Lecture Notes on Boundary Layer Meteorology

Harshit Prashant Dhanwalkar (SC21B164)[1]*

Abstract
Notes of Lectures and addional information from books:
An introduction to boundary layer meteorology ([1]).

[1] *MTech, Earth System Sciences (ESS), 1st year, Department of Physics, Indian Institute Of Spacescience and Technology (IIST)*
***email**: harshitpd1729@gamil.com*

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1. Lecture 1 09/01/2025

1.1 Introduction To Boundary Layer

The Boundary Layer can be defined as part of the troposphere that is directly influenced by the presence of the Earth's surface and responds to surface forcings with a time scale of about an hour or less.
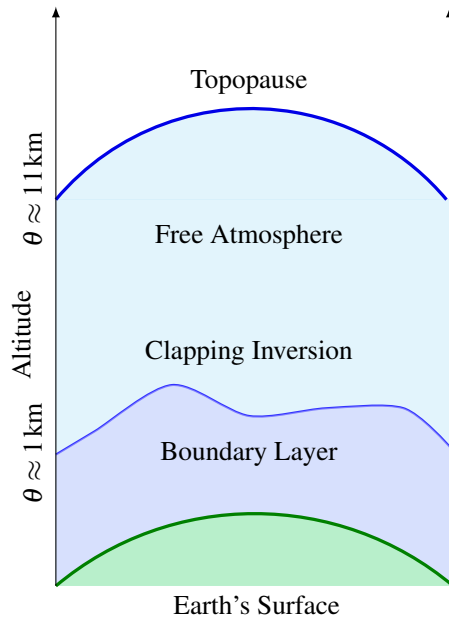
Figure 1.1. Atmosphere can be divided into 2 parts: boundary layer near surface and free atmosphere above it.

1.2 Boundary Layer Forcing Mechanism

What physical process modify boundary layer air parcel?

1. Heat transfer to.from the ground.
2. Frictional drag.
3. Evaporation/transpiration.
4. Terrain-induced flow modification.
5. Pollution emission.

1.3 Types Of Air Flow Or Wind

Air flow or wind can be decomposed into following 3 types:

1. **Mean Wind** $(\bar{u}, \bar{v}, \bar{w})$: Represents the average wind components in the horizontal $(\bar{u}, \bar{v})$ and vertical $(\bar{w})$ directions. It is important for the horizontal transport of quantities such as moisture, heat, momentum, and pollutants, a process known as advection.
2. **Waves**: Atmospheric waves, such as gravity waves, occur mostly at night in the nocturnal boundary layer (NBL). They can influence the structure of the boundary layer and the transport of energy.
3. **Turbulence**: The vertical transport of moisture, heat, momentum, and pollutants is primarily dominated by turbulence, which is characterized by chaotic and irregular motion.
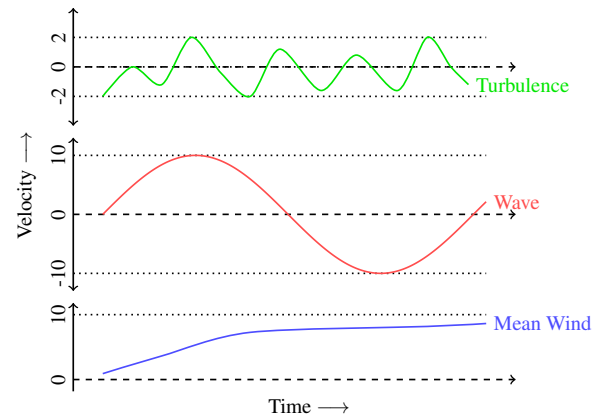
Figure 1.2. Plot showing profiles of Mean, Wave and Turbulent winds

1.4 Eddies

Eddies are formed due to the interaction of currents with obstacles like coastlines, underwater topography, or other currents, as well as from the instability of larger current systems. Eddies exhibit a rotational flow pattern, either clockwise or counterclockwise. Eddies can vary from size 100 to 3000 metres and also can exists as small as few millimetres. Small eddies might last for seconds to minutes, while larger oceanic eddies can persist for weeks, months, or even years.

