Analysis of RC Circuit Response to a Square Wave Input under Different Time Constant Scenarios

Indian Institute of Technology
Hyderabad

Lab Assignment : 02

EE1200: Electrical Circuits Lab

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1 Experiment Objectives

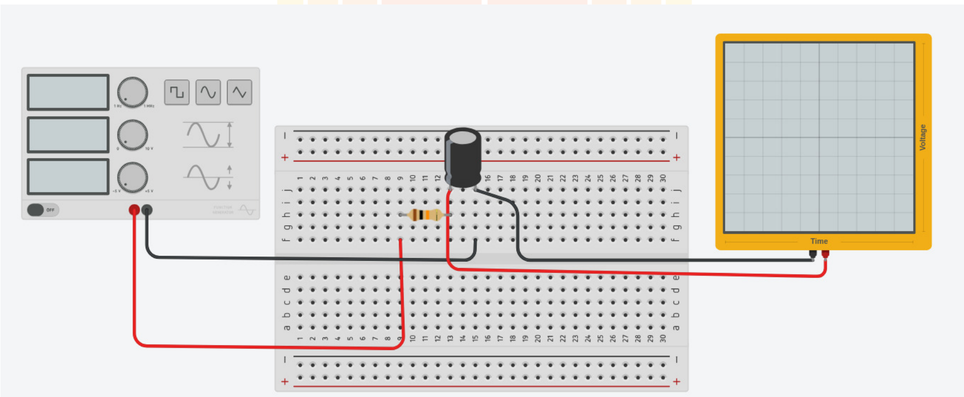
The primary objectives of this experiment are as follows

- To observe the response of an RC circuit to a square wave input, by applying a square wave input to an RC circuit.
- To plot the output and input for a transient response for the first five cycles and a steady-state response.
- Use markers from the CRO to get the voltage values and the time period and match the values you see on the CRO with your hand calculations.
- To understand the relationship between the time constant (τ)and the period of the square wave T_{wave} - $RC == T$, $RC >> T$, $RC << T$

2 Introduction to RC Circuits

- An RC circuit consists of a resistor (R) and a capacitor (C) connected in series or parallel.
- The time constant determines how quickly the circuit responds to changes in input signals. When subjected to a square wave input, the RC circuit exhibits distinct transient and steady-state behaviors depending on the relationship between the time constant and the input signal's time period (T)
- The resistor controls the rate of charge and discharge of the capacitor, creating a time-dependent behavior characterized by the time constant ($\tau = RC$).
- The capacitor stores energy in its electric field when a voltage is applied

3 Circuit Diagram



Above is the approximate circuit diagram simulated using Tinkercad
This circuit diagram represents a simple RC circuit. The key components of the circuit include

- Function Generator
- Oscilloscope

- Breadboard
- Resistor
- Capacitor

4 Components used

4.1 Function Generator

The function generator serves as the input voltage source for the RC circuit. It provides an alternating signal, typically a square wave, which causes the capacitor to alternately charge and discharge.

- Supplies a controlled AC voltage with adjustable frequency and amplitude.
- Generates a periodic waveform (such as a square wave) that forces the capacitor to charge and discharge at regular intervals.
- Allows experimentation with different signal frequencies to study their effects on the capacitor's behavior.

4.2 Oscilloscope

The oscilloscope is used to measure and visualize the voltage across the capacitor as it charges and discharges. It provides a real-time graphical representation of how the voltage varies over time.

- Captures and displays the capacitor voltage on a time scale, allowing for analysis of the waveform.
- Enables measurement of the time constant τ by determining how long it takes for the capacitor to reach approximately 63

4.3 Resistor

The resistor in the circuit controls the rate at which the capacitor charges and discharges. The value of the resistor along with the capacitor determines the value of the time constant $\tau = RC$, which governs the exponential rate of voltage change across the capacitor.

- Limits the current flowing into and out of the capacitor, preventing immediate charging/discharging.
- Influences the time constant τ which affects how fast the capacitor reaches a particular voltage.

In this experiment we used a $10k\Omega$ Resistor

4.4 Capacitor

The capacitor is the primary energy storage component in the circuit. It accumulates charge when voltage is applied and releases it when the voltage decreases.

- Stores electrical energy when voltage is applied.
- Discharges energy when the voltage drops, creating a time-dependent voltage curve.
- Demonstrates the principle of exponential charging and discharging.

In this experiment we use a $220\mu F$ capacitor

4.5 Breadboard

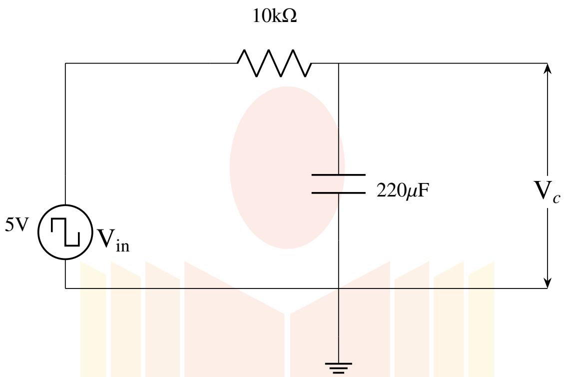
The breadboard is used as a platform for assembling the circuit without the need for soldering. It allows for easy modifications and troubleshooting of the circuit.

- Ensures proper electrical connections between the function generator, resistor, capacitor, and oscilloscope.

