

# Week-2

## NPTel CS133 — Data Science for Engineers

TA:

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### Today's Topics:

1. Linear Algebra
2. Solving linear Equations
3. Distances  
Hyperplanes & Half Space (if time permits)
4. Eigen Values  
Eigen Vectors

→ Session is being recorded

→ 3-5 min break every half hour & 10 min break every hour.

→ If you would like me to cover more topics in next session please let me know at the end.

→ Be respectful during the session, I will not hesitate to remove you from the session

### \* Linear Algebra

Recap: Vectors  
Matrices  
Basic Operations (+, -, x)  
Add: Determinants

### Example

Apples	Bananas	Bread
2/-	1/-	3/-
3	2	1
Total cost — ?		

Vector:  $[2 \ 1 \ 3] \rightarrow \textcircled{1}$

$[3 \ 2 \ 1] \rightarrow \textcircled{2}$

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}_{1 \times 3} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{3 \times 1}$$

$$= 11/-$$

A      B      C

~~~~~

Products

|   | last week | this week |
|---|-----------|-----------|
| A | 100       | 120       |
| B | 80        | 95        |
| C | 50        | 60        |

## Addition

Addition

$$\begin{bmatrix} \textcircled{1} & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} \textcircled{2} & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 4 \\ 5 & 7 & 9 \end{bmatrix}$$

## Multiplication

with 1 character

$[a][b]c] \rightarrow \begin{bmatrix} d \\ e \\ f \end{bmatrix} \downarrow \downarrow \downarrow (ad + be + cf) \quad \underline{\quad} \quad \underline{\quad}$

Diagram illustrating the associative property of matrix multiplication. It shows two ways to multiply a  $3 \times 4$  matrix by a  $4 \times 2$  matrix. The first way is  $(3 \times 4) \times (4 \times 2) = 3 \times 2$ . The second way is  $(3 \times 4) \times (4 \times 2) = 3 \times 2$ . The result is the same  $3 \times 2$  matrix.

Det

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$|A| = (1 \times 2 - 3 \times 1)$$

8-3

11

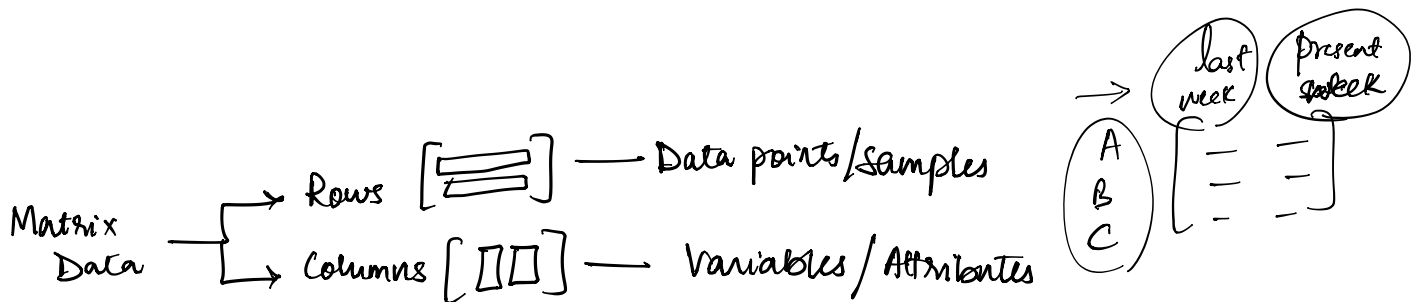
# Image as a Representation of Matrix



|    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |    |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 0  | 2   | 15  | 0   | 0   | 11  | 10  | 0   | 0   | 0   | 0   | 9   | 9   | 0   | 0   | 0  |
| 0  | 0   | 0   | 4   | 60  | 157 | 236 | 255 | 255 | 177 | 95  | 61  | 32  | 0   | 0   | 29 |
| 0  | 10  | 16  | 111 | 238 | 255 | 244 | 245 | 243 | 250 | 249 | 255 | 222 | 103 | 10  | 0  |
| 0  | 14  | 170 | 255 | 255 | 244 | 254 | 255 | 253 | 245 | 255 | 249 | 253 | 251 | 124 | 1  |