1.5 Turbulence Generation Mechanisms

- **Solar Heating**: Solar heating generates thermals, which are essentially larger eddies that drive turbulence in the atmospheric boundary layer.
- **Wind Shear**: Variations in wind speed or direction with height create wind shear, which is a significant source of turbulence.
- **Obstacle-Induced Flow**: Deflected flow around obstacles such as trees, buildings, or other structures generates turbulent eddies downstream of these obstacles, creating turbulent wakes.
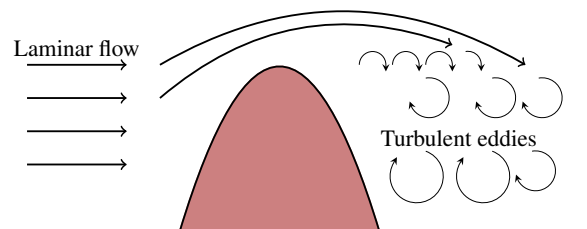
Figure 1.3. Eddy formation due to Turbulence caused by an obstacle

Large eddies will break into smaller eddies after which small eddies dissipates from K.E. to thermal energy.
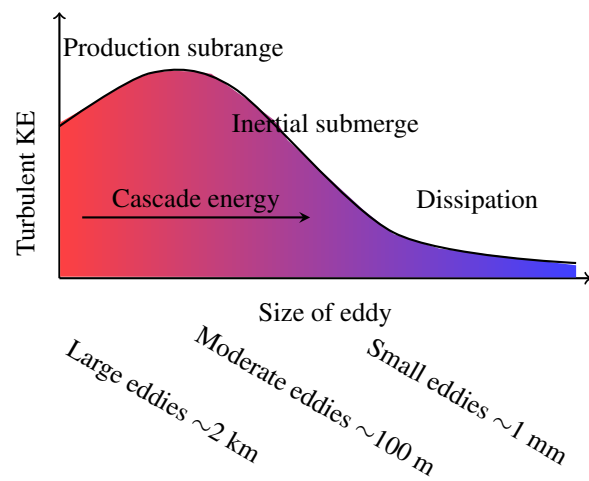
Figure 1.4. Variation of Turbulent Kinetic energy with change in Size of eddies

2. Leture 2 15/01/2025

2.1 Taylor's Hypothesis

- When studying atmospheric boundary layer (ABL), It is not easy to create a snapshot of turbulance in the Atmosphere.
- Hence it is easier and cheaper to make measurements of point in the atmoshere for a longer time, then an instantaneous snapshot.
- So we just consider the atmoshere is frozen.
- *Taylor suggested that turbulence can be considered frozen as it advects past sensor.*
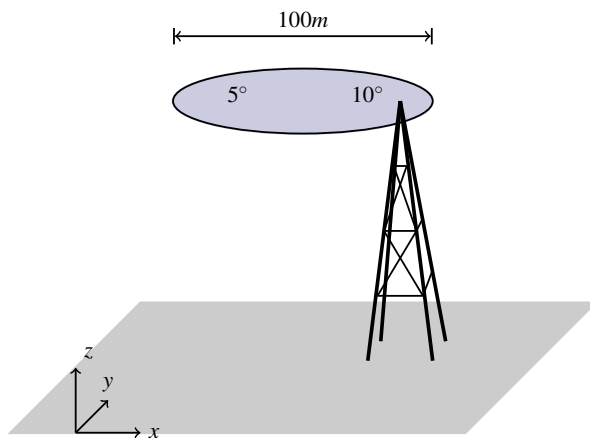
Figure 2.1. Eddy propagation
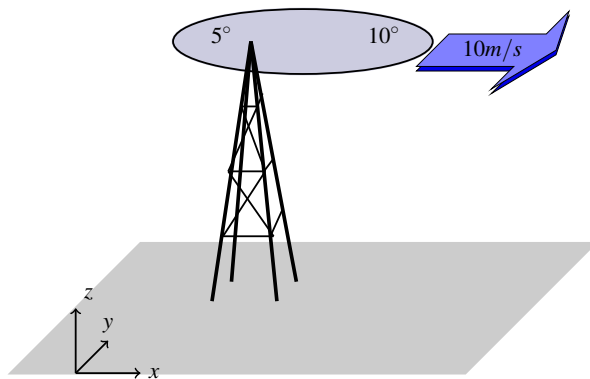
Figure 2.2. Eddy passing by the sensor mounted on tower

