chapter 3

The Relational Data Model and Relational Database Constraints

This chapter opens Part 2 of the book, which covers relational databases. The relational data model was first introduced by Ted Codd of IBM Research in 1970 in a classic paper (Codd 1970), and it attracted immediate attention due to its simplicity and mathematical foundation. The model uses the concept of a *mathematical relation*—which looks somewhat like a table of values—as its basic building block, and has its theoretical basis in set theory and first-order predicate logic. In this chapter we discuss the basic characteristics of the model and its constraints.

The first commercial implementations of the relational model became available in the early 1980s, such as the SQL/DS system on the MVS operating system by IBM and the Oracle DBMS. Since then, the model has been implemented in a large number of commercial systems. Current popular relational DBMSs (RDBMSs) include DB2 and Informix Dynamic Server (from IBM), Oracle and Rdb (from Oracle), Sybase DBMS (from Sybase) and SQLServer and Access (from Microsoft). In addition, several open source systems, such as MySQL and PostgreSQL, are available.

Because of the importance of the relational model, all of Part 2 is devoted to this model and some of the languages associated with it. In Chapters 4 and 5, we describe the SQL query language, which is the *standard* for commercial relational DBMSs. Chapter 6 covers the operations of the relational algebra and introduces the relational calculus—these are two formal languages associated with the relational model. The relational calculus is considered to be the basis for the SQL language, and the relational algebra is used in the internals of many database implementations for query processing and optimization (see Part 8 of the book).

Other aspects of the relational model are presented in subsequent parts of the book. Chapter 9 relates the relational model data structures to the constructs of the ER and EER models (presented in Chapters 7 and 8), and presents algorithms for designing a relational database schema by mapping a conceptual schema in the ER or EER model into a relational representation. These mappings are incorporated into many database design and CASE[1] tools. Chapters 13 and 14 in Part 5 discuss the programming techniques used to access database systems and the notion of connecting to relational databases via ODBC and JDBC standard protocols. We also introduce the topic of Web database programming in Chapter 14. Chapters 15 and 16 in Part 6 present another aspect of the relational model, namely the formal constraints of functional and multivalued dependencies; these dependencies are used to develop a relational database design theory based on the concept known as *normalization*.

Data models that preceded the relational model include the hierarchical and network models. They were proposed in the 1960s and were implemented in early DBMSs during the late 1960s and early 1970s. Because of their historical importance and the existing user base for these DBMSs, we have included a summary of the highlights of these models in Appendices D and E, which are available on this book's Companion Website at http://www.aw.com/elmasri. These models and systems are now referred to as *legacy database systems*.

In this chapter, we concentrate on describing the basic principles of the relational model of data. We begin by defining the modeling concepts and notation of the relational model in Section 3.1. Section 3.2 is devoted to a discussion of relational constraints that are considered an important part of the relational model and are automatically enforced in most relational DBMSs. Section 3.3 defines the update operations of the relational model, discusses how violations of integrity constraints are handled, and introduces the concept of a transaction. Section 3.4 summarizes the chapter.

3.1 Relational Model Concepts

The relational model represents the database as a collection of *relations*. Informally, each relation resembles a table of values or, to some extent, a *flat* file of records. It is called a **flat file** because each record has a simple linear or *flat* structure. For example, the database of files that was shown in Figure 1.2 is similar to the basic relational model representation. However, there are important differences between relations and files, as we shall soon see.

When a relation is thought of as a **table** of values, each row in the table represents a collection of related data values. A row represents a fact that typically corresponds to a real-world entity or relationship. The table name and column names are used to help to interpret the meaning of the values in each row. For example, the first table of Figure 1.2 is called STUDENT because each row represents facts about a particular

[1]CASE stands for computer-aided software engineering.

student entity. The column names—Name, Student_number, Class, and Major—specify how to interpret the data values in each row, based on the column each value is in. All values in a column are of the same data type.

In the formal relational model terminology, a row is called a *tuple,* a column header is called an *attribute,* and the table is called a *relation.* The data type describing the types of values that can appear in each column is represented by a *domain* of possible values. We now define these terms—*domain, tuple, attribute,* and *relation*—formally.

3.1 Domains, Attributes, Tuples, and Relations

A **domain** D is a set of atomic values. By **atomic** we mean that each value in the domain is indivisible as far as the formal relational model is concerned. A common method of specifying a domain is to specify a data type from which the data values forming the domain are drawn. It is also useful to specify a name for the domain, to help in interpreting its values. Some examples of domains follow:

- Usa_phone_numbers. The set of ten-digit phone numbers valid in the United States.
- Local_phone_numbers. The set of seven-digit phone numbers valid within a particular area code in the United States. The use of local phone numbers is quickly becoming obsolete, being replaced by standard ten-digit numbers.
- Social_security_numbers. The set of valid nine-digit Social Security numbers. (This is a unique identifier assigned to each person in the United States for employment, tax, and benefits purposes.)
- Names: The set of character strings that represent names of persons.
- Grade_point_averages. Possible values of computed grade point averages; each must be a real (floating-point) number between 0 and 4.
- Employee_ages. Possible ages of employees in a company; each must be an integer value between 15 and 80.
- Academic_department_names. The set of academic department names in a university, such as Computer Science, Economics, and Physics.
- Academic_department_codes. The set of academic department codes, such as 'CS', 'ECON', and 'PHYS'.

The preceding are called *logical* definitions of domains. A **data type** or **format** is also specified for each domain. For example, the data type for the domain Usa_phone_numbers can be declared as a character string of the form (*ddd*)*ddd*-*dddd*, where each *d* is a numeric (decimal) digit and the first three digits form a valid telephone area code. The data type for Employee_ages is an integer number between 15 and 80. For Academic_department_names, the data type is the set of all character strings that represent valid department names. A domain is thus given a name, data type, and format. Additional information for interpreting the values of a domain can also be given; for example, a numeric domain such as Person_weights should have the units of measurement, such as pounds or kilograms.

A **relation schema**[2] R, denoted by $R(A_1, A^2, \ldots, A_n)$, is made up of a relation name R and a list of attributes, $A_1, A_2, \ldots, A_n$. Each **attribute** A_i is the name of a role played by some domain D in the relation schema R. D is called the **domain** of A_i and is denoted by **dom**($\mathbf{A}_i$). A relation schema is used to *describe* a relation; R is called the **name** of this relation. The **degree** (or **arity**) of a relation is the number of attributes n of its relation schema.

A relation of degree seven, which stores information about university students, would contain seven attributes describing each student. as follows:

STUDENT(Name, Ssn, Home_phone, Address, Office_phone, Age, Gpa)

Using the data type of each attribute, the definition is sometimes written as:

STUDENT(Name: string, Ssn: string, Home_phone: string, Address: string, Office_phone: string, Age: integer, Gpa: real)

For this relation schema, STUDENT is the name of the relation, which has seven attributes. In the preceding definition, we showed assignment of generic types such as string or integer to the attributes. More precisely, we can specify the following previously defined domains for some of the attributes of the STUDENT relation: dom(Name) = Names; dom(Ssn) = Social_security_numbers; dom(HomePhone) = USA_phone_numbers[3], dom(Office_phone) = USA_phone_numbers, and dom(Gpa) = Grade_point_averages. It is also possible to refer to attributes of a relation schema by their position within the relation; thus, the second attribute of the STUDENT relation is Ssn, whereas the fourth attribute is Address.

A **relation** (or **relation state**)[4] r of the relation schema $R(A_1, A_2, \ldots, A_n)$, also denoted by $r(R)$, is a set of n-tuples $r = \{t_1, t_2, \ldots, t_m\}$. Each ***n*-tuple** t is an ordered list of n values $t = <v_1, v_2, \ldots, v_n>$, where each value v_i, $1 \le i \le n$, is an element of dom (A_i) or is a special NULL value. (NULL values are discussed further below and in Section 3.1.2.) The i^{th} value in tuple t, which corresponds to the attribute A_i, is referred to as $t[A_i]$ or $t.A_i$ (or $t[i]$ if we use the positional notation). The terms **relation intension** for the schema R and **relation extension** for a relation state $r(R)$ are also commonly used.

Figure 3.1 shows an example of a STUDENT relation, which corresponds to the STUDENT schema just specified. Each tuple in the relation represents a particular student entity (or object). We display the relation as a table, where each tuple is shown as a *row* and each attribute corresponds to a *column header* indicating a role or interpretation of the values in that column. *NULL values* represent attributes whose values are unknown or do not exist for some individual STUDENT tuple.

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[2]A relation schema is sometimes called a **relation scheme**.

[3]With the large increase in phone numbers caused by the proliferation of mobile phones, most metropolitan areas in the U.S. now have multiple area codes, so seven-digit local dialing has been discontinued in most areas. We changed this domain to Usa_phone_numbers instead of Local_phone_numbers which would be a more general choice. This illustrates how database requirements can change over time.

[4]This has also been called a **relation instance**. We will not use this term because *instance* is also used to refer to a single tuple or row.

Relation Name

Attributes

Tuples

STUDENT

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Benjamin Bayer	305-61-2435	(817)373-1616	2918 Bluebonnet Lane	NULL	19	3.21
Chung-cha Kim	381-62-1245	(817)375-4409	125 Kirby Road	NULL	18	2.89
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	(817)749-1253	25	3.53
Rohan Panchal	489-22-1100	(817)376-9821	265 Lark Lane	(817)749-6492	28	3.93
Barbara Benson	533-69-1238	(817)839-8461	7384 Fontana Lane	NULL	19	3.25

Figure 3.1
The attributes and tuples of a relation STUDENT.

The earlier definition of a relation can be *restated* more formally using set theory concepts as follows. A relation (or relation state) $r(R)$ is a **mathematical relation** of degree n on the domains $\text{dom}(A_1)$, $\text{dom}(A_2)$, ..., $\text{dom}(A_n)$, which is a **subset** of the **Cartesian product** (denoted by $\times$) of the domains that define R:

$$r(R) \subseteq (\text{dom}(A_1) \times \text{dom}(A_2) \times \ldots \times \text{dom}(A_n))$$

The Cartesian product specifies all possible combinations of values from the underlying domains. Hence, if we denote the total number of values, or **cardinality,** in a domain D by $|D|$ (assuming that all domains are finite), the total number of tuples in the Cartesian product is

$$|\text{dom}(A_1)| \times |\text{dom}(A_2)| \times \ldots \times |\text{dom}(A_n)|$$

This product of cardinalities of all domains represents the total number of possible instances or tuples that can ever exist in any relation state $r(R)$. Of all these possible combinations, a relation state at a given time—the **current relation state**—reflects only the valid tuples that represent a particular state of the real world. In general, as the state of the real world changes, so does the relation state, by being transformed into another relation state. However, the schema R is relatively static and changes *very* infrequently—for example, as a result of adding an attribute to represent new information that was not originally stored in the relation.

It is possible for several attributes to *have the same domain*. The attribute names indicate different **roles**, or interpretations, for the domain. For example, in the STUDENT relation, the same domain USA_phone_numbers plays the role of Home_phone, referring to the *home phone of a student*, and the role of Office_phone, referring to the *office phone of the student*. A third possible attribute (not shown) with the same domain could be Mobile_phone.

3.1.2 Characteristics of Relations

The earlier definition of relations implies certain characteristics that make a relation different from a file or a table. We now discuss some of these characteristics.

Ordering of Tuples in a Relation. A relation is defined as a *set* of tuples. Mathematically, elements of a set have *no order* among them; hence, tuples in a relation do not have any particular order. In other words, a relation is not sensitive to the ordering of tuples. However, in a file, records are physically stored on disk (or in memory), so there always is an order among the records. This ordering indicates first, second, *i*th, and last records in the file. Similarly, when we display a relation as a table, the rows are displayed in a certain order.

Tuple ordering is not part of a relation definition because a relation attempts to represent facts at a logical or abstract level. Many tuple orders can be specified on the same relation. For example, tuples in the STUDENT relation in Figure 3.1 could be ordered by values of Name, Ssn, Age, or some other attribute. The definition of a relation does not specify any order: There is *no preference* for one ordering over another. Hence, the relation displayed in Figure 3.2 is considered *identical* to the one shown in Figure 3.1. When a relation is implemented as a file or displayed as a table, a particular ordering may be specified on the records of the file or the rows of the table.

Ordering of Values within a Tuple and an Alternative Definition of a Relation. According to the preceding definition of a relation, an n-tuple is an *ordered list* of n values, so the ordering of values in a tuple—and hence of attributes in a relation schema—is important. However, at a more abstract level, the order of attributes and their values is *not* that important as long as the correspondence between attributes and values is maintained.

An **alternative definition** of a relation can be given, making the ordering of values in a tuple *unnecessary*. In this definition, a relation schema $R = \{A_1, A_2, \ldots, A_n\}$ is a *set* of attributes (instead of a list), and a relation state $r(R)$ is a finite set of mappings $r = \{t_1, t_2, \ldots, t_m\}$, where each tuple t_i is a **mapping** from R to D, and D is the **union** (denoted by $\cup$) of the attribute domains; that is, $D = \text{dom}(A_1) \cup \text{dom}(A_2) \cup \ldots \cup \text{dom}(A_n)$. In this definition, $t[A_i]$ must be in $\text{dom}(A_i)$ for $1 \le i \le n$ for each mapping t in r. Each mapping t_i is called a tuple.

According to this definition of tuple as a mapping, a **tuple** can be considered as a **set** of (<attribute>, <value>) pairs, where each pair gives the value of the mapping from an attribute A_i to a value v_i from $\text{dom}(A_i)$. The ordering of attributes is *not*

Figure 3.2
The relation STUDENT from Figure 3.1 with a different order of tuples.

STUDENT

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	(817)749-1253	25	3.53
Barbara Benson	533-69-1238	(817)839-8461	7384 Fontana Lane	NULL	19	3.25
Rohan Panchal	489-22-1100	(817)376-9821	265 Lark Lane	(817)749-6492	28	3.93
Chung-cha Kim	381-62-1245	(817)375-4409	125 Kirby Road	NULL	18	2.89
Benjamin Bayer	305-61-2435	(817)373-1616	2918 Bluebonnet Lane	NULL	19	3.21

important, because the *attribute name* appears with its *value*. By this definition, the two tuples shown in Figure 3.3 are identical. This makes sense at an abstract level, since there really is no reason to prefer having one attribute value appear before another in a tuple.

When a relation is implemented as a file, the attributes are physically ordered as fields within a record. We will generally use the **first definition** of relation, where the attributes and the values within tuples *are ordered,* because it simplifies much of the notation. However, the alternative definition given here is more general.[5]

Values and NULLs in the Tuples. Each value in a tuple is an **atomic** value; that is, it is not divisible into components within the framework of the basic relational model. Hence, composite and multivalued attributes (see Chapter 7) are not allowed. This model is sometimes called the **flat relational model**. Much of the theory behind the relational model was developed with this assumption in mind, which is called the **first normal form** assumption.[6] Hence, multivalued attributes must be represented by separate relations, and composite attributes are represented only by their simple component attributes in the basic relational model.[7]

An important concept is that of NULL values, which are used to represent the values of attributes that may be unknown or may not apply to a tuple. A special value, called NULL, is used in these cases. For example, in Figure 3.1, some STUDENT tuples have NULL for their office phones because they do not have an office (that is, office phone *does not apply* to these students). Another student has a NULL for home phone, presumably because either he does not have a home phone or he has one but we do not know it (value is *unknown*). In general, we can have several meanings for NULL values, such as ***value unknown, value*** exists but is ***not available***, or ***attribute does not apply*** to this tuple (also known as ***value undefined***). An example of the last type of NULL will occur if we add an attribute Visa_status to the STUDENT relation

Figure 3.3
Two identical tuples when the order of attributes and values is not part of relation definition.

t = < (Name, Dick Davidson),(Ssn, 422-11-2320),(Home_phone, NULL),(Address, 3452 Elgin Road),
(Office_phone, (817)749-1253),(Age, 25),(Gpa, 3.53)>

t = < (Address, 3452 Elgin Road),(Name, Dick Davidson),(Ssn, 422-11-2320),(Age, 25),
(Office_phone, (817)749-1253),(Gpa, 3.53),(Home_phone, NULL)>

[5]As we shall see, the alternative definition of relation is useful when we discuss query processing and optimization in Chapter 19.

[6]We discuss this assumption in more detail in Chapter 15.

[7]Extensions of the relational model remove these restrictions. For example, object-relational systems (Chapter 11) allow complex-structured attributes, as do the **non-first normal form** or **nested** relational models.

that applies only to tuples representing foreign students. It is possible to devise different codes for different meanings of NULL values. Incorporating different types of NULL values into relational model operations (see Chapter 6) has proven difficult and is outside the scope of our presentation.

The exact meaning of a NULL value governs how it fares during arithmetic aggregations or comparisons with other values. For example, a comparison of two NULL values leads to ambiguities—if both Customer A and B have NULL addresses, it *does not mean* they have the same address. During database design, it is best to avoid NULL values as much as possible. We will discuss this further in Chapters 5 and 6 in the context of operations and queries, and in Chapter 15 in the context of database design and normalization.

Interpretation (Meaning) of a Relation. The relation schema can be interpreted as a declaration or a type of **assertion**. For example, the schema of the STUDENT relation of Figure 3.1 asserts that, in general, a student entity has a Name, Ssn, Home_phone, Address, Office_phone, Age, and Gpa. Each tuple in the relation can then be interpreted as a **fact** or a particular instance of the assertion. For example, the first tuple in Figure 3.1 asserts the fact that there is a STUDENT whose Name is Benjamin Bayer, Ssn is 305-61-2435, Age is 19, and so on.

