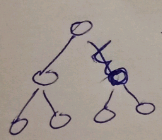
① height of complete balanced tree of n node is $\log_2(n+1)$

$h = \log_2(n+1)$



2^0

2^1

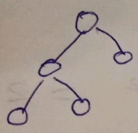
2^2

3

$n = 1$ ✓

$h = 1$

$$2^0 + 2^1 + 2^2 \dots 2^{h-1} = n$$



$$\frac{\cancel{2^0}(2^h - 1)}{\cancel{2-1}} = n$$

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$$\Rightarrow h = \log_2(n+1) \quad \underline{\text{H.P}}$$

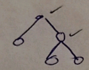
Similarly 2
leaf of the

The data can
node with m
single address

vs Array

are linear
esn't store
esn't allow
sible

② no. of int ...
0 or 2.

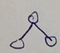
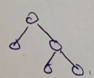


int = non-leaf = 2
leaf nodes = non leaf + 1
= 3.

o Base:
non-leaf = 0
leaf = 1. (root)

For int internal;
leaf = int + 1

For int+1 internal node
leaf = (int+1) + 1

	leaf	int
	1	0
	2	1
	3	2

int+2
leaf = (int+2) + 1

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int+n
leaf = (int+n) + 1

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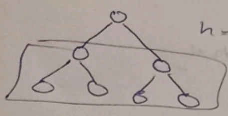
Jan 09, 2023, 23:55

✓ (3) no. of nodes in perfect BT.

$n = 2^{h+1} - 1$, h is height.



$h = 0$

$h = 0$ $\quad n = 1$ ✓

subtree $h = 1$

<u>Hypothesis</u> $\quad h = 2 \longrightarrow -1$

assuming true for h; $n = 2^{h+1} - 1$

<u>Prove that for $h+1$ $\quad n = 2^{h+1+1} - 1$</u>

so for $h+1$ subtrees are of height h

so total no. of nodes in each subtree is

$$2^{h+1} - 1$$

so total

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$$\underbrace{(2^{h+1} - 1)}_{\text{left}} + \underbrace{(2^{h+1} - \cancel{1})}_{\text{right}} + \underbrace{\cancel{1}}_{\text{parent}}$$

$= 2^{h+2} - 1$ nodes.

<u>Hence proved.</u>

(B)

(4) A BT with n internal nodes has $n+1$ external nodes

$n = 0 \rightarrow n+1 = 1$ external node

if $n+1$ internal then $n+1+1$ external

$T = n+1$ leaf nodes.   leaf nodes $+2$

former leaf becomes non-leaf

S.A.

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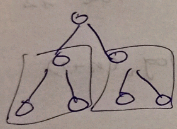
internal nodes $n+1$

leaf $\Rightarrow (n+1)+2-1 = n+2$

Hence proved.

7) No. of leaves in BT of height $h = 2$
(at most)



$h = 0 = 2^0 = 1$. (Parent root)

$h = 1 = 2$. ✓✓

<u>Hypothesis</u>:

$$n \le \cancel{2n}\ 2^h$$

1RU21A1037

S.A. For $h+1$ height. we need to prove $\le 2^{h+1}$

For each subtree max leaves 2^h

$$2^h + 2^h = 2^{h+1}$$

⑧ Any rooted tree with n nodes has $n-1$ edges

$n = 1$

$edge = 0$ ✓

let T be trees with n nodes. & E edges

and b/w 2 vertices there exists 1 edge

if we disconnect them we get n_1 & n_2 nodes

$n_1, n_2 < n$

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$\downarrow \quad \downarrow$

$n_1 - 1 \quad n_2 - 1$

$n_1 + n_2 - 2 = n-2$ edges since we removed 1 edge before so $n-1$ Hence proved

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Jan 10, 2023, 00:13

(5) $n_0 = n_2 + 1$

leaf $\downarrow$ degree 2

$n_2 = 0$ 0

$n_0 = 1$

$n_0 = 2$  3 3

$n_3 = 3$

t

$n_2 = K$ $K+1$ $t+1$

$n_0 = K+1$ $(K+1)+1$

Hence ~~p~~ proved.