9-9.2-10

EE24BTECH11064 - Harshil Rathan

Question:
Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the axis in the first quadrant.
Solution:
The parameters of the conic are

Equations
$y^2 = x$
$x = 1$
$x = 4$

TABLE 0: Given Equations

Conic	**Parameters**
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\frac{-1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
f	0

$$L : x_i = h + \kappa_i m \tag{0.1}$$

Where,

$$\kappa_i = \frac{1}{m^\top V m}(-m^\top (Vh + u) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top V m)} \tag{0.2}$$

For the line $x - 1 = 0$, the parameters are

$$h_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, m_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.3}$$

Substituting from the above in (0.2)

$$\kappa_i = 1, -1 \tag{0.4}$$

yilelding the points of intersection

$$a_o = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, a_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.5}$$

Similarly, for the line $x - 4 = 0$ (0.2)

$$h_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, m_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.6}$$

yielding

$$\kappa_i = 2, -2 \tag{0.7}$$

from which, the points of intersection are

$$a_3 = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, a_2 = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{0.8}$$

Thus, the area of the parabola in between the lines $x = 1$ and $x = 4$ is given by

$$\int_0^4 \sqrt{x}\, dx - \int_0^1 \sqrt{x}\, dx = \frac{14}{3} \tag{0.9}$$

Area enclosed is $\frac{14}{3}$.

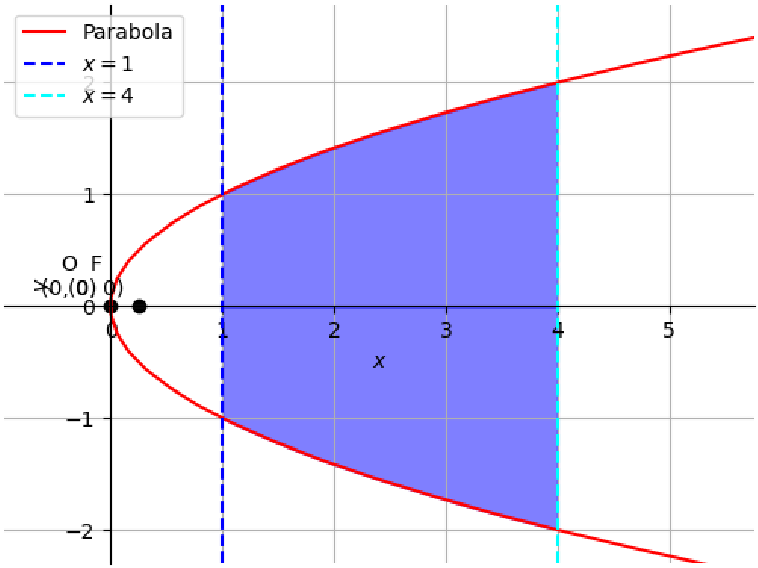

Fig. 0.1