Week-2

NPTEL CS133 — Data Science for Engineers

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Today's Topics:

1. Linear Algebra
2. Solving linear Equations
3. Distances
   Hyperplanes & Half Space (if time permits)
4. Eigen Values
   Eigen Vectors

→ Session is being recorded

→ 3-5 min break every half hour & 10 min break every hour.

→ If you would like me to cover more topics in next session please let me know at the end.

→ Be respectful during the session, I will not hesitate to remove you from the session

* Linear Algebra

Recap: Vectors
Matrices
Basic Operations (+, −, ×)
Addl: Determinants

Example

Apples	Bananas	Bread
2/-	1/-	3/-
3	2	1

Total cost — ?

Vector: [2 1 3] → (1)

[3 2 1] → (2).

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}_{1\times 3} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{3\times 1}$$

$= 11/-$

$$\underbrace{A \quad B \quad C}_{\text{Products}}$$

$$\begin{array}{c|cc} & \text{last week} & \text{this week} \\ A & & \\ B & & \\ C & & \end{array} \quad \begin{bmatrix} 100 & 120 \\ 80 & 95 \\ 50 & 60 \end{bmatrix}$$

Addition

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 \\ 5 & 7 & 9 \end{bmatrix}$$

Multiplication

$$\begin{bmatrix} a & \boxed{b} & c \end{bmatrix} \begin{bmatrix} d \\ \boxed{e} \\ f \end{bmatrix} = \begin{bmatrix} (ad + be + cf) & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

$$\underset{\boxed{m} \times n}{\begin{bmatrix} & \end{bmatrix}} \times \underset{n \times \boxed{x}}{\begin{bmatrix} & \end{bmatrix}} \longrightarrow \underset{m \times x}{\begin{bmatrix} & \end{bmatrix}}$$

Det

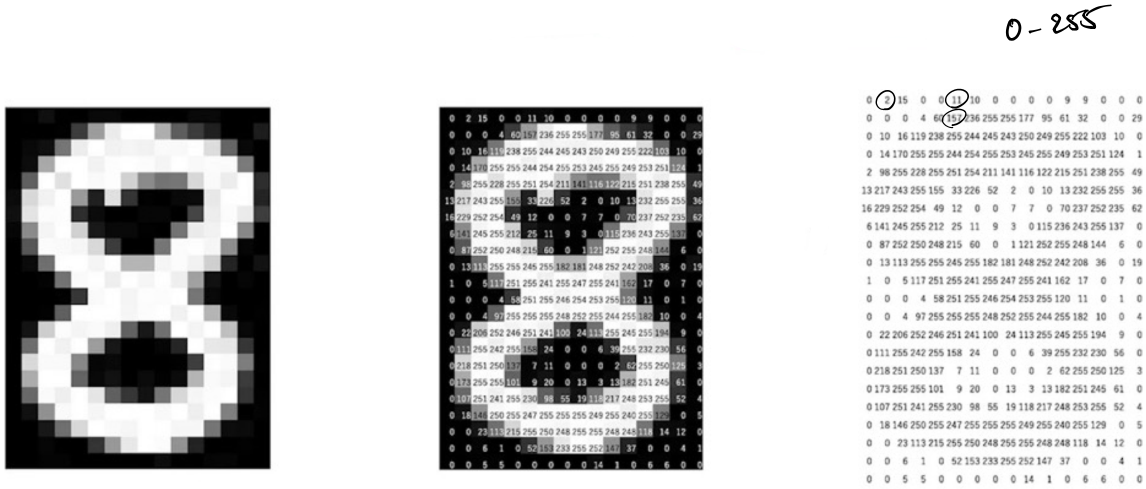
$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$|A| = (4 \times 2 - 3 \times 1)$$

$$= 8 - 3$$

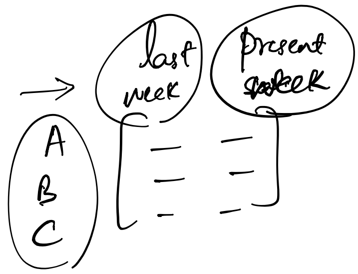
$$= 5$$

Image as a Representation of Matrix



Matrix Data → Rows [ ] — Data points / Samples

Matrix Data → Columns [ ] — Variables / Attributes



Ex: Sales Matrix.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \text{Protein} & \text{Carb} & \text{fats} \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \end{array}$$

$\boxed{\text{Rank} = 3}$

→ Rank

\# linearly independent variables.

