Back Propogation.

SID-2173839

$x_1 = 1$ ← Targets.

$x_2 = 4$ $t_1 = 0.1$

$x_3 = 5$ $t_2 = 0.05$

Initial Weights.

Input - Hidden

$w_1 = 0.1$ $w_2 = 0.2$ $w_3 = 0.3$

$w_4 = 0.4$ $w_5 = 0.5$, $w_6 = 0.6$

$b = 0.5$

Hidden - Output.

$w_7 = 0.7$. $w_9 = 0.9$

$w_8 = 0.8$ $w_{10} = 0.1$

$b_2 = 0.5$

Activation Sigmoid.

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Step1: Forward Propogation.

$zh_1 = w_1x_1 + w_3x_2 + w_5x_3 + b_1$

$= (0.1)(1) + (0.3)(4) + (0.5)(5) + 0.5$

$= 0.1 + 1.2 + 2.5 + 0.5 = 4.3$

$h_1 = \sigma(4.3)$

$h_1 = 0.9866$

$Z_{h_2} = w_2 x_1 + w_4 x_2 + w_6 x_3 + b_1$

$= (0.2)(1) + (0.4)(4) + (0.6)(5)$

$+ 0.5$

$= 0.2 + 1.6 + 3 + 0.5 = 5.3$

$h_2 = \sigma(5.3)$

$h_2 0.9950$

Output layer:

$z_{w_1} = w_7 h_1 + w_9 h_2 + b_2$

$= (0.7)(0.9866) + (0.9)(0.9950)$

$+ 0.5$

$h_2 = \sigma(5.3)$ / $0.6906 + 0.8955$

$h_2 0.9950$ / $+ 0.5$

$= 2.0861$

$b_1 = \sigma(2.0861)$

$b_1 0.8896$

$z_{w_2} = w_8 h_1 + w_{10} h_2 + b_2$

$= (0.8)(0.9866) + (0.1)(0.9950)$

$+ 0.5$

$= 1.3888$

$o_2 = \sigma(1.3888)$

$o_2 0.8003$

+w6 x3 +b)
(4) +(0.6)(5)

5 = 5.3

b2
.9)(0.9950)

.6 + 0.8955
-
61

2
(0.1)(0.9950)

Step 2: Compute Error.

$$E = \frac{1}{2}\left[(o1 - t1)^2 + (o_2 - t_2)^2\right]$$

$$= 0.5\left[(0.8896 - 0.1)^2 + (0.8003) - 0.05)^2\right]$$

$$= 0.5\left[(0.7896)^2 + (0.7503)^2\right]$$

$$= 0.5\left[0.623 + 0.563\right]$$

$$= 0.5\,(1.186)$$

E 0.593

Step 3: Backpropagation (key gradients

$$\sigma(x) = \sigma(x)(1 - \sigma(x))$$

Output layer gradient.

$$\frac{dE}{do1} = o1 - t1 = 0.8896 - 0.1$$

$$= 0.7896$$

$$\sigma(Z_{o1}) = 0.8896(1 - 0.8896)$$

$$= 0.8896(0.1) = 0.7896$$

$$\sigma(Z_{o1}) = 0.8896(1 - 0.8896)$$

$$= 0.8896\,(0.1104)$$

0.0982

$$\delta_{o1} = 0.78960.09820.0775$$

$$\frac{dE}{dw7} = \delta_{o1} h_1$$

$$= 0.07750 \cdot 9866.$$

$$= 0.0764$$

$$\frac{dE}{do_2} = o_2 - t_2$$

$$= 0.8003 - 0.05$$

$$= 0.7503$$

now Sigmoid derivative.

$$\sigma(z_{o2}) = o_2(1 - o_2)$$

$$= 0.8003(1 - 0.8003)$$

$$= 0.8003(0.1997)$$

$$0.1598$$

So.

$$\delta = 0.75030 . 1598$$

$$0.1199$$

Gradients for output Weights.

