

HEADER**BEHAVIOR**

$$\text{Stress } (\sigma) = \frac{F}{A_0}$$

$$\text{Strain } (\epsilon) = \frac{\Delta L}{L_0}$$

$$\text{Young's Modulus } (E) = E = \frac{\sigma}{\epsilon} \text{ (in elastic region)}$$

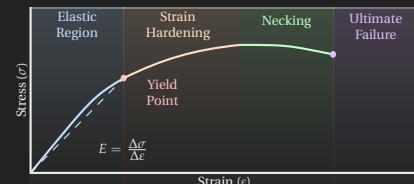
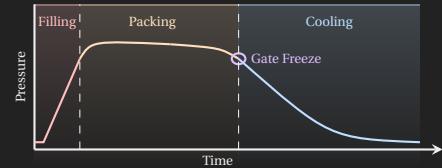
$$\text{Shear Modulus } (G) = G = \frac{\tau}{\gamma}$$

$$\text{Poisson's Ratio } (\nu) = \nu = -\frac{\epsilon_{\perp}}{\epsilon_{\parallel}}$$

$$\text{True Stress } (\sigma_{\text{true}}) = \sigma_{\text{true}} = \frac{F}{A_{\text{instantaneous}}}$$

$$\text{True Strain } (\epsilon_{\text{true}}) = \epsilon_{\text{true}} = \ln \frac{L}{L_0}$$

$$\text{Stress Relaxation: } \sigma(t) = \sigma_0 e^{-\frac{t}{\tau}}$$

**RHEOLOGY**

$$\text{Shear rate: } \dot{\gamma} = \frac{u_0}{h} \quad \text{Shear stress: } \tau = \eta \dot{\gamma}$$

$$\text{Viscosity } (\eta) = \eta = \frac{\tau}{\dot{\gamma}}$$

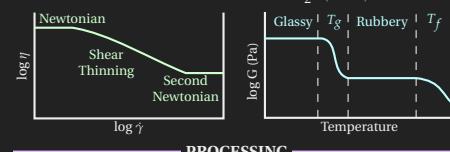
$$\text{Deborah Number (De): } De = \frac{\lambda}{t_{\text{obs}}} \quad \text{where } \lambda = \text{relaxation time}$$

$$\text{Storage Modulus } (G') = G' = \frac{\sigma_0}{\dot{\gamma}_0} \cos(\delta)$$

$$\text{Loss Modulus } (G'') = G'' = \frac{\sigma_0}{\dot{\gamma}_0} \sin(\delta)$$

$$\text{Stress Relaxation: } \sigma(t) = \sigma_0 e^{-t/\tau}$$

$$\text{Time-Temperature Superposition: } \log(a_T) = -\frac{C_1(T-T_r)}{C_2+(T-T_r)}$$

**PROCESSING**

$$\text{Capillary Rheometer: } \dot{\gamma} = \frac{4Q}{\pi R^3}, \quad \tau = \frac{(P_0 - P_L)R}{2L}$$

$$\dot{\gamma} = \text{shear rate, } Q = \text{volumetric flow rate, } R = \text{die radius, } L = \text{die length, and } (P_0 - P_L) = \text{pressure drop.}$$

$$\text{Rotational Rheometer Cone & Plate: } \dot{\gamma} = \frac{\Omega}{\theta_0}, \quad \tau = \frac{-3T}{2\pi R^3}, \quad \eta = \frac{\tau}{\dot{\gamma}} = \frac{-3T\theta_0}{2\pi R^3\Omega}$$

$$\Omega = \text{angular velocity, } \theta_0 = \text{cone angle, } T = \text{torque, } R = \text{plate radius.}$$

$$\text{Rotational Rheometer Parallel Plates: } \dot{\gamma} = \frac{T\Omega}{H}, \quad \tau = \frac{3T}{2\pi R^3}, \quad \eta = \frac{\tau}{\dot{\gamma}} = \frac{T}{\gamma} = \frac{3TH}{2\pi R^3\Omega}$$

$$r = \text{radial position, } H = \text{gap height.}$$

$$\text{Gibbs Free Energy for Blends: } \Delta G = \Delta H - T\Delta S, \quad \Delta H = \nu(\delta_1 - \delta_2)^2 \phi_1 \phi_2$$

$$\nu = \text{molar volume, } \delta = \text{solubility parameters, } \phi = \text{volume fractions.}$$

$$\text{Melt Flow Index (MFI): } \text{Mass of polymer extruded under standard conditions } [g/10 \text{ min}]$$