Notice that some relations may represent facts about *entities,* whereas other relations may represent facts about *relationships.* For example, a relation schema MAJORS (Student_ssn, Department_code) asserts that students major in academic disciplines. A tuple in this relation relates a student to his or her major discipline. Hence, the relational model represents facts about both entities and relationships *uniformly* as relations. This sometimes compromises understandability because one has to guess whether a relation represents an entity type or a relationship type. We introduce the Entity-Relationship (ER) model in detail in Chapter 7 where the entity and relationship concepts will be described in detail. The mapping procedures in Chapter 9 show how different constructs of the ER and EER (Enhanced ER model covered in Chapter 8) conceptual data models (see Part 3) get converted to relations.

An alternative interpretation of a relation schema is as a **predicate**; in this case, the values in each tuple are interpreted as values that *satisfy* the predicate. For example, the predicate STUDENT (Name, Ssn, ...) is true for the five tuples in relation STUDENT of Figure 3.1. These tuples represent five different propositions or facts in the real world. This interpretation is quite useful in the context of logical programming languages, such as Prolog, because it allows the relational model to be used within these languages (see Section 26.5). An assumption called **the closed world assumption** states that the only true facts in the universe are those present within the extension (state) of the relation(s). Any other combination of values makes the predicate false.

3.1.3 Relational Model Notation

We will use the following notation in our presentation:

- A relation schema R of degree n is denoted by $R(A_1, A_2, \ldots, A_n)$.

- The uppercase letters Q, R, S denote relation names.
- The lowercase letters q, r, s denote relation states.
- The letters t, u, v denote tuples.
- In general, the name of a relation schema such as STUDENT also indicates the current set of tuples in that relation—the *current relation state*—whereas STUDENT(Name, Ssn, ...) refers *only* to the relation schema.
- An attribute A can be qualified with the relation name R to which it belongs by using the dot notation $R.A$—for example, STUDENT.Name or STUDENT.Age. This is because the same name may be used for two attributes in different relations. However, all attribute names *in a particular relation* must be distinct.
- An n-tuple t in a relation $r(R)$ is denoted by $t = <v_1, v_2, ..., v_n>$, where v_i is the value corresponding to attribute A_i. The following notation refers to **component values** of tuples:
- Both $t[A_i]$ and $t.A_i$ (and sometimes $t[i]$) refer to the value v_i in t for attribute A_i.
- Both $t[A_u, A_w, ..., A_z]$ and $t.(A_u, A_w, ..., A_z)$, where $A_u, A_w, ..., A_z$ is a list of attributes from R, refer to the subtuple of values $<v_u, v_w, ..., v_z>$ from t corresponding to the attributes specified in the list.

As an example, consider the tuple t = <‘Barbara Benson’, ‘533-69-1238’, ‘(817)839-8461’, ‘7384 Fontana Lane’, NULL, 19, 3.25> from the STUDENT relation in Figure 3.1; we have t[Name] = <‘Barbara Benson’>, and t[Ssn, Gpa, Age] = <‘533-69-1238’, 3.25, 19>.

3.2 Relational Model Constraints and Relational Database Schemas

So far, we have discussed the characteristics of single relations. In a relational database, there will typically be many relations, and the tuples in those relations are usually related in various ways. The state of the whole database will correspond to the states of all its relations at a particular point in time. There are generally many restrictions or **constraints** on the actual values in a database state. These constraints are derived from the rules in the miniworld that the database represents, as we discussed in Section 1.6.8.

In this section, we discuss the various restrictions on data that can be specified on a relational database in the form of constraints. Constraints on databases can generally be divided into three main categories:

1. Constraints that are inherent in the data model. We call these **inherent model-based constraints** or **implicit constraints**.
2. Constraints that can be directly expressed in schemas of the data model, typically by specifying them in the DDL (data definition language, see Section 2.3.1). We call these **schema-based constraints** or **explicit constraints**.

3. Constraints that *cannot* be directly expressed in the schemas of the data model, and hence must be expressed and enforced by the application programs. We call these **application-based** or **semantic constraints** or **business rules**.

The characteristics of relations that we discussed in Section 3.1.2 are the inherent constraints of the relational model and belong to the first category. For example, the constraint that a relation cannot have duplicate tuples is an inherent constraint. The constraints we discuss in this section are of the second category, namely, constraints that can be expressed in the schema of the relational model via the DDL. Constraints in the third category are more general, relate to the meaning as well as behavior of attributes, and are difficult to express and enforce within the data model, so they are usually checked within the application programs that perform database updates.

Another important category of constraints is *data dependencies*, which include *functional dependencies* and *multivalued dependencies*. They are used mainly for testing the "goodness" of the design of a relational database and are utilized in a process called *normalization*, which is discussed in Chapters 15 and 16.

The schema-based constraints include domain constraints, key constraints, constraints on NULLs, entity integrity constraints, and referential integrity constraints.

3.2.1 Domain Constraints

Domain constraints specify that within each tuple, the value of each attribute A must be an atomic value from the domain dom(A). We have already discussed the ways in which domains can be specified in Section 3.1.1. The data types associated with domains typically include standard numeric data types for integers (such as short integer, integer, and long integer) and real numbers (float and double-precision float). Characters, Booleans, fixed-length strings, and variable-length strings are also available, as are date, time, timestamp, and money, or other special data types. Other possible domains may be described by a subrange of values from a data type or as an enumerated data type in which all possible values are explicitly listed. Rather than describe these in detail here, we discuss the data types offered by the SQL relational standard in Section 4.1.

3.2.2 Key Constraints and Constraints on NULL Values

In the formal relational model, a *relation* is defined as a *set of tuples.* By definition, all elements of a set are distinct; hence, all tuples in a relation must also be distinct. This means that no two tuples can have the same combination of values for *all* their attributes. Usually, there are other **subsets of attributes** of a relation schema R with the property that no two tuples in any relation state r of R should have the same combination of values for these attributes. Suppose that we denote one such subset of attributes by SK; then for any two *distinct* tuples t_1 and t_2 in a relation state r of R, we have the constraint that:

$$t_1[\text{SK}] \neq t_2[\text{SK}]$$

Any such set of attributes SK is called a **superkey** of the relation schema R. A superkey SK specifies a *uniqueness constraint* that no two distinct tuples in any state r of R can have the same value for SK. Every relation has at least one default superkey—the set of all its attributes. A superkey can have redundant attributes, however, so a more useful concept is that of a *key,* which has no redundancy. A **key** K of a relation schema R is a superkey of R with the additional property that removing any attribute A from K leaves a set of attributes K' that is not a superkey of R any more. Hence, a key satisfies two properties:

1. Two distinct tuples in any state of the relation cannot have identical values for (all) the attributes in the key. This first property also applies to a superkey.
2. It is a *minimal superkey*—that is, a superkey from which we cannot remove any attributes and still have the uniqueness constraint in condition 1 hold. This property is not required by a superkey.

Whereas the first property applies to both keys and superkeys, the second property is required only for keys. Hence, a key is also a superkey but not vice versa. Consider the STUDENT relation of Figure 3.1. The attribute set {Ssn} is a key of STUDENT because no two student tuples can have the same value for Ssn.[8] Any set of attributes that includes Ssn—for example, {Ssn, Name, Age}—is a superkey. However, the superkey {Ssn, Name, Age} is not a key of STUDENT because removing Name or Age or both from the set still leaves us with a superkey. In general, any superkey formed from a single attribute is also a key. A key with multiple attributes must require *all* its attributes together to have the uniqueness property.

The value of a key attribute can be used to identify uniquely each tuple in the relation. For example, the Ssn value 305-61-2435 identifies uniquely the tuple corresponding to Benjamin Bayer in the STUDENT relation. Notice that a set of attributes constituting a key is a property of the relation schema; it is a constraint that should hold on *every* valid relation state of the schema. A key is determined from the meaning of the attributes, and the property is *time-invariant:* It must continue to hold when we insert new tuples in the relation. For example, we cannot and should not designate the Name attribute of the STUDENT relation in Figure 3.1 as a key because it is possible that two students with identical names will exist at some point in a valid state.[9]

In general, a relation schema may have more than one key. In this case, each of the keys is called a **candidate key**. For example, the CAR relation in Figure 3.4 has two candidate keys: License_number and Engine_serial_number. It is common to designate one of the candidate keys as the **primary key** of the relation. This is the candidate key whose values are used to *identify* tuples in the relation. We use the convention that the attributes that form the primary key of a relation schema are underlined, as shown in Figure 3.4. Notice that when a relation schema has several candidate keys,

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[8]Note that Ssn is also a superkey.

[9]Names are sometimes used as keys, but then some artifact—such as appending an ordinal number—must be used to distinguish between identical names.

CAR

License_number	Engine_serial_number	Make	Model	Year
Texas ABC-739	A69352	Ford	Mustang	02
Florida TVP-347	B43696	Oldsmobile	Cutlass	05
New York MPO-22	X83554	Oldsmobile	Delta	01
California 432-TFY	C43742	Mercedes	190-D	99
California RSK-629	Y82935	Toyota	Camry	04
Texas RSK-629	U028365	Jaguar	XJS	04

Figure 3.4
The CAR relation, with two candidate keys: License_number and Engine_serial_number.

the choice of one to become the primary key is somewhat arbitrary; however, it is usually better to choose a primary key with a single attribute or a small number of attributes. The other candidate keys are designated as **unique keys**, and are not underlined.

Another constraint on attributes specifies whether NULL values are or are not permitted. For example, if every STUDENT tuple must have a valid, non-NULL value for the Name attribute, then Name of STUDENT is constrained to be NOT NULL.

3.2.3 Relational Databases and Relational Database Schemas

The definitions and constraints we have discussed so far apply to single relations and their attributes. A relational database usually contains many relations, with tuples in relations that are related in various ways. In this section we define a relational database and a relational database schema.

A **relational database schema** *S* is a set of relation schemas S = $\{R_1, R_2, \ldots, R_m\}$ and a set of **integrity constraints** IC. A **relational database state**[10] DB of *S* is a set of relation states DB = $\{r_1, r_2, \ldots, r_m\}$ such that each r_i is a state of R_i and such that the r_i relation states satisfy the integrity constraints specified in IC. Figure 3.5 shows a relational database schema that we call COMPANY = {EMPLOYEE, DEPARTMENT, DEPT_LOCATIONS, PROJECT, WORKS_ON, DEPENDENT}. The underlined attributes represent primary keys. Figure 3.6 shows a relational database state corresponding to the COMPANY schema. We will use this schema and database state in this chapter and in Chapters 4 through 6 for developing sample queries in different relational languages. (The data shown here is expanded and available for loading as a populated database from the Companion Website for the book, and can be used for the hands-on project exercises at the end of the chapters.)

When we refer to a relational database, we implicitly include both its schema and its current state. A database state that does not obey all the integrity constraints is

[10] A relational database *state* is sometimes called a relational database *instance*. However, as we mentioned earlier, we will not use the term *instance* since it also applies to single tuples.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date

DEPT_LOCATIONS

Dnumber	Dlocation

PROJECT

Pname	Pnumber	Plocation	Dnum

WORKS_ON

Essn	Pno	Hours

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship

Figure 3.5
Schema diagram for the COMPANY relational database schema.

called an **invalid state**, and a state that satisfies all the constraints in the defined set of integrity constraints IC is called a **valid state.**

In Figure 3.5, the Dnumber attribute in both DEPARTMENT and DEPT_LOCATIONS stands for the same real-world concept—the number given to a department. That same concept is called Dno in EMPLOYEE and Dnum in PROJECT. Attributes that represent the same real-world concept may or may not have identical names in different relations. Alternatively, attributes that represent different concepts may have the same name in different relations. For example, we could have used the attribute name Name for both Pname of PROJECT and Dname of DEPARTMENT; in this case, we would have two attributes that share the same name but represent different real-world concepts—project names and department names.

In some early versions of the relational model, an assumption was made that the same real-world concept, when represented by an attribute, would have *identical* attribute names in all relations. This creates problems when the same real-world concept is used in different roles (meanings) in the same relation. For example, the concept of Social Security number appears twice in the EMPLOYEE relation of Figure 3.5: once in the role of the employee's SSN, and once in the role of the supervisor's SSN. We are required to give them distinct attribute names—Ssn and Super_ssn, respectively—because they appear in the same relation and in order to distinguish their meaning.

Each relational DBMS must have a data definition language (DDL) for defining a relational database schema. Current relational DBMSs are mostly using SQL for this purpose. We present the SQL DDL in Sections 4.1 and 4.2.

Figure 3.6
One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Integrity constraints are specified on a database schema and are expected to hold on every valid database state of that schema. In addition to domain, key, and NOT NULL constraints, two other types of constraints are considered part of the relational model: entity integrity and referential integrity.

3.2.4 Integrity, Referential Integrity, and Foreign Keys

The **entity integrity constraint** states that no primary key value can be NULL. This is because the primary key value is used to identify individual tuples in a relation. Having NULL values for the primary key implies that we cannot identify some tuples. For example, if two or more tuples had NULL for their primary keys, we may not be able to distinguish them if we try to reference them from other relations.

Key constraints and entity integrity constraints are specified on individual relations. The **referential integrity constraint** is specified between two relations and is used to maintain the consistency among tuples in the two relations. Informally, the referential integrity constraint states that a tuple in one relation that refers to another relation must refer to an *existing tuple* in that relation. For example, in Figure 3.6, the attribute Dno of EMPLOYEE gives the department number for which each employee works; hence, its value in every EMPLOYEE tuple must match the Dnumber value of some tuple in the DEPARTMENT relation.

To define referential integrity more formally, first we define the concept of a *foreign key*. The conditions for a foreign key, given below, specify a referential integrity constraint between the two relation schemas R_1 and R_2. A set of attributes FK in relation schema R_1 is a **foreign key** of R_1 that **references** relation R_2 if it satisfies the following rules:

1. The attributes in FK have the same domain(s) as the primary key attributes PK of R_2; the attributes FK are said to **reference** or **refer to** the relation R_2.
2. A value of FK in a tuple t_1 of the current state $r_1(R_1)$ either occurs as a value of PK for some tuple t_2 in the current state $r_2(R_2)$ *or is NULL*. In the former case, we have $t_1[\text{FK}] = t_2[\text{PK}]$, and we say that the tuple t_1 **references** or **refers to** the tuple t_2.

In this definition, R_1 is called the **referencing relation** and R_2 is the **referenced relation**. If these two conditions hold, a **referential integrity constraint** from R_1 to R_2 is said to hold. In a database of many relations, there are usually many referential integrity constraints.

To specify these constraints, first we must have a clear understanding of the meaning or role that each attribute or set of attributes plays in the various relation schemas of the database. Referential integrity constraints typically arise from the *relationships among the entities* represented by the relation schemas. For example, consider the database shown in Figure 3.6. In the EMPLOYEE relation, the attribute Dno refers to the department for which an employee works; hence, we designate Dno to be a foreign key of EMPLOYEE referencing the DEPARTMENT relation. This means that a value of Dno in any tuple t_1 of the EMPLOYEE relation must match a value of the primary key of DEPARTMENT—the Dnumber attribute—in some tuple t_2 of the DEPARTMENT relation, or the value of Dno *can be NULL* if the employee does not belong to a department or will be assigned to a department later. For example, in Figure 3.6 the tuple for employee 'John Smith' references the tuple for the 'Research' department, indicating that 'John Smith' works for this department.

Notice that a foreign key can *refer to its own relation.* For example, the attribute Super_ssn in EMPLOYEE refers to the supervisor of an employee; this is another employee, represented by a tuple in the EMPLOYEE relation. Hence, Super_ssn is a foreign key that references the EMPLOYEE relation itself. In Figure 3.6 the tuple for employee 'John Smith' references the tuple for employee 'Franklin Wong,' indicating that 'Franklin Wong' is the supervisor of 'John Smith.'

We can *diagrammatically display referential integrity constraints* by drawing a directed arc from each foreign key to the relation it references. For clarity, the arrowhead may point to the primary key of the referenced relation. Figure 3.7 shows the schema in Figure 3.5 with the referential integrity constraints displayed in this manner.

All integrity constraints should be specified on the relational database schema (i.e., defined as part of its definition) if we want to enforce these constraints on the database states. Hence, the DDL includes provisions for specifying the various types of constraints so that the DBMS can automatically enforce them. Most relational DBMSs support key, entity integrity, and referential integrity constraints. These constraints are specified as a part of data definition in the DDL.

3.2.5 Other Types of Constraints

The preceding integrity constraints are included in the data definition language because they occur in most database applications. However, they do not include a large class of general constraints, sometimes called *semantic integrity constraints,* which may have to be specified and enforced on a relational database. Examples of such constraints are *the salary of an employee should not exceed the salary of the employee's supervisor* and *the maximum number of hours an employee can work on all projects per week is 56*. Such constraints can be specified and enforced within the application programs that update the database, or by using a general-purpose **constraint specification language**. Mechanisms called **triggers** and **assertions** can be used. In SQL, CREATE ASSERTION and CREATE TRIGGER statements can be used for this purpose (see Chapter 5). It is more common to check for these types of constraints within the application programs than to use constraint specification languages because the latter are sometimes difficult and complex to use, as we discuss in Section 26.1.

