NUMERICAL ANALYSIS OF A WASTE-WATER TREATMENT PLANT

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Abstract

Thorough analysis of the wastewater treatment process of an oxidation pond to present a mathematical model and obtain the numerical solution. The model investigates the consequences of mPHO (a biological product) on the influent stream. The different variables of effluent streams are calculated using Ordinary Differential Equations (ODE). The solution was implemented by employing Euler's Explicit Method (Level-1) and Runge-Kutta Method (Level-2) of solving the given ODE. The result obtained shows that the proposed equations provide a good approximation of the processes occurring between mPHO and the influent stream. A comparison is made between the influent and effluent stream and dynamics of variables involved are plotted against time (in interval of 70 days).

1. INTRODUCTION:

Day to day human ventures (domestic, industrial) generate plenty of waste that are a cause of effluence and pollution especially in accessible water bodies. To prevent harm to people and the environment, this type of sewage needs to be tackled cautiously. Our primary focus is to propose a mathematical model for the oxidation pond technique which is one of the most popular treatment methods for medium-sized populations and incurs low maintenance cost as compared to other alleged treatment systems.

The use of mathematical modelling to resolve such real-world issues has been around for a while. Examining the water quality of rivers and stabilisation ponds in human settlements is the primary concern. Streeter and Phelps created one of the earliest mathematical models of water quality in 1925 to investigate the relationship between BOD and DO on the River Cam in Eastern England. This model shows how BOD and DO can change over the course of an observation. One study examined the effects of chemicals contaminated with dissolved oxygen on the Tha Chin river stream in Thailand (Pimpunchat et al., 2009). In addition, research has been done to create a mathematical model that will be able to forecast the precise growth rate and biomass concentration of microbes used in wastewater treatment (Abdulkareem, 2004). To examine the coexisting interaction between DO and BOD at river Suswa, India, a Beck modified Khanna Bhutiani model (BMKB model) has been proposed. (Bhutiani & Khanna, 2007). Even though a lot of literature and research work has been compiled on sewage treatment, only few findings compare simulation results to field data such as the one on which our study is based (A Hamzah et al., 2015), cited Taman Timor oxidation pond, the possible reason being the challenges encountered in locating reliable data for the simulation procedure.

This study compares the results generated by the mathematical model utilising experimental data as the foundation. Within the time interval of cited study, a biologically based product called mPHO was injected on a regular basis to increase the efficiency of the oxidation pond technique. Samples were taken at the CP1 (influent and mPHO application) and CP2 (effluent) points. Data comparison (collected at CP1 and CP2) is in agreement that mPHO has a positive impact on lowering the concentration of BOD, COD, and pollutants while raising the concentration of phototrophic bacteria and DO.

2. PROBLEM FORMULATION

For the wastewater treatment process, first order ODEs can be used for mathematical modelling.

Numerous parameters and variables are used for this formulation given as:

$M(t)$ is the concentration of PSB in the pond (mg/litre)

$P(t)$ is the concentration of microbes (Coliform) in the pond (mg/litre).

$D(t)$ is the concentration of chemical oxygen demand in the pond (mg/litre).

$X(t)$ is the concentration of dissolved oxygen in the pond (mg/litre).

m is the concentration of PSB in one litre of mPHO (mg/litre).

$U(t)$ is the amount of mPHO applied to the pond according to the JBMI schedule per litre in 70 days (considered constant = 20).

P_0 is the concentration of microbes (Coliform) at CP1 (mg/litre).

D_0 is the concentration of chemical oxygen demand at CP1 (mg/litre).

X_0 is the concentration of dissolved oxygen at CP1 (mg/litre).

X_{atm} is the saturated oxygen concentration=10 mg/litre.

ν_{in} is the average amount of sewage coming in (litre/day).

ν_p is the volume of the pond in litres.

