Shear Force and Bending Moment Analysis of a Beam Under Moving Load Using Influence Line Diagrams

Harshit

April 7, 2025

Objective

To develop a Python-based program to calculate the shear force and bending moment for a simply supported beam under a moving load using the influence line diagram concept.

Problem Statement

A simply supported beam is subjected to two moving point loads W_1 and W_2, spaced by a distance x. The program must:

- Accept user-defined values for beam length L, loads W_1 and W_2, and spacing x
- Calculate:
  - Maximum reactions at supports A and B
  - Bending moment BM_{01} when W_1 is at 0 m
  - Shear force SF_{01} at mid-span ($0.5L$)
  - Maximum shear force $SF_{\max}$ and its location y
  - Maximum bending moment $BM_{\max}$ and its location z

Inputs

- Length of beam: $L\,\mathrm{m}$
- Load values: $W_1\,\mathrm{kN}$, $W_2\,\mathrm{kN}$
- Distance between loads: $x\,\mathrm{m}$

Methodology

We use influence line theory to determine the variation of support reactions, shear force, and bending moment due to moving loads. The beam is analyzed at multiple positions of the moving loads to capture critical values.

Reaction Influence Lines

For a simply supported beam:

$$R_A = \frac{L-a}{L}, \quad R_B = \frac{a}{L}$$

where a is the position of the point load from A.

Shear Force Influence Line at Point p

$$IL_{SF}(x) = \begin{cases} \frac{L-p}{L}, & \text{if } x < p \\ -\frac{p}{L}, & \text{if } x > p \\ \text{undefined}, & \text{if } x = p \end{cases}$$

Bending Moment Calculation

$$BM(p) = R_A \cdot p - \sum (w_i \cdot (p - a_i)), \quad \text{for } a_i < p$$

1 Design Calculations

Given Data

- Length of Beam, $L =$ **...** m
- Load 1, $W_1 =$ **...** kN
- Load 2, $W_2 =$ **...** kN
- Distance between Loads, $x =$ **...** m

Step 1: Maximum Reactions at Supports

Using Influence Line Diagram (ILD):

$$R_A^{\max} = \max_{position} \left(W_1 \cdot \frac{L - a}{L} + W_2 \cdot \frac{L - (a + x)}{L} \right)$$
$$R_B^{\max} = \max_{position} \left(W_1 \cdot \frac{a}{L} + W_2 \cdot \frac{a + x}{L} \right)$$

Step 2: Bending Moment at $W_1 = 0$ m

$$BM_{01} = R_B \cdot L$$

(All the load acts at left support, so moment about A is zero, and max at B)

Step 3: Shear Force at Mid-span

$$SF_{0.5L} = R_A - \sum W_i \quad \text{(where } W_i \text{ are to the left of mid-span)}$$

Step 4: Maximum Bending Moment and Shear Force

$$BM_{\max} = \max_{positions} \left(R_A \cdot a - W_1 \cdot (a - d_1) - W_2 \cdot (a - d_2) \right)$$
$$SF_{\max} = \max_{positions} \left| R_A - \sum W_i \right|$$

- a = location from support A
- d_1, d_2 = distances of W_1, W_2 from point of interest

Step 5: Final Results

- Maximum Reaction at A: $\boxed{R_A^{\max} = \ldots \text{ kN}}$
- Maximum Reaction at B: $\boxed{R_B^{\max} = \ldots \text{ kN}}$
- Bending Moment at $W_1 = 0$: $\boxed{BM_{01} = \ldots \text{ kNm}}$
- Shear Force at mid-span: $\boxed{SF_{0.5L} = \ldots \text{ kN}}$
- Maximum Shear Force: $\boxed{SF_{\max} = \ldots \text{ kN at } y \text{ m from A}}$
- Maximum Bending Moment: $\boxed{BM_{\max} = \ldots \text{ kNm at } z \text{ m from A}}$