5 Case I : $RC == T$

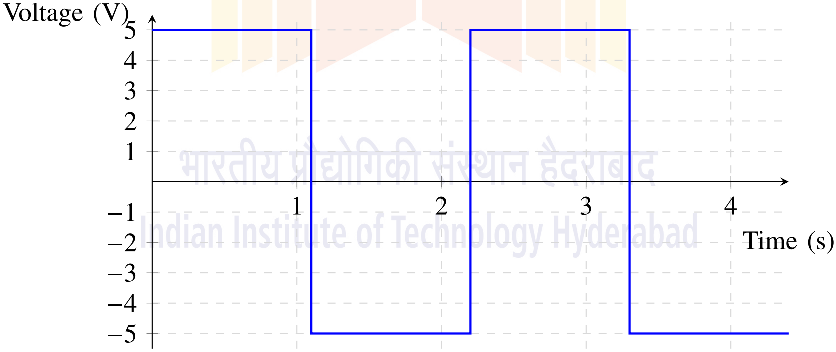
In this case the time constant RC matches with the time period of the curve

5.1 Diagram



5.2 Input signal

The input signal is a square wave with a time period $T = 2.2\,\text{s}$, alternating between +5V and -5V.



5.3 *Input parameters*

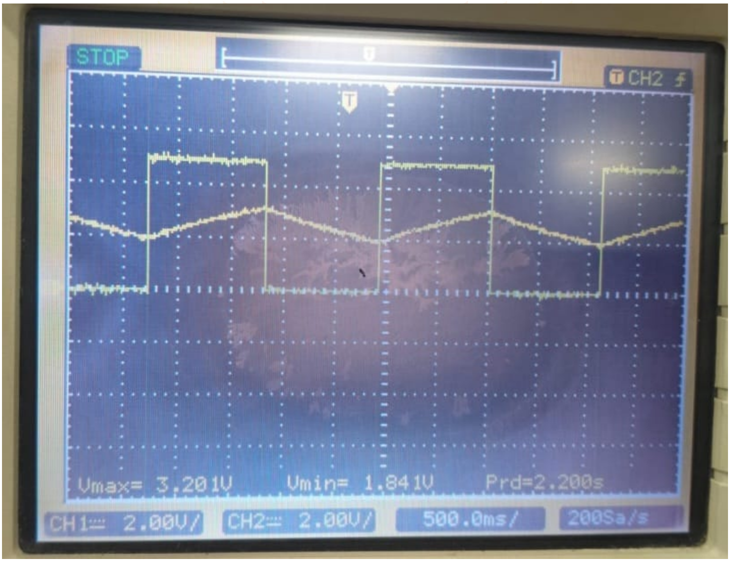
- $R = 10k\Omega$
- $C = 220\mu F$
- Square wave input
  - Amplitude = ±5
  - Frequency = 0.454545Hz
  - Time Period = $2.2s$

The value of the time constant in our experiment is $10 \times 10^3 \times 220 \times 10^{-6} = 2.2$

5.4 *Observations*

- Connect all the components according to the circuit diagram
- Input the signals and parameters mentioned above
- First we get a **periodic(continuous)waveform** and then we fix the number of cycles to get a **single(captured) waveform**

5.4.1 *Continuous Waveform*

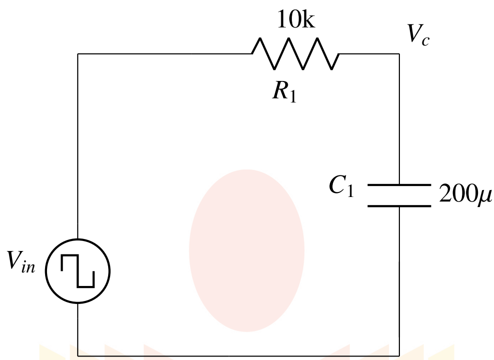


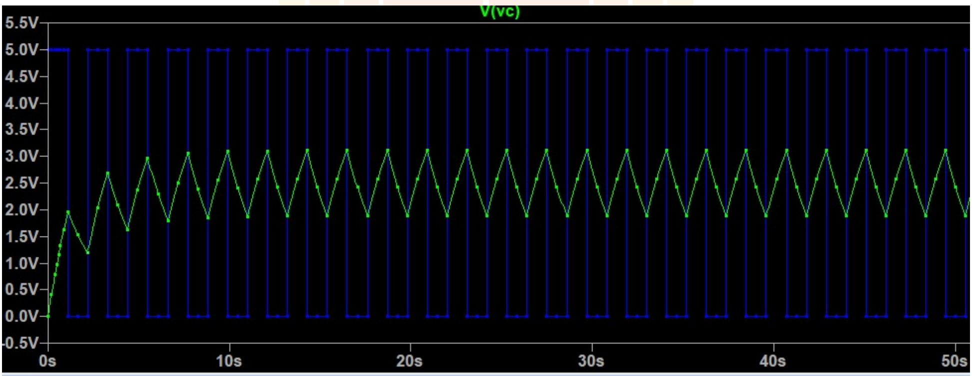
The above table are the set of values inputted on the function generator

5.4.2 *Plotting using LTspice*

On simulating the below circuit on LTspice with $T = 2.2$s, we can verify the figure that we obtained on the CRO for steady state response

Parameter	Value
Waveform Type	Square Wave
Period	2.200 s
Frequency	$\frac{1}{2.2}$ = 0.4545 Hz
High Level Voltage	5.000 V
Low Level Voltage	0 mV (0 V)
Start Phase	0Â°





The above figure shows that the circuits eventually attains a steady state response and the values for V_{max}, V_{min} obtained also match after approx 10 cycles The values obtained according to the simualtion are