Four coupled differential equations formulate the model. These equations were used to account for the time-dependent development of the concentrations of four state variables (pollutant – P(t), Dissolved Oxygen – D(t), Chemical Oxygen Demand – D(t) and PSB – M(t)). The rates of change of the concentration of these state variables with time t, are expressed as

$$\frac{dP(t)}{dt} = (c_1 - c_2P(t) - c_3M(t))P(t) + \frac{c_4X(t)P(t)}{X(t) + c_5} + \frac{\nu_{in}}{\nu_p}P_0(t) \qquad \sim(1)$$

$$\frac{dM(t)}{dt} = (c_6 - c_7M(t) - c_8P(t))M(t) + \frac{c_9X(t)M(t)}{X(t) + c_{10}} + c_{11}\frac{\nu_{in}}{\nu_p}mU(t) \qquad \sim(2)$$

$$\frac{dD(t)}{dt} = -(c_{12} - c_{13}M(t))D(t) + \frac{c_{14}X(t)D(t)}{D(t) + c_{15}} + \frac{\nu_{in}}{\nu_p}D_0(t) \qquad \sim(3)$$

$$\frac{dX(t)}{dt} = c_{16}(X_{atm} - X(t) - c_{17}\,X(t)M(t) - c_{18}X(t)P(t) - c_{19}X(t)D(t) + \frac{\nu_{in}}{\nu_p}X_0(t) \qquad \sim(4)$$

the interval of research being (0-70 days).

3. NUMERICAL ANALYSIS

We propose to solve the simultaneous ODEs using the Euler's Explicit method. Each of the four differential equations are coupled, hence slope of function in the formula is initialized using the initial values of the state variables.

In general, the modelled differential equation can be written as (Gear, n.d.):

$$\frac{dY_1(t)}{dt} = S(Y_1, Y_2, Y_3, Y_4)$$

Applying the Euler's explicit method (Jayati Sarkar, n.d.-a),

$$Y_{1,i+1} = Y_{1,i} + hS(Y_{1,i}, Y_{2,i}, Y_{3,i}, Y_{4,i})$$

The values of constants (c_1-c_{19}) present in the proposed mathematical equation were estimated using the data of influent and effluent streams which were collected through sampling from the oxidation pond in 70 days, Using that data, the unknown constants present in the equations (1-4) have been determined. An optimisation algorithm is used in calculating approximated values of the constants (c_1-c_{19}). An initial guess value for each parameter is assigned and the procedure is iteratively repeated till we reach a point of good approximation

$$f(c_1, \ldots, c_{19}) = \sum_{i=1}^{12} \left| P(t_i) - P^*(t_i) \right| + \sum_{i=1}^{12} \left| M(t_i) - M^*(t_i) \right| + \sum_{i=1}^{10} \left| D(t_i) - D^*(t_i) \right| + \sum_{i=1}^{10} \left| X(t_i) - X^*(t_i) \right|. \quad \sim(5)$$

$c_1 = 0.018335$
$c_2 = 0.021041$
$c_3 = 0.024755$
$c_4 = 0.018643$

$c_{11} = 0.000056$
$c_{12} = 0.198528$
$c_{13} = 0.014884$
$c_{14} = 0.025081$

$c_5 = 0.012740$
$c_6 = 0.012418$
$c_7 = 0.026238$
$c_8 = 0.028729$
$c_9 = 0.018214$
$c_{10} = 0.018177$

$c_{15} = 0.018056$
$c_{16} = 0.015532$
$c_{17} = 0.025985$
$c_{18} = 0.015218$
$c_{19} = 0.000853$

