

Algorithm: Compute the Row-Reduced Echelon (RRE) Form of a Matrix

Input:

A real matrix (A) of order (mxn).

Output:

The row-reduced echelon (RRE) form of (A).

1. Read the matrix

- 1.1. Input the dimensions (m) (number of rows) and (n) (number of columns).
- 1.2. Input all the entries of the matrix (A).

2. Initialize counters

- 2.1. Set `sortedRow = 0`.
(This will count how many rows from the top already contain pivots.)
- 2.2. Set `sortedColumn = 0`.
(This will keep track of the last pivot column we have processed.)

3. Repeat the following steps:

3.1. Find the next pivot position

- Starting from the column `sortedColumn`, move to the right, column by column, until you find a column that has at least one non-zero entry in some row `r` with `r ≥ sortedRow`.
- If no such column exists (i.e., all remaining columns are zero in rows `sortedRow` to `m-1`), **stop** the process.
- Let the first such row be called `currentRow`, and the corresponding column be the new `sortedColumn`.

4. 3.2. Swap rows if necessary

- If `currentRow ≠ sortedRow`, interchange row `currentRow` with row `sortedRow`.
(Now the pivot will be located in row `sortedRow` and column `sortedColumn`.)

5. 3.3. Normalize the pivot row

- Let the pivot element be the entry in row `sortedRow` and column `sortedColumn`.
- Divide all entries of row `sortedRow` from column `sortedColumn` to column `n` by this pivot value, so that the pivot becomes 1.

6. 3.4. Eliminate the pivot column in all other rows

- For every row index `i` from 0 to `m - 1`, with `i ≠ sortedRow`:

- Let **factor** be the entry in row **i** and column **sortedColumn**.
- Replace row **i** by:

$$\text{row } i = \text{row } i - \text{factor} \times \text{row } \text{sortedRow}$$
- After this operation, the entry in column **sortedColumn** of row **i** becomes 0.

7. 3.5. Move to the next row

- Increase **sortedRow** by 1.
- Go back to Step 3.1 to search for the next pivot in a later column.

8. Output the result

- 4.1. After the loop terminates, the matrix (A) is in **row-reduced echelon form**.
- 4.2. Print the final matrix as the RRE form of the original matrix.