| 2  | 98  | 255 | 228 | 255 | 251 | 254 | 211 | 141 | 116 | 122 | 215 | 251 | 238 | 255 | 49 |
| 13 | 217 | 243 | 255 | 155 | 33  | 226 | 52  | 2   | 0   | 10  | 13  | 232 | 255 | 255 | 36 |
| 16 | 229 | 252 | 254 | 49  | 12  | 0   | 0   | 7   | 7   | 0   | 70  | 237 | 252 | 235 | 62 |
| 6  | 141 | 245 | 255 | 212 | 25  | 11  | 9   | 3   | 0   | 115 | 236 | 243 | 255 | 137 | 0  |
| 0  | 87  | 252 | 250 | 248 | 215 | 60  | 0   | 1   | 121 | 252 | 255 | 248 | 144 | 6   | 0  |
| 0  | 13  | 113 | 255 | 255 | 245 | 255 | 182 | 181 | 248 | 252 | 242 | 208 | 36  | 0   | 19 |
| 1  | 0   | 5   | 117 | 251 | 255 | 241 | 255 | 247 | 255 | 241 | 162 | 17  | 0   | 7   | 0  |
| 0  | 0   | 0   | 4   | 58  | 251 | 255 | 246 | 254 | 253 | 255 | 120 | 11  | 0   | 1   | 0  |
| 0  | 0   | 4   | 97  | 255 | 255 | 255 | 248 | 252 | 255 | 244 | 255 | 182 | 10  | 4   | 0  |
| 0  | 22  | 206 | 252 | 246 | 251 | 241 | 100 | 24  | 113 | 255 | 245 | 255 | 194 | 9   | 0  |
| 0  | 111 | 255 | 247 | 255 | 158 | 24  | 0   | 0   | 6   | 39  | 255 | 232 | 230 | 56  | 0  |
| 0  | 218 | 251 | 250 | 137 | 7   | 11  | 0   | 0   | 0   | 2   | 62  | 255 | 250 | 125 | 3  |
| 0  | 173 | 255 | 255 | 101 | 9   | 20  | 0   | 13  | 3   | 13  | 182 | 251 | 245 | 61  | 0  |
| 0  | 107 | 251 | 241 | 255 | 230 | 98  | 55  | 19  | 118 | 217 | 248 | 253 | 255 | 52  | 4  |
| 0  | 18  | 146 | 250 | 255 | 247 | 255 | 255 | 255 | 249 | 255 | 240 | 255 | 129 | 0   | 5  |
| 0  | 0   | 23  | 113 | 215 | 255 | 250 | 248 | 255 | 255 | 248 | 248 | 118 | 14  | 12  | 0  |
| 0  | 0   | 6   | 1   | 0   | 52  | 153 | 233 | 255 | 252 | 147 | 37  | 0   | 0   | 4   | 1  |
| 0  | 0   | 5   | 5   | 0   | 0   | 0   | 0   | 14  | 1   | 0   | 6   | 6   | 0   | 0   | 0  |

0-255

|    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |    |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 0  | 2   | 15  | 0   | 0   | 11  | 10  | 0   | 0   | 0   | 0   | 9   | 9   | 0   | 0   | 0  |
| 0  | 0   | 0   | 4   | 60  | 157 | 236 | 255 | 255 | 177 | 95  | 61  | 32  | 0   | 0   | 29 |
| 0  | 10  | 16  | 119 | 238 | 255 | 244 | 245 | 243 | 250 | 249 | 255 | 222 | 103 | 10  | 0  |
| 0  | 14  | 170 | 255 | 255 | 244 | 254 | 255 | 253 | 245 | 255 | 249 | 253 | 251 | 124 | 1  |
| 2  | 98  | 255 | 228 | 255 | 251 | 254 | 211 | 141 | 116 | 122 | 215 | 251 | 238 | 255 | 49 |
| 13 | 217 | 243 | 255 | 155 | 33  | 226 | 52  | 2   | 0   | 10  | 13  | 232 | 255 | 255 | 36 |
| 16 | 229 | 252 | 254 | 49  | 12  | 0   | 0   | 7   | 7   | 0   | 70  | 237 | 252 | 235 | 62 |