Another type of constraint is the *functional dependency* constraint, which establishes a functional relationship among two sets of attributes X and Y. This constraint specifies that the value of X determines a unique value of Y in all states of a relation; it is denoted as a functional dependency $X \rightarrow Y$. We use functional depen-dencies and other types of dependencies in Chapters 15 and 16 as tools to analyze the quality of relational designs and to "normalize" relations to improve their quality.

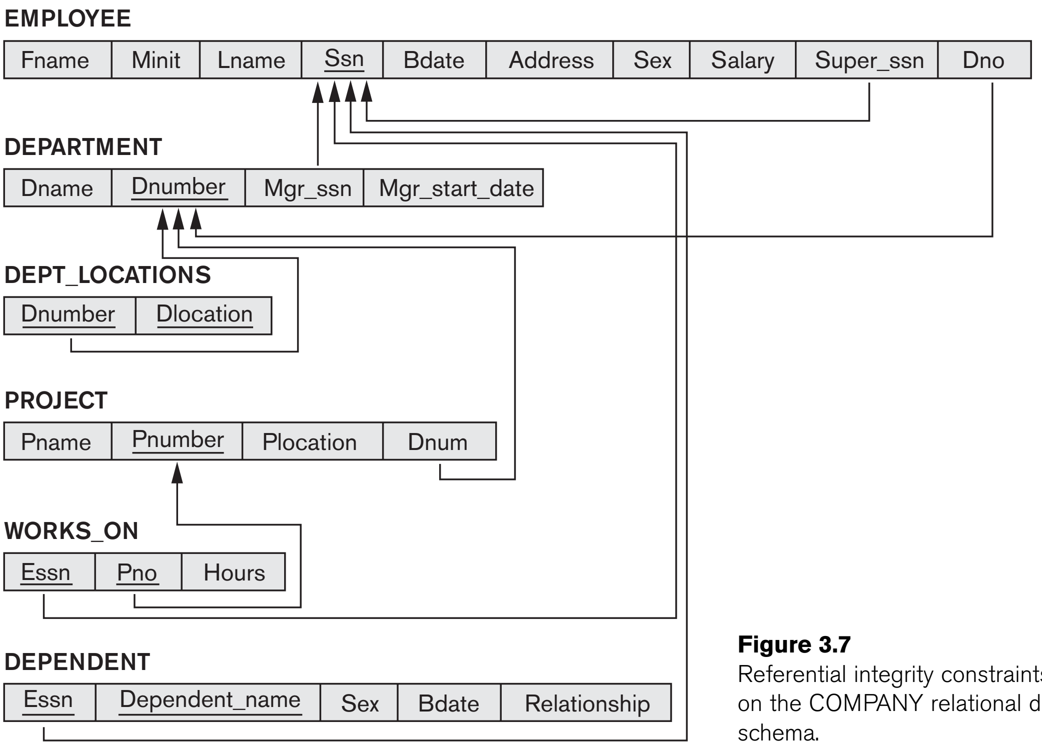

Figure 3.7
Referential integrity constraints displayed on the COMPANY relational database schema.

The types of constraints we discussed so far may be called **state constraints** because they define the constraints that a *valid state* of the database must satisfy. Another type of constraint, called **transition constraints**, can be defined to deal with state changes in the database.[11] An example of a transition constraint is: "the salary of an employee can only increase." Such constraints are typically enforced by the application programs or specified using active rules and triggers, as we discuss in Section 26.1.

3.3 Update Operations, Transactions, and Dealing with Constraint Violations

The operations of the relational model can be categorized into *retrievals* and *updates*. The relational algebra operations, which can be used to specify **retrievals**, are discussed in detail in Chapter 6. A relational algebra expression forms a new relation after applying a number of algebraic operators to an existing set of relations; its main use is for querying a database to retrieve information. The user formulates a query that specifies the data of interest, and a new relation is formed by applying relational operators to retrieve this data. That **result relation** becomes the

[11]State constraints are sometimes called *static constraints*, and transition constraints are sometimes called *dynamic constraints*.

answer to (or result of) the user's query. Chapter 6 also introduces the language called relational calculus, which is used to define the new relation declaratively without giving a specific order of operations.

In this section, we concentrate on the database **modification** or **update** operations. There are three basic operations that can change the states of relations in the database: Insert, Delete, and Update (or Modify). They insert new data, delete old data, or modify existing data records. **Insert** is used to insert one or more new tuples in a relation, **Delete** is used to delete tuples, and **Update** (or **Modify**) is used to change the values of some attributes in existing tuples. Whenever these operations are applied, the integrity constraints specified on the relational database schema should not be violated. In this section we discuss the types of constraints that may be violated by each of these operations and the types of actions that may be taken if an operation causes a violation. We use the database shown in Figure 3.6 for examples and discuss only key constraints, entity integrity constraints, and the referential integrity constraints shown in Figure 3.7. For each type of operation, we give some examples and discuss any constraints that each operation may violate.

3.3.1 The Insert Operation

The **Insert** operation provides a list of attribute values for a new tuple *t* that is to be inserted into a relation *R*. Insert can violate any of the four types of constraints discussed in the previous section. Domain constraints can be violated if an attribute value is given that does not appear in the corresponding domain or is not of the appropriate data type. Key constraints can be violated if a key value in the new tuple *t* already exists in another tuple in the relation *r*(*R*). Entity integrity can be violated if any part of the primary key of the new tuple *t* is NULL. Referential integrity can be violated if the value of any foreign key in *t* refers to a tuple that does not exist in the referenced relation. Here are some examples to illustrate this discussion.

- *Operation*:
  Insert <'Cecilia', 'F', 'Kolonsky', NULL, '1960-04-05', '6357 Windy Lane, Katy, TX', F, 28000, NULL, 4> into EMPLOYEE.
  Result: This insertion violates the entity integrity constraint (NULL for the primary key Ssn), so it is rejected.
- *Operation*:
  Insert <'Alicia', 'J', 'Zelaya', '999887777', '1960-04-05', '6357 Windy Lane, Katy, TX', F, 28000, '987654321', 4> into EMPLOYEE.
  Result: This insertion violates the key constraint because another tuple with the same Ssn value already exists in the EMPLOYEE relation, and so it is rejected.
- *Operation*:
  Insert <'Cecilia', 'F', 'Kolonsky', '677678989', '1960-04-05', '6357 Windswept, Katy, TX', F, 28000, '987654321', 7> into EMPLOYEE.
  Result: This insertion violates the referential integrity constraint specified on Dno in EMPLOYEE because no corresponding referenced tuple exists in DEPARTMENT with Dnumber = 7.

- *Operation*:
  Insert <‘Cecilia’, ‘F’, ‘Kolonsky’, ‘677678989’, ‘1960-04-05’, ‘6357 Windy Lane, Katy, TX’, F, 28000, NULL, 4> into EMPLOYEE.
  Result: This insertion satisfies all constraints, so it is acceptable.

If an insertion violates one or more constraints, the default option is to *reject the insertion.* In this case, it would be useful if the DBMS could provide a reason to the user as to why the insertion was rejected. Another option is to attempt to *correct the reason for rejecting the insertion,* but this is typically not used for violations caused by Insert; rather, it is used more often in correcting violations for Delete and Update. In the first operation, the DBMS could ask the user to provide a value for Ssn, and could then accept the insertion if a valid Ssn value is provided. In operation 3, the DBMS could either ask the user to change the value of Dno to some valid value (or set it to NULL), or it could ask the user to insert a DEPARTMENT tuple with Dnumber = 7 and could accept the original insertion only after such an operation was accepted. Notice that in the latter case the insertion violation can **cascade** back to the EMPLOYEE relation if the user attempts to insert a tuple for department 7 with a value for Mgr_ssn that does not exist in the EMPLOYEE relation.

3.3.2 The Delete Operation

The **Delete** operation can violate only referential integrity. This occurs if the tuple being deleted is referenced by foreign keys from other tuples in the database. To specify deletion, a condition on the attributes of the relation selects the tuple (or tuples) to be deleted. Here are some examples.

- *Operation*:
  Delete the WORKS_ON tuple with Essn = ‘999887777’ and Pno = 10.
  Result: This deletion is acceptable and deletes exactly one tuple.
- *Operation*:
  Delete the EMPLOYEE tuple with Ssn = ‘999887777’.
  Result: This deletion is not acceptable, because there are tuples in WORKS_ON that refer to this tuple. Hence, if the tuple in EMPLOYEE is deleted, referential integrity violations will result.
- *Operation*:
  Delete the EMPLOYEE tuple with Ssn = ‘333445555’.
  Result: This deletion will result in even worse referential integrity violations, because the tuple involved is referenced by tuples from the EMPLOYEE, DEPARTMENT, WORKS_ON, and DEPENDENT relations.

Several options are available if a deletion operation causes a violation. The first option, called **restrict,** is to *reject the deletion.* The second option, called **cascade,** is to *attempt to cascade (or propagate) the deletion* by deleting tuples that reference the tuple that is being deleted. For example, in operation 2, the DBMS could automatically delete the offending tuples from WORKS_ON with Essn = ‘999887777’. A third option, called **set null** or **set default,** is to *modify the referencing attribute values* that cause the violation; each such value is either set to NULL or changed to reference another default valid tuple. Notice that if a referencing attribute that causes a violation is *part of the primary key,* it *cannot* be set to NULL; otherwise, it would violate entity integrity.

Combinations of these three options are also possible. For example, to avoid having operation 3 cause a violation, the DBMS may automatically delete all tuples from WORKS_ON and DEPENDENT with Essn = '333445555'. Tuples in EMPLOYEE with Super_ssn = '333445555' and the tuple in DEPARTMENT with Mgr_ssn = '333445555' can have their Super_ssn and Mgr_ssn values changed to other valid values or to NULL. Although it may make sense to delete automatically the WORKS_ON and DEPENDENT tuples that refer to an EMPLOYEE tuple, it may not make sense to delete other EMPLOYEE tuples or a DEPARTMENT tuple.

In general, when a referential integrity constraint is specified in the DDL, the DBMS will allow the database designer to *specify which of the options* applies in case of a violation of the constraint. We discuss how to specify these options in the SQL DDL in Chapter 4.

3.3.3 The Update Operation

The **Update** (or **Modify**) operation is used to change the values of one or more attributes in a tuple (or tuples) of some relation *R*. It is necessary to specify a condition on the attributes of the relation to select the tuple (or tuples) to be modified. Here are some examples.

- *Operation*:
  Update the salary of the EMPLOYEE tuple with Ssn = '999887777' to 28000.
  Result: Acceptable.
- *Operation*:
  Update the Dno of the EMPLOYEE tuple with Ssn = '999887777' to 1.
  Result: Acceptable.
- *Operation*:
  Update the Dno of the EMPLOYEE tuple with Ssn = '999887777' to 7.
  Result: Unacceptable, because it violates referential integrity.
- *Operation*:
  Update the Ssn of the EMPLOYEE tuple with Ssn = '999887777' to '987654321'.
  Result: Unacceptable, because it violates primary key constraint by repeating a value that already exists as a primary key in another tuple; it violates referential integrity constraints because there are other relations that refer to the existing value of Ssn.

Updating an attribute that is *neither part of a primary key nor of a foreign key* usually causes no problems; the DBMS need only check to confirm that the new value is of the correct data type and domain. Modifying a primary key value is similar to deleting one tuple and inserting another in its place because we use the primary key to identify tuples. Hence, the issues discussed earlier in both Sections 3.3.1 (Insert) and 3.3.2 (Delete) come into play. If a foreign key attribute is modified, the DBMS must make sure that the new value refers to an existing tuple in the referenced relation (or is set to NULL). Similar options exist to deal with referential integrity violations caused by Update as those options discussed for the Delete operation. In fact, when a referential integrity constraint is specified in the DDL, the DBMS will allow the user to choose separate options to deal with a violation caused by Delete and a violation caused by Update (see Section 4.2).

3.3.4 The Transaction Concept

A database application program running against a relational database typically executes one or more *transactions*. A **transaction** is an executing program that includes some database operations, such as reading from the database, or applying insertions, deletions, or updates to the database. At the end of the transaction, it must leave the database in a valid or consistent state that satisfies all the constraints specified on the database schema. A single transaction may involve any number of retrieval operations (to be discussed as part of relational algebra and calculus in Chapter 6, and as a part of the language SQL in Chapters 4 and 5), and any number of update operations. These retrievals and updates will together form an atomic unit of work against the database. For example, a transaction to apply a bank withdrawal will typically read the user account record, check if there is a sufficient balance, and then update the record by the withdrawal amount.

A large number of commercial applications running against relational databases in **online transaction processing** (**OLTP**) systems are executing transactions at rates that reach several hundred per second. Transaction processing concepts, concurrent execution of transactions, and recovery from failures will be discussed in Chapters 21 to 23.

3.4 Summary

In this chapter we presented the modeling concepts, data structures, and constraints provided by the relational model of data. We started by introducing the concepts of domains, attributes, and tuples. Then, we defined a relation schema as a list of attributes that describe the structure of a relation. A relation, or relation state, is a set of tuples that conforms to the schema.

Several characteristics differentiate relations from ordinary tables or files. The first is that a relation is not sensitive to the ordering of tuples. The second involves the ordering of attributes in a relation schema and the corresponding ordering of values within a tuple. We gave an alternative definition of relation that does not require these two orderings, but we continued to use the first definition, which requires attributes and tuple values to be ordered, for convenience. Then, we discussed values in tuples and introduced NULL values to represent missing or unknown information. We emphasized that NULL values should be avoided as much as possible.

We classified database constraints into inherent model-based constraints, explicit schema-based constraints, and application-based constraints, otherwise known as semantic constraints or business rules. Then, we discussed the schema constraints pertaining to the relational model, starting with domain constraints, then key constraints, including the concepts of superkey, candidate key, and primary key, and the NOT NULL constraint on attributes. We defined relational databases and relational database schemas. Additional relational constraints include the entity integrity constraint, which prohibits primary key attributes from being NULL. We described the interrelation referential integrity constraint, which is used to maintain consistency of references among tuples from different relations.

The modification operations on the relational model are Insert, Delete, and Update. Each operation may violate certain types of constraints (refer to Section 3.3). Whenever an operation is applied, the database state after the operation is executed must be checked to ensure that no constraints have been violated. Finally, we introduced the concept of a transaction, which is important in relational DBMSs because it allows the grouping of several database operations into a single atomic action on the database.

Review Questions

3.1. Define the following terms as they apply to the relational model of data: *domain*, *attribute*, *n-tuple*, *relation schema*, *relation state*, *degree of a relation*, *relational database schema*, and *relational database state*.

3.2. Why are tuples in a relation not ordered?

3.3. Why are duplicate tuples not allowed in a relation?

3.4. What is the difference between a key and a superkey?

3.5. Why do we designate one of the candidate keys of a relation to be the primary key?

3.6. Discuss the characteristics of relations that make them different from ordinary tables and files.

3.7. Discuss the various reasons that lead to the occurrence of NULL values in relations.

3.8. Discuss the entity integrity and referential integrity constraints. Why is each considered important?

3.9. Define *foreign key*. What is this concept used for?

3.10. What is a transaction? How does it differ from an Update operation?

Exercises

3.11. Suppose that each of the following Update operations is applied directly to the database state shown in Figure 3.6. Discuss *all* integrity constraints violated by each operation, if any, and the different ways of enforcing these constraints.

a. Insert <'Robert', 'F', 'Scott', '943775543', '1972-06-21', '2365 Newcastle Rd, Bellaire, TX', M, 58000, '888665555', 1> into EMPLOYEE.

b. Insert <'ProductA', 4, 'Bellaire', 2> into PROJECT.

c. Insert <'Production', 4, '943775543', '2007-10-01'> into DEPARTMENT.

d. Insert <'677678989', NULL, '40.0'> into WORKS_ON.

e. Insert <'453453453', 'John', 'M', '1990-12-12', 'spouse'> into DEPENDENT.

f. Delete the WORKS_ON tuples with Essn = '333445555'.

g. Delete the EMPLOYEE tuple with Ssn = '987654321'.

h. Delete the PROJECT tuple with Pname = 'ProductX'.

i. Modify the Mgr_ssn and Mgr_start_date of the DEPARTMENT tuple with Dnumber = 5 to '123456789' and '2007-10-01', respectively.

j. Modify the Super_ssn attribute of the EMPLOYEE tuple with Ssn = '999887777' to '943775543'.

k. Modify the Hours attribute of the WORKS_ON tuple with Essn = '999887777' and Pno = 10 to '5.0'.

3.12. Consider the AIRLINE relational database schema shown in Figure 3.8, which describes a database for airline flight information. Each FLIGHT is identified by a Flight_number, and consists of one or more FLIGHT_LEGs with Leg_numbers 1, 2, 3, and so on. Each FLIGHT_LEG has scheduled arrival and departure times, airports, and one or more LEG_INSTANCEs—one for each Date on which the flight travels. FAREs are kept for each FLIGHT. For each FLIGHT_LEG instance, SEAT_RESERVATIONs are kept, as are the AIRPLANE used on the leg and the actual arrival and departure times and airports. An AIRPLANE is identified by an Airplane_id and is of a particular AIRPLANE_TYPE. CAN_LAND relates AIRPLANE_TYPEs to the AIRPORTs at which they can land. An AIRPORT is identified by an Airport_code. Consider an update for the AIRLINE database to enter a reservation on a particular flight or flight leg on a given date.

a. Give the operations for this update.

b. What types of constraints would you expect to check?

c. Which of these constraints are key, entity integrity, and referential integrity constraints, and which are not?

d. Specify all the referential integrity constraints that hold on the schema shown in Figure 3.8.