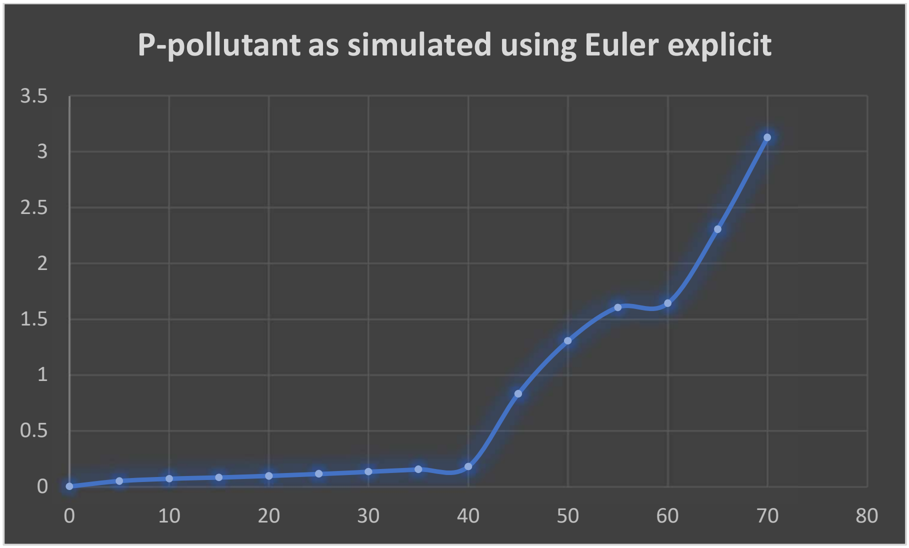
4. RESULTS AND DISCUSSION

t (in days)	Pollutant	PSB	COD	DO
0	0.4	0	180	2.1
5	0.1	0.0015	120	3.6
10	0	0.002	160	3.9
15	0	0.0015	200	3.9
20	0	0	240	3.9
25	0	0.001	210	4.3
30	0	0.0015	150	4.9
35	0	0	200	3.9
40	5.2	0.005	195	4.1
45	3.3	0.005	193	4.3
50	2	0.002	190	4.4
55	0.1	0.016	140	4.6
60	5.4	0.0175	200	4.8
65	8	0.01	310	4.6
70	3.7	0.002	600	4.9

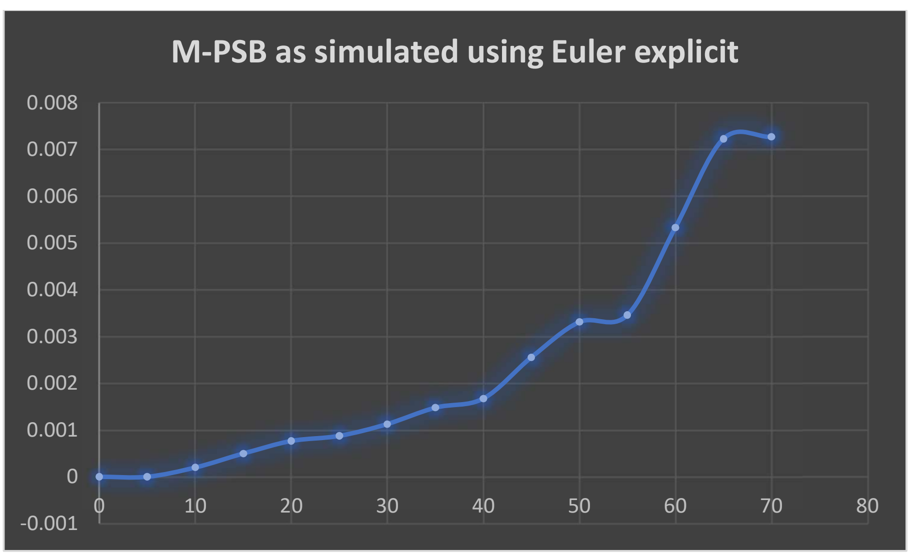
Values of the different state variables as obtained from the experimental data analysed for aforementioned oxidation pond

t (in days)	Pollutant	PSB	COD	DO
0	0	0	100	2.9
5	0.04782	0	216.29	4.72509
10	0.068364	0.000201	145.644	5.06257
15	0.080494	0.000497	192.985	6.93726
20	0.094677	0.000768	241.383	6.08471
25	0.111211	0.000875	289.446	4.74258
30	0.130432	0.001129	253.761	4.3962
35	0.152704	0.001481	181.723	5.8871
40	0.178425	0.001675	241.156	6.23664
45	0.829654	0.002557	235.649	4.92986
50	1.30469	0.003312	233.039	5.19668
55	1.6051	0.003463	229.454	5.14673
60	1.64172	0.005335	169.645	5.35533
65	2.30583	0.007232	240.953	6.90697
70	3.12673	0.007274	373.115	4.33011