| 6  | 141 | 245 | 255 | 212 | 25  | 11  | 9   | 3   | 0   | 115 | 236 | 243 | 255 | 137 | 0  |
| 0  | 87  | 252 | 250 | 248 | 215 | 60  | 0   | 1   | 121 | 252 | 255 | 248 | 144 | 6   | 0  |
| 0  | 13  | 113 | 255 | 255 | 245 | 255 | 182 | 181 | 248 | 252 | 242 | 208 | 36  | 0   | 19 |
| 1  | 0   | 5   | 117 | 251 | 255 | 241 | 255 | 247 | 255 | 241 | 162 | 17  | 0   | 7   | 0  |
| 0  | 0   | 0   | 4   | 58  | 251 | 255 | 246 | 254 | 253 | 255 | 120 | 11  | 0   | 1   | 0  |
| 0  | 0   | 4   | 97  | 255 | 255 | 255 | 248 | 252 | 255 | 244 | 255 | 182 | 10  | 4   | 0  |
| 0  | 22  | 206 | 252 | 246 | 251 | 241 | 100 | 24  | 113 | 255 | 245 | 255 | 194 | 9   | 0  |
| 0  | 111 | 255 | 247 | 255 | 158 | 24  | 0   | 0   | 6   | 39  | 255 | 232 | 230 | 56  | 0  |
| 0  | 218 | 251 | 250 | 137 | 7   | 11  | 0   | 0   | 0   | 2   | 62  | 255 | 250 | 125 | 3  |
| 0  | 173 | 255 | 255 | 101 | 9   | 20  | 0   | 13  | 3   | 13  | 182 | 251 | 245 | 61  | 0  |
| 0  | 107 | 251 | 241 | 255 | 230 | 98  | 55  | 19  | 118 | 217 | 248 | 253 | 255 | 52  | 4  |
| 0  | 18  | 146 | 250 | 255 | 247 | 255 | 255 | 255 | 249 | 255 | 240 | 255 | 129 | 0   | 5  |
| 0  | 0   | 23  | 113 | 215 | 255 | 250 | 248 | 255 | 255 | 248 | 248 | 118 | 14  | 12  | 0  |
| 0  | 0   | 6   | 1   | 0   | 52  | 153 | 233 | 255 | 252 | 147 | 37  | 0   | 0   | 4   | 1  |
| 0  | 0   | 5   | 5   | 0   | 0   | 0   | 0   | 14  | 1   | 0   | 6   | 6   | 0   | 0   | 0  |



→ Rank

# linearly independent variables.

|   | Protein | Carb | Fats |
|---|---------|------|------|
| A | 2       | 3    | 1    |
| B | 4       | 1    | 2    |
| C | 3       | 2    | 1    |

Rank = 3

→ Null Space

Multipled by a  $\lambda$ -matrix results zero vector.

→ Rank-Nullity Theorem

$$\text{Rank}(\text{Matrix}) + \dim(\text{null space}) = \text{no. of col.}$$

— R Code —

# Null Space of a Matrix

Multiplied by a  $n$ -matrix results zero vector.

$$\underbrace{A} \quad \underbrace{x} = 0$$

$$Ax = 0$$

$$\begin{array}{c} c_1 \quad c_2 \\ R_1 \quad \begin{bmatrix} 2 & 1 \end{bmatrix} \\ R_2 \quad \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$R_2 \leftrightarrow R_2 - 1.$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$0 + y = 0 \Rightarrow y = 0.$$

$$2x + y = 0$$

$$2x + 0 = 0$$

$$2x = 0$$

$$\boxed{x = 0}$$

$$\boxed{Ax = 0}$$

$x \rightarrow$  null space of  $A$ .

$$\begin{array}{l} x, y = 0. \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

## Rank-Nullify Theorem

$$A = \begin{bmatrix} & & \end{bmatrix} \longrightarrow$$

