Assignment-1

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I. Section A

1) The maximum distance from origin of a point on the curve
$x = a\sin(t-b)\sin(\frac{at}{b})$
$y = a\cos(t-b)\cos(\frac{at}{b})$, both a, b > 0 [2002]
   a) a-b
   b) a+b
   c) $\sqrt{a^2+b^2}$
   d) $\sqrt{a^2-b^2}$

2) If $2a+3b+6c=0$, $(a,b,c \in R)$ then the quadratic equation ax^2+bx+c has [2002]
   a) at least one root in $[0,1]$
   b) at least one root in $[2,3]$
   c) at least one root in $[4,5]$
   d) none of these

3) If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a >0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals [2003]
   a) $\frac{1}{2}$
   b) 3
   c) 1
   d) 2

4) A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is [2004]
   a) $(\frac{9}{8}, \frac{9}{2})$
   b) $(2, -4)$
   c) $(\frac{-9}{8}, \frac{9}{2})$
   d) $(2, 4)$

5) A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$.If its graph passes through the point (2,1) and at that point the tangent to the graph $y = 3x-5$, then the function is [2004]
   a) $(x+1)^2$
   b) $(x-1)^3$
   c) $(x+1)^3$
   d) $(x-1)^2$

6) The normal to the curve $x = a(1+cos\theta)$,y=a sinθ at θ always passes through the fixed point [2004]
   a) (a, a)
   b) $(0, a)$
   c) $(0, 0)$
   d) $(a, 0)$

7) If $2a+3b+6c = 0$, then atleast one root of the equation ax^2+bx+clies in the interval [2004]
   a) $(1, 3)$
   b) $(1, 2)$
   c) $(2, 3)$
   d) $(0, 1)$

8) Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$is
   a) $2ab$
   b) ab
   c) $\sqrt{ab}$
   d) $\frac{a}{b}$

9) The normal to the curve $x = a\cos\theta +\ \sin\theta$, y=$a\sin\theta - \cos\theta$ at any point $\theta$is such that
   a) it passes through the origin
   b) it makes an angle $\frac{\pi}{2}+\theta$ with the x axis
   c) it passes through $(a\frac{\pi}{2}, -a)$
   d) it is at a constant distance from the origin

II. Section B

1) Let f be a function defined on R (the set of all real numbers) such that $f'(x)$=2010 $(x-2009)$ $(x-2010)^2$ $\left(x-2011^3\right)$ $\left(x-2012^4\right)$ for all x $\in R$
If g is a function defined on R with values in the interval $(0, \infty)$ such that
$f(x)$=ln $g(x)$, for all x $\in R$
then the number of points in R at which g has a local maximum is (2010)

2) let f:$IR \rightarrow IR$ be defined as $f(x)$= $|x| + |x^2-1|$. The total number of points at which $f$attains either a local maximum or a local minimum is
(2012)

3) Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and local minimum at $x = 3$.If $p(1)$=6 and $p(3)$=2, then $p'(0)$ is (2012)

4) A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q .Let the tangents to the ellipse at P and Q meet at the point R. If Δ(h)=area of the triangle PQR, $\Delta_1 = \max_{1/2 \leq h \leq 1} \Delta(h)$ and $\Delta_2 = min_{1/2 \leq h \leq 1} \Delta(h)$,then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2$= $(JEEAdv.2013)$

5) The slope of the tangent to the curve $(y - x^5)^2 = x(1+x^2)^2$ at the point $(1, 3)$ is $(JEEAdv.2014)$

6) A cylindrical container is to be made from a certain solid material with the following constraints:It has a fixed inner volume of V mm^3, has a 2mm thick solid wall and is open at the top.The bottom of the container is a solid circular disc of thickness 2mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the value of $\frac{V}{250\pi}$ is $(JEEAdv.2015)$