DIMENSIONAL ANALYSIS

Buckingham Pi Theorem: Every system with m physical quantities reduced to $m-n$ dimensionless groups, $n = \text{number of basic dimensions}$

Basic Dimensions [MLT⁰]:

Length (L): m Mass (M): kg Time (T): s Temp (Θ): K

Matrix Method: (1) Select n dimension core matrix (repeating vars)

(2) Form [L,M,T] rows x vars cols matrix

(3) Solve for Pi groups: $\Pi_1 = f(\Pi_2, \Pi_3, \dots)$

Key Dimensionless Numbers:

Deborah Number: $De = \frac{\text{Material relax time}}{\text{Process time}}$

$De \rightarrow 0$: Viscous fluid, $De \rightarrow \infty$: Elastic solid

Biot Number: $Bi = \frac{\text{Surface convection}}{\text{Internal conduction}} = \frac{hL}{k}$

$Bi \ll 1$: $T_c \approx T_s$ (uniform temp)

$Bi \gg 1$: $T_s \approx T_\infty$ (surface controlled)

Scaling Laws:

Length: $L \propto \text{size}$

Area: $A \propto L^2$

Volume/Weight: $V, W \propto L^3$

Stress: $\sigma = W/A \propto L$

Similarity Types:

Geometric: shape ratios same

Kinematic: velocity ratios same

Dynamic: force ratios same

MATRIX TRANSFORMATION METHOD EXAMPLE

Consider pipe flow with pressure drop (Δp), diameter (D), length (L), viscosity (η), density (ρ), velocity (u)

Step 1: List Variables with Dimensions

Variable	Dimension	Variable	Dimension
D	L	Δp	$ML^{-1}T^{-2}$
ρ	ML^{-3}	L	L
u	LT^{-1}	η	$ML^{-1}T^{-1}$

Step 2: Select Core Matrix (n=3 basic dimensions)

Choose D, ρ, u as repeating variables

Step 3: Form Dimensional Matrix

	D	ρ	u	Δp	L	η
L	1	-3	1	-1	1	-1
M	0	1	0	1	0	1
T	0	0	-1	-2	0	-1

Step 4: Solve for Π Groups

For Π_1 using Δp :

$$[D^a \rho^b u^c \Delta p] = [M^0 L^0 T^0]$$

$$L : \quad a - 3b + c - 1 = 0$$

$$M : \quad b + 1 = 0$$

$$T : \quad -c - 2 = 0$$

Similarly for remaining groups:

$$\Pi_2 = \frac{L}{D}$$

$$\Pi_3 = \frac{\eta}{D \rho u} = \frac{1}{Re}$$

Final Result:

$$f\left(\frac{\Delta p}{\eta}, \frac{L}{D}, \frac{\eta}{D \rho u}\right) = 0$$

Solving gives: $\Pi_1 = \frac{\Delta p}{\eta}$

DESIGN OF EXPERIMENTS**Seven-Step Process:**

- Identify Factors/Metrics
- Formulate Objective
- Design Experiment
- Run Trials
- Analyze Results
- Select Setpoints
- Iterate/Validate

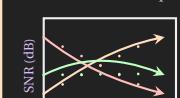
Control Factors:

Temperature, Pressure, Time
Material Properties, Geometry
Process Parameters

Noise Factors:

Manufacturing Variance
Environmental Conditions
User/Operation Variations

Signal-to-Noise Ratio Analysis: Signal-to-Noise Ratio (SNR) measures the relationship between response values and their variation across sample size.



$$\eta_L = -10 \log_{10} \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right)$$

$$\eta_S = -10 \log_{10} \left(\frac{1}{n} \sum_{i=1}^n y_i^2 \right)$$

$$\eta_N = 10 \log_{10} \left(\frac{y^2}{s^2} \right)$$

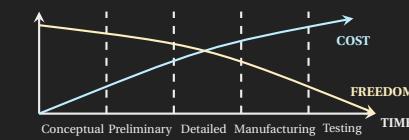
L9 Orthogonal Array Experimental design matrix enabling efficient study of multiple factor effects with minimal trials.