$3 \times 3$

$$\textcircled{N} = \begin{bmatrix} \\ \\ \end{bmatrix}_{3 \times 1}$$

$$\underline{\text{Rank}(A)} + \text{ncol}(N) = \text{ncol}(A)$$

|      |   |                                                                                    |   |
|------|---|------------------------------------------------------------------------------------|---|
| (i)  | 3 | <div style="border: 1px solid black; padding: 2px; display: inline-block;">0</div> | 3 |
| (ii) | 2 | 1                                                                                  | 3 |

# \* Linear Equations

$$A x = b$$

$A \rightarrow (m \times n)$   $\frac{m}{n}$  Equations  
 $x \rightarrow (n \times 1)$   $\frac{n}{m}$  variables  
 $b \rightarrow (m \times 1)$

$$y = 8x + 9$$

$$y = mx^2$$

$$y = x^3 + 2$$

} Not linear

Ex:

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \phantom{0} \end{bmatrix}$$

$3 \times 3$   $3 \times 2$   $3 \times 2$

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 3z &= 3 \\ 3x + 2y + z &= -1 \end{aligned}$$

$(3) \rightarrow (3)$

$(m, n) \rightarrow \begin{cases} m = n \rightarrow \text{easiest} \\ m > n \rightarrow \text{No sol.} \\ m < n \rightarrow \text{Multiple solutions} \end{cases}$

$m \rightarrow \# \text{ eq}^n$   
 $n \rightarrow \text{variables}$

Matrix Eq<sup>n</sup>  
 $[m = n]$

— Unique sol.

— Consistent  $\rightarrow \infty$  sols.

— Inconsistent  $\rightarrow$  No sol.

$|A| \neq 0$  &  $A$  is full rank

}  $A$  is not full rank.

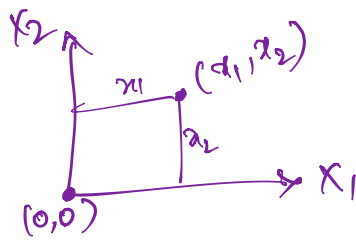
Full Rank  $\rightarrow$  # linearly independent rows = # rows in the matrix.

## \* Distance

$x \rightarrow$  Datapoint

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_1, x_2 \rightarrow$  distances from  $x_1, x_2$  axes



$$d = ? \sqrt{x_1^2 + x_2^2}$$

$$d \text{ of } x_1 = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} \quad x_2 = \begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix}$$

$$l = ?$$

$$l = \sqrt{(x_1' - x_1'')^2 + (x_2' - x_2'')^2}$$

Try: In  $\mathbb{R}^n$  console ?? distance

## \* Unit Vector

Magnitude  $\geq 1$

$$\text{Mag}(A) = \frac{A}{|A|}$$

$$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

## \* Orthogonal Vectors

dot product = 0

$$A^T B = 0$$

$$A = \begin{bmatrix} \quad \end{bmatrix}_{3 \times 1} \quad B = \begin{bmatrix} \quad \end{bmatrix}_{3 \times 1}$$

$$A^T = \begin{bmatrix} \quad \end{bmatrix}_{1 \times 3} \quad B = \begin{bmatrix} \quad \end{bmatrix}_{3 \times 1}$$

$$A^T B = \begin{bmatrix} \quad \end{bmatrix}_{1 \times 1}$$