Lecture Notes on Numerical Weather Prediction

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Abstract
Notes of Lectures and addional information from books.

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1. Lecture 1 06/01/2025

Numerical Weathering Problem (NWP) was first proposed by Bjerkives around 1900. It is mathematical initial value problem (IVP).

Initial value Problem (IVP) $\rightarrow$ simple pendulum.

$$\ddot{\theta} + \omega^2 \theta = 0 \tag{1}$$

$$\frac{d^2\theta}{dt^2} + \omega^2 \theta = 0 \tag{2}$$

$$\theta(t) = A\cos(\omega t) + B\sin(\omega t) \tag{3}$$

Eq.(1) and (2) are second order linear ordinary differential equation, whose solution Eq.(3) has 2 constants of integration A and B. Here θ and t are tge dependent and independent variable since Eq.(1) and (2) have only one independent variable.

Values of A and B will depend on initial condition.

Since ODE is second order, 2 initial condition are needed at initial time, say $t = 0$. Which are:

$$\left.\begin{aligned} \theta(t=0) &= 1 \\ \frac{\theta(t=0)}{dt} &= 0 \end{aligned}\right\} \tag{4}$$

Eq.(2) and initial conditions Eq.(4) are together called **Mathematical IVP**. For any physical system the following two requirements are needed:

1. The equation (ODE or PDE) that governs the evolution of the above system.
2. The initial state of the system.

7 independent variables **(u,v,w,T,ρ,p,q)**.

Surface area of Earth = $4\pi R^2$ = $4\pi(6.37 \times 10^{12}) \approx 5.1 \times 10^{14}$ m^2

2. Lecture 2 07/01/2025

7 independent variables **(u,v,w,T,ρ,p,q)** therefore we need 7 Governing equations (system of 7 coupled non-linear partial differential equations):

1. Conservation of masss (continuity equation).
2. Conservation of momentum in rotating frame of refence (3 scalar equations, one each corresponding to scalar component of velocity).
3. Conservation of energy (Thermodynamic energy equation).
4. Conservation of moisture (moisture continuity equation).
5. Equation of state (Ideal gas equation).

Euler discription of fluid motion is more convinent becasue of dependance on time and above 7 equations.

Total advective and convective time of lagrangian is given by:

$$\underbrace{\frac{DT}{Dt}}_{\text{Lagrangian Derivative}} = \underbrace{\underbrace{\frac{\partial T}{\partial t}}_{\text{Local derivative}} + \underbrace{\vec{V}\cdot\nabla T}_{\text{Advective Term}}}_{\text{Euler Derivative}}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \tag{5}$$