- $V_{max} = 3.11V$
- $V_{min} = 1.88V$

The values match approximately with the ones on the CRO, there is some error due to noise and other practical factors
Hence the plot is verified

5.4.3 Verification using Python

It is difficult to prove the steady state values mathematically hence we use python. Below are code snippets that help verify the steady state value
The input variables for the code are given below

```
V_high = 5 # High voltage of the square wave (V)
V_low = 0 # Low voltage of the square wave (V)
T = 2.2 # Time period of the square wave (s)
RC = 2.2 # RC time constant (s)
duty_cycle = 0.5 # Duty cycle (50%)
t_high = duty_cycle * T # Time duration of high phase
t_low = (1 - duty_cycle) * T # Time duration of low phase
```

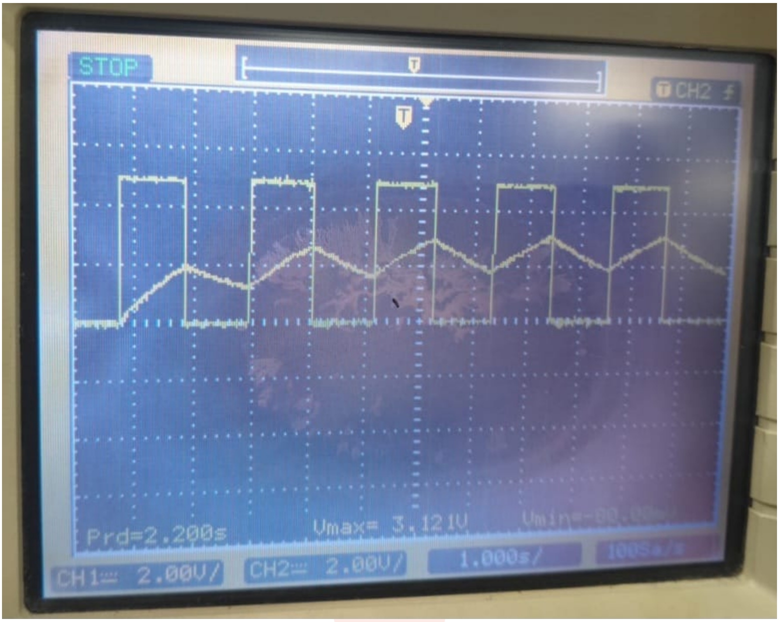
The below snippet iteratively calculates the steady-state crest and trough voltages of an RC circuit under a square wave input

```
while True:
    # Calculate new crest voltage during charging
    V_crest_new = V_high + (V_trough_prev - V_high) * np.exp(-t_high / RC)

    # Calculate new trough voltage during discharging
    V_trough_new = V_low + (V_crest_new - V_low) * np.exp(-t_low / RC)

    # Check for convergence
    if abs(V_crest_new - V_crest_prev) < tolerance and abs(V_trough_new -
        V_trough_prev) < tolerance:
        break

    # Update previous values for the next iteration
    V_crest_prev = V_crest_new
    V_trough_prev = V_trough_new
```

On executing this python code we get the values of $V_{max} = 3.112296$ and $V_{min} = 1.887703$
Which is close to the values obtained on simulation via LTspice and the CRO
Hence the steady state response for $RC == T$ is verified

5.4.4 Single Waveform

The single waveform obtained for 5 cycles is below



The transient response is obtained for the below parameters, no. of cycles is set to 5

5.4.5 Mathematical Proof

During charging of capacitor

$$V_c = V_{max}(1 - e^{\frac{-t}{\tau}})$$

Where t goes from 0 to $\frac{t}{2}$ During discharging of capacitor

$$V_c = V_{initial}(e^{\frac{-t}{\tau}})$$

where $V_{initial}$ = voltage at $\frac{t}{2}$
The RC value for this experiment is 2.2
Time period = 2.2 s

- Where the time period is 2.2s, the half-period is

$$\frac{T}{2} = 1.1$$

During the charging phase the voltage across the capacitor is given by

$$V_c(\frac{T}{2}) = V_{max}(1 - e^{\frac{-T/2}{\tau}})$$

$\tau = 2.2$ and $\frac{T}{2} = 1.1$, on substituting

$$V_c(\frac{T}{2}) = 5(1 - e^{-0.5})$$

$$V_c(\frac{T}{2}) = 5(1 - 0.6065)$$

$$V_c(\frac{T}{2}) \approx 1.9675$$

The voltage per division is recorded as 2.2 on the CRO which is approximately equal to the value obtained, there is always some error due to noise and other factors
During discharge of the capacitor, the capacitor discharges from $V_{initial} = 1.9675$

