Experiment : set of procedures & observation.

Outcome : any possible observation of expt.

Each outcome is
distinguishable
from other outcomes.

Sample space $\Omega$ $S$ : set of all possible outcomes

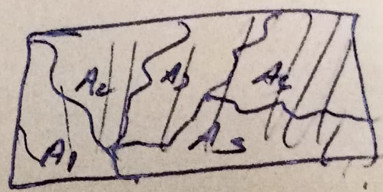
① mutually exclusive

If 1 outcome occurs, then no other outcome can occur. $\boxed{P(A_1 \cap A_2) = 0}$

② collectively exhaustive

Every outcome must be in sample space.

$\boxed{P(A_1 \cup A_2 \cdots) = S} = 1.$




Axioms of Probability :

① $p(A) \geq 0$

② $p(S) = 1$

} b/w 0 & 1.

③ $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) \cdots$ $\boxed{\text{mutually exclusive}}$

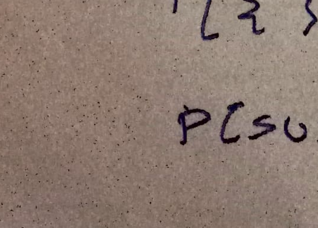
$P(A_1 \cap A_2) = 0$ $\boxed{\text{mutually exclusive}}$

$P[\{\ \}] = 0$ or $P[S'] = 0$

$P[S \cup S^c] = P(S) + P(S^c)$

$= P(S)$

If not mutually exclusive




$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If $A \subseteq B$

$\Rightarrow \boxed{P(A) \leq P(B)}$

# Conditional Probability

$P(A) \rightarrow$ prior prob. of A

knowledge of an occurrence of event A before conducting experiment.

$P(A|B)$ : probability of occurrence of A given B has already occured.

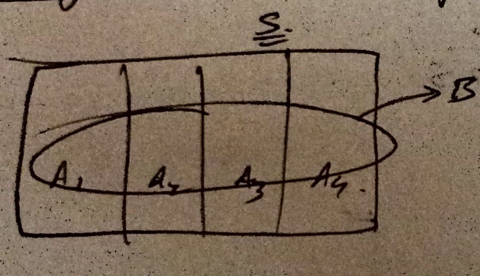
$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

Inference :

① B is sample space rather than S ($B \subset S$)

July 26.

## Law of total probability




$P(B) = ?$

under $A_1$ what is $P(B) = ?$

$P(B) = \frac{P(B \cap A_1)}{\cancel{P(A_1)}} \ldots$

$$\therefore \boxed{P(B) = \sum_{i=1}^{n} (B \cap A_i)}$$ where A are mutually exclusive.

now ~~P(B/A)~~

$\cancel{P(A/B)} = \frac{\cancel{P(A \cap B)}}{\cancel{P(B)}} \Rightarrow$ $\cancel{P(B)} =$

$P(B/A_i) = \frac{P(B \cap A_i)}{P(A_i)}$ ($A_i$ has already occured)

conditional probability

$$\therefore \Rightarrow \boxed{P(B) = \sum_{i=1}^{n} P(B/A_i)\, P(A_i)}$$

$\boxed{P(A \cap B) = P(A, B) = P(A}$

Eg: In a sample space all the occurrences must be equal likely. $\frac{3 \times 4}{184}$

Eg: $B_1$ $B_2$ $B_3$

80% 90% 60%

3000 4000 3000 resistors

$$\begin{array}{r} 2400 \\ 3600 \\ +1800 \\ \hline 7800 \end{array}$$

~~80 × 3000~~

$$\frac{0.8 \times 3000 + 0.9 \times 4000 + 0.6 \times 3000}{3000 + 4000 + 3000} = \frac{7800}{10000}$$

$$\frac{.18}{0.78}$$

$$\frac{\frac{.60 \times 3000}{3000}}{\frac{78}{100}}$$

$$\frac{6 \times 300 \times 100}{78}$$

$$\frac{.6 \times 3000}{.78}$$

$$\frac{.6}{0.78}$$

Eg:

$P(E) = 2/3$. $P(C_2/E_2) =$

3 2 4 3 4 2 $2/2 = 1$

Eg:

10 boxes.