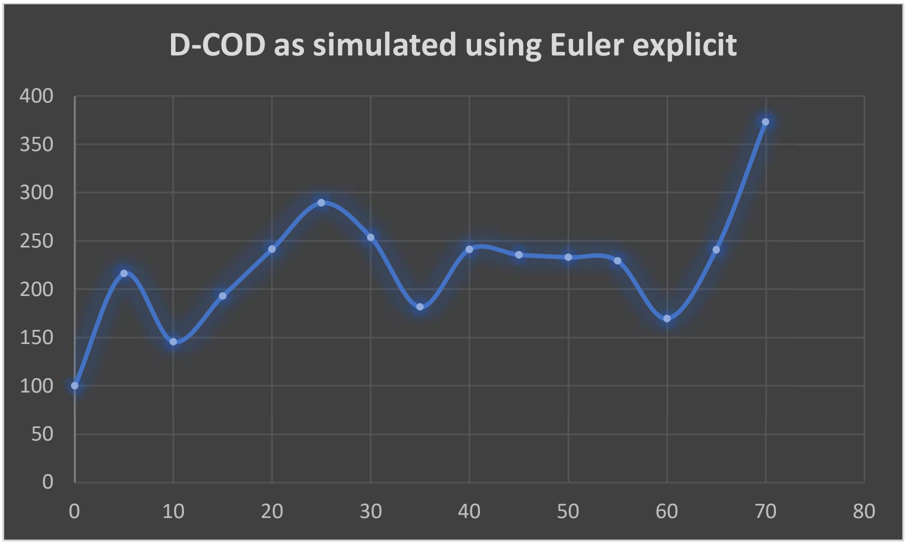
Calculated values of the different state variables as obtained from the Euler's explicit method taking step size as h = 5 (in days)



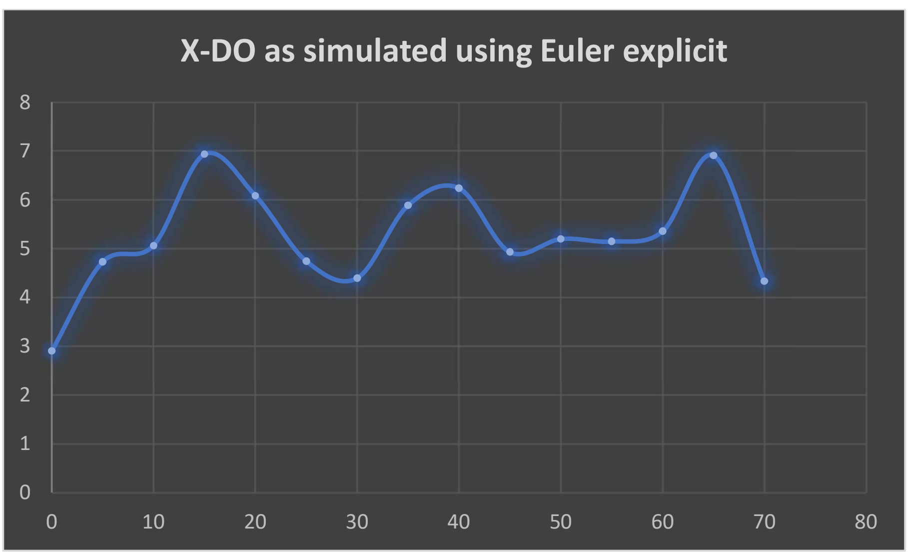
Dynamics of pollutant (coliform) from simulation