$$V_C(T) = V_{initial}(e^{\frac{-T/2}{\tau}})$$

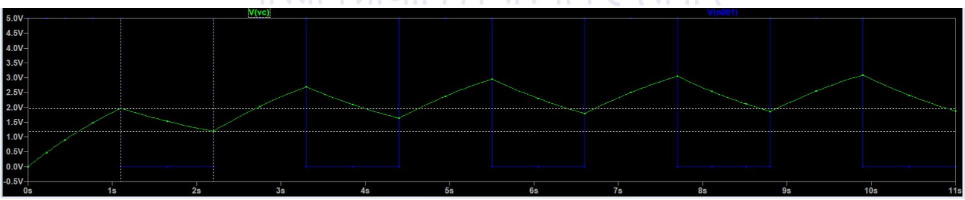
$$V_C(T) = 1.9675(e^{-0.5})$$

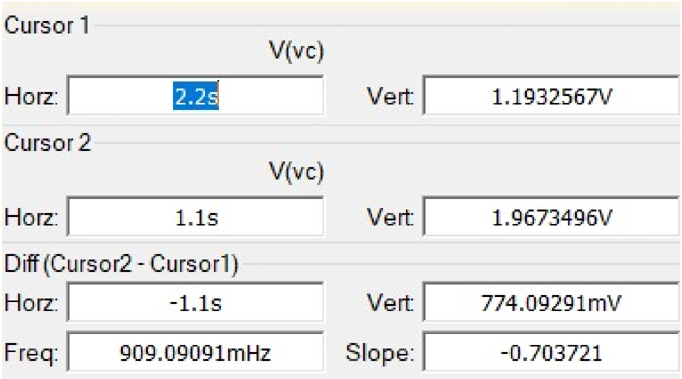
On substituting we get

$$V_C(T) = 1.9675(0.6065) = 1.1932$$

The values of $V_C(T)$ and $V_C(T/2)$ match with the values on the oscilloscope hence the transient state response is proved mathematically

5.4.6 Plotting usig LTspice





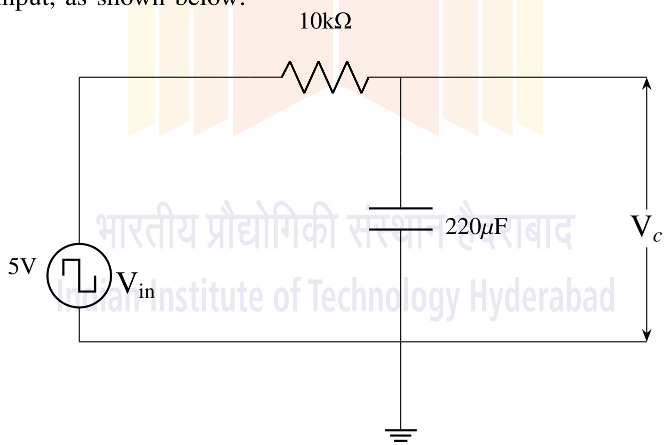
As we can see from the figure above, the cursor 1 is placed at T=2.2, V_C = 1.1932567 and the cursor 2 is placed at T=1.1, V_C = 1.9673496
Which is approximately same as the values obtained via mathematical proof
Hence the transient response for $RC == T$ for 5 cycles is verified both by simulation and mathematical methods

6 Case II : $RC \ll T$

In this case, the time constant RC is much smaller than the time period of the input signal, meaning that the capacitor charges and discharges relatively slowly compared to the input waveform's period. With a time period of $T = 22\,\text{s}$, which is much larger than the time constant $\tau = RC = 2.2\,\text{s}$, the voltage across the capacitor will have sufficient time to reach a significant portion of the maximum voltage during each cycle.

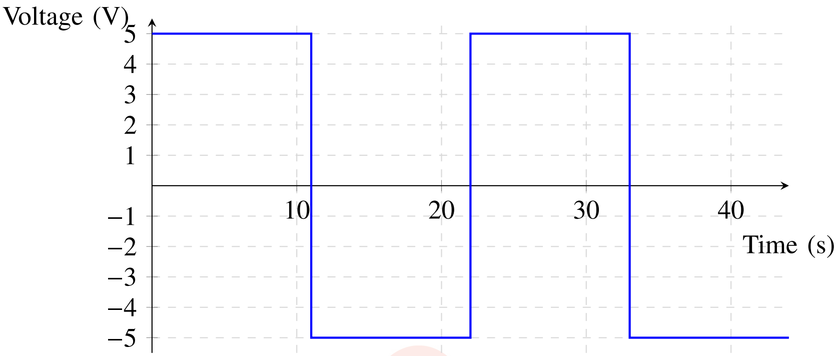
6.1 Diagram

The circuit used for this analysis consists of a resistor and capacitor in series with a square wave input, as shown below:



6.2 *Input Signal*

The input signal is a square wave with a time period $T = 22\,\mathrm{s}$, alternating between +5V and -5V.



6.3 *Input Parameters*

The input signal parameters are summarized in the following table:

Parameter	**Value**
Resistance R	$10\,k\Omega$
Capacitance C	$220\,\mu F$
Amplitude of input signal	$\pm 5\,V$
Frequency of input signal	$\frac{1}{22}$ Hz
Time Period T	$22\,\mathrm{s}$

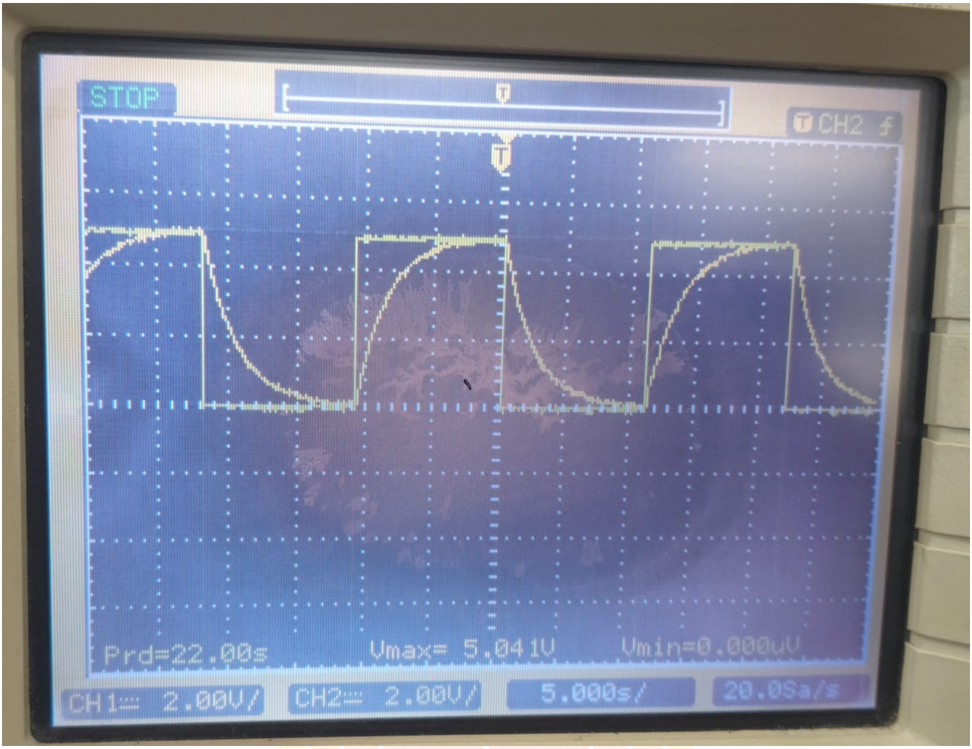
The time constant τ is calculated as:

$$\tau = R \cdot C = 10 \times 10^3\,\Omega \times 220 \times 10^{-6}\,\mathrm{F} = 2.2\,\mathrm{s}$$

6.4 *Observations*

6.4.1 *Continuous Waveform*

For the continuous waveform, the input signal repeats indefinitely, maintaining the same characteristics as the square wave. This is shown below.



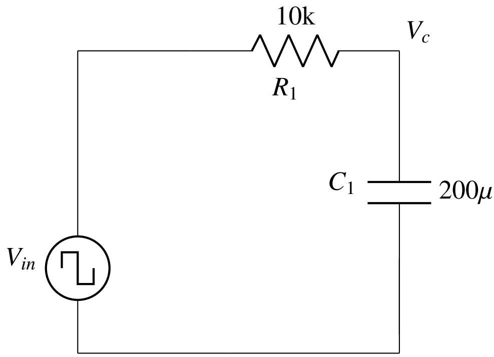
Parameter	Value
High Level Voltage	5.000 V
Low Level Voltage	0 mV (0 V)
Start Phase	0Â°

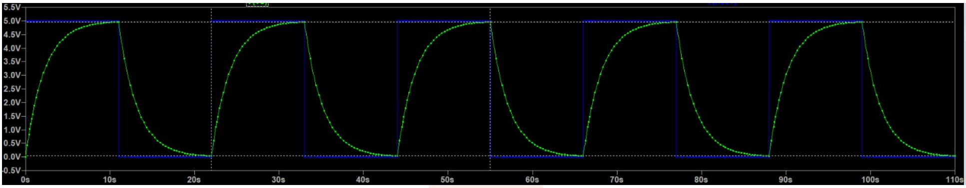
The above table and the parameters mentioned are the set of values inputted on the function generator

6.4.2 Plotting using LTspice

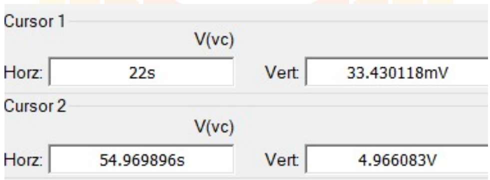
On simulating the below circuit on LTspice with T = 22s, we can verify the figure that we obtained on the CRO for steady state response

Since the time period $T = 22\,\text{s}$ is much greater than $\tau = 2.2\,\text{s}$, the capacitor will have enough time to charge and discharge significantly during each half-cycle of the square wave input.





From the figure we can clearly see that the simulation matches the figure that we obtained on the CRO



On placing cursors at the crest and trough, we get the following values

- $V_{max} = 4.9660$
- $V_{min} = 33mV$

The values match approximately with the ones on the CRO, hence the plot is verified

6.4.3 Verification using python

It is difficult to prove the steady state values mathematically hence we use python. Below are code snippets that help verify the steady state value

The input variables for the code are given below

```
V_high = 5 # High voltage of the square wave (V)
V_low = 0 # Low voltage of the square wave (V)
T = 22 # Time period of the square wave (s)
RC = 2.2 # RC time constant (s)
duty_cycle = 0.5 # Duty cycle (50%)
t_high = duty_cycle * T # Time duration of high phase
t_low = (1 - duty_cycle) * T # Time duration of low phase
```

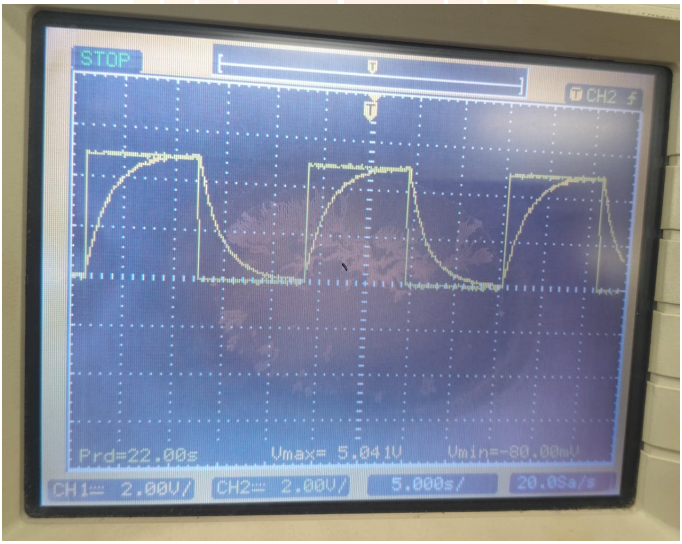
The below snippet iteratively calculates the steady-state response of an RC circuit under a square wave input

```
while True:
    # Calculate new crest voltage during charging
    V_crest_new = V_high + (V_trough_prev - V_high) * np.exp(-t_high / RC)

    # Calculate new trough voltage during discharging
    V_trough_new = V_low + (V_crest_new - V_low) * np.exp(-t_low / RC)

    # Check for convergence
    if abs(V_crest_new - V_crest_prev) < tolerance and abs(V_trough_new -
         V_trough_prev) < tolerance:
        break

    # Update previous values for the next iteration
    V_crest_prev = V_crest_new
    V_trough_prev = V_trough_new
```