1st 2nd 3 4 ... 10th box

10 cups

→ In each box exactly cup is broken with $P = 1/2$

→ contestant choose box (1/101)

→ given that no broken cups is found, find probability he choose 1st box.

$$\frac{\frac{1}{10}\left(\frac{1}{2}\right)^1}{\frac{1}{10}\left(\frac{1}{2}\right)^1+\frac{1}{10}\left(\frac{1}{2}\right)^2+\dots\frac{1}{10}\left(\frac{1}{2}\right)^{10}}$$

$$\boxed{=0.5}$$

$r=\frac{1}{2}$

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$=\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{10}\right)}{\frac{1}{2}}$$

---

Qu 1 random dog is bitter : 0.2 non bitter = 0.8

P(Barker | not bite) = p P(Barker | bite) = 3p.

~~P(A|B)~~ = p ~~P(B|A)~~ = 3p. 3p = ?

probability

$p=\frac{A\cap B}{0.8}$ $3p=\frac{A\cap B}{A}$

P(bite | barker) = ?

$$P(\text{Barker} \mid \text{not bite}) = \frac{P(\text{Barker} \cap \text{not bite})}{\cancel{P(\text{no bite})}\ 0.8} = p.$$

$$P(\text{Barker} \mid \text{bite}) = \frac{P(\text{barker} \cap \text{bite})}{\cancel{P(\text{bite})}\ 0.2} = 3p.$$

$$P(\text{bite} \mid \text{barker}) = \frac{P(\text{bite} \cap \text{barker})}{P(\text{barker})}$$

$$\frac{0.2\cdot 3p}{0.8(p)+0.2(3p)} = \frac{0.6}{\cancel{0.14}\ 1.4} = \frac{6}{14}$$

Q.2

Thursday
exercise = 0.7
no exercise = 0.3
eats pizza = 0.2
no pizza = 0.8

Tuesday
exercise → 0.15 pizza.

P(~~suppose exercise~~) = 0·15

P(~~... A NP~~) = ?

## Discrete Random Variable ( distinct values)

X : random variable (observation)

$S_x$ : range of random variable X

outcomes → are quantified

$$X = \begin{cases} 1 & \text{Heads} \\ 0 & \text{tail.} \end{cases} \qquad Y = \}$$

depending on situation it is possible it be either DRV or CRV

DRV → nearest integer

CRV → within range.

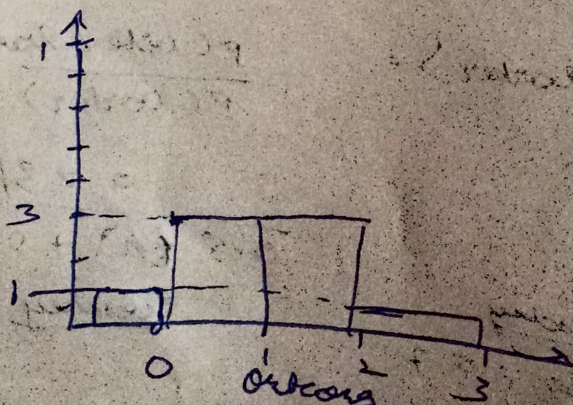
$P(Y \leq 30)$

P(sums of upper) → quantified

X = "heads" over 3 ~~coins~~ flips.

| | |
|---|---|
| X = 0 | = 1/8 |
| X = 1 | = 3/8 |
| X = 2 | = 3/8 |
| X = 3 | = 1/8 |

→ Probability Mass function.




Discrete probability distribution

theoretical mean

$$\boxed{E(X) = \sum x\, p(x)} \quad \mu$$

$$\text{Var} = \boxed{E(x^2) - (E(x))^2}$$

$$\text{Var}(X) = E[(X-\mu)^2] = \sigma^2$$

B **P.M.F** ← (D.R.V)

① $P(x) \geq 0$

② $\sum P(x) = 1$.

**P.D.F** ← (C.R.V)

① $\int_{-\infty}^{\infty} f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Density fun

probability only.

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1/8 | 3/8 | 3/8 | 1/8. |

← $\boxed{\text{P.M.F}}$

## Distribution function.

$$F(x) = \begin{cases} 1/8 & x \leq 0 \\ 4/8 & x \leq 1 \\ 7/8 & x \leq 2 \\ 1 & x \leq 3 \end{cases}$$

$$h(x) = \begin{cases} \frac{1}{2^x} & x = 0,1,2 \\ 0. & \text{otherwise} \end{cases}$$

is this P.M.F?

$\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} > 1$ NO

$-8 + + + + 2\ 7$

## Continuous RV

$\infty$ no. of values in an interval.