→ Null Space

Multiplied by a matrix results zero vector.

→ Rank-Nullity Theorem

Rank(Matrix) + dim (null space) = no. of col.

R Code

Null Space of a Matrix

Multiplied by a matrix results zero vector.

$$\underbrace{A \quad x}\quad = \quad 0$$

$$Ax = 0$$

$$\begin{array}{c|c} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array} \begin{bmatrix} \cancel{x} \\ \cancel{y} \end{bmatrix}$$

0

$R_2 \longleftrightarrow R_2 - 1.$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$0 + y = 0 \longrightarrow y = 0.$$

$$2x + y = 0$$

$$2x + 0 = 0$$

$$2x = 0$$

$$\boxed{x = 0}$$

$x, y = 0.$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{Ax = 0}$$

$x \rightarrow$ null space of A.

Rank - Nullify Theorem

$A = \begin{bmatrix} & & \end{bmatrix}_{3\times 3}$

$N = \begin{bmatrix} \end{bmatrix}_{3\times 1}$

$$\boxed{\text{Rank}(A) + \text{ncol}(N) = \text{ncol}(A)}$$

(i)	3	$\boxed{0}$	3
(ii)	2	1	3

* Linear Equations

$$Ax = b$$

$A \rightarrow (m \times n)$ — m Equations

$x \rightarrow (n \times 1)$ — n variables

$b \rightarrow (m \times 1)$

$y = 8x + 9$ — linear

$y = mx^2$
$y = x^3 + 2$ } Not linear

Ex:

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 1 & 2 & 3 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} & \\ & \end{bmatrix}_{3 \times 2}$$

$x + y + z = 1$
$x + 2y + 3z = 3$
$3x + 2y + z = -1$
$3 \leftrightarrow 3$

(m, n)
- $m = n$ → easiest.
- $m > n$ → No sol.
- $m < n$ → Multiple solutions.

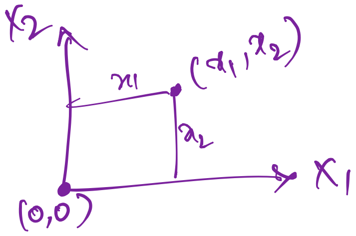
$m \rightarrow$ # eq^n
$n \rightarrow$ variables.

Matrix Eq^n $[m = n]$
- Unique sol. → $|A| \neq 0$ & A is full rank
- Consistent → ∞ sols.
- Inconsistent → No sol.

} A is not full rank.

Full Rank → # linearly independent rows = # rows in the matrix.

* Distance

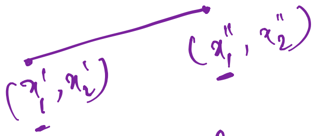


$X \rightarrow$ Datapoint

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x_1, x_2 \rightarrow$ distances from X_1, X_2 axes

$d = ?$ $\sqrt{x_1^2 + x_2^2}$



If $x' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$ $\quad x_2 = \begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix}$

$l = ?$

$l = \sqrt{(x_1' - x_1'')^2 + (x_2' - x_2'')^2}$

Try: In $\boxed{R, \ ?? \text{distance}}$

console

* Unit Vector

Magnitude = 1

$Mag(A) = \dfrac{A}{|A|}$

$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

* Orthogonal Vectors

dot product = 0

$A^T B = 0$

$A = [\quad]_{3\times 1} \quad B = [\quad]_{3\times 1}$

$A^T = [\quad]_{1\times 3} \quad B = [\quad]_{3\times 1}$

$A^T B = [\quad]_{1\times 1}$