On executing this python code we get the values of V_{max} = 4.966536 and V_{min} = 0.033464
Which is close to the values obtained on simulation via LTspice and the CRO
Hence the steady state response for $RC \ll T$ is verified

6.4.4 Single Waveform

The single waveform obtained for 5 cycles is below



The transient response is obtained for the below parameters, no. of cycles is set to 5. But the no of cycles visible are only 3, set the zoom to low to see all the 5 cycles

6.4.5 Mathematical Proof

For this case, $t \gg RC$, meaning the time constant is much smaller than the period of the square wave, so the capacitor will have enough time to charge and discharge during the input's cycle.

During the charging phase, the voltage across the capacitor increases according to the equation:

$$V_c(t) = V_{\text{max}}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Where: - $V_c(t)$ is the voltage across the capacitor at time t, - $V_{\text{max}} = 5.041\,\text{V}$ is the maximum voltage across the capacitor, - $\tau = 2.2\,\text{s}$ is the time constant, - t is the time elapsed during the charging phase.

During the Charging Phase:

At the beginning of the charging phase, $t = 0$, and the capacitor voltage is zero. As time passes, the capacitor charges according to the exponential equation.

For instance, after one time constant ($t = \tau$):

$$V_c(\tau) = 5.041\left(1 - e^{-1}\right) = 5.041\,(1 - 0.3679) = 5.041 \times 0.6321 = 3.18\,\text{V}$$

After two time constants ($t = 2\tau$):

$$V_c(2\tau) = 5.041\left(1 - e^{-2}\right) = 5.041\,(1 - 0.1353) = 5.041 \times 0.8647 = 4.36\,\text{V}$$

As the charging phase continues, the capacitor voltage approaches the maximum voltage $V_{\text{max}} = 5.041\,\text{V}$.

During the Discharging Phase:

During the discharging phase, the voltage across the capacitor decreases exponentially according to the equation:

$$V_c(t) = V_{\text{initial}}e^{-\frac{t}{\tau}}$$

Where: - V_{initial} is the voltage across the capacitor at the start of the discharging phase.

For example, if the capacitor is charged to $5\,\text{V}$ at the end of the charging phase, the voltage across the capacitor at the end of one time constant during discharging is:

$$V_c(\tau) = 5 \times e^{-1} = 5 \times 0.3679 = 1.84\,\text{V}$$

Time for the Voltage to Change:

The time required for the capacitor to charge up to a certain voltage V_c during the charging phase is given by:

$$t = -\tau \ln\left(1 - \frac{V_c}{V_{\max}}\right)$$

Substituting $V_c = 5.041\,\text{V}$, the voltage across the capacitor reaches its maximum value after a significant period of charging.

Since $t \gg RC$, the capacitor will charge and discharge in each cycle, with the voltage staying within a range close to 5.041 V (maximum) and 0 V (minimum) after many cycles.

The voltage across the capacitor after a time $T/2 = 11\,\text{s}$ during the charging phase is approximately:

$$V_c(11\,\text{s}) = 5.041\left(1 - e^{-\frac{11}{2.2}}\right)$$

$$V_c(11\,\text{s}) = 5.041\left(1 - e^{-5}\right)$$

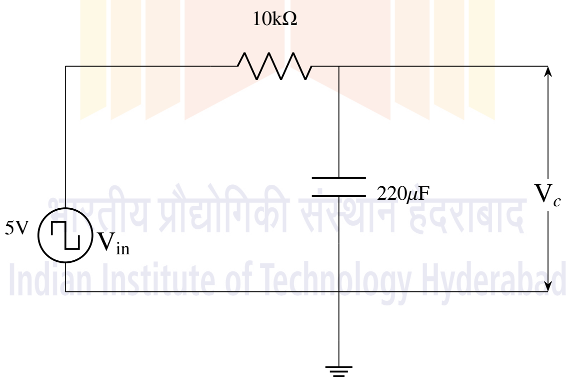
$$V_c(11\,\text{s}) \approx 5.041 \times (1 - 0.0067) = 5.041 \times 0.9933 = 5.009\,\text{V}$$

Thus, the voltage at $T/2$ is approximately 5.009 V, which is very close to the maximum voltage.
Hence the values for $RC \ll T$ are verified both mathematically and via simulation

7 Case III : $RC \gg T$

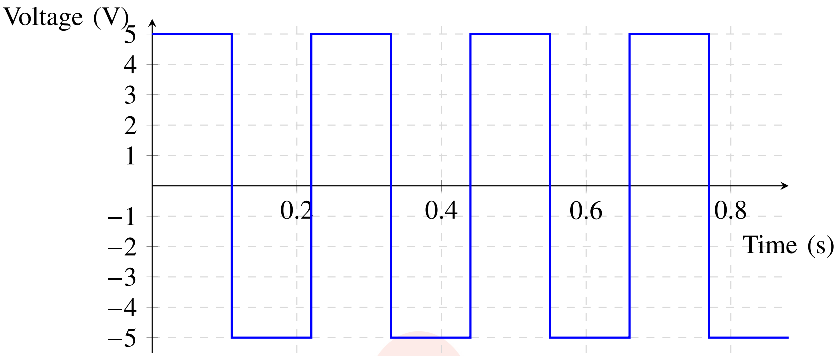
In this case the time constant RC is much greater than the time period of the curve

7.1 Diagram



7.2 Input signal

The input signal is a square wave with a time period $T = 0.22\,\mathrm{s}$, alternating between +5V and -5V.