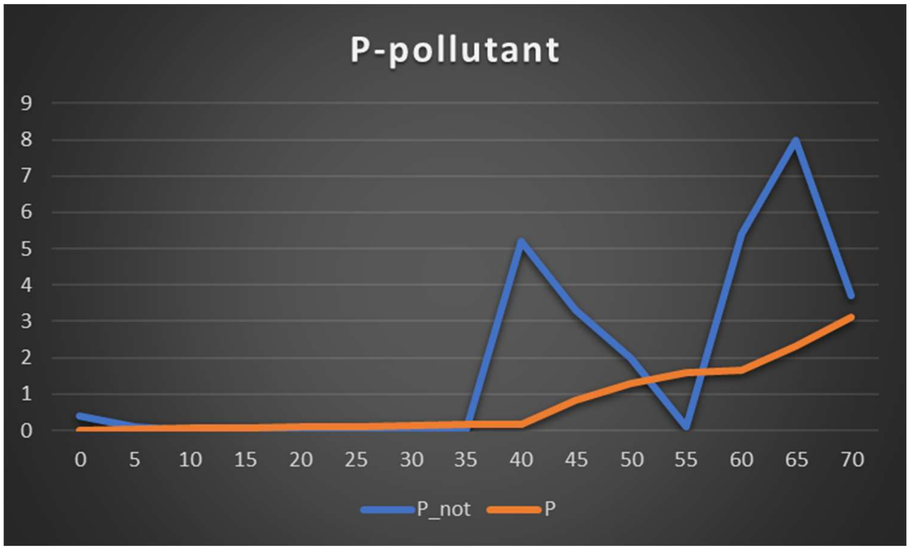
Dynamics of PSB from simulation



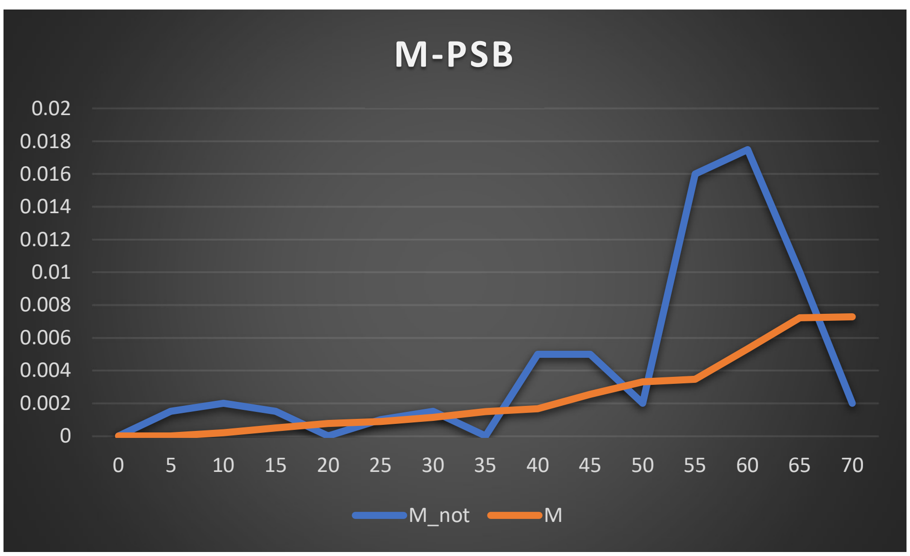
Dynamics of COD from simulation



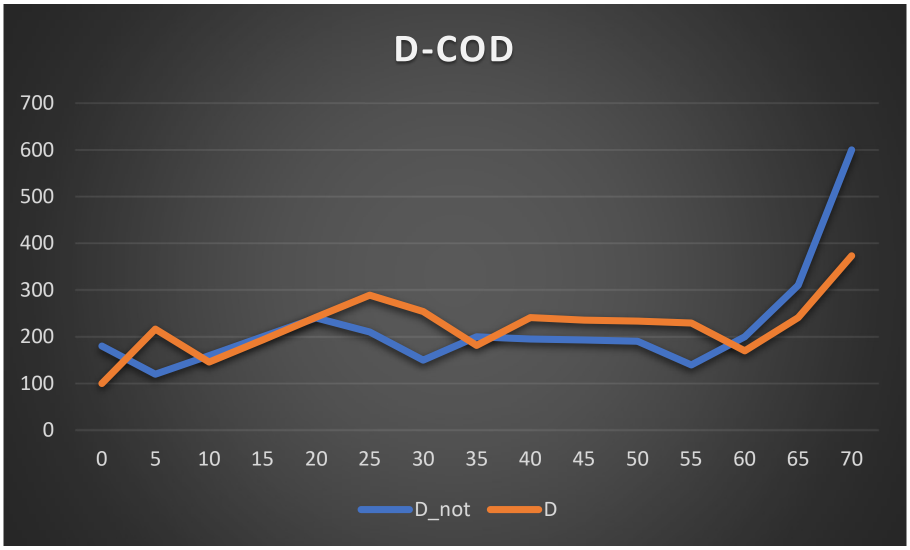
Dynamics of DO from simulation



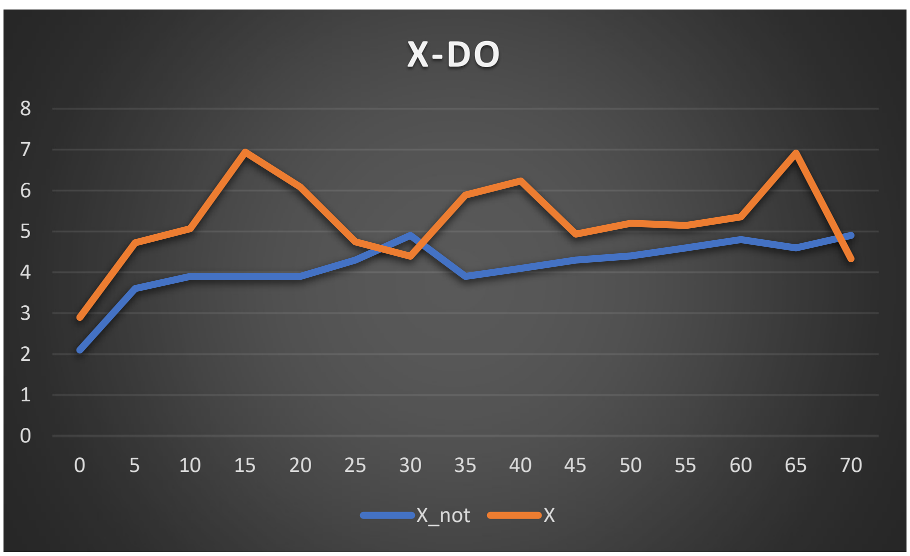
Comparison between dynamics of pollutant for influent stream and effluent stream



Comparison between dynamics of PSB for influent stream and effluent stream



Comparison between dynamics of COD for influent stream and effluent stream



Comparison between dynamics of DO for influent stream and effluent stream

The results that we conclude from comparison graphs of influent and effluent stream for various parameters are mentioned below:

- In the effluent stream, the concentration level of pollutant is less when compared with the concentration of effluent stream which proves the effectiveness of m-PHO in this wastewater treatment model.
- The demand oxygen concentration generally increases throughout the graph, though a sharp dip is being observed which can be explained by the within experimental errors
- The graph of concentration of PSB gives an increasing function which is undesirable. The desired trend is increment of PSB initially and then decrement but observed trend is continuous increment. Euler's explicit method uses previous values for producing newer data points which is the cause of this trend shift.

5. PATH FORWARD

We have solved the four given coupled differential equation using Euler's explicit method taking an interval of five days. From the theory we know that Euler's explicit method has an error of order two, so, for a better approximation we can solve the given four ODE's by using some implicit methods like Crank-Nicholson Method which require further use of some numerical methods to find the next probable value. Also, we can use some better explicit methods like the family of Runge Kutta Methods. The most accurate and efficient of them being the RK-4 method which is of order five. We can also decrease the value of step-size(h) i.e., the time interval between the two observation which eventually result in a more accurate value.

We take the study to the next level by introducing the Runge-Kutta methods (RK-4) which promises to give more accurate results than Euler's explicit method.