3.13. Consider the relation CLASS(Course#, Univ_Section#, Instructor_name, Semester, Building_code, Room#, Time_period, Weekdays, Credit_hours). This represents classes taught in a university, with unique Univ_section#s. Identify what you think should be various candidate keys, and write in your own words the conditions or assumptions under which each candidate key would be valid.

AIRPORT

Airport_code	Name	City	State

FLIGHT

Flight_number	Airline	Weekdays

FLIGHT_LEG

Flight_number	Leg_number	Departure_airport_code	Scheduled_departure_time	Arrival_airport_code	Scheduled_arrival_time

LEG_INSTANCE

Flight_number	Leg_number	Date	Number_of_available_seats	Airplane_id	Departure_airport_code	Departure_time	Arrival_airport_code	Arrival_time

FARE

Flight_number	Fare_code	Amount	Restrictions

AIRPLANE_TYPE

Airplane_type_name	Max_seats	Company

CAN_LAND

Airplane_type_name	Airport_code

AIRPLANE

Airplane_id	Total_number_of_seats	Airplane_type

SEAT_RESERVATION

Flight_number	Leg_number	Date	Seat_number	Customer_name	Customer_phone

Figure 3.8
The AIRLINE relational database schema.

3.14. Consider the following six relations for an order-processing database application in a company:

CUSTOMER(Cust#, Cname, City)
ORDER(Order#, Odate, Cust#, Ord_amt)
ORDER_ITEM(Order#, Item#, Qty)

ITEM(Item#, Unit_price)
SHIPMENT(Order#, Warehouse#, Ship_date)
WAREHOUSE(Warehouse#, City)

Here, Ord_amt refers to total dollar amount of an order; Odate is the date the order was placed; and Ship_date is the date an order (or part of an order) is shipped from the warehouse. Assume that an order can be shipped from several warehouses. Specify the foreign keys for this schema, stating any assumptions you make. What other constraints can you think of for this database?

3.15. Consider the following relations for a database that keeps track of business trips of salespersons in a sales office:

SALESPERSON(Ssn, Name, Start_year, Dept_no)
TRIP(Ssn, From_city, To_city, Departure_date, Return_date, Trip_id)
EXPENSE(Trip_id, Account#, Amount)

A trip can be charged to one or more accounts. Specify the foreign keys for this schema, stating any assumptions you make.

3.16. Consider the following relations for a database that keeps track of student enrollment in courses and the books adopted for each course:

STUDENT(Ssn, Name, Major, Bdate)
COURSE(Course#, Cname, Dept)
ENROLL(Ssn, Course#, Quarter, Grade)
BOOK_ADOPTION(Course#, Quarter, Book_isbn)
TEXT(Book_isbn, Book_title, Publisher, Author)

Specify the foreign keys for this schema, stating any assumptions you make.

3.17. Consider the following relations for a database that keeps track of automobile sales in a car dealership (OPTION refers to some optional equipment installed on an automobile):

CAR(Serial_no, Model, Manufacturer, Price)
OPTION(Serial_no, Option_name, Price)
SALE(Salesperson_id, Serial_no, Date, Sale_price)
SALESPERSON(Salesperson_id, Name, Phone)

First, specify the foreign keys for this schema, stating any assumptions you make. Next, populate the relations with a few sample tuples, and then give an example of an insertion in the SALE and SALESPERSON relations that *violates* the referential integrity constraints and of another insertion that does not.

3.18. Database design often involves decisions about the storage of attributes. For example, a Social Security number can be stored as one attribute or split into three attributes (one for each of the three hyphen-delineated groups of numbers in a Social Security number—XXX-XX-XXXX). However, Social Security numbers are usually represented as just one attribute. The decision is based on how the database will be used. This exercise asks you to think about specific situations where dividing the SSN is useful.

3.19. Consider a STUDENT relation in a UNIVERSITY database with the following attributes (Name, Ssn, Local_phone, Address, Cell_phone, Age, Gpa). Note that the cell phone may be from a different city and state (or province) from the local phone. A possible tuple of the relation is shown below:

Name	Ssn	Local_phone	Address	Cell_phone	Age	Gpa
George Shaw William Edwards	123-45-6789	555-1234	123 Main St., Anytown, CA 94539	555-4321	19	3.75

a. Identify the critical missing information from the Local_phone and Cell_phone attributes. (*Hint*: How do you call someone who lives in a different state or province?)

b. Would you store this additional information in the Local_phone and Cell_phone attributes or add new attributes to the schema for STUDENT?

c. Consider the Name attribute. What are the advantages and disadvantages of splitting this field from one attribute into three attributes (first name, middle name, and last name)?

d. What general guideline would you recommend for deciding when to store information in a single attribute and when to split the information?

e. Suppose the student can have between 0 and 5 phones. Suggest two different designs that allow this type of information.

3.20. Recent changes in privacy laws have disallowed organizations from using Social Security numbers to identify individuals unless certain restrictions are satisfied. As a result, most U.S. universities cannot use SSNs as primary keys (except for financial data). In practice, Student_id, a unique identifier assigned to every student, is likely to be used as the primary key rather than SSN since Student_id can be used throughout the system.

a. Some database designers are reluctant to use generated keys (also known as *surrogate keys*) for primary keys (such as Student_id) because they are artificial. Can you propose any natural choices of keys that can be used to identify the student record in a UNIVERSITY database?

b. Suppose that you are able to guarantee uniqueness of a natural key that includes last name. Are you guaranteed that the last name will not change during the lifetime of the database? If last name can change, what solutions can you propose for creating a primary key that still includes last name but remains unique?

c. What are the advantages and disadvantages of using generated (surrogate) keys?

chapter 6

The Relational Algebra and Relational Calculus

In this chapter we discuss the two *formal languages* for the relational model: the relational algebra and the relational calculus. In contrast, Chapters 4 and 5 described the *practical language* for the relational model, namely the SQL standard. Historically, the relational algebra and calculus were developed before the SQL language. In fact, in some ways, SQL is based on concepts from both the algebra and the calculus, as we shall see. Because most relational DBMSs use SQL as their language, we presented the SQL language first.

Recall from Chapter 2 that a data model must include a set of operations to manipulate the database, in addition to the data model's concepts for defining the database's structure and constraints. We presented the structures and constraints of the formal relational model in Chapter 3. The basic set of operations for the relational model is the **relational algebra**. These operations enable a user to specify basic retrieval requests as *relational algebra expressions*. The result of a retrieval is a new relation, which may have been formed from one or more relations. The algebra operations thus produce new relations, which can be further manipulated using operations of the same algebra. A sequence of relational algebra operations forms a **relational algebra expression**, whose result will also be a relation that represents the result of a database query (or retrieval request).

The relational algebra is very important for several reasons. First, it provides a formal foundation for relational model operations. Second, and perhaps more important, it is used as a basis for implementing and optimizing queries in the query processing and optimization modules that are integral parts of relational database management systems (RDBMSs), as we shall discuss in Chapter 19. Third, some of its concepts are incorporated into the SQL standard query language for RDBMSs.

Although most commercial RDBMSs in use today do not provide user interfaces for relational algebra queries, the core operations and functions in the internal modules of most relational systems are based on relational algebra operations. We will define these operations in detail in Sections 6.1 through 6.4 of this chapter.

Whereas the algebra defines a set of operations for the relational model, the **relational calculus** provides a higher-level *declarative* language for specifying relational queries. A relational calculus expression creates a new relation. In a relational calculus expression, there is *no order of operations* to specify how to retrieve the query result—only what information the result should contain. This is the main distinguishing feature between relational algebra and relational calculus. The relational calculus is important because it has a firm basis in mathematical logic and because the standard query language (SQL) for RDBMSs has some of its foundations in a variation of relational calculus known as the tuple relational calculus.[1]

The relational algebra is often considered to be an integral part of the relational data model. Its operations can be divided into two groups. One group includes set operations from mathematical set theory; these are applicable because each relation is defined to be a set of tuples in the *formal* relational model (see Section 3.1). Set operations include UNION, INTERSECTION, SET DIFFERENCE, and CARTESIAN PRODUCT (also known as CROSS PRODUCT). The other group consists of operations developed specifically for relational databases—these include SELECT, PROJECT, and JOIN, among others. First, we describe the SELECT and PROJECT operations in Section 6.1 because they are **unary operations** that operate on single relations. Then we discuss set operations in Section 6.2. In Section 6.3, we discuss JOIN and other complex **binary operations**, which operate on two tables by combining related tuples (records) based on *join conditions*. The COMPANY relational database shown in Figure 3.6 is used for our examples.

Some common database requests cannot be performed with the original relational algebra operations, so additional operations were created to express these requests. These include **aggregate functions**, which are operations that can *summarize* data from the tables, as well as additional types of JOIN and UNION operations, known as OUTER JOINs and OUTER UNIONs. These operations, which were added to the original relational algebra because of their importance to many database applications, are described in Section 6.4. We give examples of specifying queries that use relational operations in Section 6.5. Some of these same queries were used in Chapters 4 and 5. By using the same query numbers in this chapter, the reader can contrast how the same queries are written in the various query languages.

In Sections 6.6 and 6.7 we describe the other main formal language for relational databases, the **relational calculus**. There are two variations of relational calculus. The *tuple* relational calculus is described in Section 6.6 and the *domain* relational calculus is described in Section 6.7. Some of the SQL constructs discussed in Chapters 4 and 5 are based on the tuple relational calculus. The relational calculus is a formal language, based on the branch of mathematical logic called predicate cal-

[1]SQL is based on tuple relational calculus, but also incorporates some of the operations from the relational algebra and its extensions, as illustrated in Chapters 4, 5, and 9.

culus.[2] In tuple relational calculus, variables range over *tuples*, whereas in domain relational calculus, variables range over the *domains* (values) of attributes. In Appendix C we give an overview of the Query-By-Example (QBE) language, which is a graphical user-friendly relational language based on domain relational calculus. Section 6.8 summarizes the chapter.

For the reader who is interested in a less detailed introduction to formal relational languages, Sections 6.4, 6.6, and 6.7 may be skipped.

6.1 Unary Relational Operations: SELECT and PROJECT

6.1.1 The SELECT Operation

The SELECT operation is used to choose a *subset* of the tuples from a relation that satisfies a **selection condition**.[3] One can consider the SELECT operation to be a *filter* that keeps only those tuples that satisfy a qualifying condition. Alternatively, we can consider the SELECT operation to *restrict* the tuples in a relation to only those tuples that satisfy the condition. The SELECT operation can also be visualized as a *horizontal partition* of the relation into two sets of tuples—those tuples that satisfy the condition and are selected, and those tuples that do not satisfy the condition and are discarded. For example, to select the EMPLOYEE tuples whose department is 4, or those whose salary is greater than $30,000, we can individually specify each of these two conditions with a SELECT operation as follows:

$$\sigma_{\text{Dno}=4}(\text{EMPLOYEE})$$
$$\sigma_{\text{Salary}>30000}(\text{EMPLOYEE})$$

In general, the SELECT operation is denoted by

$$\sigma_{<\text{selection condition}>}(R)$$

where the symbol σ (sigma) is used to denote the SELECT operator and the selection condition is a Boolean expression (condition) specified on the attributes of relation R. Notice that R is generally a *relational algebra expression* whose result is a relation—the simplest such expression is just the name of a database relation. The relation resulting from the SELECT operation has the *same attributes* as R.

The Boolean expression specified in <selection condition> is made up of a number of **clauses** of the form

<attribute name> <comparison op> <constant value>

or

<attribute name> <comparison op> <attribute name>

---

[2]In this chapter no familiarity with first-order predicate calculus—which deals with quantified variables and values—is assumed.

[3]The SELECT operation is different from the SELECT clause of SQL. The SELECT operation chooses tuples from a table, and is sometimes called a RESTRICT or FILTER operation.

where <attribute name> is the name of an attribute of R, <comparison op> is normally one of the operators $\{=, <, \leq, >, \geq, \neq\}$, and <constant value> is a constant value from the attribute domain. Clauses can be connected by the standard Boolean operators *and*, *or*, and *not* to form a general selection condition. For example, to select the tuples for all employees who either work in department 4 and make over \$25,000 per year, or work in department 5 and make over \$30,000, we can specify the following SELECT operation:

$$\sigma_{(\text{Dno}=4 \textbf{ AND } \text{Salary}>25000) \textbf{ OR } (\text{Dno}=5 \textbf{ AND } \text{Salary}>30000)}(\text{EMPLOYEE})$$

The result is shown in Figure 6.1(a).

Notice that all the comparison operators in the set $\{=, <, \leq, >, \geq, \neq\}$ can apply to attributes whose domains are *ordered values*, such as numeric or date domains. Domains of strings of characters are also considered to be ordered based on the collating sequence of the characters. If the domain of an attribute is a set of *unordered values*, then only the comparison operators in the set $\{=, \neq\}$ can be used. An example of an unordered domain is the domain Color = { 'red', 'blue', 'green', 'white', 'yellow', ...}, where no order is specified among the various colors. Some domains allow additional types of comparison operators; for example, a domain of character strings may allow the comparison operator SUBSTRING_OF.

In general, the result of a SELECT operation can be determined as follows. The <selection condition> is applied independently to each *individual tuple t* in R. This is done by substituting each occurrence of an attribute A_i in the selection condition with its value in the tuple $t[A_i]$. If the condition evaluates to TRUE, then tuple t is

Figure 6.1
Results of SELECT and PROJECT operations. (a) $\sigma_{(\text{Dno}=4 \textbf{ AND } \text{Salary}>25000) \text{ OR } (\text{Dno}=5 \textbf{ AND } \text{Salary}>30000)}$ (EMPLOYEE). (b) $\pi_{\text{Lname, Fname, Salary}}$(EMPLOYEE). (c) $\pi_{\text{Sex, Salary}}$(EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

selected. All the selected tuples appear in the result of the SELECT operation. The Boolean conditions AND, OR, and NOT have their normal interpretation, as follows:

- (cond1 **AND** cond2) is TRUE if both (cond1) and (cond2) are TRUE; otherwise, it is FALSE.
- (cond1 **OR** cond2) is TRUE if either (cond1) or (cond2) or both are TRUE; otherwise, it is FALSE.
- (**NOT** cond) is TRUE if cond is FALSE; otherwise, it is FALSE.

The SELECT operator is **unary**; that is, it is applied to a single relation. Moreover, the selection operation is applied to *each tuple individually*; hence, selection conditions cannot involve more than one tuple. The **degree** of the relation resulting from a SELECT operation—its number of attributes—is the same as the degree of R. The number of tuples in the resulting relation is always *less than or equal to* the number of tuples in R. That is, $|\sigma_c (R)| \leq |R|$ for any condition C. The fraction of tuples selected by a selection condition is referred to as the **selectivity** of the condition.

Notice that the SELECT operation is **commutative**; that is,

$$\sigma_{<\text{cond1}>}(\sigma_{<\text{cond2}>}(R)) = \sigma_{<\text{cond2}>}(\sigma_{<\text{cond1}>}(R))$$

Hence, a sequence of SELECTs can be applied in any order. In addition, we can always combine a **cascade** (or **sequence**) of SELECT operations into a single SELECT operation with a conjunctive (AND) condition; that is,

$$\sigma_{<\text{cond1}>}(\sigma_{<\text{cond2}>}(...(\sigma_{<\text{cond}n>}(R)) ...)) = \sigma_{<\text{cond1}>\ \textbf{AND}<\text{cond2}>\ \textbf{AND}...\textbf{AND}\ <\text{cond}n>}(R)$$

In SQL, the SELECT condition is typically specified in the WHERE clause of a query. For example, the following operation:

$$\sigma_{\text{Dno}=4\ \textbf{AND}\ \text{Salary}>25000}\ (\text{EMPLOYEE})$$

would correspond to the following SQL query:

```
SELECT   *
FROM     EMPLOYEE
WHERE    Dno=4 AND Salary>25000;
```

6.1.2 The PROJECT Operation

If we think of a relation as a table, the SELECT operation chooses some of the *rows* from the table while discarding other rows. The **PROJECT** operation, on the other hand, selects certain *columns* from the table and discards the other columns. If we are interested in only certain attributes of a relation, we use the PROJECT operation to *project* the relation over these attributes only. Therefore, the result of the PROJECT operation can be visualized as a *vertical partition* of the relation into two relations: one has the needed columns (attributes) and contains the result of the operation, and the other contains the discarded columns. For example, to list each employee's first and last name and salary, we can use the PROJECT operation as follows:

$$\pi_{\text{Lname, Fname, Salary}}(\text{EMPLOYEE})$$

The resulting relation is shown in Figure 6.1(b). The general form of the PROJECT operation is

$$\pi_{<\text{attribute list}>}(R)$$

where π (pi) is the symbol used to represent the PROJECT operation, and <attribute list> is the desired sublist of attributes from the attributes of relation R. Again, notice that R is, in general, a *relational algebra expression* whose result is a relation, which in the simplest case is just the name of a database relation. The result of the PROJECT operation has only the attributes specified in <attribute list> *in the same order as they appear in the list*. Hence, its **degree** is equal to the number of attributes in <attribute list>.