$$\frac{\partial T}{\partial x} = 0.05 \text{K/m}, \quad \frac{\partial T}{\partial t} = -0.5 \text{K/s}$$

$$\underbrace{\frac{DT}{Dt}}_{\substack{\text{Total derivative = 0} \\ \text{(Taylor's hypothesis)}}} = \underbrace{\frac{\partial T}{\partial t}}_{\text{Local derivative}} + \underbrace{u\frac{\partial T}{\partial x}}_{\text{Advective term}} \qquad (2.1)$$

2.2 Virtual Potential Temperature

Virtual potential temperature:

$$\theta_v = \theta(1 + 0.61r) \qquad (2.2)$$

Virtual temperature:

$$T_v = T(1 + 0.61r) \qquad (2.3)$$
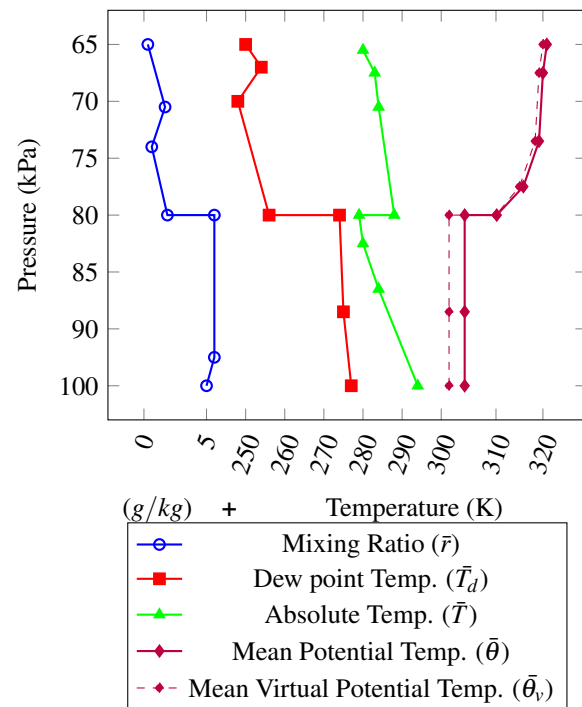
Figure 2.3. Pressure v/s Termperature

Question 2.1:

Given 25°C termperature, mixing ratio $\bar{r}$ is $20g/kg$, measured Pressurere at $900hPa$, find virtual potential temperature.

Answer 2.1:

Solution:

$$\theta = T \times \left(\frac{1000}{P}\right)^{0.286}$$

$$= 298 \times \left(\frac{1000}{900}\right)^{0.286}$$

$$= 332.222K$$

$$\theta_v = \theta \times (1 + 0.61r)$$

$$= 332.22 \times (1 + 0.61 \times 0.025)$$

$$= 336.273K$$

$$\theta_v - \theta \approx 4.05K$$

2.3 Boundary Layer Depth and Structure

- Mixed layer
- Residual layer
- Stable Boundary layer

- Capping Inversion
- Nauturnal Boundary layer
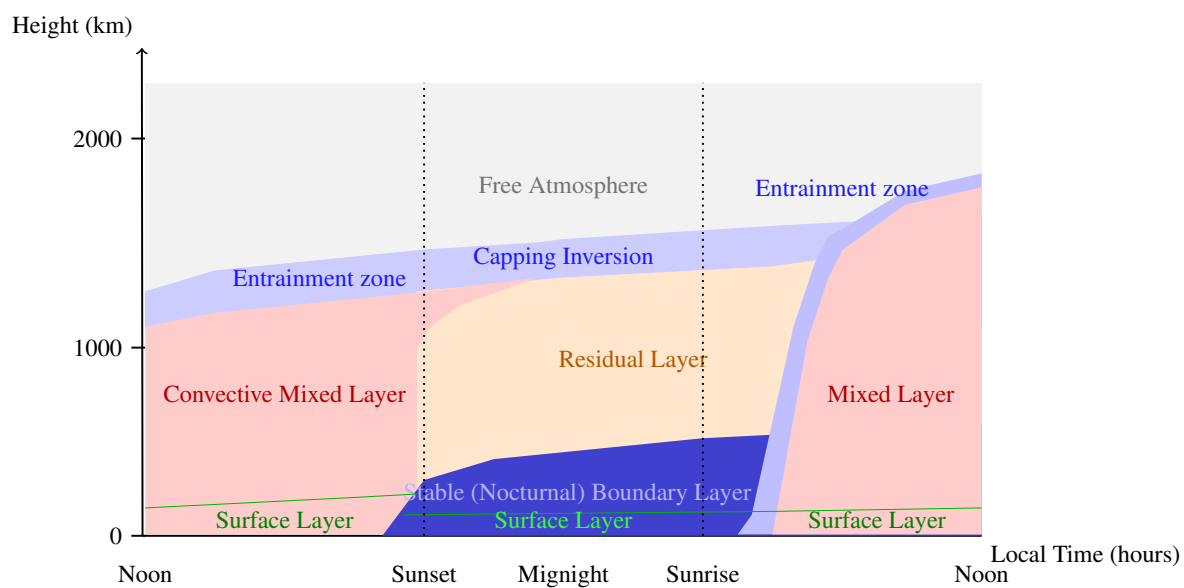
Figure 2.4. Height vs Local Time Diagram of Atmospheric Boundary Layers.

3. Lecture 3 16/01/2025

Ocean: Variations are minimal, with only 10% changes observed over 1000 km. Significant variations occur primarily during weather phenomena.

Land: Day-to-day and diurnal variations are prominent, with distinct boundary layer structures:

1. **Convective Mixed Layer:** Thermodynamically unstable with intense vertical mixing.
2. **Residual Layer:** Neutral stratification with turbulence of equal intensity in all directions.
3. **Stable Boundary Layer (Nocturnal B.L.):**
   - Neutral stratification with nocturnal jets (30 m/s, 200 m width).
   - Sporadic turbulence and internal gravity waves transporting air parcels vertically.
4. **Capping Inversion:** Found at altitudes between 1.5–3 km, acting as a barrier to upward mixing.
5. **Entrainment Zone:** Transition region from stable to unstable conditions, facilitating energy and mass exchange.

3.1 Stability and Plume Behaviour

1. **Looping plumes:** Occur in highly unstable conditions, usually during the day, when strong surface heating causes warm air to rise and interact turbulently with cooler air. This leads to an oscillatory motion that disperses pollutants in multiple directions, often seen in arid regions.

   Fanning plumes: Form in stable conditions, typically at night, when surface cooling creates temperature inversions. The plume spreads horizontally, concentrating pollutants close to the surface, which can impact air quality in urban or industrial areas.
3. **Coning plumes:** Develop under neutral conditions, where vertical and horizontal mixing is balanced. The plume takes a cone-like shape, often observed on overcast days or in the early morning and evening, with moderate turbulence.
4. **Lofting plumes:** Occur when the atmosphere is stable near the ground but unstable above. Pollutants rise and disperse above the stable layer, reducing ground-level pollution and minimizing surface concentrations.
5. **Fumigation plumes:** Happen when pollutants are trapped in a stable layer and then forced downward due to rising turbulence. This leads to high concentrations at the surface, posing risks to air quality, especially in industrial areas.

FIGURE for each to be added later.

