, then $\int_{a}^{-a} (x + [x])\, dx$ is equal to

7) Let three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be such that $\mathbf{c}$ is coplanar with $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a} \cdot \mathbf{c} = 7$ and $\mathbf{b}$ is perpendicular to $\mathbf{c}$, where $\mathbf{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{k}$, then the value of $2|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2$ is

8) Let $A = \{n \in N : \text{n is a 3-digit number}\}$, $B = \{9k + 2 : k \in N\}$ and $C = \{9k + I : k \in N\}$ for some I $(0 < I < 9)$. If the sum of all the elements of the set $A \cap (B \cup C)$ is $274x400$, then I is equal to

9) If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ $\left(z \in C \text{ and } i = \sqrt{-1}\right)$ has a solution, are p and q respectively; then $4\left(p^2 + q^2\right)$ is equal to

10) Let M be any $3x3$ matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is