RK-4:

(Jayati Sarkar, n.d.-b)

$$Y_{1,i+1} = Y_{1,i} + hS(Y_{1,i}, Y_{2,i}, Y_{3,i}, Y_{4,i})$$

where,

$$S\left(Y_{1,i}, Y_{2,i}, Y_{3,i}, Y_{4,i}\right) = w_1 k_1 + \ w_2 k_2 + \ w_3 k_3 + \ w_4 k_4$$

$$w_1 = \frac{1}{6}, w_2 = \frac{1}{3}, w_3 = \frac{1}{3}, w_4 = \frac{1}{6}$$

$$k_1 = \ S(Y_{1,0}, Y_{2,0}, Y_{3,0}, Y_{4,0})$$
$$k_3 = \ S\left(Y_{1,0} + \frac{h}{2}, Y_{2,0} + \frac{i_1}{2}, Y_{3,0} + \frac{k_1}{2}, Y_{4,0} + \frac{l_1}{2}\right)$$
$$k_3 = \ S\left(Y_{1,0} + \frac{h}{2}, Y_{2,0} + \frac{i_2}{2}, Y_{3,0} + \frac{k_2}{2}, Y_{4,0} + \frac{l_2}{2}\right)$$
$$k_4 = \ S\left(Y_{1,0} + h, Y_{2,0} + i_3, Y_{3,0} + k_3, Y_{4,0} + l_3\right)$$

where i, k, l are corresponding values in Y_2, Y_3, and Y_4 for k_2, k_3 and k_4.(Atkinson et al., n.d.)

6. <u>CONCLUSION</u>

We have studied the wastewater treatment process and solved the mathematical model for numerical solution. The treatment model examines the effectiveness of mPHO (a biological product) on the influent stream. The different variables of effluent streams are calculated using Ordinary Differential Equations. The solution implemented was solved using Euler's Explicit Method of solving the given ODE. From the numerical analysis, we conclude that the biological treatment using mPHO is effective in improving the quality of water as it decreases the concentration of pollutant in effluent stream and increases the concentration of dissolved oxygen (though presence of sharp edge at conclusion time).

Here, we have considered only four state variables, namely – pollutants, PBS, chemical oxygen demand and dissolved oxygen but for the simulation of an actual wastewater treatment plant, many more contaminants and dissolved minerals like concentration of bacteria, protozoa, chlorine, zinc, sodium, potassium etc. can be considered and consequently included in the ODE analysis.

7. SELF ASSESSMENT

LEVEL – 1:

We think we have worked really hard on this project.
This assignment has definitely introduced us to the huge world of scientific methodology and research papers.
We were able to make the perfectly working code which solved the four coupled equations using Euler's explicit method. Solutions of these equations would predict the concentration of various substances in effluent. For instant we thought that Euler explicit method would not be ideal as it is not the best method to solve the differential equation but results were far better from expectation as we were able to get the results similar to results obtained by experiments, though it was not able to register very sharp changes as our step size was 5 days (not the smallest).

LEVEL – 2:

From our understanding of this term paper, we can extend this study to Level – 2 by considering the following:

The step size used (h = 5 days) can be reduced significantly by increasing experimental data values.
Adaptive step size can be used for data points where sharp changes in values are observed.
Euler's explicit method gives $O(h^2)$ order of error. We can therefore use:

- Richardson's extrapolation to increase the order of error and accuracy.
- Runge-Kutta (RK-4) which can increase the order of accuracy to $O(h^5)$, which along with Richardson's extrapolation can increase the accuracy to $O(h^6)$

As for the RK methods, the intermediate slope values are required but source of these are the experimental values, therefore, we can employ Newton-Raphson method to obtain the intermediate values from the initial values.

Even the original graph results we obtained in Euler's explicit method have not been previously published or used in any research articles as we ventured to use this method for numerical analysis of the analytical solution of the ODEs.

The results of the extension of this study as mentioned are definitely worth publishing in a high quality International Journal.

WE, AS A TEAM WOULD LIKE TO THANK YOU FOR IGNITING THE FIRE OF RESEARCH IN US.

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