3.2 Importance of Boundary Layer

The boundary layer plays a critical role in regulating interactions between the Earth's surface and the atmosphere. Its study is important in various fields, including:

1. **Agricultural meteorology:** Understanding microclimates within the boundary layer aids in crop management, irrigation planning, and predicting the effects of extreme weather on agriculture.
2. **Air pollution meteorology:** Dispersion and concentration of pollutants are governed by boundary layer processes, making it crucial for air quality monitoring and pollution control strategies.
3. **Cloud nuclei meteorology:** The boundary layer provides a reservoir of aerosols and moisture that act as cloud condensation nuclei, influencing cloud formation, precipitation, and local weather patterns.
4. **Thunderstorms and hurricanes physics:** The exchange of heat, moisture, and momentum in the boundary layer drives the development and intensity of thunderstorms and hurricanes, making it essential for improving weather prediction models.
5. **Urban meteorology:** The boundary layer's interactions with urban landscapes affect local climate, energy balance, and pollutant dispersion, aiding in city planning and sustainability efforts.
6. **Renewable energy:** Wind energy potential and efficiency are heavily dependent on boundary layer dynamics, which dictate wind speed profiles and turbulence levels near the surface.

Property	Boundary Layer	Free Atmosphere
Turbulence	Almost continuously turbulent	Sporadic, CAT, turbulence within clouds
Friction	Strong drag due to surface interaction	Small viscous dissipation
Dispersion	Rapid turbulent mixing	Small molecular diffusion
Winds	Near logarithmic profile	Geostrophic winds
Vertical Transport	Turbulent vertical motion	Horizontal transport by mean wind
Thickness	100m - 3km (variable)	8-16km (less variable)

Table 1. Comparison of Boundary Layer and Free Atmosphere Properties

4. Lecture 4 23/01/2025

4.1 Statistical Tools Required For Turbulence
Turbulence is characteristiced by randomness.

$$U = \bar{u} + u'$$
$$V = \bar{v} + v'$$
$$W = \bar{w} + w'$$
$$C = \bar{c} + c'$$

4.1.1 Mean

$$\bar{A} = \frac{1}{N}\sum_{i=1}^{N} A(i,s)$$

$$\bar{A} = \frac{1}{T}\int_{t=0}^{T} A(t,s)\,dt$$

4.1.2 Rules for averaging
If A and B are variables dependent on time, then:

$$\overline{A+B} = \bar{A} + \bar{B},$$
$$\bar{\bar{A}} = A,$$
$$\overline{\bar{A}\cdot B} = \bar{A}\cdot\bar{B},$$
$$\overline{\frac{dA}{dt}} = \frac{d\bar{A}}{dt}.$$

Reynold's averaging rule

$$\bar{\bar{A}} = \overline{\bar{A}+a'} = \bar{A}, \text{since } \bar{a'}=0$$
$$\overline{A\cdot B} = \overline{(\bar{A}+a')(\bar{B}+b')} = \bar{A}\bar{B} + \overline{(a'b')},$$
$$\overline{(a'b')} \neq \overline{a'b'}$$
$$\overline{a'^2} \neq 0$$
$$\overline{b'^2} \neq 0$$

4.1.3 Variance

$$\sigma_A^2 = \frac{1}{N}\sum_{i=1}^{N}(A_i - \bar{A})^2 = \overline{a'^2}$$

4.1.4 Standard deviation

$$\sigma_A = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(A_i - \bar{A})^2} = \left(\overline{a'^2}\right)^2$$

4.1.5 Covariance

$$\sigma_{A,B} = \frac{1}{N}\sum_{i=1}^{N}(A_i - \bar{A})(B_i - \bar{B}) = \overline{a'b'}.$$

4.1.6 Correlation

$$\gamma_{A,B} = \frac{\overline{a'b'}}{\sigma_A\sigma_B}$$

Mean Kinetic Energy (MKE) $= \frac{1}{2}(\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$

Turbulent Kinetic Energy (TKE) $= \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$

> **Question 4.1:**
>
> Suppose we erect instruments with an anemometer to measure u and w components, recording wind speeds every 6 seconds for a minute, resulting in the following 10 points of wind observations shown in Table 2. Calculate the mean, variance, and standard deviation for each component. Also, find the covariance and correlation between them.

u (m/s)	5	6	5	4	7	5	3	5	4	6
w (m/s)	0	-1	1	0	-2	1	2	-1	1	-1

Table 2. Wind observations for u and w components.