If the attribute list includes only nonkey attributes of R, duplicate tuples are likely to occur. The PROJECT operation *removes any duplicate tuples*, so the result of the PROJECT operation is a set of distinct tuples, and hence a valid relation. This is known as **duplicate elimination**. For example, consider the following PROJECT operation:

$$\pi_{\text{Sex, Salary}}(\text{EMPLOYEE})$$

The result is shown in Figure 6.1(c). Notice that the tuple <'F', 25000> appears only once in Figure 6.1(c), even though this combination of values appears twice in the EMPLOYEE relation. Duplicate elimination involves sorting or some other technique to detect duplicates and thus adds more processing. If duplicates are not eliminated, the result would be a **multiset** or **bag** of tuples rather than a set. This was not permitted in the formal relational model, but is allowed in SQL (see Section 4.3).

The number of tuples in a relation resulting from a PROJECT operation is always less than or equal to the number of tuples in R. If the projection list is a superkey of R—that is, it includes some key of R—the resulting relation has the *same number* of tuples as R. Moreover,

$$\pi_{<\text{list1}>}(\pi_{<\text{list2}>}(R)) = \pi_{<\text{list1}>}(R)$$

as long as <list2> contains the attributes in <list1>; otherwise, the left-hand side is an incorrect expression. It is also noteworthy that commutativity *does not* hold on PROJECT.

In SQL, the PROJECT attribute list is specified in the SELECT clause of a query. For example, the following operation:

$$\pi_{\text{Sex, Salary}}(\text{EMPLOYEE})$$

would correspond to the following SQL query:

```
SELECT   DISTINCT Sex, Salary
FROM     EMPLOYEE
```

Notice that if we remove the keyword **DISTINCT** from this SQL query, then duplicates will not be eliminated. This option is not available in the formal relational algebra.

6.1.3 Sequences of Operations and the RENAME Operation

The relations shown in Figure 6.1 that depict operation results do not have any names. In general, for most queries, we need to apply several relational algebra operations one after the other. Either we can write the operations as a single **relational algebra expression** by nesting the operations, or we can apply one operation at a time and create intermediate result relations. In the latter case, we must give names to the relations that hold the intermediate results. For example, to retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a SELECT and a PROJECT operation. We can write a single relational algebra expression, also known as an **in-line expression**, as follows:

$$\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$$

Figure 6.2(a) shows the result of this in-line relational algebra expression. Alternatively, we can explicitly show the sequence of operations, giving a name to each intermediate relation, as follows:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{Dno}=5}(\text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Salary}}(\text{DEP5\_EMPS})$$

It is sometimes simpler to break down a complex sequence of operations by specifying intermediate result relations than to write a single relational algebra expression. We can also use this technique to **rename** the attributes in the intermediate and

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)
TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2
Results of a sequence of operations. (a) $\pi_{\text{Fname, Lname, Salary}}$ $(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$. (b) Using intermediate relations and renaming of attributes.

result relations. This can be useful in connection with more complex operations such as UNION and JOIN, as we shall see. To rename the attributes in a relation, we simply list the new attribute names in parentheses, as in the following example:

$$\text{TEMP} \leftarrow \sigma_{\text{Dno}=5}(\text{EMPLOYEE})$$
$$R(\text{First\_name, Last\_name, Salary}) \leftarrow \pi_{\text{Fname, Lname, Salary}}(\text{TEMP})$$

These two operations are illustrated in Figure 6.2(b).

If no renaming is applied, the names of the attributes in the resulting relation of a SELECT operation are the same as those in the original relation and in the same order. For a PROJECT operation with no renaming, the resulting relation has the same attribute names as those in the projection list and in the same order in which they appear in the list.

We can also define a formal **RENAME** operation—which can rename either the relation name or the attribute names, or both—as a unary operator. The general RENAME operation when applied to a relation R of degree n is denoted by any of the following three forms:

$$\rho_{S(B1,\, B2,\, \ldots,\, Bn)}(R) \quad \text{or} \quad \rho_S(R) \quad \text{or} \quad \rho_{(B1,\, B2,\, \ldots,\, Bn)}(R)$$

where the symbol ρ (rho) is used to denote the RENAME operator, S is the new relation name, and $B_1, B_2, \ldots, B_n$ are the new attribute names. The first expression renames both the relation and its attributes, the second renames the relation only, and the third renames the attributes only. If the attributes of R are $(A_1, A_2, \ldots, A_n)$ in that order, then each A_i is renamed as B_i.

In SQL, a single query typically represents a complex relational algebra expression. Renaming in SQL is accomplished by aliasing using **AS**, as in the following example:

```
SELECT    E.Fname AS First_name, E.Lname AS Last_name, E.Salary AS Salary
FROM      EMPLOYEE AS E
WHERE     E.Dno=5,
```

6.2 Relational Algebra Operations from Set Theory

6.2.1 The UNION, INTERSECTION, and MINUS Operations

The next group of relational algebra operations are the standard mathematical operations on sets. For example, to retrieve the Social Security numbers of all employees who either work in department 5 or directly supervise an employee who works in department 5, we can use the UNION operation as follows:[4]

[4]As a single relational algebra expression, this becomes Result $\leftarrow \pi_{\text{Ssn}}\ (\sigma_{\text{Dno}=5}\ (\text{EMPLOYEE})\ ) \cup \pi_{\text{Super\_ssn}}\ (\sigma_{\text{Dno}=5}\ (\text{EMPLOYEE}))$

$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{Dno}=5}(\text{EMPLOYEE})$
$\text{RESULT1} \leftarrow \pi_{\text{Ssn}}(\text{DEP5\_EMPS})$
$\text{RESULT2(Ssn)} \leftarrow \pi_{\text{Super\_ssn}}(\text{DEP5\_EMPS})$
$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$

The relation RESULT1 has the Ssn of all employees who work in department 5, whereas RESULT2 has the Ssn of all employees who directly supervise an employee who works in department 5. The UNION operation produces the tuples that are in either RESULT1 or RESULT2 or both (see Figure 6.3), while eliminating any duplicates. Thus, the Ssn value '333445555' appears only once in the result.

Several set theoretic operations are used to merge the elements of two sets in various ways, including **UNION**, **INTERSECTION**, and **SET DIFFERENCE** (also called **MINUS** or **EXCEPT**). These are **binary** operations; that is, each is applied to two sets (of tuples). When these operations are adapted to relational databases, the two relations on which any of these three operations are applied must have the same **type of tuples**; this condition has been called *union compatibility* or *type compatibility*. Two relations $R(A_1, A_2, ..., A_n)$ and $S(B_1, B_2, ..., B_n)$ are said to be **union compatible** (or **type compatible**) if they have the same degree n and if $\text{dom}(A_i) = \text{dom}(B_i)$ for $1 \leq i \leq n$. This means that the two relations have the same number of attributes and each corresponding pair of attributes has the same domain.

We can define the three operations UNION, INTERSECTION, and SET DIFFERENCE on two union-compatible relations R and S as follows:

- UNION: The result of this operation, denoted by $R \cup S$, is a relation that includes all tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated.
- INTERSECTION: The result of this operation, denoted by $R \cap S$, is a relation that includes all tuples that are in both R and S.
- SET DIFFERENCE (or MINUS): The result of this operation, denoted by $R - S$, is a relation that includes all tuples that are in R but not in S.

We will adopt the convention that the resulting relation has the same attribute names as the *first* relation R. It is always possible to rename the attributes in the result using the rename operator.

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Figure 6.3
Result of the UNION operation RESULT ← RESULT1 ∪ RESULT2.

Figure 6.4 illustrates the three operations. The relations STUDENT and INSTRUCTOR in Figure 6.4(a) are union compatible and their tuples represent the names of students and the names of instructors, respectively. The result of the UNION operation in Figure 6.4(b) shows the names of all students and instructors. Note that duplicate tuples appear only once in the result. The result of the INTERSECTION operation (Figure 6.4(c)) includes only those who are both students and instructors.

Notice that both UNION and INTERSECTION are *commutative operations*; that is,

$$R \cup S = S \cup R \quad \text{and} \quad R \cap S = S \cap R$$

Both UNION and INTERSECTION can be treated as *n*-ary operations applicable to any number of relations because both are also *associative operations;* that is,

$$R \cup (S \cup T) = (R \cup S) \cup T \quad \text{and} \quad (R \cap S) \cap T = R \cap (S \cap T)$$

The MINUS operation is *not commutative;* that is, in general,

$$R - S \neq S - R$$

Figure 6.4
The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT – INSTRUCTOR. (e) INSTRUCTOR – STUDENT.

(a) **STUDENT**

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

Figure 6.4(d) shows the names of students who are not instructors, and Figure 6.4(e) shows the names of instructors who are not students.

Note that INTERSECTION can be expressed in terms of union and set difference as follows:

$$R \cap S = ((R \cup S) - (R - S)) - (S - R)$$

In SQL, there are three operations—UNION, INTERSECT, and EXCEPT—that correspond to the set operations described here. In addition, there are multiset operations (UNION ALL, INTERSECT ALL, and EXCEPT ALL) that do not eliminate duplicates (see Section 4.3.4).

6.2.2 The CARTESIAN PRODUCT (CROSS PRODUCT) Operation

Next, we discuss the **CARTESIAN PRODUCT** operation—also known as **CROSS PRODUCT** or **CROSS JOIN**–which is denoted by $\times$. This is also a binary set operation, but the relations on which it is applied do *not* have to be union compatible. In its binary form, this set operation produces a new element by combining every member (tuple) from one relation (set) with every member (tuple) from the other relation (set). In general, the result of $R(A_1, A_2, \ldots, A_n) \times S(B_1, B_2, \ldots, B_m)$ is a relation Q with degree $n + m$ attributes $Q(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_m)$, in that order. The resulting relation Q has one tuple for each combination of tuples—one from R and one from S. Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then $R \times S$ will have $n_R * n_S$ tuples.

The n-ary CARTESIAN PRODUCT operation is an extension of the above concept, which produces new tuples by concatenating all possible combinations of tuples from n underlying relations.

In general, the CARTESIAN PRODUCT operation applied by itself is generally meaningless. It is mostly useful when followed by a selection that matches values of attributes coming from the component relations. For example, suppose that we want to retrieve a list of names of each female employee's dependents. We can do this as follows:

$\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{Sex='F'}}(\text{EMPLOYEE})$
$\text{EMPNAMES} \leftarrow \pi_{\text{Fname, Lname, Ssn}}(\text{FEMALE\_EMPS})$
$\text{EMP\_DEPENDENTS} \leftarrow \text{EMPNAMES} \times \text{DEPENDENT}$
$\text{ACTUAL\_DEPENDENTS} \leftarrow \sigma_{\text{Ssn=Essn}}(\text{EMP\_DEPENDENTS})$
$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Dependent\_name}}(\text{ACTUAL\_DEPENDENTS})$

The resulting relations from this sequence of operations are shown in Figure 6.5. The EMP_DEPENDENTS relation is the result of applying the CARTESIAN PRODUCT operation to EMPNAMES from Figure 6.5 with DEPENDENT from Figure 3.6. In EMP_DEPENDENTS, every tuple from EMPNAMES is combined with every tuple from DEPENDENT, giving a result that is not very meaningful (every dependent is combined with *every* female employee). We want to combine a female employee tuple only with her particular dependents—namely, the DEPENDENT tuples whose

Figure 6.5
The Cartesian Product (Cross Product) operation.

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1986-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

Essn value match the Ssn value of the EMPLOYEE tuple. The ACTUAL_DEPENDENTS relation accomplishes this. The EMP_DEPENDENTS relation is a good example of the case where relational algebra can be correctly applied to yield results that make no sense at all. It is the responsibility of the user to make sure to apply only meaningful operations to relations.

The CARTESIAN PRODUCT creates tuples with the combined attributes of two relations. We can SELECT *related tuples only* from the two relations by specifying an appropriate selection condition after the Cartesian product, as we did in the preceding example. Because this sequence of CARTESIAN PRODUCT followed by SELECT is quite commonly used to combine *related tuples* from two relations, a special operation, called JOIN, was created to specify this sequence as a single operation. We discuss the JOIN operation next.

In SQL, CARTESIAN PRODUCT can be realized by using the CROSS JOIN option in joined tables (see Section 5.1.6). Alternatively, if there are two tables in the WHERE clause and there is no corresponding join condition in the query, the result will also be the CARTESIAN PRODUCT of the two tables (see Q10 in Section 4.3.3).

6.3 Binary Relational Operations: JOIN and DIVISION

6.3.1 The JOIN Operation

The **JOIN** operation, denoted by $\bowtie$, is used to combine *related tuples* from two relations into single "longer" tuples. This operation is very important for any relational database with more than a single relation because it allows us to process relationships among relations. To illustrate JOIN, suppose that we want to retrieve the name of the manager of each department. To get the manager's name, we need to combine each department tuple with the employee tuple whose Ssn value matches the Mgr_ssn value in the department tuple. We do this by using the JOIN operation and then projecting the result over the necessary attributes, as follows:

$$\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE}$$
$$\text{RESULT} \leftarrow \pi_{\text{Dname, Lname, Fname}}(\text{DEPT\_MGR})$$

The first operation is illustrated in Figure 6.6. Note that Mgr_ssn is a foreign key of the DEPARTMENT relation that references Ssn, the primary key of the EMPLOYEE relation. This referential integrity constraint plays a role in having matching tuples in the referenced relation EMPLOYEE.

The JOIN operation can be specified as a CARTESIAN PRODUCT operation followed by a SELECT operation. However, JOIN is very important because it is used very frequently when specifying database queries. Consider the earlier example illustrating CARTESIAN PRODUCT, which included the following sequence of operations:

$$\text{EMP\_DEPENDENTS} \leftarrow \text{EMPNAMES} \times \text{DEPENDENT}$$
$$\text{ACTUAL\_DEPENDENTS} \leftarrow \sigma_{\text{Ssn=Essn}}(\text{EMP\_DEPENDENTS})$$

DEPT_MGR

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

Figure 6.6
Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\bowtie_{\text{Mgr\_ssn=Ssn}}$ EMPLOYEE.

These two operations can be replaced with a single JOIN operation as follows:

ACTUAL_DEPENDENTS ← EMPNAMES $\bowtie_{\text{Ssn=Essn}}$ DEPENDENT

The general form of a JOIN operation on two relations[5] $R(A_1, A_2, \ldots, A_n)$ and $S(B_1, B_2, \ldots, B_m)$ is

$R \bowtie_{\text{<join condition>}} S$

The result of the JOIN is a relation Q with $n + m$ attributes $Q(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_m)$ in that order; Q has one tuple for each combination of tuples—one from R and one from S—*whenever the combination satisfies the join condition*. This is the main difference between CARTESIAN PRODUCT and JOIN. In JOIN, only combinations of tuples *satisfying the join condition* appear in the result, whereas in the CARTESIAN PRODUCT *all* combinations of tuples are included in the result. The join condition is specified on attributes from the two relations R and S and is evaluated for each combination of tuples. Each tuple combination for which the join condition evaluates to TRUE is included in the resulting relation Q *as a single combined tuple*.

A general join condition is of the form

<condition> **AND** <condition> **AND**...**AND** <condition>

where each <condition> is of the form $A_i \ \theta \ B_j$, A_i is an attribute of R, B_j is an attribute of S, A_i and B_j have the same domain, and θ (theta) is one of the comparison operators $\{=, <, \leq, >, \geq, \neq\}$. A JOIN operation with such a general join condition is called a **THETA JOIN**. Tuples whose join attributes are NULL or for which the join condition is FALSE *do not* appear in the result. In that sense, the JOIN operation does *not* necessarily preserve all of the information in the participating relations, because tuples that do not get combined with matching ones in the other relation do not appear in the result.

[5]Again, notice that R and S can be any relations that result from general *relational algebra expressions*.

6.3.2 Variations of JOIN: The EQUIJOIN and NATURAL JOIN

The most common use of JOIN involves join conditions with equality comparisons only. Such a JOIN, where the only comparison operator used is =, is called an **EQUIJOIN**. Both previous examples were EQUIJOINs. Notice that in the result of an EQUIJOIN we always have one or more pairs of attributes that have *identical values* in every tuple. For example, in Figure 6.6, the values of the attributes Mgr_ssn and Ssn are identical in every tuple of DEPT_MGR (the EQUIJOIN result) because the equality join condition specified on these two attributes *requires the values to be identical* in every tuple in the result. Because one of each pair of attributes with identical values is superfluous, a new operation called **NATURAL JOIN**—denoted by *—was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.[6] The standard definition of NATURAL JOIN requires that the two join attributes (or each pair of join attributes) have the same name in both relations. If this is not the case, a renaming operation is applied first.

Suppose we want to combine each PROJECT tuple with the DEPARTMENT tuple that controls the project. In the following example, first we rename the Dnumber attribute of DEPARTMENT to Dnum—so that it has the same name as the Dnum attribute in PROJECT—and then we apply NATURAL JOIN:

$$\text{PROJ\_DEPT} \leftarrow \text{PROJECT} * \rho_{(\text{Dname, Dnum, Mgr\_ssn, Mgr\_start\_date})}(\text{DEPARTMENT})$$

The same query can be done in two steps by creating an intermediate table DEPT as follows:

$$\text{DEPT} \leftarrow \rho_{(\text{Dname, Dnum, Mgr\_ssn, Mgr\_start\_date})}(\text{DEPARTMENT})$$
$$\text{PROJ\_DEPT} \leftarrow \text{PROJECT} * \text{DEPT}$$

The attribute Dnum is called the **join attribute** for the NATURAL JOIN operation, because it is the only attribute with the same name in both relations. The resulting relation is illustrated in Figure 6.7(a). In the PROJ_DEPT relation, each tuple combines a PROJECT tuple with the DEPARTMENT tuple for the department that controls the project, but *only one join attribute value* is kept.