	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1



Response Surface Analysis Maps relationship between input factors and system response for optimization.

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j} \beta_{ij} x_i x_j + \epsilon$$



From continuity: $(U/L_x) \sim (V/L_y)$. If $L_x \gg L_y$, then $U \gg V$.

Momentum in x-dir: $\rho U^2 / L_x \sim \Delta p / L_x \sim \mu U / L_x^2 \sim \mu U / L_y^2$.

Dominance depends on $\frac{L_x}{L_y}$ and $Re = \frac{\rho UL_x}{\mu}$. For small Re and large L_x/L_y , viscous terms ($\mu U / L_y^2$) dominate over inertial ($\rho U^2 / L_x$).

Second derivative: $\partial^2 U / \partial x^2 \sim U / L_x^2$ vs. squared gradient: $(\partial U / \partial x)^2 \sim (U / L_x)^2$. Scaling guides term retention.

Geometric Parameters:**Width Ratio:**

$$\frac{W}{L} \sim O(1)$$

Channel Aspect:

$$\frac{h}{W} \ll 1$$

Flow Analysis:**Continuity:**

$$\frac{U}{L_x} \sim \frac{V}{L_y}$$

Process-Specific Scaling:**Injection Molding:**

$$Re \ll 1, \epsilon \ll 1$$

Film Casting:

$$We = \frac{\rho u^2 h}{\sigma} \gg 1$$

Critical Numbers:

$$Re \left(\frac{L_y}{L_x} \right)$$

$$Re \left(\frac{L_y}{L_x} \right)$$

Process-Specific Scaling:**Injection Molding:**

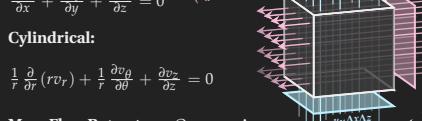
$$Re \ll 1, \epsilon \ll 1$$

Film Casting:

$$We = \frac{\rho u^2 h}{\sigma} \gg 1$$

CONTINUITY**MASS CONSERVATION****Cartesian:**

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

**Mass Flow Rate:**

$$\dot{m} = \rho Q = \rho u \cdot A$$

Continuity (Incompressible):

$$\nabla \cdot \mathbf{u} = 0$$

Continuity (Compressible):

$$\frac{\partial}{\partial x} (\rho u_x) + \frac{\partial}{\partial y} (\rho u_y) + \frac{\partial}{\partial z} (\rho u_z) = 0$$

MOMENTUM CONSERVATION

$$\rho c_v \left(\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right)$$

$$= k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + 2\mu \left(\left(\frac{\partial u_x}{\partial x} \right)^2 + \left(\frac{\partial u_y}{\partial y} \right)^2 + \left(\frac{\partial u_z}{\partial z} \right)^2 \right)$$

$$+ \mu \left(\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right)^2 + \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)^2 + \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right)^2 \right) + Q$$

$$\rho C_p \frac{dT}{dt} = -\nabla \cdot \mathbf{q} + Q + \dot{Q}_{\text{viscous heating}}$$

Material Derivative:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T$$

$$\mathbf{q} = -k \nabla T$$

Heat Conduction:

$$-\nabla \cdot \mathbf{q} = k \nabla^2 T = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

Simple Shear Heating:

$$\dot{Q}_{\text{viscous heating}} = \eta \left(\frac{\partial u}{\partial y} \right)^2$$