> **Answer 4.1:**
>
> Solution:
>
> $$\bar{u} = 5, \qquad \bar{v} = 0$$
> $$\sigma_U^2 = 1.2, \qquad \sigma_W^2 = 1.1$$
> $$\sigma_U = \sqrt{1.2}, \qquad \sigma_W = \sqrt{1.1}$$
> $$\sigma_{U,W} = \overline{u'w'} = -1.1, \qquad \gamma_{U,W} =$$

5. Lecture 5 24/01/2025

5.1 Fluxes

Mass, Heat, Moisture, Momentum, Pollutent, etc.

Quantity	Unit
Mass	kg_{air}/m^2s
Heat	J/m^2s
Moisture	kg_{wv}/m^2s
Momentum	$kgms^{-1}/m^2s$
Pollutant	$kg_{pollutant}/m^2s$

5.2 Kinematic Flux

Note: We asssume atmoshpere to be of constant density (ρ_{air}).

Flux Quantity	Formula	Unit
Mass flux	$\frac{mass}{\rho_{air}}$	kg_{air}/m^2s
Heat flux	$\frac{heat}{\rho C_p}$	J/m^2s
Moisture flux	-	kg_{wv}/m^2s
Momentum flux	-	$kgms^{-1}/m^2s$
Pollutant flux	-	$kg_{pollutant}/m^2s$

Examples:-

$$\text{Veritcal advective heat flux} = \overline{w}\,\overline{\theta}$$

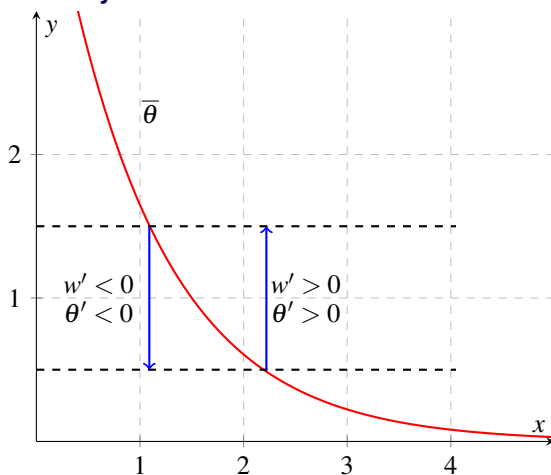$$\text{x-direction advective heat flux} = \overline{u}\,\overline{\theta}$$

$$\text{x-direction moisture flux} = \overline{u}\,\overline{q}$$

$$\text{Veritcal eddy heat flux} = \overline{w'\theta'}$$
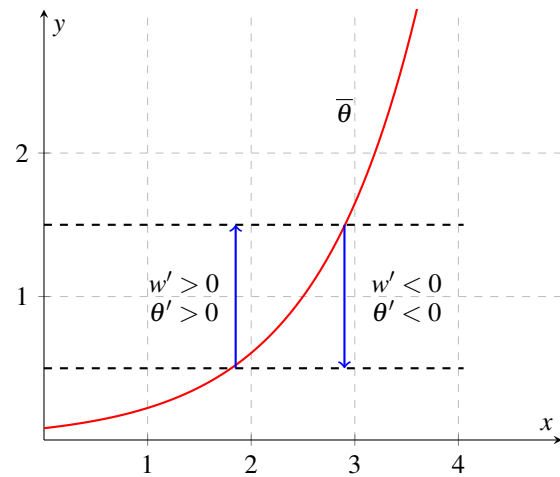
$$\text{x-direction eddy flux} = \overline{u'q'}$$

$$\text{x-direction solar flux} = \left(\frac{1/4 S_0}{c_p \rho}\right) = 0.2773 \text{ Km/s}$$

5.3 Eddy Flux

$$\Delta w' = 0$$
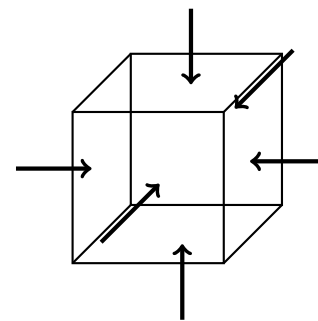$$\Delta(\overline{w'\theta'}) > 0$$
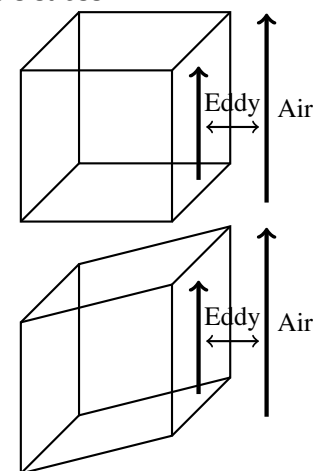

$$\Delta w' = 0$$
$$\Delta(\overline{w'\theta'}) < 0$$

5.4 Stress

Force tending to produce deformation in a body is called **Stress**.