① weight

$$\boxed{P(x>1) \Rightarrow \int_1^{\infty} f(x)\,dx}$$

## Probability density function

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$g(x)\ h(x)$

Eg: $x$ is CRV

$$f(x) = \begin{cases} \alpha(2x - x^2) & 0 \leq x \leq 2 \\ 0 & \text{o/w}. \end{cases}$$

① $\alpha \to 3/4$

② $P(X > 1)$

$\int_{-\infty}^{\infty} = \cancel{\int_{-\infty}^{0}} + \int_0^{2} + \cancel{\int_{2}^{\infty}} = 1$

$\alpha\left(\frac{2x^2}{2} - \frac{x^3}{3}\right)\Big|_0^2 = 1$ $\quad 4/3$

$\alpha\left(\frac{4}{\cancel{2}} - \frac{8}{3}\right) = 1$ $\quad 0.3$

Q) $f(x) = \begin{cases} kx^2 & -3 < x < 3 \\ 0 & o/w \end{cases}$

$k = ?$

$P(1 \le x \le 2)$

$P\{x \le 2\}$

$P(x > 1)$

$$\int_{-3}^{3} kx^2 = 1 \Rightarrow \boxed{k = \frac{1}{18}}$$

mean life time $(E(x)) = \int_{-\infty}^{\infty} xf$

$$P(1 \le x \le 2) = \frac{1}{18}\int_{1}^{2} x^2 dx =$$

$$P(x \le 2) = \frac{1}{18}\int_{-3}^{2} x^2 dx .$$

$$P(x > 1) = \frac{1}{18}\int_{1}^{3} kx^2 dx$$

## Poisson R.V

k : DRV

$$\boxed{P(x = k) = \frac{e^{-\lambda}\lambda^k}{k!} \quad k = 0, 1, 2 \ldots \infty}$$

$\lambda$ = expected no...

Q10 expected no. of things = 3.

Poisson ..

a) probability of 5 = ? $= \dfrac{e^{-3}3^5}{5!}$

b). forget 5 things if know atleast 4 thi $\dfrac{5 \cap 4}{4}$

$$P(x = 5 \mid x \ge 4) = \frac{P(\{x = 5\} \cap x(5)}{1 - P(x < 4)} = \frac{\frac{e^{-3}3^5}{5!}}{}$$

$P(x=0) + (x=1) + (x=2) + P(x=3)$

Q12 m hooks on ur line

Binomial

$1-p \rightarrow p$

not swallow, swallow

PMF

Bernoulli

(m)

$$K = \begin{cases} {}^mC_k \, p^k (1-p)^{m-k} & k = 0 \dots m \\ 0 & \text{otherwise} \end{cases}$$

Q13

catches $\rightarrow p$

$$PMF(x) = P(X=k) = \begin{cases} (1-p)^{k-1} \, p^k & p = 1, 2 \dots \\ 0 & o/w \end{cases}$$

k : success

---

$$\boxed{\text{Mean life Time} = \int_{-\infty}^{\infty} x \, f(x) \, dx}$$

$$\int_{10}^{\infty} z \cdot \frac{10}{z^2} \, dz = \infty$$

$(x+y)^0$

$(x+y)^1$

| Pascal's | Prob. |
|---|---|
| 1 | 1 |
| 1 1 | 1/2 1/2 |
| 1 2 1 | 1/4 1/2 1/4 |
| 1 3 3 1 | 1/8 3/8 3/8 1/8 |
| 1 4 6 4 1 | |

Probabilistic Inferencing

Non-Probabilistic Inferencing

| | Mean | S.D |
|---|---|---|
| entry ~~mean~~ +3 | +3 | No change |
| entry x2 | x2 | x2 |
| entry x -1 | x -1 | no change |

## CRV

$$\int f(x)\,dx = 1$$

$$E(x) = \int x\, f(x)\,dx$$

$$E(x^2) = \int x^2 f(x)\,dx$$

$$Var(x) = E(x^2) - [E(x)]^2$$

NOTE: ALWAYS CHECK if function is +ve only then go perform $\int$

Eg: P(catching) $\to p$

P(run | catch)

$$P(x) = \begin{cases} (1-p)^{x-1}\, p & x = 1, 2, \ldots \\ 0 & o/w \end{cases}$$

Eg: flip a coin until heads. X be no. of flips you made up to & including first heads. Probability $X = 1$ given $X < 3$?

$P(H) = P(T) = 1/2$

$P(X=1) = 1/2$

$$P(X<3) = P(X=1) + P(X=2)$$
$$= \frac{1}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$
T H
$$= 3/4$$

$$P(X=1 \mid X<3) = \frac{P(X=1 \cap X<3)}{P(X<3)}$$

$$= \frac{P(X=1)}{P(X<3)} = 2/3$$

Eg:

P( exercise ) = 0.7

P( pizza ) = 0.2

P( pizza | exercise ) = 0.15

P( no pizza and no exercise ) = ~~P((A∪B))~~

$\cap$

$A^c \cap B^c$

$= (A \cup B)^c$

$(A \cap B)^c$

$P(A) = 0.7$

$P(B) = 0.2$

$P(B|A) = 0.15 = \frac{P(A \cap B)}{P(A)}$ —?