ORDER OF MAGNITUDE**FUNDAMENTAL ASSUMPTIONS AND REDUCTIONS****Assumption / Approach**

$$\text{Steady State: } \frac{\partial}{\partial t} = 0$$

$$\frac{\partial}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u}$$

$$\text{Constant Material Properties: } \rho = \text{const}, \mu = \text{const}$$

$$\frac{\partial \rho}{\partial t} = 0, \frac{\partial \mu}{\partial t} = 0$$

$$\Rightarrow \nabla \cdot \mathbf{u} = 0$$

$$\text{Fully Developed Flow: } \frac{\partial u_x}{\partial x} = 0$$

$$\frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

$$\frac{\partial u_x}{\partial z} = 0, u_z = 0$$

$$\frac{\partial u_x}{\partial z} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0 \text{ with } u_z = 0$$

$$\text{Symmetry (Independent of Width): } \frac{\partial u_x}{\partial z} = 0$$

$$\frac{\partial u_x}{\partial z} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = 0 \Rightarrow 0 =$$

$$-\nabla p + \mu \nabla^2 \mathbf{u}$$

$$\text{Constant Material Properties (Reiterated): } \nabla \cdot \mathbf{u} = 0, 0 = -\nabla p + \mu \nabla^2 \mathbf{u}$$

$$\text{Lubrication Approximation: } \frac{\partial p}{\partial x} \gg \frac{\partial p}{\partial y}$$

$$-\nabla p = -\frac{\partial p}{\partial x} \hat{i} - \frac{\partial p}{\partial y} \hat{j}$$

LUBRICATION**Velocity Profile:**

$$u(y) = \frac{1}{2} \left(\frac{dp}{dx} \right) (y^2 - hy)$$

Flow Rate:

$$q = -\frac{h^3}{12\mu} \left(\frac{dp}{dx} \right)$$

TEXTBOOK

EXAMPLE 9.1: FLOW IN A TUBE

Consider the classical problem of pressure drop during flow in a smooth straight pipe, ignoring the inlet effects. The first step is to list all possible variables or quantities that are related to the problem under consideration. In this case, we have:

Target quantity: Pressure drop Δp

Geometric variables: Pipe diameter D , and pipe length L

Physical or material properties: viscosity η , density ρ

Process variable: average fluid velocity u

If we choose D , u , and ρ as the repeating variables, the dimensional matrix is written as:

$$\begin{matrix} M & D & u & \rho & \Delta p & L & \eta \\ L & 0 & 1 & -3 & -1 & 1 & -1 \\ T & 0 & -1 & 0 & -2 & 0 & -1 \end{matrix} \quad (1)$$

After reducing the core matrix to an identity matrix, the dimensional matrix becomes:

$$\begin{matrix} M & D & u & \rho & \Delta p & L & \eta \\ M & 1 & 0 & 0 & 0 & 1 & 1 \\ L & 0 & 1 & 0 & 2 & 0 & 1 \\ T & 0 & 0 & 1 & 1 & 0 & 1 \end{matrix} \quad (9.8)$$

which results in the 3 dimensionless groups,

$$\Pi_1 = \frac{\Delta p}{u^2 \rho} = Eu \quad (\text{Euler number}) \quad (9.9)$$

$$\Pi_2 = \frac{L}{D} \quad (2)$$

$$\Pi_3 = \frac{\eta}{Du\rho} = Re^{-1} \quad (\text{Reynolds number}) \quad (3)$$

The following relationship can be written:

$$f\left(Eu, Re, \frac{L}{D}\right) = 0 \quad (9.10)$$

which, of course, by itself cannot produce the nature of the relation; however, the form of the function f can be generated experimentally.

DIMENSIONLESS VARIABLES

Name	Symbol	Definition	Meaning
Biot	Bi	$\frac{hL}{k}$	Convection Conduction
Brinkman	Br	$\frac{\eta u^2}{k\Delta T}$	Viscous Conduction
Capillary	Ca	$\frac{\tau R}{\sigma_s}$	Deviatoric Surface
Damköhler	Da	$\frac{cAHr}{\rho c_p T_0}$	Reaction Internal
Deborah	De	$\frac{\lambda}{t}$	Relaxation Process
Fourier	Fo	$\frac{\alpha t}{L^2}$	Process Thermal
Graetz	Gz	$\frac{uL}{\alpha} \left(\frac{d}{L} \right)$	Convection Conduction
Nusselt	Nu	$\frac{hL}{k_f luid}$	Convective Conductive
Péclet	Pe	$\frac{UL}{\alpha}$	Advection Diffusion
Prandtl	Pr	$\frac{v}{\alpha}$	Momentum Thermal
Reynolds	Re	$\frac{\rho u L}{\eta}$	Inertia Viscous
Schmidt	Sc	$\frac{v}{D}$	Mechanical Diffusion
Weissenberg	We	$\lambda \gamma$ or $\frac{N_1}{\tau}$	Elastic Viscous

