MA - 2010

EE24BTECH11064 - Harshil Rathan

Single Correct 2 Marks each

1) For the linear programming problem

$$\text{Minimize } z = x - y \text{ , subject to } 2x + 3y \leq 6, 0 \leq x \leq 3, 0 \leq y \leq 3,$$

the number of extreme points of its feasible region and the number of basic feasible solutions respectively, are

a) 3 and 3
b) 4 and 4
c) 3 and 5
d) 4 and 5

2) Which one of the following statements is correct?
a) If a Linear Programming Problem (LPP) is infeasible, then its dual is also infeasible
b) If an LPP is infeasible, then its dual always has unbounded solution
c) If an LPP has unbounded solution, then its dual also has unbounded solution
d) If an LPP has unbounded solution, then its dual is infeasible

3) Which one of the following groups is simple?

a) S_3
b) $GL(2, R)$
c) $Z_2 \times Z_2$
d) A_5

4) Consider the algebraic extension $E = Q\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ of the field $\mathbb{Q}$ of rational numbers. Then $[E \colon Q]$, the degree of E over Q, is

a) 3
b) 4
c) 7
d) 8

5) The general solution of the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = x + y$ is of the form

a) $\frac{1}{2}xy(x+y) + F(x) + G(y)$
b) $\frac{1}{2}xy(x-y) + F(x) + G(y)$
c) $\frac{1}{2}xy(x-y) + F(x)G(y)$
d) $\frac{1}{2}xy(x+y) + F(x)G(y)$

6) The numerical value obtained by applying the two-point trapezoidal rule to the integral $\int_0^1 \frac{\ln(1+x)}{x}, dx$ is

a) $\frac{1}{2}(\ln 2 + 1)$
b) $\frac{1}{2}$
c) $\frac{1}{2}(\ln 2 - 1)$
d) $\frac{1}{2}\ln 2$

7) Let $l_k(x) =, k = 0, 1, \ldots, n$ denote the Lagrange's fundamental polynomials of degree n for the nodes $x_0, x_1, \ldots, x_n$. Then the value of $\sum_{k=0}^{n} l_k(x)$ is

a) 0
b) 1
c) $x^n + 1$
d) $x^n - 1$

8) Let X and Y be normed linear spaces and $\{T_n\}$ be a sequence of bounded linear operators from X to Y. Consider the statements:

$$P\colon\ \{\|T_n x\| : n \in \mathbb{N}\}\ \text{ is bounded for each } x \in X$$
$$Q\colon\ \{\|T_n\| : n \in \mathbb{N}\}\ \text{ is bounded}$$

a) If P implies Q, then both X and Y are Banach spaces
b) If P implies Q, then only one of X and Y are Banach space
c) If X is a Banach space, then P implies Q
d) If Y is a Banach space, then P implies Q

9) Let $X = C[0,1]$ with the norm $\|x\|_t = \int_0^1 |x(t)|\,,dt$, $x \in C[0,1]$ and $\Omega = \{f \in X'\colon \|f\| = 1\}$, where X' denotes the dual space of X. Let $C(\Omega)$ be the linear space of continuous functions on Ω with the norm $\|u\| = sup_{s\in\Omega} |u(s)|$, $u \in C(\Omega)$. Then

a) X is linearly isometric with $C(\Omega)$
b) X is linearly isometric with a proper subspace of $C(\Omega)$
c) there does not exist a linear isometry from X into $C(\Omega)$
d) every linear isometry from X to $C(\Omega)$ is onto

10) Let $X = R$ equipped with the topology generated by open intervals of the form $|braa, b$ and sets of the form $(a,b) \cup Q$. The which one of the following statements is correct?

a) X is regular
b) X is normal
c) $\frac{X}{Q}$ is dense in X
d) Q is dense in X

11) Let H, T and V denote the Hamiltonian, the kinetic energy and the potential energy respectively of a mechanical system at time t. If H contains t explicitly, then $\frac{\partial H}{\partial t}$ is equal to

a) $\frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$
b) $\frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$
c) $\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$
d) $-\frac{\partial V}{\partial t} - \frac{\partial V}{\partial t}$

12) The Euler's equation for the variational problem: Minimize
$I[y(x)] = \int_0^1 (2x - xy - y')\,y', dx$, is

a) $2y'' - y = 2$
b) $2y'' + y = 2$
c) $y'' + 2y = 0$
d) $2y'' - y = 0$

13) Let X have a binomial distribution with parameter n and p, $n = 3$. For testing the hypothesis $H_0 : p = \frac{2}{3}$ against $H_1 : p = \frac{1}{3}$, let a test be :"Reject H_0 if $X \geq 2$ and accept H_0 if $X \leq 1$". Then the probabilities of Type 1 and Type 2 errors respectively are

a) $\frac{20}{27}$ and $\frac{20}{27}$

b) $\frac{7}{27}$ and $\frac{20}{27}$

c) $\frac{20}{27}$ and $\frac{7}{27}$

d) $\frac{7}{27}$ and $\frac{7}{27}$