If the attributes on which the natural join is specified already *have the same names in both relations*, renaming is unnecessary. For example, to apply a natural join on the Dnumber attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write

$$\text{DEPT\_LOCS} \leftarrow \text{DEPARTMENT} * \text{DEPT\_LOCATIONS}$$

The resulting relation is shown in Figure 6.7(b), which combines each department with its locations and has one tuple for each location. In general, the join condition for NATURAL JOIN is constructed by equating *each pair of join attributes* that have the same name in the two relations and combining these conditions with **AND**. There can be a list of join attributes from each relation, and each corresponding pair must have the same name.

[6]NATURAL JOIN is basically an EQUIJOIN followed by the removal of the superfluous attributes.

(a)

PROJ_DEPT

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Figure 6.7
Results of two NATURAL JOIN operations. (a) PROJ_DEPT ← PROJECT * DEPT.
(b) DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS.

A more general, *but nonstandard* definition for NATURAL JOIN is

$$Q \leftarrow R *_{(\text{<list1>}),(\text{<list2>})} S$$

In this case, <list1> specifies a list of i attributes from R, and <list2> specifies a list of i attributes from S. The lists are used to form equality comparison conditions between pairs of corresponding attributes, and the conditions are then ANDed together. Only the list corresponding to attributes of the first relation R—<list1>—is kept in the result Q.

Notice that if no combination of tuples satisfies the join condition, the result of a JOIN is an empty relation with zero tuples. In general, if R has n_R tuples and S has n_S tuples, the result of a JOIN operation $R \bowtie_{\text{<join condition>}} S$ will have between zero and $n_R * n_S$ tuples. The expected size of the join result divided by the maximum size $n_R * n_S$ leads to a ratio called **join selectivity**, which is a property of each join condition. If there is no join condition, all combinations of tuples qualify and the JOIN degenerates into a CARTESIAN PRODUCT, also called CROSS PRODUCT or CROSS JOIN.

As we can see, a single JOIN operation is used to combine data from two relations so that related information can be presented in a single table. These operations are also known as **inner joins**, to distinguish them from a different join variation called *outer joins* (see Section 6.4.4). Informally, an *inner join* is a type of match and combine operation defined formally as a combination of CARTESIAN PRODUCT and SELECTION. Note that sometimes a join may be specified between a relation and itself, as we will illustrate in Section 6.4.3. The NATURAL JOIN or EQUIJOIN operation can also be specified among multiple tables, leading to an *n-way join*. For example, consider the following three-way join:

$$((\text{PROJECT} \bowtie_{\text{Dnum=Dnumber}} \text{DEPARTMENT}) \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE})$$

This combines each project tuple with its controlling department tuple into a single tuple, and then combines that tuple with an employee tuple that is the department manager. The net result is a consolidated relation in which each tuple contains this project-department-manager combined information.

In SQL, JOIN can be realized in several different ways. The first method is to specify the <join conditions> in the WHERE clause, along with any other selection conditions. This is very common, and is illustrated by queries Q1, Q1A, Q1B, Q2, and Q8 in Sections 4.3.1 and 4.3.2, as well as by many other query examples in Chapters 4 and 5. The second way is to use a nested relation, as illustrated by queries Q4A and Q16 in Section 5.1.2. Another way is to use the concept of joined tables, as illustrated by the queries Q1A, Q1B, Q8B, and Q2A in Section 5.1.6. The construct of joined tables was added to SQL2 to allow the user to specify explicitly all the various types of joins, because the other methods were more limited. It also allows the user to clearly distinguish join conditions from the selection conditions in the WHERE clause.

6.3.3 A Complete Set of Relational Algebra Operations

It has been shown that the set of relational algebra operations $\{\sigma, \pi, \cup, \rho, -, \times\}$ is a **complete** set; that is, any of the other original relational algebra operations can be expressed as a *sequence of operations from this set*. For example, the INTERSECTION operation can be expressed by using UNION and MINUS as follows:

$$R \cap S \equiv (R \cup S) - ((R - S) \cup (S - R))$$

Although, strictly speaking, INTERSECTION is not required, it is inconvenient to specify this complex expression every time we wish to specify an intersection. As another example, a JOIN operation can be specified as a CARTESIAN PRODUCT followed by a SELECT operation, as we discussed:

$$R \bowtie_{<\text{condition}>} S \equiv \sigma_{<\text{condition}>}(R \times S)$$

Similarly, a NATURAL JOIN can be specified as a CARTESIAN PRODUCT preceded by RENAME and followed by SELECT and PROJECT operations. Hence, the various JOIN operations are also *not strictly necessary* for the expressive power of the relational algebra. However, they are important to include as separate operations because they are convenient to use and are very commonly applied in database applications. Other operations have been included in the basic relational algebra for convenience rather than necessity. We discuss one of these—the DIVISION operation—in the next section.

6.3.4 The DIVISION Operation

The DIVISION operation, denoted by ÷, is useful for a special kind of query that sometimes occurs in database applications. An example is *Retrieve the names of employees who work on **all** the projects that 'John Smith' works on*. To express this query using the DIVISION operation, proceed as follows. First, retrieve the list of project numbers that 'John Smith' works on in the intermediate relation SMITH_PNOS:

$$\text{SMITH} \leftarrow \sigma_{\text{Fname='John' AND Lname='Smith'}}(\text{EMPLOYEE})$$
$$\text{SMITH\_PNOS} \leftarrow \pi_{\text{Pno}}(\text{WORKS\_ON} \bowtie_{\text{Essn=Ssn}} \text{SMITH})$$

Next, create a relation that includes a tuple <Pno, Essn> whenever the employee whose Ssn is Essn works on the project whose number is Pno in the intermediate relation SSN_PNOS:

$$\text{SSN\_PNOS} \leftarrow \pi_{\text{Essn, Pno}}(\text{WORKS\_ON})$$

Finally, apply the DIVISION operation to the two relations, which gives the desired employees' Social Security numbers:

$$\text{SSNS(Ssn)} \leftarrow \text{SSN\_PNOS} \div \text{SMITH\_PNOS}$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname}}(\text{SSNS} * \text{EMPLOYEE})$$

The preceding operations are shown in Figure 6.8(a).

Figure 6.8
The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

In general, the DIVISION operation is applied to two relations $R(Z) \div S(X)$, where the attributes of R are a subset of the attributes of S; that is, $X \subseteq Z$. Let Y be the set of attributes of R that are not attributes of S; that is, $Y = Z - X$ (and hence $Z = X \cup Y$). The result of DIVISION is a relation $T(Y)$ that includes a tuple t if tuples t_R appear in R with $t_R [Y] = t$, and with $t_R [X] = t_S$ for *every* tuple t_S in S. This means that, for a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every tuple* in S. Note that in the formulation of the DIVISION operation, the tuples in the denominator relation S restrict the numerator relation R by selecting those tuples in the result that match all values present in the denominator. It is not necessary to know what those values are as they can be computed by another operation, as illustrated in the SMITH_PNOS relation in the above example.

Figure 6.8(b) illustrates a DIVISION operation where $X = \{A\}$, $Y = \{B\}$, and $Z = \{A, B\}$. Notice that the tuples (values) b_1 and b_4 appear in R in combination with all three tuples in S; that is why they appear in the resulting relation T. All other values of B in R do not appear with all the tuples in S and are not selected: b_2 does not appear with a_2, and b_3 does not appear with a_1.

The DIVISION operation can be expressed as a sequence of π, $\times$, and $-$ operations as follows:

$$T1 \leftarrow \pi_Y(R)$$
$$T2 \leftarrow \pi_Y((S \times T1) - R)$$
$$T \leftarrow T1 - T2$$

The DIVISION operation is defined for convenience for dealing with queries that involve *universal quantification* (see Section 6.6.7) or the *all* condition. Most RDBMS implementations with SQL as the primary query language do not directly implement division. SQL has a roundabout way of dealing with the type of query illustrated above (see Section 5.1.4, queries Q3A and Q3B). Table 6.1 lists the various basic relational algebra operations we have discussed.

6.3.5 Notation for Query Trees

In this section we describe a notation typically used in relational systems to represent queries internally. The notation is called a *query tree* or sometimes it is known as a *query evaluation tree* or *query execution tree*. It includes the relational algebra operations being executed and is used as a possible data structure for the internal representation of the query in an RDBMS.

A **query tree** is a tree data structure that corresponds to a relational algebra expression. It represents the input relations of the query as *leaf nodes* of the tree, and represents the relational algebra operations as internal nodes. An execution of the query tree consists of executing an internal node operation whenever its operands (represented by its child nodes) are available, and then replacing that internal node by the relation that results from executing the operation. The execution terminates when the root node is executed and produces the result relation for the query.

Table 6.1 Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{(\text{<join attributes 1>}),\, (\text{<join attributes 2>})} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{(\text{<join attributes 1>}),\, (\text{<join attributes 2>})} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Figure 6.9 shows a query tree for Query 2 (see Section 4.3.1): *For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.* This query is specified on the relational schema of Figure 3.5 and corresponds to the following relational algebra expression:

$$\pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(((\sigma_{\text{Plocation='Stafford'}}(\text{PROJECT})) \bowtie_{\text{Dnum=Dnumber}}(\text{DEPARTMENT})) \bowtie_{\text{Mgr\_ssn=Ssn}}(\text{EMPLOYEE}))$$

In Figure 6.9, the three leaf nodes P, D, and E represent the three relations PROJECT, DEPARTMENT, and EMPLOYEE. The relational algebra operations in the expression

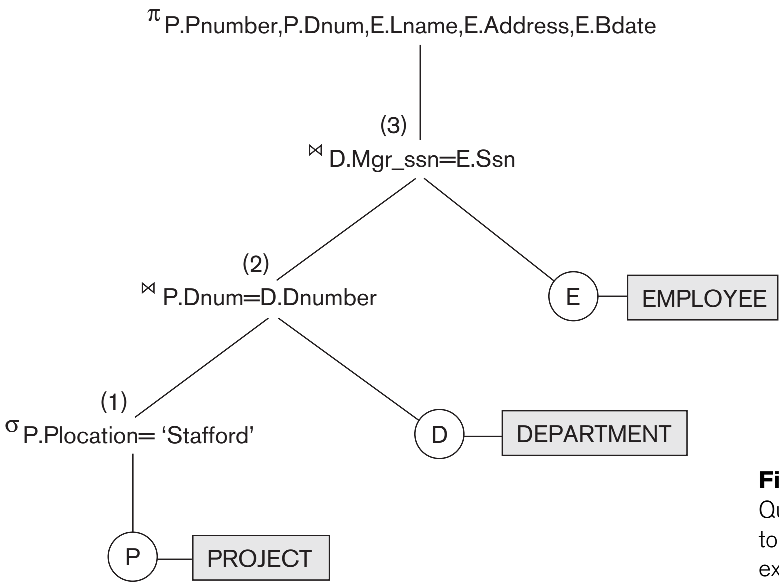

Figure 6.9
Query tree corresponding to the relational algebra expression for Q2.

are represented by internal tree nodes. The query tree signifies an explicit order of execution in the following sense. In order to execute Q2, the node marked (1) in Figure 6.9 must begin execution before node (2) because some resulting tuples of operation (1) must be available before we can begin to execute operation (2). Similarly, node (2) must begin to execute and produce results before node (3) can start execution, and so on. In general, a query tree gives a good visual representation and understanding of the query in terms of the relational operations it uses and is recommended as an additional means for expressing queries in relational algebra. We will revisit query trees when we discuss query processing and optimization in Chapter 19.

6.4 Additional Relational Operations

Some common database requests—which are needed in commercial applications for RDBMSs—cannot be performed with the original relational algebra operations described in Sections 6.1 through 6.3. In this section we define additional operations to express these requests. These operations enhance the expressive power of the original relational algebra.

6.4.1 Generalized Projection

The generalized projection operation extends the projection operation by allowing functions of attributes to be included in the projection list. The generalized form can be expressed as:

$$\pi_{F1,\, F2,\, \ldots,\, Fn}\,(R)$$

where F_1, F_2, ..., F_n are functions over the attributes in relation R and may involve arithmetic operations and constant values. This operation is helpful when developing reports where computed values have to be produced in the columns of a query result.

As an example, consider the relation

EMPLOYEE (Ssn, Salary, Deduction, Years_service)

A report may be required to show

Net Salary = Salary – Deduction,
Bonus = 2000 * Years_service, and
Tax = 0.25 * Salary.

Then a generalized projection combined with renaming may be used as follows:

$$\text{REPORT} \leftarrow \rho_{(\text{Ssn, Net\_salary, Bonus, Tax})}(\pi_{\text{Ssn, Salary} - \text{Deduction}, 2000 * \text{Years\_service}, 0.25 * \text{Salary}}(\text{EMPLOYEE})).$$

6.4.2 Aggregate Functions and Grouping

Another type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database. Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples. These functions are used in simple statistical queries that summarize information from the database tuples. Common functions applied to collections of numeric values include SUM, AVERAGE, MAXIMUM, and MINIMUM. The COUNT function is used for counting tuples or values.

Another common type of request involves grouping the tuples in a relation by the value of some of their attributes and then applying an aggregate function *independently to each group*. An example would be to group EMPLOYEE tuples by Dno, so that each group includes the tuples for employees working in the same department. We can then list each Dno value along with, say, the average salary of employees within the department, or the number of employees who work in the department.

We can define an AGGREGATE FUNCTION operation, using the symbol $\Im$ (pronounced *script F*)[7], to specify these types of requests as follows:

$$_{\text{<grouping attributes>}} \Im _{\text{<function list>}} (R)$$

where <grouping attributes> is a list of attributes of the relation specified in R, and <function list> is a list of (<function> <attribute>) pairs. In each such pair, <function> is one of the allowed functions—such as SUM, AVERAGE, MAXIMUM, MINIMUM, COUNT—and <attribute> is an attribute of the relation specified by R. The

[7]There is no single agreed-upon notation for specifying aggregate functions. In some cases a "script A" is used.

resulting relation has the grouping attributes plus one attribute for each element in the function list. For example, to retrieve each department number, the number of employees in the department, and their average salary, while renaming the resulting attributes as indicated below, we write:

$$\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}}\ \mathfrak{I}\ _{\text{COUNT Ssn, AVERAGE Salary}}\ (\text{EMPLOYEE}))$$

The result of this operation on the EMPLOYEE relation of Figure 3.6 is shown in Figure 6.10(a).

In the above example, we specified a list of attribute names—between parentheses in the RENAME operation—for the resulting relation *R*. If no renaming is applied, then the attributes of the resulting relation that correspond to the function list will each be the concatenation of the function name with the attribute name in the form <function>_<attribute>.[8] For example, Figure 6.10(b) shows the result of the following operation:

$$_{\text{Dno}}\ \mathfrak{I}\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$$

If no grouping attributes are specified, the functions are applied to *all the tuples* in the relation, so the resulting relation has a *single tuple only*. For example, Figure 6.10(c) shows the result of the following operation:

$$\mathfrak{I}\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$$

It is important to note that, in general, duplicates are *not eliminated* when an aggregate function is applied; this way, the normal interpretation of functions such as

Figure 6.10
The aggregate function operation.
a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}}\ \mathfrak{I}\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE}))$.
b. $_{\text{Dno}}\ \mathfrak{I}\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.
c. $\mathfrak{I}\ _{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

(a) **R**

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

[8]Note that this is an arbitrary notation we are suggesting. There is no standard notation.

SUM and AVERAGE is computed.[9] It is worth emphasizing that the result of applying an aggregate function is a relation, not a scalar number—even if it has a single value. This makes the relational algebra a closed mathematical system.

6.4.3 Recursive Closure Operations

Another type of operation that, in general, cannot be specified in the basic original relational algebra is **recursive closure.** This operation is applied to a **recursive relationship** between tuples of the same type, such as the relationship between an employee and a supervisor. This relationship is described by the foreign key Super_ssn of the EMPLOYEE relation in Figures 3.5 and 3.6, and it relates each employee tuple (in the role of supervisee) to another employee tuple (in the role of supervisor). An example of a recursive operation is to retrieve all supervisees of an employee *e* at all levels—that is, all employees e' directly supervised by *e*, all employees $e'\Im$ directly supervised by each employee e', all employees e''' directly supervised by each employee e'', and so on.

It is relatively straightforward in the relational algebra to specify all employees supervised by *e at a specific level* by joining the table with itself one or more times. However, it is difficult to specify all supervisees at *all* levels. For example, to specify the Ssns of all employees e' directly supervised—*at level one*—by the employee *e* whose name is 'James Borg' (see Figure 3.6), we can apply the following operation:

$$\text{BORG\_SSN} \leftarrow \pi_{\text{Ssn}}(\sigma_{\text{Fname='James' } \textbf{AND} \text{ Lname='Borg'}}(\text{EMPLOYEE}))$$
$$\text{SUPERVISION(Ssn1, Ssn2)} \leftarrow \pi_{\text{Ssn,Super\_ssn}}(\text{EMPLOYEE})$$
$$\text{RESULT1(Ssn)} \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{BORG\_SSN})$$

To retrieve all employees supervised by Borg at level 2—that is, all employees e'' supervised by some employee e' who is directly supervised by Borg—we can apply another **JOIN** to the result of the first query, as follows:

$$\text{RESULT2(Ssn)} \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{RESULT1})$$

To get both sets of employees supervised at levels 1 and 2 by 'James Borg', we can apply the UNION operation to the two results, as follows:

$$\text{RESULT} \leftarrow \text{RESULT2} \cup \text{RESULT1}$$

The results of these queries are illustrated in Figure 6.11. Although it is possible to retrieve employees at each level and then take their UNION, we cannot, in general, specify a query such as "retrieve the supervisees of 'James Borg' at all levels" without utilizing a looping mechanism unless we know the maximum number of levels.[10] An operation called the *transitive closure* of relations has been proposed to compute the recursive relationship as far as the recursion proceeds.

