

## **Algorithm: Compute the Row-Reduced Echelon (RRE) Form of a Matrix**

### **Input:**

A real matrix ( $A$ ) of order ( $m \times n$ ).

### **Output:**

The row-reduced echelon (RRE) form of ( $A$ ).

#### **1. Read the matrix**

- 1.1. Input the dimensions ( $m$ ) (number of rows) and ( $n$ ) (number of columns).
- 1.2. Input all the entries of the matrix ( $A$ ).

#### **2. Initialize counters**

- 2.1. Set `sortedRow` = 0.

(This will count how many rows from the top already contain pivots.)

- 2.2. Set `sortedColumn` = 0.

(This will keep track of the last pivot column we have processed.)

#### **3. Repeat the following steps:**

##### **3.1. Find the next pivot position**

- Starting from the column `sortedColumn`, move to the right, column by column, until you find a column that has at least one non-zero entry in some row  $r$  with  $r \geq \text{sortedRow}$ .
- If no such column exists (i.e., all remaining columns are zero in rows `sortedRow` to  $m-1$ ), **stop** the process.
- Let the first such row be called `currentRow`, and the corresponding column be the new `sortedColumn`.

##### **4. 3.2. Swap rows if necessary**

- If `currentRow`  $\neq \text{sortedRow}$ , interchange row `currentRow` with row `sortedRow`.  
(Now the pivot will be located in row `sortedRow` and column `sortedColumn`.)

##### **5. 3.3. Normalize the pivot row**

- Let the pivot element be the entry in row `sortedRow` and column `sortedColumn`.
- Divide all entries of row `sortedRow` from column `sortedColumn` to column  $n$  by this pivot value, so that the pivot becomes 1.

##### **6. 3.4. Eliminate the pivot column in all other rows**

- For every row index  $i$  from 0 to  $m - 1$ , with  $i \neq \text{sortedRow}$ :

- Let **factor** be the entry in row **i** and column **sortedColumn**.
- Replace row **i** by:  

$$\text{row } i = \text{row } i - \text{factor} \times \text{row sortedRow}$$
- After this operation, the entry in column **sortedColumn** of row **i** becomes 0.

## 7. 3.5. Move to the next row

- Increase **sortedRow** by 1.
- Go back to Step 3.1 to search for the next pivot in a later column.

## 8. Output the result

- 4.1. After the loop terminates, the matrix (A) is in **row-reduced echelon form**.
- 4.2. Print the final matrix as the RRE form of the original matrix.