7.3 Input parameters

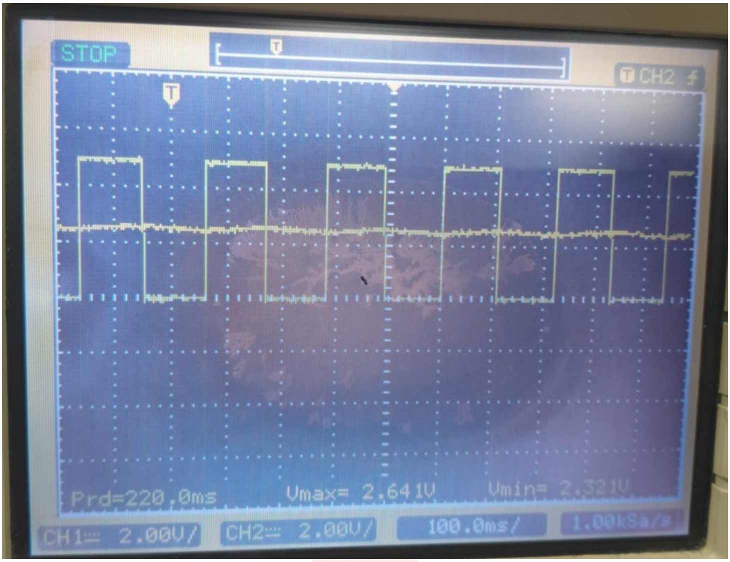
- $R = 10k\Omega$
- $C = 220\mu F$
- Square wave input
  - Amplitude = ±5
  - Frequency = 454545Hz
  - Time Period = $0.22s$

The value of the time constant in our experiment is $10 \times 10^3 \times 220 \times 10^{-6} = 2.2$

7.4 Observations

- Connect all the components according to the circuit diagram
- Input the signals and parameters mentioned above
- First we get a **periodic(continuous)waveform** and then we fix the number of cycles to get a **single(captured) waveform**

7.4.1 Continuous Waveform

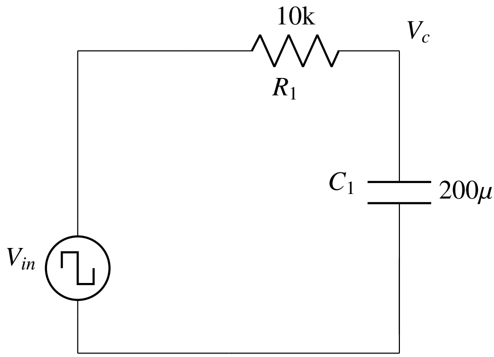


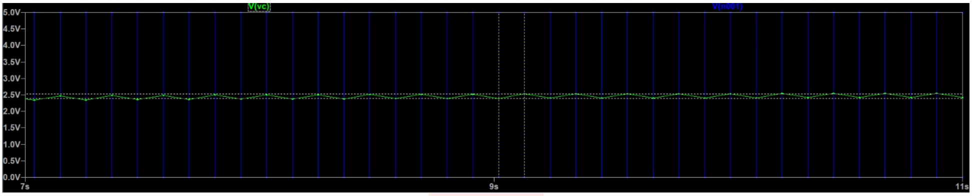
Parameter	**Value**
Waveform Type	Square Wave
Period	0.220 s
Frequency	$\frac{1}{0.22} = 4.5454$ Hz
High Level Voltage	5.000 V
Low Level Voltage	0 mV (0 V)
Start Phase	0Â°

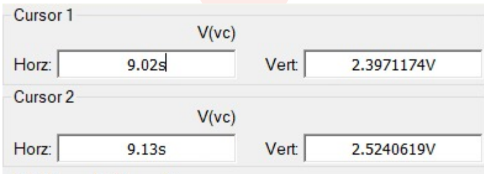
The above table are the set of values inputted on the function generator

7.4.2 Plotting using LTspice

On simulating the below circuit on LTspice with $T = 0.22$s, we can verify the figure that we obtained on the CRO for steady state response







The steady state values obtained on simulation are

- $V_{max} = 2.5240619V$
- $V_{min} = 2.3971174V$

The values match approximately with the ones on the CRO, hence the plot is verified by simulation

7.4.3 Verification using Python

It is difficult to prove the steady state values mathematically hence we use python. Below are code snippets that help verify the steady state value

The input variables for the code are given below

```
V_high = 5 # High voltage of the square wave (V)
V_low = 0 # Low voltage of the square wave (V)
T = 0.22 # Time period of the square wave (s)
RC = 2.2 # RC time constant (s)
duty_cycle = 0.5 # Duty cycle (50%)
t_high = duty_cycle * T # Time duration of high phase
t_low = (1 - duty_cycle) * T # Time duration of low phase
```