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[9]In SQL, the option of eliminating duplicates before applying the aggregate function is available by including the keyword DISTINCT (see Section 4.4.4).

[10]The SQL3 standard includes syntax for recursive closure.

SUPERVISION

(Borg's Ssn is 888665555)

Ssn1 (Ssn)	Ssn2 (Super_ssn)
123456789	333445555
333445555	888665555
999887777	987654321
987654321	888665555
666884444	333445555
453453453	333445555
987987987	987654321
888665555	null

RESULT1

Ssn
333445555
987654321

(Supervised by Borg)

RESULT2

Ssn
123456789
999887777
666884444
453453453
987987987

(Supervised by Borg's subordinates)

RESULT

Ssn
123456789
999887777
666884444
453453453
987987987
333445555
987654321

(RESULT1 ∪ RESULT2)

Figure 6.11
A two-level recursive query.

6.4.4 OUTER JOIN Operations

Next, we discuss some additional extensions to the JOIN operation that are necessary to specify certain types of queries. The JOIN operations described earlier match tuples that satisfy the join condition. For example, for a NATURAL JOIN operation $R * S$, only tuples from R that have matching tuples in S—and vice versa—appear in the result. Hence, tuples without a *matching* (or *related*) tuple are eliminated from the JOIN result. Tuples with NULL values in the join attributes are also eliminated. This type of join, where tuples with no match are eliminated, is known as an **inner join**. The join operations we described earlier in Section 6.3 are all inner joins. This amounts to the loss of information if the user wants the result of the JOIN to include all the tuples in one or more of the component relations.

A set of operations, called **outer joins**, were developed for the case where the user wants to keep all the tuples in R, or all those in S, or all those in both relations in the result of the JOIN, regardless of whether or not they have matching tuples in the other relation. This satisfies the need of queries in which tuples from two tables are to be combined by matching corresponding rows, but without losing any tuples for lack of matching values. For example, suppose that we want a list of all employee names as well as the name of the departments they manage *if they happen to manage a department*; if they do not manage one, we can indicate it with a NULL value. We can apply an operation **LEFT OUTER JOIN**, denoted by ⟕, to retrieve the result as follows:

$$\text{TEMP} \leftarrow (\text{EMPLOYEE} ⟕_{\text{Ssn=Mgr\_ssn}} \text{DEPARTMENT})$$

$$\text{RESULT} \leftarrow \pi_{\text{Fname, Minit, Lname, Dname}}(\text{TEMP})$$

The LEFT OUTER JOIN operation keeps every tuple in the *first*, or *left*, relation *R* in *R* ⟕ *S*; if no matching tuple is found in *S*, then the attributes of *S* in the join result are filled or *padded* with NULL values. The result of these operations is shown in Figure 6.12.

A similar operation, **RIGHT OUTER JOIN**, denoted by ⟖, keeps every tuple in the *second*, or right, relation *S* in the result of *R* ⟖ *S*. A third operation, **FULL OUTER JOIN**, denoted by ⟗, keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with NULL values as needed. The three outer join operations are part of the SQL2 standard (see Section 5.1.6). These operations were provided later as an extension of relational algebra in response to the typical need in business applications to show related information from multiple tables exhaustively. Sometimes a complete reporting of data from multiple tables is required whether or not there are matching values.

6.4.5 The OUTER UNION Operation

The **OUTER UNION** operation was developed to take the union of tuples from two relations that have some common attributes, but are *not union (type) compatible*. This operation will take the UNION of tuples in two relations $R(X, Y)$ and $S(X, Z)$ that are **partially compatible**, meaning that only some of their attributes, say X, are union compatible. The attributes that are union compatible are represented only once in the result, and those attributes that are not union compatible from either

Figure 6.12
The result of a LEFT OUTER JOIN operation.

RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

relation are also kept in the result relation $T(X, Y, Z)$. It is therefore the same as a FULL OUTER JOIN on the common attributes.

Two tuples t_1 in R and t_2 in S are said to **match** if $t_1[X]=t_2[X]$. These will be combined (unioned) into a single tuple in t. Tuples in either relation that have no matching tuple in the other relation are padded with NULL values. For example, an OUTER UNION can be applied to two relations whose schemas are STUDENT(Name, Ssn, Department, Advisor) and INSTRUCTOR(Name, Ssn, Department, Rank). Tuples from the two relations are matched based on having the same combination of values of the shared attributes—Name, Ssn, Department. The resulting relation, STUDENT_OR_INSTRUCTOR, will have the following attributes:

STUDENT_OR_INSTRUCTOR(Name, Ssn, Department, Advisor, Rank)

All the tuples from both relations are included in the result, but tuples with the same (Name, Ssn, Department) combination will appear only once in the result. Tuples appearing only in STUDENT will have a NULL for the Rank attribute, whereas tuples appearing only in INSTRUCTOR will have a NULL for the Advisor attribute. A tuple that exists in both relations, which represent a student who is also an instructor, will have values for all its attributes.[11]

Notice that the same person may still appear twice in the result. For example, we could have a graduate student in the Mathematics department who is an instructor in the Computer Science department. Although the two tuples representing that person in STUDENT and INSTRUCTOR will have the same (Name, Ssn) values, they will not agree on the Department value, and so will not be matched. This is because Department has two different meanings in STUDENT (the department where the person studies) and INSTRUCTOR (the department where the person is employed as an instructor). If we wanted to apply the OUTER UNION based on the same (Name, Ssn) combination only, we should rename the Department attribute in each table to reflect that they have different meanings and designate them as not being part of the union-compatible attributes. For example, we could rename the attributes as MajorDept in STUDENT and WorkDept in INSTRUCTOR.

6.5 Examples of Queries in Relational Algebra

The following are additional examples to illustrate the use of the relational algebra operations. All examples refer to the database in Figure 3.6. In general, the same query can be stated in numerous ways using the various operations. We will state each query in one way and leave it to the reader to come up with equivalent formulations.

Query 1. Retrieve the name and address of all employees who work for the 'Research' department.

[11]Note that OUTER UNION is equivalent to a FULL OUTER JOIN if the join attributes are *all* the common attributes of the two relations.

$\text{RESEARCH\_DEPT} \leftarrow \sigma_{\text{Dname}=\text{'Research'}}(\text{DEPARTMENT})$
$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{\text{Dnumber}=\text{Dno}} \text{EMPLOYEE})$
$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Address}}(\text{RESEARCH\_EMPS})$

As a single in-line expression, this query becomes:

$\pi_{\text{Fname, Lname, Address}}\ (\sigma_{\text{Dname}=\text{'Research'}}(\text{DEPARTMENT} \bowtie_{\text{Dnumber}=\text{Dno}}(\text{EMPLOYEE}))$

This query could be specified in other ways; for example, the order of the JOIN and SELECT operations could be reversed, or the JOIN could be replaced by a NATURAL JOIN after renaming one of the join attributes to match the other join attribute name.

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$\text{STAFFORD\_PROJS} \leftarrow \sigma_{\text{Plocation}=\text{'Stafford'}}(\text{PROJECT})$
$\text{CONTR\_DEPTS} \leftarrow (\text{STAFFORD\_PROJS} \bowtie_{\text{Dnum}=\text{Dnumber}} \text{DEPARTMENT})$
$\text{PROJ\_DEPT\_MGRS} \leftarrow (\text{CONTR\_DEPTS} \bowtie_{\text{Mgr\_ssn}=\text{Ssn}} \text{EMPLOYEE})$
$\text{RESULT} \leftarrow \pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(\text{PROJ\_DEPT\_MGRS})$

In this example, we first select the projects located in Stafford, then join them with their controlling departments, and then join the result with the department managers. Finally, we apply a project operation on the desired attributes.

Query 3. Find the names of employees who work on *all* the projects controlled by department number 5.

$\text{DEPT5\_PROJS} \leftarrow \rho_{(\text{Pno})}(\pi_{\text{Pnumber}}(\sigma_{\text{Dnum}=5}(\text{PROJECT})))$
$\text{EMP\_PROJ} \leftarrow \rho_{(\text{Ssn, Pno})}(\pi_{\text{Essn, Pno}}(\text{WORKS\_ON}))$
$\text{RESULT\_EMP\_SSNS} \leftarrow \text{EMP\_PROJ} \div \text{DEPT5\_PROJS}$
$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{RESULT\_EMP\_SSNS} * \text{EMPLOYEE})$

In this query, we first create a table DEPT5_PROJS that contains the project numbers of all projects controlled by department 5. Then we create a table EMP_PROJ that holds (Ssn, Pno) tuples, and apply the division operation. Notice that we renamed the attributes so that they will be correctly used in the division operation. Finally, we join the result of the division, which holds only Ssn values, with the EMPLOYEE table to retrieve the desired attributes from EMPLOYEE.

Query 4. Make a list of project numbers for projects that involve an employee whose last name is 'Smith', either as a worker or as a manager of the department that controls the project.

$\text{SMITHS(Essn)} \leftarrow \pi_{\text{Ssn}}\ (\sigma_{\text{Lname}=\text{'Smith'}}(\text{EMPLOYEE}))$
$\text{SMITH\_WORKER\_PROJS} \leftarrow \pi_{\text{Pno}}(\text{WORKS\_ON} * \text{SMITHS})$
$\text{MGRS} \leftarrow \pi_{\text{Lname, Dnumber}}(\text{EMPLOYEE} \bowtie_{\text{Ssn}=\text{Mgr\_ssn}} \text{DEPARTMENT})$
$\text{SMITH\_MANAGED\_DEPTS(Dnum)} \leftarrow \pi_{\text{Dnumber}}\ (\sigma_{\text{Lname}=\text{'Smith'}}(\text{MGRS}))$
$\text{SMITH\_MGR\_PROJS(Pno)} \leftarrow \pi_{\text{Pnumber}}(\text{SMITH\_MANAGED\_DEPTS} * \text{PROJECT})$
$\text{RESULT} \leftarrow (\text{SMITH\_WORKER\_PROJS} \cup \text{SMITH\_MGR\_PROJS})$

In this query, we retrieved the project numbers for projects that involve an employee named Smith as a worker in SMITH_WORKER_PROJS. Then we retrieved the project numbers for projects that involve an employee named Smith as manager of the department that controls the project in SMITH_MGR_PROJS. Finally, we applied the **UNION** operation on SMITH_WORKER_PROJS and SMITH_MGR_PROJS. As a single in-line expression, this query becomes:

$$\pi_{\text{Pno}} (\text{WORKS\_ON} \bowtie_{\text{Essn=Ssn}}(\pi_{\text{Ssn}} (\sigma_{\text{Lname='Smith'}}(\text{EMPLOYEE}))) \cup \pi_{\text{Pno}} ((\pi_{\text{Dnumber}} (\sigma_{\text{Lname='Smith'}}(\pi_{\text{Lname, Dnumber}}(\text{EMPLOYEE}))) \bowtie_{\text{Ssn=Mgr\_ssn}}\text{DEPARTMENT})) \bowtie_{\text{Dnumber=Dnum}}\text{PROJECT})$$

Query 5. List the names of all employees with two or more dependents.

Strictly speaking, this query cannot be done in the *basic* (*original*) *relational algebra*. We have to use the AGGREGATE FUNCTION operation with the COUNT aggregate function. We assume that dependents of the *same* employee have *distinct* Dependent_name values.

$$T1(\text{Ssn, No\_of\_dependents}) \leftarrow {}_{\text{Essn}} \Im_{\text{COUNT Dependent\_name}}(\text{DEPENDENT})$$
$$T2 \leftarrow \sigma_{\text{No\_of\_dependents}>2}(T1)$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(T2 * \text{EMPLOYEE})$$

Query 6. Retrieve the names of employees who have no dependents.

This is an example of the type of query that uses the MINUS (SET DIFFERENCE) operation.

$$\text{ALL\_EMPS} \leftarrow \pi_{\text{Ssn}}(\text{EMPLOYEE})$$
$$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$$
$$\text{EMPS\_WITHOUT\_DEPS} \leftarrow (\text{ALL\_EMPS} - \text{EMPS\_WITH\_DEPS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{EMPS\_WITHOUT\_DEPS} * \text{EMPLOYEE})$$

We first retrieve a relation with all employee Ssns in ALL_EMPS. Then we create a table with the Ssns of employees who have at least one dependent in EMPS_WITH_DEPS. Then we apply the SET DIFFERENCE operation to retrieve employees Ssns with no dependents in EMPS_WITHOUT_DEPS, and finally join this with EMPLOYEE to retrieve the desired attributes. As a single in-line expression, this query becomes:

$$\pi_{\text{Lname, Fname}}((\pi_{\text{Ssn}}(\text{EMPLOYEE}) - \rho_{\text{Ssn}}(\pi_{\text{Essn}}(\text{DEPENDENT}))) * \text{EMPLOYEE})$$

Query 7. List the names of managers who have at least one dependent.

$$\text{MGRS(Ssn)} \leftarrow \pi_{\text{Mgr\_ssn}}(\text{DEPARTMENT})$$
$$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$$
$$\text{MGRS\_WITH\_DEPS} \leftarrow (\text{MGRS} \cap \text{EMPS\_WITH\_DEPS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{MGRS\_WITH\_DEPS} * \text{EMPLOYEE})$$

In this query, we retrieve the Ssns of managers in MGRS, and the Ssns of employees with at least one dependent in EMPS_WITH_DEPS, then we apply the SET INTERSECTION operation to get the Ssns of managers who have at least one dependent.

As we mentioned earlier, the same query can be specified in many different ways in relational algebra. In particular, the operations can often be applied in various orders. In addition, some operations can be used to replace others; for example, the INTERSECTION operation in Q7 can be replaced by a NATURAL JOIN. As an exercise, try to do each of these sample queries using different operations.[12] We showed how to write queries as single relational algebra expressions for queries Q1, Q4, and Q6. Try to write the remaining queries as single expressions. In Chapters 4 and 5 and in Sections 6.6 and 6.7, we show how these queries are written in other relational languages.

6.6 The Tuple Relational Calculus

In this and the next section, we introduce another formal query language for the relational model called **relational calculus**. This section introduces the language known as **tuple relational calculus**, and Section 6.7 introduces a variation called **domain relational calculus**. In both variations of relational calculus, we write one **declarative** expression to specify a retrieval request; hence, there is no description of how, or *in what order*, to evaluate a query. A calculus expression specifies *what* is to be retrieved rather than *how* to retrieve it. Therefore, the relational calculus is considered to be a **nonprocedural** language. This differs from relational algebra, where we must write a *sequence of operations* to specify a retrieval request *in a particular order* of applying the operations; thus, it can be considered as a **procedural** way of stating a query. It is possible to nest algebra operations to form a single expression; however, a certain order among the operations is always explicitly specified in a relational algebra expression. This order also influences the strategy for evaluating the query. A calculus expression may be written in different ways, but the way it is written has no bearing on how a query should be evaluated.

It has been shown that any retrieval that can be specified in the basic relational algebra can also be specified in relational calculus, and vice versa; in other words, the **expressive power** of the languages is *identical*. This led to the definition of the concept of a *relationally complete* language. A relational query language *L* is considered **relationally complete** if we can express in *L* any query that can be expressed in relational calculus. Relational completeness has become an important basis for comparing the expressive power of high-level query languages. However, as we saw in Section 6.4, certain frequently required queries in database applications cannot be expressed in basic relational algebra or calculus. Most relational query languages are relationally complete but have *more expressive power* than relational algebra or relational calculus because of additional operations such as aggregate functions, grouping, and ordering. As we mentioned in the introduction to this chapter, the relational calculus is important for two reasons. First, it has a firm basis in mathematical logic. Second, the standard query language (SQL) for RDBMSs has some of its foundations in the tuple relational calculus.

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[12]When queries are optimized (see Chapter 19), the system will choose a particular sequence of operations that corresponds to an execution strategy that can be executed efficiently.

are applied be union (or type) compatible; these include UNION, INTERSECTION, and SET DIFFERENCE. The CARTESIAN PRODUCT operation is a set operation that can be used to combine tuples from two relations, producing all possible combinations. It is rarely used in practice; however, we showed how CARTESIAN PRODUCT followed by SELECT can be used to define matching tuples from two relations and leads to the JOIN operation. Different JOIN operations called THETA JOIN, EQUIJOIN, and NATURAL JOIN were introduced. Query trees were introduced as a graphical representation of relational algebra queries, which can also be used as the basis for internal data structures that the DBMS can use to represent a query.

We discussed some important types of queries that *cannot* be stated with the basic relational algebra operations but are important for practical situations. We introduced GENERALIZED PROJECTION to use functions of attributes in the projection list and the AGGREGATE FUNCTION operation to deal with aggregate types of statistical requests that summarize the information in the tables. We discussed recursive queries, for which there is no direct support in the algebra but which can be handled in a step-by-step approach, as we demonstrated. Then we presented the OUTER JOIN and OUTER UNION operations, which extend JOIN and UNION and allow all information in source relations to be preserved in the result.

