HUL315: Econometrics Methods

Assignment 2

Indian Institute of Technology Delhi
Maximum Marks: 10 Marks

Instructions:

1. Deadline for submission is **January 28, 2024**.
2. You need to upload your assignment on Moodle in the following format (Name_Entry Number).
3. Assignments have to be submitted only in PDF generated using LaTeX.

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1. The t-distribution with r degrees of freedom can be defined as the ratio of two independent random variables. The numerator being a N (0, 1) random variable and the denominator being the square- root of a χ^2 random variable divided by its degrees of freedom. The t-distribution is a symmetric distribution like the Normal distribution but with fatter tails. As r $\rightarrow \infty$, the t-distribution approaches the Normal distribution. [5 Marks]

   a) Verify that if X1,...,Xn are a random sample drawn from a N(μ, σ^2) distribution, then z $= (\bar{X} - \mu)/(\sigma/\sqrt{n})$ is N(0,1).

   b) Use the fact that $(n-1)s^2/\sigma^2 \sim \chi^2_{n-1}$ to show that $t = z/\sqrt{s^2/\sigma^2} = (\bar{X} - \mu)/(s/\sqrt{n})$ has a t-distribution with $(n-1)$ degrees of freedom.

   c) For n $= 16$, $\bar{x} = 20$ and $s^2 = 4$, construct a 95 % confidence interval for μ.

2. Let $\bar{Y}$ denote the sample average from a random sample with mean μ and variance σ^2. Consider two alternative estimators of μ : $W_1 = [(n-1)/n]\bar{Y}$ and $W_2 = \bar{Y}/2$. [3 Marks]

   a) Show that W_1 and W_2 are both biased estimators of μ and find the biases. What happens to the biases as n $\rightarrow \infty$?

   b) Find the probability limits of W_1 and W_2. Which estimator is consistent?

3. Suppose that you have two independent unbiased estimators of the same parameter α, say $\hat{a_1}$ and $\hat{a_2}$, with different standard deviations σ_1 and σ_2. What linear combination of $\hat{a_1}$ and $\hat{a_2}$ is the minimum variance unbiased estimator of α? [2 Marks]