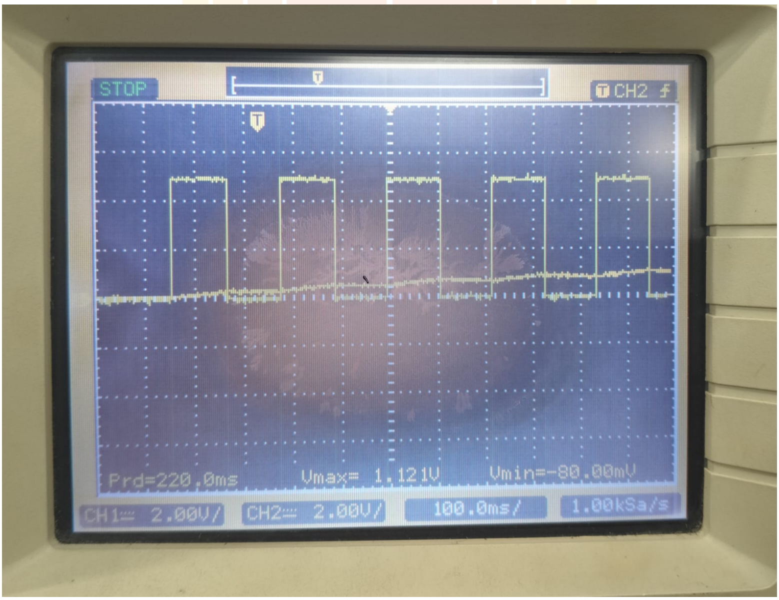
The below snippet iteratively calculates the steady-state crest and trough voltages of an RC circuit under a square wave input

```python
while True:
    # Calculate new crest voltage during charging
    V_crest_new = V_high + (V_trough_prev - V_high) * np.exp(-t_high / RC)

    # Calculate new trough voltage during discharging
    V_trough_new = V_low + (V_crest_new - V_low) * np.exp(-t_low / RC)

    # Check for convergence
    if abs(V_crest_new - V_crest_prev) < tolerance and abs(V_trough_new -
         V_trough_prev) < tolerance:
        break

    # Update previous values for the next iteration
    V_crest_prev = V_crest_new
    V_trough_prev = V_trough_new
```

On executing this python code we get the values of V_{max} = 2.562478 and V_{min} = 2.437505 Which are close to the values obtained on simulation via simulation and the CRO Hence the steady state response for $RC \gg T$ is verified

7.4.4 Single Waveform

The single waveform obtained for 6 cycles is below



The transient response is obtained for the below parameters, no. of cycles is set to 6

7.4.5 Mathematical Proof

During the charging of the capacitor, the voltage across it is given by:

$$V_c = V_{max}\left(1 - e^{\frac{-t}{\tau}}\right)$$

where t goes from 0 to $\frac{T}{2}$.

During the discharging phase, the voltage follows:

$$V_c = V_{initial}e^{\frac{-t}{\tau}}$$

where $V_{initial}$ is the voltage at $t = \frac{T}{2}$.

Given values: - $\tau = 2.2s$ (RC time constant) - $T = 0.22s$ (Time period) - $\frac{T}{2} = 0.11s$ - $V_{max} \approx 2.641V$

Step 1: Calculating $V_c(\frac{T}{2})$ (Voltage at half-period during charging)

$$\begin{aligned} V_c(\frac{T}{2}) &= V_{max}\left(1 - e^{-\frac{T/2}{\tau}}\right) \\ &= 2.641\left(1 - e^{-\frac{0.11}{2.2}}\right) \\ &= 2.641\left(1 - e^{-0.05}\right) \end{aligned}$$

Using $e^{-0.05} \approx 0.9512$:

$$\begin{aligned} V_c(\frac{T}{2}) &= 2.641 \times (1 - 0.9512) \\ &= 2.641 \times 0.0488 \\ &\approx 0.129V \end{aligned}$$

Step 2: Calculating $V_C(T)$ (Voltage at full period during discharging)

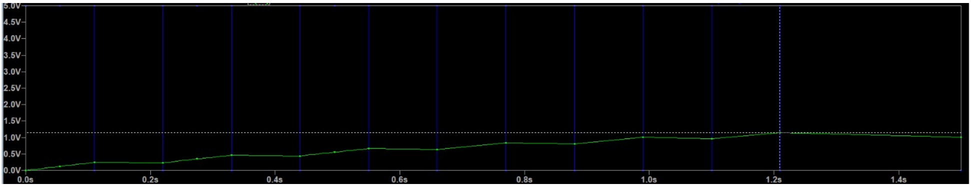
$$\begin{aligned} V_C(T) &= V_{initial}e^{-\frac{T/2}{\tau}} \\ &= 0.129e^{-\frac{0.11}{2.2}} \\ &= 0.129e^{-0.05} \end{aligned}$$

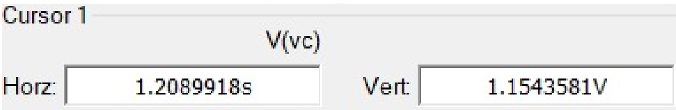
Using $e^{-0.05} \approx 0.9512$:

$$
\begin{aligned}
V_C(T) &= 0.129 \times 0.9512 \\
&\approx 0.123V
\end{aligned}
$$

The values of $V_C(T)$ and $V_C(T/2)$ closely match the oscilloscope readings, confirming the transient response behavior mathematically.

7.4.6 Plotting using LTspice





The value for V_{max} for transient response via simulation is 0.129 and the value for V_{max} obtained on the CRO is 0.123. Since these values are approximately equal, the error might be because of noise and other practical factors.

Hence, the transient response for $RC >> T$ over 6 cycles is successfully verified both mathematically and through simulation.