The last two sections described the basic concepts behind relational calculus, which is based on the branch of mathematical logic called predicate calculus. There are two types of relational calculi: (1) the tuple relational calculus, which uses tuple variables that range over tuples (rows) of relations, and (2) the domain relational calculus, which uses domain variables that range over domains (columns of relations). In relational calculus, a query is specified in a single declarative statement, without specifying any order or method for retrieving the query result. Hence, relational calculus is often considered to be a higher-level *declarative* language than the relational algebra, because a relational calculus expression states *what* we want to retrieve regardless of *how* the query may be executed.

We discussed the syntax of relational calculus queries using both tuple and domain variables. We introduced query graphs as an internal representation for queries in relational calculus. We also discussed the existential quantifier ($\exists$) and the universal quantifier ($\forall$). We saw that relational calculus variables are bound by these quantifiers. We described in detail how queries with universal quantification are written, and we discussed the problem of specifying safe queries whose results are finite. We also discussed rules for transforming universal into existential quantifiers, and vice versa. It is the quantifiers that give expressive power to the relational calculus, making it equivalent to the basic relational algebra. There is no analog to grouping and aggregation functions in basic relational calculus, although some extensions have been suggested.

Review Questions

6.1. List the operations of relational algebra and the purpose of each.

6.2. What is union compatibility? Why do the UNION, INTERSECTION, and DIFFERENCE operations require that the relations on which they are applied be union compatible?

6.3. Discuss some types of queries for which renaming of attributes is necessary in order to specify the query unambiguously.

6.4. Discuss the various types of *inner join* operations. Why is theta join required?

6.5. What role does the concept of *foreign key* play when specifying the most common types of meaningful join operations?

6.6. What is the FUNCTION operation? What is it used for?

6.7. How are the OUTER JOIN operations different from the INNER JOIN operations? How is the OUTER UNION operation different from UNION?

6.8. In what sense does relational calculus differ from relational algebra, and in what sense are they similar?

6.9. How does tuple relational calculus differ from domain relational calculus?

6.10. Discuss the meanings of the existential quantifier ($\exists$) and the universal quantifier ($\forall$).

6.11. Define the following terms with respect to the tuple calculus: *tuple variable*, *range relation*, *atom*, *formula*, and *expression*.

6.12. Define the following terms with respect to the domain calculus: *domain variable*, *range relation*, *atom*, *formula*, and *expression*.

6.13. What is meant by a *safe expression* in relational calculus?

6.14. When is a query language called relationally complete?

Exercises

6.15. Show the result of each of the sample queries in Section 6.5 as it would apply to the database state in Figure 3.6.

6.16. Specify the following queries on the COMPANYrelational database schema shown in Figure 5.5, using the relational operators discussed in this chapter. Also show the result of each query as it would apply to the database state in Figure 3.6.

a. Retrieve the names of all employees in department 5 who work more than 10 hours per week on the ProductX project.

b. List the names of all employees who have a dependent with the same first name as themselves.

c. Find the names of all employees who are directly supervised by 'Franklin Wong'.

d. For each project, list the project name and the total hours per week (by all employees) spent on that project.

e. Retrieve the names of all employees who work on every project.

f. Retrieve the names of all employees who do not work on any project.

g. For each department, retrieve the department name and the average salary of all employees working in that department.

h. Retrieve the average salary of all female employees.

i. Find the names and addresses of all employees who work on at least one project located in Houston but whose department has no location in Houston.

j. List the last names of all department managers who have no dependents.

6.17. Consider the AIRLINE relational database schema shown in Figure 3.8, which was described in Exercise 3.12. Specify the following queries in relational algebra:

a. For each flight, list the flight number, the departure airport for the first leg of the flight, and the arrival airport for the last leg of the flight.

b. List the flight numbers and weekdays of all flights or flight legs that depart from Houston Intercontinental Airport (airport code 'IAH') and arrive in Los Angeles International Airport (airport code 'LAX').

c. List the flight number, departure airport code, scheduled departure time, arrival airport code, scheduled arrival time, and weekdays of all flights or flight legs that depart from some airport in the city of Houston and arrive at some airport in the city of Los Angeles.

d. List all fare information for flight number 'CO197'.

e. Retrieve the number of available seats for flight number 'CO197' on '2009-10-09'.

6.18. Consider the LIBRARY relational database schema shown in Figure 6.14, which is used to keep track of books, borrowers, and book loans. Referential integrity constraints are shown as directed arcs in Figure 6.14, as in the notation of Figure 3.7. Write down relational expressions for the following queries:

a. How many copies of the book titled *The Lost Tribe* are owned by the library branch whose name is 'Sharpstown'?

b. How many copies of the book titled *The Lost Tribe* are owned by each library branch?

c. Retrieve the names of all borrowers who do not have any books checked out.

d. For each book that is loaned out from the Sharpstown branch and whose Due_date is today, retrieve the book title, the borrower's name, and the borrower's address.

e. For each library branch, retrieve the branch name and the total number of books loaned out from that branch.

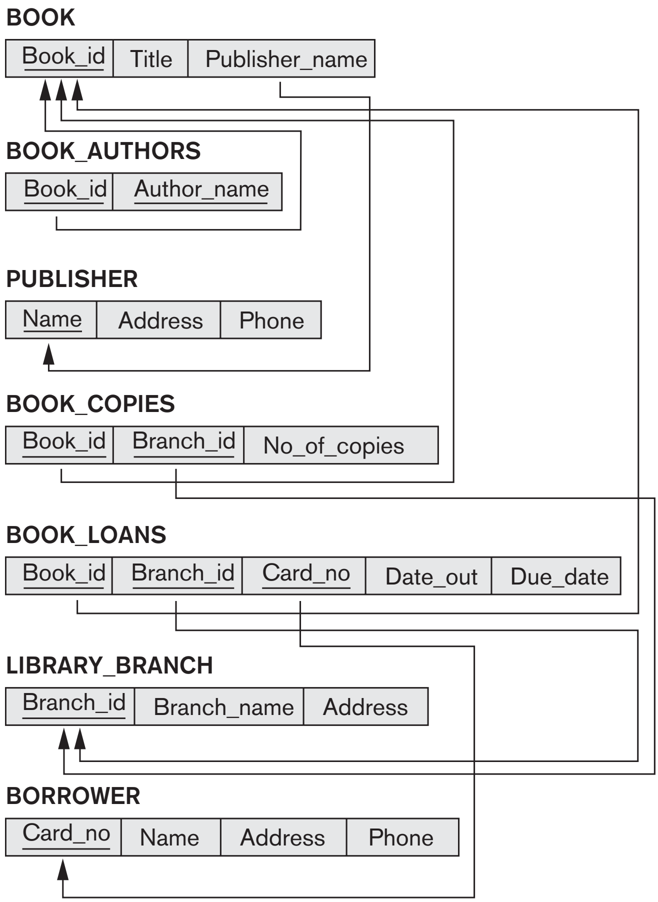

Figure 6.14
A relational database schema for a LIBRARY database.

f. Retrieve the names, addresses, and number of books checked out for all borrowers who have more than five books checked out.

g. For each book authored (or coauthored) by Stephen King, retrieve the title and the number of copies owned by the library branch whose name is Central.

6.19. Specify the following queries in relational algebra on the database schema given in Exercise 3.14:

a. List the Order# and Ship_date for all orders shipped from Warehouse# W2.

b. List the WAREHOUSE information from which the CUSTOMER named Jose Lopez was supplied his orders. Produce a listing: Order#, Warehouse#.

c. Produce a listing Cname, No_of_orders, Avg_order_amt, where the middle column is the total number of orders by the customer and the last column is the average order amount for that customer.

d. List the orders that were not shipped within 30 days of ordering.

e. List the Order# for orders that were shipped from *all* warehouses that the company has in New York.

6.20. Specify the following queries in relational algebra on the database schema given in Exercise 3.15:

a. Give the details (all attributes of trip relation) for trips that exceeded $2,000 in expenses.

b. Print the Ssns of salespeople who took trips to Honolulu.

c. Print the total trip expenses incurred by the salesperson with SSN = '234-56-7890'.

6.21. Specify the following queries in relational algebra on the database schema given in Exercise 3.16:

a. List the number of courses taken by all students named John Smith in Winter 2009 (i.e., Quarter=W09).

b. Produce a list of textbooks (include Course#, Book_isbn, Book_title) for courses offered by the 'CS' department that have used more than two books.

c. List any department that has all its adopted books published by 'Pearson Publishing'.

6.22. Consider the two tables $T1$ and $T2$ shown in Figure 6.15. Show the results of the following operations:

a. $T1 \bowtie_{T1.P = T2.A} T2$

b. $T1 \bowtie_{T1.Q = T2.B} T2$

c. $T1 ⟕_{T1.P = T2.A} T2$

d. $T1 ⟖_{T1.Q = T2.B} T2$

e. $T1 \cup T2$

f. $T1 \bowtie_{(T1.P = T2.A \textbf{ AND } T1.R = T2.C)} T2$

Figure 6.15
A database state for the relations $T1$ and $T2$.

TABLE T1

P	Q	R
10	a	5
15	b	8
25	a	6

TABLE T2

A	B	C
10	b	6
25	c	3
10	b	5

6.23. Specify the following queries in relational algebra on the database schema in Exercise 3.17:

a. For the salesperson named 'Jane Doe', list the following information for all the cars she sold: Serial#, Manufacturer, Sale_price.

b. List the Serial# and Model of cars that have no options.

c. Consider the NATURAL JOIN operation between SALESPERSON and SALE. What is the meaning of a left outer join for these tables (do not change the order of relations)? Explain with an example.

d. Write a query in relational algebra involving selection and one set operation and say in words what the query does.

6.24. Specify queries a, b, c, e, f, i, and j of Exercise 6.16 in both tuple and domain relational calculus.

6.25. Specify queries a, b, c, and d of Exercise 6.17 in both tuple and domain relational calculus.

6.26. Specify queries c, d, and f of Exercise 6.18 in both tuple and domain relational calculus.

6.27. In a tuple relational calculus query with n tuple variables, what would be the typical minimum number of join conditions? Why? What is the effect of having a smaller number of join conditions?

6.28. Rewrite the domain relational calculus queries that followed Q0 in Section 6.7 in the style of the abbreviated notation of Q0A, where the objective is to minimize the number of domain variables by writing constants in place of variables wherever possible.

6.29. Consider this query: Retrieve the Ssns of employees who work on at least those projects on which the employee with Ssn=123456789 works. This may be stated as (**FORALL** x) (**IF** P **THEN** Q), where

- x is a tuple variable that ranges over the PROJECT relation.
- $P \equiv$ EMPLOYEE with Ssn=123456789 works on PROJECT x.
- $Q \equiv$ EMPLOYEE e works on PROJECT x.

Express the query in tuple relational calculus, using the rules

- $(\forall x)(P(x)) \equiv$ **NOT**$(\exists x)($**NOT**$(P(x)))$.
- (**IF** P **THEN** Q) $\equiv$ (**NOT**(P) **OR** Q).

6.30. Show how you can specify the following relational algebra operations in both tuple and domain relational calculus.

a. $\sigma_{A=C}(R(A, B, C))$

b. $\pi_{<A, B>}(R(A, B, C))$

c. $R(A, B, C) * S(C, D, E)$

d. $R(A, B, C) \cup S(A, B, C)$

e. $R(A, B, C) \cap S(A, B, C)$

f. $R(A, B, C) = S(A, B, C)$

g. $R(A, B, C) \times S(D, E, F)$

h. $R(A, B) \div S(A)$

6.31. Suggest extensions to the relational calculus so that it may express the following types of operations that were discussed in Section 6.4: (a) aggregate functions and grouping; (b) OUTER JOIN operations; (c) recursive closure queries.

6.32. A nested query is a query within a query. More specifically, a nested query is a parenthesized query whose result can be used as a value in a number of places, such as instead of a relation. Specify the following queries on the database specified in Figure 3.5 using the concept of nested queries and the relational operators discussed in this chapter. Also show the result of each query as it would apply to the database state in Figure 3.6.

a. List the names of all employees who work in the department that has the employee with the highest salary among all employees.

b. List the names of all employees whose supervisor's supervisor has '888665555' for Ssn.

c. List the names of employees who make at least $10,000 more than the employee who is paid the least in the company.

6.33. State whether the following conclusions are true or false:

a. **NOT** $(P(x)$ **OR** $Q(x)) \rightarrow$ (**NOT** $(P(x))$ **AND** (**NOT** $(Q(x)))$

b. **NOT** $(\exists x)\ (P(x)) \rightarrow \forall\, x$ (**NOT** $(P(x))$

c. $(\exists x)\ (P(x)) \rightarrow \forall\, x\ ((P(x))$

Laboratory Exercises

6.34. Specify and execute the following queries in relational algebra (RA) using the RA interpreter on the COMPANY database schema in Figure 3.5.

a. List the names of all employees in department 5 who work more than 10 hours per week on the ProductX project.

b. List the names of all employees who have a dependent with the same first name as themselves.

c. List the names of employees who are directly supervised by Franklin Wong.

d. List the names of employees who work on every project.

e. List the names of employees who do not work on any project.

f. List the names and addresses of employees who work on at least one project located in Houston but whose department has no location in Houston.

g. List the names of department managers who have no dependents.

6.35. Consider the following MAILORDER relational schema describing the data for a mail order company.

PARTS(<u>Pno</u>, Pname, Qoh, Price, Olevel)
CUSTOMERS(<u>Cno</u>, Cname, Street, Zip, Phone)
EMPLOYEES(<u>Eno</u>, Ename, Zip, Hdate)
ZIP_CODES(<u>Zip</u>, City)
ORDERS(<u>Ono</u>, Cno, Eno, Received, Shipped)
ODETAILS(<u>Ono</u>, <u>Pno</u>, Qty)

Qoh stands for *quantity on hand*: the other attribute names are self-explanatory. Specify and execute the following queries using the RA interpreter on the MAILORDER database schema.

a. Retrieve the names of parts that cost less than $20.00.
b. Retrieve the names and cities of employees who have taken orders for parts costing more than $50.00.
c. Retrieve the pairs of customer number values of customers who live in the same ZIP Code.
d. Retrieve the names of customers who have ordered parts from employees living in Wichita.
e. Retrieve the names of customers who have ordered parts costing less than $20.00.
f. Retrieve the names of customers who have not placed an order.
g. Retrieve the names of customers who have placed exactly two orders.

6.36. Consider the following GRADEBOOK relational schema describing the data for a grade book of a particular instructor. (*Note*: The attributes A, B, C, and D of COURSES store grade cutoffs.)

CATALOG(<u>Cno</u>, Ctitle)
STUDENTS(<u>Sid</u>, Fname, Lname, Minit)
COURSES(<u>Term</u>, <u>Sec_no</u>, Cno, A, B, C, D)
ENROLLS(<u>Sid</u>, <u>Term</u>, <u>Sec_no</u>)

Specify and execute the following queries using the RA interpreter on the GRADEBOOK database schema.

a. Retrieve the names of students enrolled in the Automata class during the fall 2009 term.
b. Retrieve the Sid values of students who have enrolled in CSc226 and CSc227.
c. Retrieve the Sid values of students who have enrolled in CSc226 or CSc227.
d. Retrieve the names of students who have not enrolled in any class.
e. Retrieve the names of students who have enrolled in all courses in the CATALOG table.

6.37. Consider a database that consists of the following relations.

SUPPLIER(Sno, Sname)
PART(Pno, Pname)
PROJECT(Jno, Jname)
SUPPLY(Sno, Pno, Jno)

The database records information about suppliers, parts, and projects and includes a ternary relationship between suppliers, parts, and projects. This relationship is a many-many-many relationship. Specify and execute the following queries using the RA interpreter.

a. Retrieve the part numbers that are supplied to exactly two projects.
b. Retrieve the names of suppliers who supply more than two parts to project 'J1'.
c. Retrieve the part numbers that are supplied by every supplier.
d. Retrieve the project names that are supplied by supplier 'S1' only.
e. Retrieve the names of suppliers who supply at least two different parts each to at least two different projects.

6.38. Specify and execute the following queries for the database in Exercise 3.16 using the RA interpreter.

a. Retrieve the names of students who have enrolled in a course that uses a textbook published by Addison-Wesley.
b. Retrieve the names of courses in which the textbook has been changed at least once.
c. Retrieve the names of departments that adopt textbooks published by Addison-Wesley only.
d. Retrieve the names of departments that adopt textbooks written by Navathe and published by Addison-Wesley.
e. Retrieve the names of students who have never used a book (in a course) written by Navathe and published by Addison-Wesley.

6.39. Repeat Laboratory Exercises 6.34 through 6.38 in domain relational calculus (DRC) by using the DRC interpreter.

Selected Bibliography

Codd (1970) defined the basic relational algebra. Date (1983a) discusses outer joins. Work on extending relational operations is discussed by Carlis (1986) and Ozsoyoglu et al. (1985). Cammarata et al. (1989) extends the relational model integrity constraints and joins.

Codd (1971) introduced the language Alpha, which is based on concepts of tuple relational calculus. Alpha also includes the notion of aggregate functions, which goes beyond relational calculus. The original formal definition of relational calculus