$$w8(h_1 - o_2)$$

$$\frac{dE}{dw8} = \delta_{o2} h_1$$

$$= 0.11990.9866.$$

Update.

$$w8_{new} = 0.8 - 0.01(0.1183)$$

$$= 0.798817$$

$$w10(h_2 - o_2)$$

$$\frac{dE}{dw10} = \delta_{o2} h_2$$

$$= 0.11990.9950.$$

$$w8_{new} = 0.8 - 0.001183$$

$$= 0.798817.$$

Step 4. Update Weights

$$w = w - a\frac{dE}{dw}.$$

Learning rate -

$$a = 0.01$$

$$w7_{new} = 0.7 - 0.01(0.0764)$$

$$= 0.7 - 0.000764$$

$$= 0.699236.$$

Gradient for w9

$$\frac{\partial E}{\partial w9} = \delta_{o1} \cdot h_2$$

$$\delta_{o1} = 0.0775$$

So $\frac{\partial E}{\partial w9} = 0.0775 \times 0.9950$

$$= 0.0771$$

Update.

$$w9_{new} = 0.9 - 0.01(0.0771)$$

$$= 0.899229$$

Update Bias b2.

$$\frac{\partial E}{\partial b2} = \delta_{o1} + \delta_{o2}$$

$$b2_{new} = 0.5 - 0.01(0.1973)$$

$$= 0.498027$$

3) Hidden Layer Gradient.

For hidden neuron h_1:

$$\delta_{h_1} = (\delta_{o_1} w_7 + \delta_{o_2} w_8) \times h_1 (1 - h_1)$$

For hidden neuron h_2

$$\delta_{h_2} = (\delta_{o_1} w_9 + \delta_{o_2} w_{10}) \times h_2 (1 - h_2)$$

$$\delta_{h_1} = (0.07750.7 + 0.11990.8)(0.9866)(1 - 0.9866).$$

$$= (0.05425 + 0.09592)(0.9866)(0.0134)$$

$$= (0.15017)(0.01322)$$

$$\delta_{h_1} = 0.00199$$

For the hidden neuron h_2

$w_9 = 0.9$

$w_{10} = 0.1$

$h_2 = 0.9950$

$$\delta_{h_2} = (0.07750.99 + 0.11990.1)(0.9950)(1 - 0.9950)$$

$$= (0.06975 + 0.01199)(0.9950)(0.0050)$$

$$= (0.08174)(0.004975)$$

$$\delta_{h_2} = 0.000406.$$

$$\frac{dE}{dw_1} = \delta_{h_1} x_1$$

Since $x_1 = 1$

$$\frac{dE}{dw_1} = 0.00199$$

Similarity $\frac{dE}{dw_3} = 0.00199 \times 4 = 0.00796$

$$\frac{dE}{dw_5} = 0.00199 \times 5 = 0.00995$$

For h_2 side: $\frac{dE}{dw_2} = 0.000406 \times 1 = 0.000406$

$$\frac{dE}{dw_4} = 0.000406 \times 4 = 0.001624$$

$$\frac{dE}{dw_6} = 0.000406 \times 5 = 0.00203$$

Updated all Weights.

$w_1 = 0.1 - 0.0_1(0.00199)$

$w_1 = 0.0999801$

Updated w_3

$w_3 = 0.3 - 0.01(0.00796)$

$w_3 = 0.2999204$

Updated w_5.

$w_5 = 0.5 - 0.01(0.00995)$

$w_5 = 0.499905.$

Updated w_2

$w_2 = 0.2 - 0.01(0.000406)$

$w_2 = 0.1999$

Updated W_4.

$W_4 = 0.4 - 0.01(0.001624)$

$W_4 = 0.39998376$

Updated W_6.

$W_6 = 0.6 - 0.01(0.00203)$

$W_6 = 0.59997797$

$\frac{dE}{db_1} = \delta h_1 + \delta h_2$

$= 0.00199 + 0.000406$

$= 0.002396$

Update

$b_1 = 0.5 - 0.01(0.002396)$

$b_1 = 0.49997604$