HUL315: Econometric Methods

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Problem 1

a) What is meant by Type I and Type II errors in the context of statistical hypothesis testing?

In context of Hypothesis Testing, we define a **Null Hypothesis** H_1(which is presumed to be true until the data strongly suggests otherwise) and **Alternative Hypothesis** H_1. While testing, two types of errors are possible.

1) Type I error: Rejecting the null hypothesis when it is true.
2) Type II error: Failing to reject the null hypothesis when it is false.

b) The recommended daily dietary allowance for zinc among males older than 50 years is 15 mg/day. The article "Nutrient Intakes and Dietary Patterns of Older Americans: A National Study" reports the following summary data on intake for a sample of males aged 65 - 74 years: $n = 115, \overline{x} = 11.3$ and $s = 6.43$. Does this data indicate that average daily zinc intake in the population of all males aged 65 - 74 falls bellow the recommended allowance?

State the Null Hypothesis and Alternative Hypothesis as,

$$H_0 : \mu = 15$$
$$H_1 : \mu < 15$$

Assuming H_0 is true, the population mean $\mu = 15$ and sample has, $n = 115$, $\overline{x} = 11.3$, $s = 6.43$. Since n is very large, z-score and t-score will be same (approximately). So we use s as σ for the z-score. This is a **lower-tailed test** and rejection region will be of type $z \leq -z_\alpha$.

$$\begin{aligned} z &= \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \\ &= \frac{11.3 - 15}{6.43/\sqrt{115}} \\ &= -6.1707 \end{aligned}$$

In **one-tailed test** for, $\alpha = 0.05$, rejection region is $z \leq -1.65$ and for $\alpha = 0.01$, rejection region is $z \leq -2.33$. In both cases, the **Test Statistic** falls in the **Rejection Region**. so we can reject H_0.

Conclusion: The average daily zinc intake in the population of all males aged 65 - 74 falls bellow the recommended allowance.

Problem 2

Have you ever been frustrated because you could not get a container of some sort to release the last bit of its contents? The article "Shake, Rattle and Squeeze: How Much Is Left In That Container?" (Consumer Reports, May 2009 : 8) reported on an investigation of this issue for various consumer products. Suppose five 6 oz tubes of toothpaste of a particular brand are randomly selected and squeezed until no more toothpaste will come out. Then each tube is cut open and the amount remaining is weighted, resulting in the following data (consistent with what the cited article reported): $0.53, 0.65, 0.46, 0.50, 0.37$. Does it appear that the true average amount left is less than 10% of the advertised net contents?

1. Check the validity of any assumptions necessary for testing the appropriate hypothesis.

   The population here refers to the weight of toothpaste in tubes. The sample size is very small, so we can't estimate the population variance from the sample. Consequently, $\frac{x-\mu}{s/\sqrt{x}}$ need not follow $\mathcal{N}(0,1)$ distribution so we can't use the z-test. In order to use the t-test, we assume that the population is Normally distributed (this is reasonable as "defects" that the amount is much larger or smaller than the mean should be very few) and that sampling is done independently.

2. Carry out a test of the appropriate hypothesis using a significance level of 0.05. Would your conclusion change if a significance level of 0.01 had been used?

   We convert the data into % age of 6 oz

$$\frac{100 \times 0.53}{6}, \frac{100 \times 0.65}{6}, \frac{100 \times 0.46}{6}, \frac{100 \times 50}{6}, \frac{100 \times 37}{6}$$
$$= 8.833, 10.833, 7.667, 8.333, 6.167$$

   So we have

$$\begin{aligned}
\bar{x} &= \frac{\sum_{i=1}^{n} x_i}{n} = \frac{8.833 + 10.833 + 7.667 + 8.333 + 6.167}{5} = 8.367 \\
s^2 &= \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \\
&= \frac{(8.833 - 8.367)^2 + (10.8338.367)^2 + (7.667 - 8.367)^2 + (8.333 - 8.367)^2 + (6.167 - 8.367)^2}{4} \\
&= 2.90833 \\
\implies s &= 1.7054
\end{aligned}$$

   Define the sample as, "Weight of contents of tube, as % age of 6 oz", denote the population mean by μ.

$$H_0 : \mu = 10$$
$$H_1 : \mu < 10$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{8.367 - 10}{1.7054/\sqrt{5}} = -2.141$$

   This is a **lower-tailed test** and rejection region will be of type $t \leq -t_{\alpha,n-1}$. We look at **t-table** with 4 Degrees of Freedom. In **one-tailed test** $t_{0.95} = 2.132$ and $t_{0.99} = 3.747$.

   (a) For $\alpha = 0.05$, rejection region is $t \leq -2.132$. The **Test Statistic** falls in the rejection region so we reject H_0.
   Conclusion: The true average amount left in is less than 10% of the advertised net contents.

   (b) For $\alpha = 0.01$ rejection region is $t \leq -3.747$. The **Test Statistic** doesn't fall in the rejection region so we fail to reject H_0.
   Conclusion: The true average amount left in is NOT less than 10% of the advertised net contents.