2 Outcomes

The analysis of a simply supported beam subjected to a moving load using the Influence Line Diagram (ILD) approach led to the following key outcomes:

- Successfully calculated the maximum reactions at supports A and B based on the relative positions of moving loads.
- Accurately computed the bending moment at the start of the beam ($W_1 = 0$ m) and the shear force at the midpoint of the beam.
- Determined the maximum bending moment and shear force values along with their positions from support A, which are critical for structural design.
- Developed a modular and well-commented Python program capable of simulating moving load effects using influence lines, shear force and bending moment envelopes.
- Enabled easy visualization of loading, shear force, bending moment diagrams, and ILD using matplotlib.

This implementation aids in optimizing structural design by identifying critical points under dynamic loading conditions, thus ensuring safer and more cost-effective engineering solutions.

Code Structure

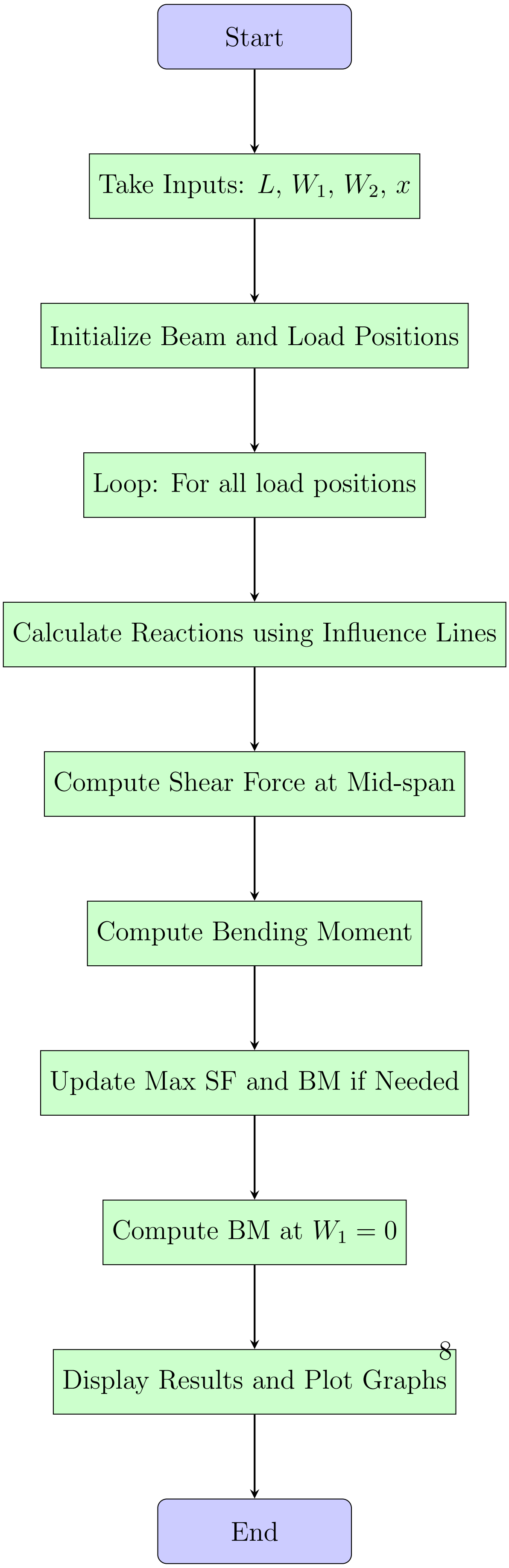
- **analyze_ss_movingload.py**: Main module that coordinates analysis and outputs results.
- **influence_line.py**: Contains influence line logic and envelope calculation methods.
- **visualization.py**: Responsible for plotting beam, shear force, and bending moment diagrams.
- **beam_calculator.py**: (Optional) Modular functions to support calculations.