5.4.1 Reynold's stress

6. Lecture 6 29/01/2025

6.1 Summation Notation

Definition: Integer variables m, n, and q can take values $1, 2, 3$. Then:

- A_m is a velocity vector.
- X_m represents distance.

where,

$$m = 1, 2, 3,$$
$$n = 1, 2, 3,$$
$$q = 1, 2, 3.$$

Hence,

$A_1 = u,$	$A_2 = v,$	$A_3 = w,$
$X_1 = x,$	$X_2 = y,$	$X_3 = z.$

6.2 Unit vector

Unit vector is represented by δ_m,
where,

$$\delta_1 = \hat{i},$$
$$\delta_2 = \hat{j},$$
$$\delta_3 = \hat{k}$$

6.3 Kronecker Delta

Kronecker Delta is represented by δ_{mn}, such that,

$$\delta_{mn} = \begin{cases} 1, & \text{for } m = n, \\ 0, & \text{for } m \neq n \end{cases}$$

6.4 Alternating Unit Tensor

Alternating Unit Tensor is represented by ε_{mn}, such that,

$$\delta_{mn} = \begin{cases} +1, & \text{for } mnq = 123, 231, 312 \\ -1, & \text{for } mnq = 321, 132, 213 \\ 0, & \text{for any 2 or more indices alike} \end{cases}$$

6.5 Rules For Summation Notation

Rule I:

Wherever two identical appear same in one term, it is implied that there is a sum of that term over each value of repreated index.

For example:

1.
$$A_n \frac{\partial B_m}{\partial X_n} = A_1 \frac{\partial B_m}{\partial X_1} + A_2 \frac{\partial B_m}{\partial X_2} + A_3 \frac{\partial B_m}{\partial X_3}$$
$$= u \frac{\partial B_m}{\partial x} + v \frac{\partial B_m}{\partial y} + w \frac{\partial B_m}{\partial z}$$

2.
$$\delta_{2n} A_n = \delta_{21} A_1 + \delta_{22} A_3 + \delta_{23} A_3$$
$$= A_2$$
$$= v$$

Rule II:

Wherever one index appear unsummed in one term, then the same index appear unsummed in all 3 terms in that equation. Hence that equation effectively represents 3 equations for each value of unsummed index.

$$A_m = B_m + \delta_{mn} C_n$$
$$A_1 = B_1 + \delta_{1n} C_n = B_1 + C_1$$
$$A_2 = B_2 + \delta_{2n} C_n = B_2 + C_2$$
$$A_2 = B_3 + \delta_{3n} C_n = B_3 + C_3$$

For example:

1.
$$\frac{\partial A_m}{\partial t} + B_n \frac{\partial A_m}{\partial X_n} = -\delta_{m3} g + f \varepsilon_{mn3} B_n$$
$$- \frac{1}{\rho} \frac{\partial P}{\partial X_m} + \frac{1}{\rho} \left[\frac{\tau_{mn}}{\partial X_n} \right]$$

Expanding with $n = 1, 2$ and 3, we get:

$$\frac{\partial A_m}{\partial t} + B_1 \frac{\partial A_m}{\partial X_1} + B_2 \frac{\partial A_m}{\partial X_2} + B_3 \frac{\partial A_m}{\partial X_3} =$$
$$- \delta_{m3} g - f [\varepsilon_{m13} B_1 + \varepsilon_{m23} B_2 + \varepsilon_{m33} B_3]$$
$$- \frac{1}{\rho} \left[\frac{\partial P}{\partial X_m} \right] + \frac{1}{\rho} \left[\frac{\tau_{m1}}{\partial X_1} + \frac{\tau_{m2}}{\partial X_2} + \frac{\tau_{m3}}{\partial X_3} \right]$$