EXAMPLES

PRESSURE-DRIVEN FLOW THROUGH SLIT

Assumptions

The following assumptions are made:

1. The flow is steady, fully developed, and entrance effects are ignored.
2. The fluid is Newtonian and incompressible.
3. The flow is unidirectional, with only one non-zero velocity component u_z .

Governing Equations

The continuity equation for an incompressible flow reduces to

$$\frac{\partial u_z}{\partial z} = 0.$$

The z -momentum equation for a Newtonian, incompressible flow (Navier-Stokes equations) is given by

$$-\frac{\partial p}{\partial z} + \mu \frac{\partial^2 u_z}{\partial y^2} = 0.$$

The x - and y -momentum components reduce to

$$-\frac{\partial p}{\partial x} = -\frac{\partial p}{\partial y} = 0.$$

This indicates that the pressure depends only on z . Since u_z does not vary with z , the pressure gradient $\frac{\partial p}{\partial z}$ is constant:

$$\frac{\partial p}{\partial z} = \frac{\Delta p}{L}.$$

Simplified Momentum Equation

Substituting the constant pressure gradient into the z -momentum equation:

$$\frac{1}{\mu} \frac{\Delta p}{L} = \frac{\partial^2 u_z}{\partial y^2}.$$

Boundary Conditions

No-slip boundary conditions are applied:

$$u_z \left(\pm \frac{h}{2} \right) = 0.$$

Solution for Velocity Profile

Integrating the simplified momentum equation twice and applying the boundary conditions, the velocity profile is obtained as

$$u_z(y) = \frac{h^2}{8\mu} \frac{\Delta p}{L} \left[1 - \left(\frac{2y}{h} \right)^2 \right].$$

Mean Velocity

The mean velocity in the channel is calculated as

$$\bar{u}_z = \frac{2}{h} \int_0^{h/2} u_z(y) dy = \frac{h^2}{12\mu} \frac{\Delta p}{L}.$$

The volumetric flow rate is then given by

$$Q = hW\bar{u}_z = \frac{Wh^3}{12\mu} \frac{\Delta p}{L},$$

where W is the width of the channel.

SHEAR FLOW AND VISCOSITY HEATING

Assumptions

The following assumptions are made:

1. The material is at 210°C .
2. The plate has a surface area of 100 cm^2 and is moving at a constant speed of $u_{\text{top}} = 1.0 \text{ cm/s}$.
3. The gap between the plates is $h = 0.1 \text{ cm}$.
4. The fluid follows the apparent shear viscosity ($\eta_a = \mu$) vs. shear rate ($\dot{\gamma}$) relationship provided.

Governing Equations

The viscous heating per unit volume is governed by the equation:

$$\Phi_v = \mu \left(\frac{\partial u}{\partial y} \right)^2,$$

where:

Φ_v is the viscous heating per unit volume,

μ is the shear viscosity,

$\frac{\partial u}{\partial y}$ is the velocity gradient.

Shear Rate Calculation

The velocity gradient (shear rate) is given by:

$$\dot{\gamma} = \frac{\partial u}{\partial y} = \frac{u_{\text{top}}}{h}.$$

Substituting the given values:

$$\dot{\gamma} = \frac{1.0 \text{ cm/s}}{0.1 \text{ cm}} = 10 \text{ s}^{-1}.$$

Viscosity from Graph

From the given graph, the viscosity μ corresponding to $\dot{\gamma} = 10 \text{ s}^{-1}$ is approximately:

$$\mu = 0.2 \text{ Pa} \cdot \text{s}.$$

Viscous Heating Calculation

Substituting μ and $\dot{\gamma}$ into the viscous heating formula:

$$\Phi_v = \mu \dot{\gamma}^2 = (0.2)(10)^2 = 20 \text{ W/m}^3.$$

Adiabatic Boundary Condition

With adiabatic boundaries, the heat generated internally is retained, leading to a uniform temperature rise throughout the fluid domain due to symmetry and constant thermal properties.