3 Solution in Python

```python
"""
Main analysis module: analyze_ss_movingload.py
Performs moving load analysis on a simply supported beam using hybrid approach
(beam calculation + influence lines + visualization)
"""

from beam_calculator import Beam
from influence_line import InfluenceLine1, InfluenceLine
from visualization import Visualization

def analyze_beam(L, W1, W2, x):
    """
    Analyze a simply supported beam under two moving point loads using influence lines and direct analysis.

    Parameters:
        L (float): Length of the beam in meters (L > 0)
        W1 (float): Magnitude of first moving load in kN (W1 > 0)
        W2 (float): Magnitude of second moving load in kN (W2 > 0)
        x (float): Spacing between W1 and W2 (0 < x < L)

    Returns:
        dict: Contains results including max reactions, shear force, bending moment, envelopes, and critical positions
    """

    # Input Validation
    if L <= 0:
        raise ValueError("Beam length must be positive.")
    if W1 <= 0 or W2 <= 0:
        raise ValueError("Loads must be positive.")
    if x <= 0 or x >= L:
        raise ValueError("Spacing x must be positive and less than beam length.")

    # Setup
    beam = Beam(L)
    infl = InfluenceLine1(L)
    infl2 = InfluenceLine(L)
    viz = Visualization(L)

    # Initialize result storage
    results = {}

```

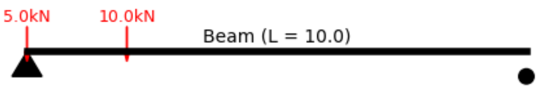
```python
    # Maximum Reactions
    max_RA = infl2.reaction_at_A(W1, W2, x)
    max_RB = infl2.reaction_at_B(W1, W2, x)
    results['Max Reaction at A'] = round(max_RA, 3)
    results['Max Reaction at B'] = round(max_RB, 3)

    # Specific Bending Moment when W1 is at 0 and W2 at x
    loads = [(0, W1), (x, W2)]
    # W1 at 0.0 and W2 at x
    # Specific Point Analysis: BM_01 (when W1 at 0.0 and W2 at x)
    # Specific Point Analysis: BM_01 (when W1 at 0.0 and W2 at x)
    BM_01_position = (0.0 * W1 + x * W2) / (W1 + W2)  # effective position of combined load
    BM_01 = beam.bending_moment_two(W1, 0.0, W2, x, BM_01_position)

    results['BM when W1 at 0'] = round(BM_01, 3)

    # Shear Force at midspan for critical position
    midspan = L / 2
    SF_mid = beam.shear_force_at(loads, midspan)
    results['Shear Force at 0.5L'] = round(SF_mid, 3)

    # Maximum Effects via influence line
    sf_envelope, sf_max_val, sf_max_pos = infl.max_shear_force(W1, W2, x)
    bm_envelope, bm_max_val, bm_max_pos = infl.max_bending_moment(W1, W2, x)

    results['Max SF'] = round(sf_max_val, 3)
    results['SF Location from A'] = round(sf_max_pos, 3)
    results['Max BM'] = round(bm_max_val, 3)
    results['BM Location from A'] = round(bm_max_pos, 3)

    # Visualization
    viz.plot_beam_with_loads(loads)
    viz.plot_shear_force_envelope(sf_envelope)
    viz.plot_bending_moment_envelope(bm_envelope)

    return results


# Example usage (remove or comment during deployment)
if __name__ == "__main__":
    result = analyze_beam(L=10.0, W1=5.0, W2=10.0, x=2.0)
    for key, val in result.items():
        print(f"{key}: {val}")
```