P(.

$A^c \cap B^c = (A \cup B)^c$

$= 1 - (A \cup B)$

0.105.

$P(A \cup B) = PA + PB - P A \cap B)$

Eg 10 floors

# Bernoulli Distribution

→ ONLY 2 possible outcomes $\nearrow$ 1 success, $\searrow$ 0 failure. DSC

$P(X) = \begin{cases} 1-p & x=0 \\ p & x=1 \end{cases}$

$\mu = np$ mean
$\sigma^2 = np(1-p)$

$\boxed{P(X=x) = p^{x}(1-p)^{1-x}}$

$p$ → happening
$1-p$ → not happens.

$x = 0, 1$

$P(X=1) = p$
$P(X=0) = 1-p$

Binomial → distribution of success
geometric → distribution of first success
-ve binom → $r^{th}$ success.

Var $= E(x^2) - [E(x)]^2$
$= p - p^2$

$\boxed{E(x) = 1 * p + 0(1-p) = p}$

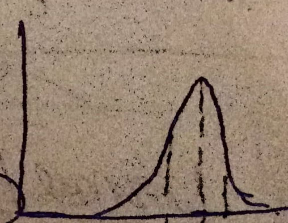
# Binomial distribution

DSC

$\boxed{p(x) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}}$

$x$: no. of success $p$
$n-x$: no. of failure $1-p$

Eg: probabi a 5 comes out exactly 2 times out of 3.
either a 5 or not a 5. $3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1$



# Geometric distribution "first" success. DSC

→ $x^{th}$ is success
→ $(1-x)$ is failure.

$x = 1, 2, 3 \dots$

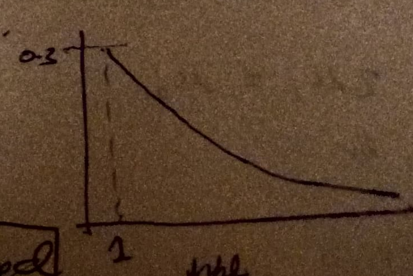
$\mu = \frac{1}{p}$

$var = \frac{1-p}{p^2}$

$\boxed{p(x) = (1-p)^{x-1}\, p}$

Q: 30% success 6th person success first.

$p(x) = (0.7)^5\, 0.3$

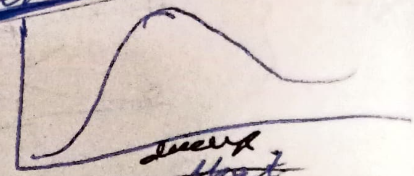



$\boxed{\text{right skewed}}$

## Poisson :

① no. of occurrence of an event in a given unit of distance / area / volume.

→ no. of car accident in a day.



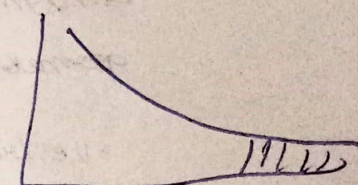

$$\boxed{P(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}}$$

λ : mean in ~~that~~ period

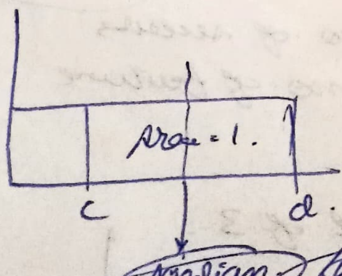
per year → total no.

## Exponential:

when next.

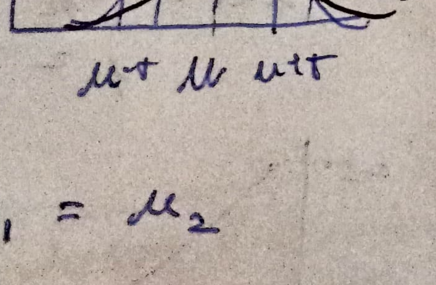


## Uniform.




→ constant probability of all outcomes within range.

## Normal / Gaussian




→ symmetric, bell shaped

μ : mean / median.

σ : SD

±σ 68%

±2σ 95%

±3σ 99%

$\boxed{\text{mean} = 0 \quad \text{var} = 1}$

$2\mu_1 = \mu_2$

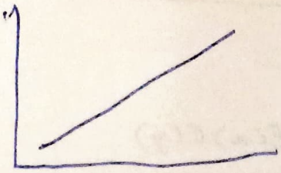
# Correlation Coefficient.