Now expanding with $m = 1$, we get following equation:

$$\frac{\partial A_1}{\partial t} + B_1 \frac{\partial A_1}{\partial X_1} + B_2 \frac{\partial A_1}{\partial X_2} + B_3 \frac{\partial A_1}{\partial X_3} =$$
$$- \delta_{13} g + f [\varepsilon_{113} B_1 + \varepsilon_{123} B_2 + \varepsilon_{133} B_3]$$
$$- \frac{1}{\rho} \left[\frac{\partial P}{\partial X_1} \right] + \frac{1}{\rho} \left[\frac{\tau_{11}}{\partial X_1} + \frac{\tau_{12}}{\partial X_2} + \frac{\tau_{13}}{\partial X_3} \right]$$
$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = +fv - \frac{1}{\rho} \left[\frac{\partial P}{\partial x} \right]$$
$$+ \frac{1}{\rho} \left[\frac{\tau_{xx}}{\partial x} + \frac{\tau_{xy}}{\partial y} + \frac{\tau_{xz}}{\partial z} \right]$$

Now expanding with $m = 2$, we get following equation:

$$\frac{\partial A_2}{\partial t} + B_1 \frac{\partial A_2}{\partial X_1} + B_2 \frac{\partial A_2}{\partial X_2} + B_3 \frac{\partial A_2}{\partial X_3} =$$
$$- \delta_{23} g + f [\varepsilon_{213} B_1 + \varepsilon_{223} B_2 + \varepsilon_{233} B_3]$$
$$- \frac{1}{\rho} \left[\frac{\partial P}{\partial X_2} \right] + \frac{1}{\rho} \left[\frac{\tau_{21}}{\partial X_1} + \frac{\tau_{22}}{\partial X_2} + \frac{\tau_{23}}{\partial X_3} \right]$$
$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -fu - \frac{1}{\rho} \left[\frac{\partial P}{\partial y} \right]$$
$$+ \frac{1}{\rho} \left[\frac{\tau_{yx}}{\partial x} + \frac{\tau_{yy}}{\partial y} + \frac{\tau_{yz}}{\partial z} \right]$$

Now expanding with $m = 3$, we get following equation:

$$\frac{\partial A_3}{\partial t} + B_1 \frac{\partial A_3}{\partial X_1} + B_2 \frac{\partial A_3}{\partial X_2} + B_3 \frac{\partial A_3}{\partial X_3} =$$

$$- \delta_{33} g + f \left[\varepsilon_{313} B_1 + \varepsilon_{323} B_2 + \varepsilon_{333} B_3 \right]$$

$$- \frac{1}{\rho} \left[\frac{\partial P}{\partial X_3} \right] + \frac{1}{\rho} \left[\frac{\tau_{31}}{\partial X_1} + \frac{\tau_{32}}{\partial X_2} + \frac{\tau_{33}}{\partial X_3} \right]$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -g - \frac{1}{\rho} \left[\frac{\partial P}{\partial z} \right]$$

$$+ \frac{1}{\rho} \left[\frac{\tau_{zx}}{\partial x} + \frac{\tau_{zy}}{\partial y} + \frac{\tau_{zz}}{\partial z} \right]$$

7. Lecture 7 30/01/2025

7.1 Governing Equation For Turbulent Flow
1. Mean variable $(\overline{u}, \overline{v}, \overline{w})$.
2. Flux variable $(\overline{u'w'}, \overline{w'\theta'})$.
3. Total Kinetic Energy $(\overline{u'^2}, \overline{v'^2}, \overline{w'^2})$.

Steps:
1. Identify the basic equations.
2. Expand the total derivative into local and advective term.
3. Expand dependent variables into mean and turbulent parts.
4. Apply Reynold's averaging.
5. Add continuity equation to put the result in flux part.

References

[1] R. B. Stull. *An Introduction to Boundary Layer Meteorology*. Atmospheric Sciences Library. Kluwer Academic Publishers, 1988.