Conclusion on Temperature Rise

The temperature rise is uniform throughout the fluid domain due to the adi-

abatic boundary conditions and uniform generation of viscous heating.

VISCOSUS HEATING

First, let's find μ from the plot and known parameters.

What is $\dot{\gamma}$ (shear rate)?

$$\dot{\gamma} = \frac{u_0}{h} = \frac{1.0 \text{ (cm/sec)}}{0.1 \text{ (cm)}} = 10 \text{ sec}^{-1}$$

Say,

$$\eta = 1.2 \times 10^5 \text{ g/cm.sec.}$$

Now let's simplify Φ_v :

$$\Phi_v = 2 \left[\left(\frac{\partial u_x}{\partial x} \right)^2 + \left(\frac{\partial u_y}{\partial y} \right)^2 + \left(\frac{\partial u_z}{\partial z} \right)^2 \right] + \left[\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right)^2 + \left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \right)^2 + \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)^2 \right].$$

Only u_x is not zero, and it is a function of y .

$$\Rightarrow \Phi_v = 2 \left(\frac{\partial u_x}{\partial y} \right)^2 \Rightarrow \Phi_v = 2 \left(\frac{u_0}{h} \right)^2 = 2 \times 10^2 \text{ sec}^{-2}.$$

$$\Rightarrow \mu \Phi_v = 1.2 \times 10^7 \left[1.2 \times 10^5 \times 2 \times 10^2 \right] \text{ g/cm.sec.}$$

Units:

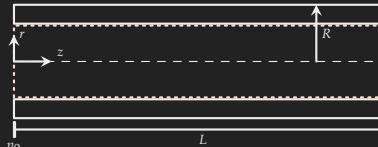
$$\text{Energy per unit time per unit volume: } \frac{\text{g.cm}}{\text{sec}^2} \cdot \frac{\text{cm}}{\text{sec}} \cdot \frac{1}{\text{cm}^3}$$

UNIFORM TEMPERATURE RISE

Since $\frac{\partial u_x}{\partial y}$ (or $\dot{\gamma}$) is uniform throughout the flow domain, so is the viscosity (μ). Thus, $\mu \Phi_v$ should be uniform $\Rightarrow T$ is uniform.

FLOW THROUGH CYLINDRICAL PIPE

Tube flow is encountered in several polymer processes, such as in extrusion dies and sprue and runner systems inside injection molds. When deriving the equations for pressure-driven flow in tubes, also known as Hagen-Poiseuille flow, we assume that the flow is steady, fully developed, with no entrance effects, and axis-symmetric.



Assuming $u_z = u_z(r)$, $u_r = u_\theta = 0$ and $p = p(z)$, the only non-vanishing component of the rate-of-deformation tensor is the rz -component. For generalized Newtonian flow, τ_{rz} is the only non-zero component of the viscous stress. The z -momentum equation then reduces to:

$$\frac{1}{r} \frac{d}{dr} (r \tau_{rz}) = \frac{dp}{dz},$$

where τ_{rz} is a function of r .

From the symmetry argument, $\tau_{rz}(r)$ satisfies:

$$\tau_{rz} = -m \left| \frac{du_z}{dr} \right|^{n-1} \frac{du_z}{dr},$$

where m and n are material-specific parameters for the power-law fluid. The velocity profile is derived as:

$$u_z(r) = \left(\frac{3n+1}{n+1} \right) \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \bar{u}_z,$$

where \bar{u}_z is the mean velocity defined as:

$$\bar{u}_z = \frac{2}{R^2} \int_0^R u_z r dr.$$

Finally, the volumetric flow rate is expressed as:

$$Q = \pi R^2 \bar{u}_z = \left(\frac{n\pi}{3n+1} \right) \left[\frac{R^{n+1}}{2m} \frac{dp}{dz} \right]^{1/n}.$$

PRACTICE PROBLEMS