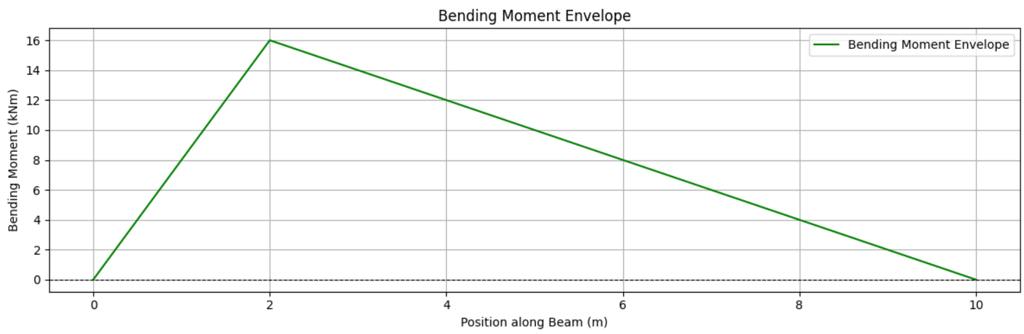
4 Flowchart

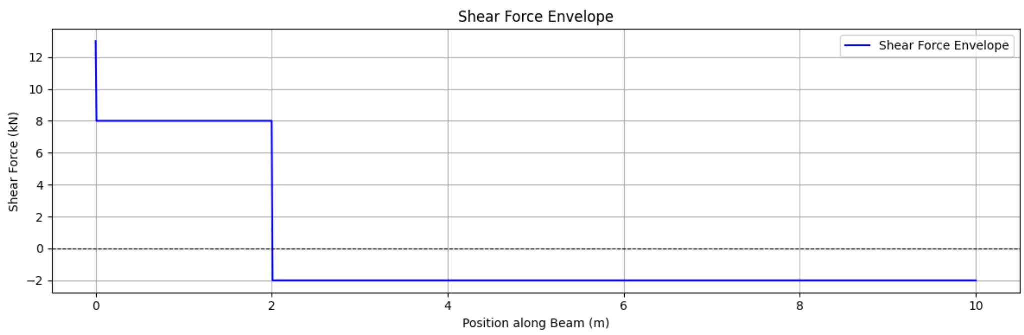

Start
Take Inputs: L, W_1, W_2, x
Initialize Beam and Load Positions
Loop: For all load positions
Calculate Reactions using Influence Lines
Compute Shear Force at Mid-span
Compute Bending Moment
Update Max SF and BM if Needed
Compute BM at $W_1 = 0$
Display Results and Plot Graphs
End

Visualizations

- Influence Line for Reactions at A and B
- Influence Line for Shear Force at p
- Shear Force Envelope
- Bending Moment Envelope







Results

The results of the beam analysis under a moving load using the Influence Line Diagram (ILD) method are as follows:

- **Maximum Reaction at Support A:** $R_A =$ `<value>` kN
- **Maximum Reaction at Support B:** $R_B =$ `<value>` kN
- **Bending Moment at** $W_1 = 0\,\mathrm{m}: \quad M_{W_1=0} =$ `<value>` kNm
- **Shear Force at Mid-span** $\left(x = \frac{L}{2}\right): \quad V_{x=L/2} =$ `<value>` kN
- **Maximum Shear Force:** $V_{\max} =$ `<SF_max>` kN at $y =$ `<location>` m
- **Maximum Bending Moment:** $M_{\max} =$ `<BM_max>` kNm at $z =$ `<location>` m

Conclusion

The program efficiently calculates shear force and bending moment using influence line theory. It also visualizes critical structural behavior under moving loads, ensuring design safety and accuracy.

5 References

- IS 800:2007 – *General Construction in Steel – Code of Practice*
- Hibbeler, R. C. (2017). *Structural Analysis.* Pearson Education.
- Negi, L. S. (2008). *Structural Analysis.* Tata McGraw-Hill Education.
- Bhavikatti, S. S. (2014). *Structural Analysis Vol I and II.* Vikas Publishing House.
- Influence Line Diagrams – Lecture Notes and Examples, NPTEL Courses
- Online resources:

– https://en.wikipedia.org/wiki/Influence$_{l}inehttps : //www.engineeringintro.com/structur$