2-variable relation.

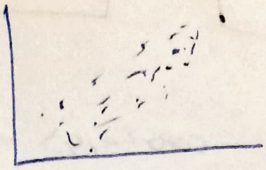
→ 1   +ve correlation

→ 0   independent data.

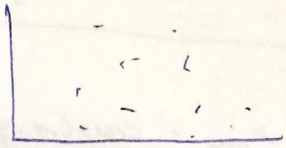
→ -1   -ve correlation.



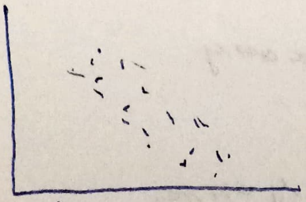
$r = +1$



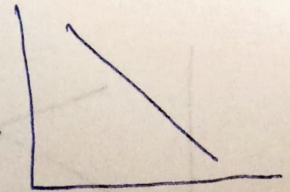
$r < 1$



$r = 0$.



~~$0 < r$~~ $-1 < r < 0$



$r = -1$

$\boxed{-1 \le r \le 1}$

$\boxed{-1 \le \rho_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y} \le 1}$

$cov(x,y) = E(xy) - E(x)E(y)$

$= \frac{\Sigma xy}{n} - \frac{\Sigma x}{n} \frac{\Sigma y}{n}$

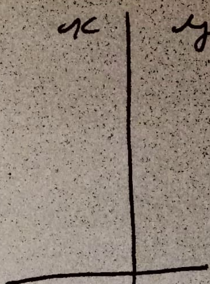
$\sigma_x = \sqrt{\Sigma(x^2) - [E(x)]^2}$

E : expected value

$\sigma_x$ : SD of x

on moving data points.
r DOES NOT change.

ques: calculate correlation constant.




$r = \frac{cov(x,y)}{\sigma_x \sigma_y}$

$cov(x,y) = E(xy) - E(x)E(y)$

$= \frac{\Sigma xy}{n} - \frac{\Sigma x}{n} \frac{\Sigma y}{n}$

$\sigma_x = \sqrt{E(x^2) - [E(x)]^2}$

$\frac{\Sigma x^2}{n} \quad \left(\frac{\Sigma x}{n}\right)^2$

$\Sigma x = 553$

$\Sigma y = 552$

$\Sigma xy = 37580$

$\Sigma x^2 = 37028$

$\Sigma y^2 = 38132$

$\boxed{r = 0.6032}$

Q.205

$x$ on $y$

$$\boxed{x-\bar{x} = r_{xy}\frac{\sigma_x}{\sigma_y}(y-\bar{y})}$$

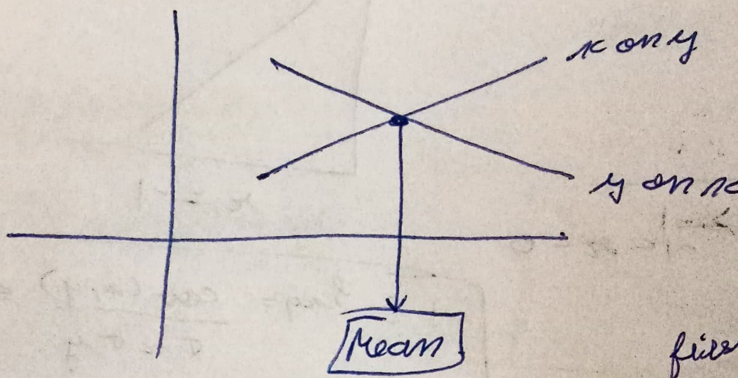
$y$ on $x$

$$\boxed{y-\bar{y} = r_{yx}\frac{\sigma_y}{\sigma_x}(x-\bar{x})}$$

$$r_{xy} = \frac{cov(x,y)}{\sigma_x\sigma_y}$$

$$cov(x,y) = E(xy) - E(x)E(y)$$

$$\sigma_x = \sqrt{E(x^2) - [E(x)]^2}$$




If $x$ on $y$ from first eq & $y$ on $x$ second eq. doesn't satisfy then reverse the eqs.

NOTE:

$$cov(x,y) = E(xy) - E(x)E(y)$$

$$= \frac{\Sigma xy}{n} - \frac{\Sigma x}{n}\frac{\Sigma y}{n}$$

$$\approx \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} - \frac{\Sigma(x-\bar{x})}{n}\frac{\Sigma(y-\bar{y})}{n}.$$